The Design of Apparatus for Measuring Ocean Wave Parameters

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Abstract

This document describes the design of a low-cost apparatus for the measurement of ocean wave parameters. The design is implemented using magnetometers to measure attitude of a free-floating buoy, and accelerometers to measure motion of the buoy, and a microprocessor to analyse the data. Calibration of the sensors is described, as well as the use of digital filtering in the analysis of the signals to determine attitude and motion. Testing of the system, both on land and on water, is described, including protection of the devices from the corrosive effects of salt water.

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Acknowledgments

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1 Introduction

Measurement of ocean waves is important for

- **Engineering purposes**
  - Ship design. Pitching and rolling can slow down the progress of the ship, as well as causing cargo to move and causing other damage. Knowledge of the size and frequency of waves likely to be encountered allows the ship designer to minimise the negative effects of waves.
  - Design of structures to be erected in the sea (eg oil drilling platforms, wharves).Waves are the most important environmental factor producing forces on offshore structures. Not only can there be high stress loads from individual waves, but fatigue from significant wave activity over a long period must be designed for.
  - Wave-driven electricity generation. An average of 25MW of electricity could be generated per km of coastline. However, this is variable, so knowledge of wave climate is needed to properly estimate the return from investing in a wave-driven power station.
  - Military. During World War II, much effort was put into oceanography to predict wave characteristics for beach landings. Pressure-operated mines, which need to be set off by ships but not wave action is another example.

- **Environmental reasons**
  - Study of erosion in coastal areas. This is becoming increasingly important with the predicted rise in sea level due to greenhouse effects. Not only do waves erode cliffs and beaches, but material is also carried by ‘longshore drift’ to be deposited elsewhere.
  - Changed ocean climate. Measurements suggest a significant worsening of the eastern North Atlantic wave climate between 1960 and 1977. [Draper 1986]
  - Study of ocean/atmosphere interaction and weather prediction. The oceans have a great effect on the atmosphere and vice versa. It is increasingly becoming recognised that ocean waves are an important mechanism for energy exchange between the ocean and air, and an important part of ocean currents. [Melville 2002]

The height of ocean waves is the property of the most interest because it gives a measure of the amount of energy being carried by the wave, and therefore its destructive power. Also of interest is the direction of the wave.

A central problem to ocean wave study is that observations need to be made over an extended period of time (months to years) to gain an understanding of the wave climate. In addition, the location of interest is often in a remote part of the ocean, rarely visited by people.

Often, data needs to be gathered from locations in deep water. In these circumstances, it is necessary for the sensors to be mounted on a buoy that floats on the ocean surface. Typically, the motion of the buoy is measured as it follows the ocean surface. In many cases, mooring a buoy in the correct location is prohibitively expensive, so buoys are deployed and allowed to drift. This in turn is expensive, since the time that buoy is providing the required data is limited. Only 37% of drifting buoys are still fully operational after 18 months [Meindl 1992].

The following properties are therefore desirable in a device to measure ocean waves in deep water:

1. Autonomy. It is desirable to place the device at the desired location and record ocean wave data over long time periods without human intervention.
2. Measurement of wave height.
3. Measurement of wave direction.
4. Self-powered. To allow this, all components must have minimal power requirements so that batteries or solar cells can power the system.
5. Stable sensors or a self-calibrating system.
6. Data transmission. Data should be transmitted by the device to a remote base station, possibly in batches, or at least stored for later retrieval.
7. Data analysis. Analysing data in situ will reduce the amount of data transmission/storage required. Data may be analysed in real time (eg. for weather prediction) or batch mode (for longer-term studies).
8. Robustness. Due to remoteness of data gathering locations, it may be very expensive to repair faults. This suggests a minimum of mechanical devices.
9. Corrosion resistance. Sea water is extremely corrosive to metallic devices.

The School of Electronic and Software Engineering at the Central Institute of Technology, Trentham, New Zealand has been developing a solar-powered boat as a research project [van der Hulst 2001]. It is an autonomous vessel designed to operate far from land for long periods of time. This capability makes it suitable for use as a self-positioning data buoy for monitoring weather or ocean conditions, if suitable sensors are constructed.

This report will begin by outlining the current theory of ocean wave motion, and the current techniques used to measure ocean waves (Chapter 2: Context). The following chapters will describe alternative techniques that can be applied (Chapter 3: This Project), the hardware and software implemented during this research project (Chapters 4 and 5), and the outcome of testing of the system (Chapter 6: Results). Finally, possible directions for further research based on the outcome of this project are outlined in Chapter 7, and conclusions discussed in Chapter 8.

2 Context

2.1 Ocean Waves

Waves on the ocean are produced by surface friction with wind. Once a small unevenness appears, it feeds back positively to the wind, causing the wind to speed up beyond the crest causing a low-pressure area. This in turn causes the wave to increase in size, and further increases the effect on the wind.

Wave sizes vary from capillary ripples (centimetres in length, millimetres in amplitude, surface tension is a significant aspect) to storm surges (kilometres in length, metres in amplitude, days in period).

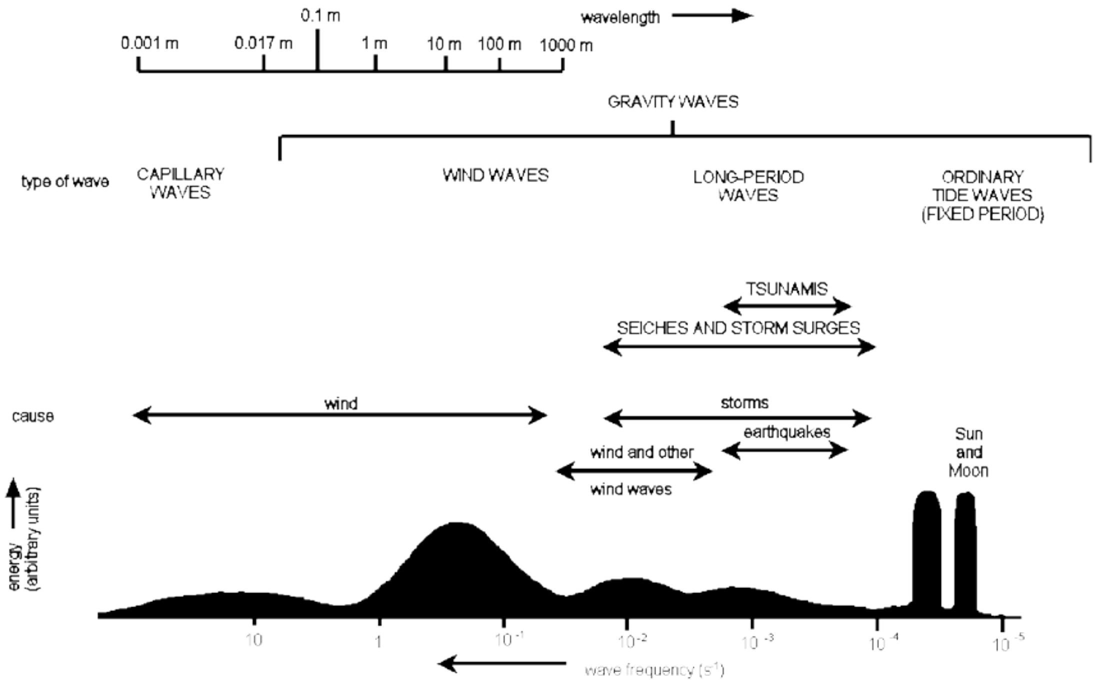

Figure 2-1: Ocean Wave Spectrum

Waves with a shorter period (less than 10 seconds) are known as seas. These tend to be non-symmetrical, are caused by local influences, and tend to dissipate quickly. Longer-period waves are known as swells. These are produced by wind action on the large scale (pressure changes from storms), are relatively symmetrical, and travel great distances from the area where they originate. Collectively, these large motion waves are known as gravity waves.

The first study of a wave system and its component frequencies by [Barber 1948] identified swells with a period of 20 seconds which had travelled over 1500 miles across the North Atlantic, taking 2 days to do so.

For small waves, the hydrodynamic equations are linear and can be modelled as sinusoidal motion [Tucker 1991, Chapter 11]. In a sinusoidal wave train in deep water there is no significant interaction with the seabed, and the water particles travel in circular orbits.

However, the motion is somewhat complex, with wave trains of different frequencies and amplitudes travelling in different directions. Waves of different frequency travel at different speeds; in general, those with lower frequency travel faster and further and transport more energy than higher frequency waves.

*Trochoidal moti*on models the movement water particles in ocean waves better than a sinusoidal model; it allows a small net displacement of water particles in the direction of the wave train. A *trochoid* is the locus of points traced by a fixed point *P* on the radius of a circle that rolls on a straight line without slipping. If the straight line is the x-axis, the circle has radius *B*, and *P* is at a distance *A* from the centre of the circle, then parametric equations for the trochoid are given by $x = B*Z - A \sin(Z)$ and $y = A \cos(Z)$ where *Z* is the angle of rotation of the circle. In general, trochoidal waves have sharper peaks and flatter troughs than sinusoidal waves.

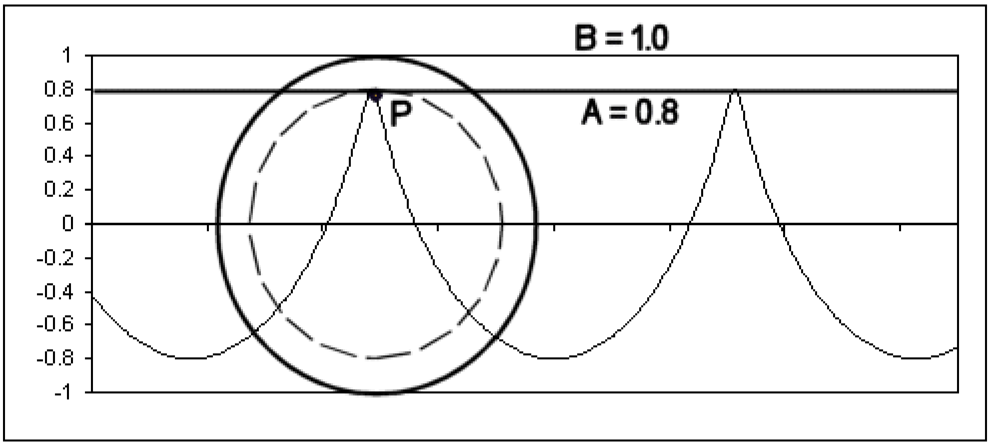

Figure 2-2: Trochoidal Motion: Locus of $x = Z - 0.8 \sin(Z)$ and $y = 0.8 \cos(Z)$

As waves become steeper (the ratio of height to wavelength increases), they tend to become less sinusoidal and more trochoidal. The extreme case is a breaking wave, where the steepness exceeds the ability of the water particles to maintain the surface.

Wave heights follow a Rayleigh probability distribution; about a third of waves are 1.60 times the mean height, and 1% are 2.66 times the mean height.

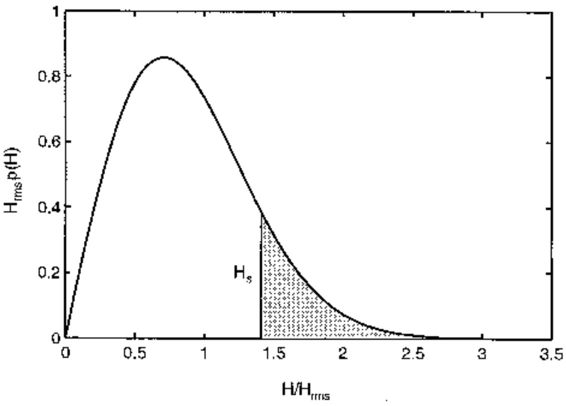

Figure 2-3: The probability function for wave height as defined by the Rayleigh distribution

The range of wave heights varies with latitude and season; highest waves occur in July at about 50°S with one third being over 6m in height. [Young 1999, chapter 3] Extreme waves may reach heights of 27m (South Atlantic, Indian Ocean, TR = 30 years) [Massel 1996, chapter 8]

2.2 Theory of Ocean Waves

Ocean wave theory has been developed since the mid-1800s [Airy 1845; Jeffreys 1924; Jeffreys 1925]. Due to the complexity of the mathematics involved, many simplifying assumptions have been made:

- Water is of constant depth.
- Water is incompressible.
- Viscosity, turbulence, and surface tension have negligible effect.
- Waves have constant wavelength and period.
- Waves have constant form.
- Wave height is small compared to water depth and wavelength.
- No forcing of waves by wind.

Given these assumptions, formulae have been developed to describe the initial formation of waves, and to describe the behaviour of these waves – *Linear Wave Theory*. This predicts sinusoidal motion and consequently assumes that water particles move in closed orbits (there is no net transport of water) and that crest and trough heights are equal (the wave is distributed evenly about the still water level).

For periodic motion, a Fourier series can calculate water surface position as the sum of a number of sine waves:

$$z(t) = \sum_{i=1}^{N} a_i \sin(k_i - 2\pi f_i t + \phi_i)$$

where $z(t)$ represents the surface elevation at time t, and a_i, f_i, and ϕ_i are the amplitude, frequency, and phase of the ith wave in the summation. a_i^2 is related to the energy content of the wave. Although the number of frequencies measured is discrete, as $N \to \infty$ the amplitude spectrum can be transformed into a continuous spectrum and it is common to describe wave data in this manner.

[Borgman 1977] describes a further extension to the Fourier model to cover wave components propagating in different directions:

$$z(x, y, t) = \sum_{i=1}^{N} a_i \cos[k_i (x \cos\theta_i + y \sin\theta_i) - 2\pi f_i t + \phi_i]$$

where θ_i is the angle between the x axis and direction of propagation of the wave and k_i is the wave number = $2\pi/L_i$

The limitations of Linear Wave Theory have been investigated and attempts to reduce one or more have been made, resulting in *Finite Amplitude Wave Theories*. These theories consider the influence of the wave itself on its own properties – phase speed, wavelength, water surface shape, and other properties are considered to be functions of the actual wave height. These theories can predict peaked crests, and flat troughs and a small net fluid transport, in accordance with trochoidal particle motion.

Gerstner Wave Theory (also called trochoidal wave theory since the elevation profile takes the form of a trochoidal curve) was developed for periodic waves of finite height. The solutions are exact and satisfy continuity as well as the pressure conditions at the water surface, and experimental studies have shown that the theory closely approximates the profiles of real waves on a horizontal bottom. Drawbacks include that mass transport is not predicted, the velocity field is rotational, and the particle movements are opposite to that expected in real waves (and found in other theories) [LeMehaute 1976].

The two most commonly adopted theories are *Stokes' Wave Theory* [Stokes 1847; Stokes 1880; Miche 1944] which is applicable to deep water, and *Cnoidal Wave Theory* [Korteweg and De Vries 1895; Keulegan and Patterson 1940; Wiegel 1960] which is applicable to shallow water. These theories require that waves be of a single period and length.

Water is considered to be 'deep' if the water depth is more than ¼ of the wavelength. In water deeper than this, waves are not significantly affected by interaction with the bottom.

The predictions of both Gerstner and Stokes wave theories agree equally well with measured wave profiles. This is explained by the fact that if the Gerstner wave equations are expanded into a series the first three terms are identical to those in the Stokes solution. This similarity in predictive ability and greater ease of use means that Gerstner wave theory is preferred in many engineering applications.

2.3 Limiting Wave Parameters

In a simple wave, the limiting wave has a sharp angle at the crest of 120° [Bascom 1980]; each side slopes down at 30° to the horizontal. At the crest, the particles are moving forward at exactly the phase speed of the wave. If the wave is any steeper, the particle speed exceeds the wave speed and water particles spill down the front of the wave. For this wave, the downward acceleration of particles at the crest is –0.5G.

However, it has been shown [Longuet-Higgins 1985] using Stokes Wave Theory that the minimum crest acceleration tends towards –0.39G and maximum trough acceleration is +0.30G.

The highest wave has a steepness (height/wavelength ratio) of 1/7 (0.14) [Michell 1893]. The phase velocity (speed) of this wave is 1.2 times that of a low-amplitude wave of the same wavelength.

[Hidy 1971] states that $c = \sqrt{gh}$ for waves with a steepness less than 0.05

Where c = the wave's speed, g = gravitational constant, h = wave height

The minimum speed for gravity waves in water is 0.23 m/sec [Hidy 1971, p131].

The theoretical minimum and maximum frequencies for wind-induced gravity waves are 0.03Hz and 13.6Hz [Massel 1996]

In standing waves, water particles only move vertically; there is no horizontal component. When the downward acceleration at the crest exceeds –1G, water leaves the top of the wave and it breaks. At the limiting height, the water surface angle is 45°. Standing waves are not found in a real wind-formed sea. However, fast-moving storms can produce high wave energies moving in opposite directions. This produces waves similar to standing waves, but with short crest lengths, known as 'pyramidal waves'.

2.4 Measurement

Historically, human visual observation and estimation has been the standard technique for recording wave information. However, this is subjective and consequently inaccurate and unreliable. People tend to remember extraordinary rather than ordinary waves. Another disadvantage is the need for a human presence; this has resulted in a rather patchy record of wave data.

Many different automated wave measurement techniques have been described [Bascom 1980, Chapter 8; Young 1999, Chapter 9; Massel 1996, Chapter 9; Tucker 1991, Chapter 3-4]. They may be divided into two categories: in situ and remote sensing.

2.4.1 In Situ Instruments

In situ instruments are instruments that are physically located on or in the sea. They may be further categorised as Fixed or Surface Following.

2.4.1.1 Fixed Instruments

These are physically connected to the seabed in some way and measure the motion of the waves as they pass by. Instruments may either pierce the surface and use the difference in properties between air and seawater to determine the location of the water surface, or they may measure the distance to the surface remotely.

2.4.1.1.1 Surface piercing

Early wave recorders were based on the tide gauge invented by Lord Kelvin in 1882. This consisted of a float constrained in a pipe mounted vertically in the sea, and open to it. The float is connected mechanically to a pencil, which draws the record on paper on a chart recorder moving at a constant speed (Figure 2-4).

By varying the size of the opening, the system is 'tuneable' to record waves in a desired frequency range. By measuring the difference between a 'low frequency' instrument and a 'high frequency' instrument, it is possible to remove low frequency (tides, etc) information.

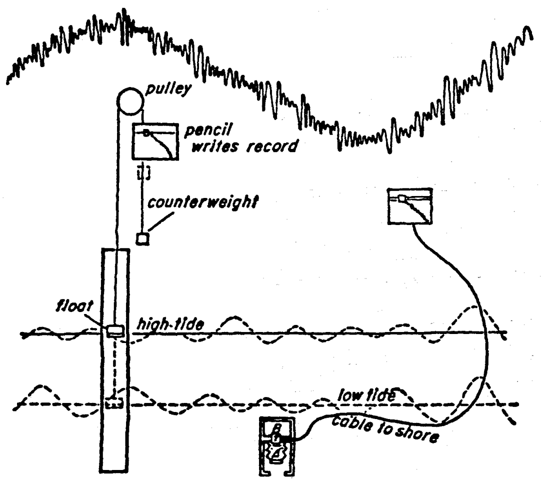

Figure 2-4: Simple Wave recording mechanisms [Bascom 1980, p166]

Electrical variations of this consist of

- Resistive Gauge: A pair of wires of known resistivity is mounted vertically in the water. As the water level rises and falls, it ‘shorts’ the resistive circuit. The voltage drop will therefore vary depending on the water level. The resistance varies depending on the salinity of the water.
- Wave staff: A series of contacts mounted on a pole. As the water level rises, the contacts are closed by water. The contacts are connected to a series of resistors.
- Capacitance: Water forms the dielectric in a capacitor consisting of two wires or plates mounted vertically in the water. As the water level rises, the capacitance changes. The capacitance is measured, and is proportional to water level.
- Zwarts Pole: A pole is mounted vertically in the water. An electromagnetic wave transmitted down the pole reflects off the water surface, and the time for the signal to return is a direct measure of water surface elevation.

All of these have the disadvantage that they are subject to fouling by marine organisms because they must be at the ocean surface where much biological activity occurs.

2.4.1.1.2 Sub-surface

- The weight of a column of seawater varies with depth of the column. A pressure transducer at the bottom of the sea, measuring this weight, can therefore measure the height of waves as they pass over. However, wave bottom pressure decreases rapidly with depth… at depths greater than $\lambda/2$ they are less than one tenth of the surface pressure. Therefore bottom-mounted sensors are useful in relatively shallow water only. In addition, they have poor HF performance, being useful only for frequencies below 1.2Hz. Movement of sand may cover the sensor, rendering it unreliable or even inoperative.
- An echo-sounder (sonar) may be mounted on the sea floor and look upward to measure the distance to the sea surface. However, when the water aerates during a storm, it will produce inaccurate measurements.

- A horizontal current meter mounted below the surface may measure the orbital motion of water particles. However, measurement of wave direction is difficult.
- In deeper water, a buoy may be anchored to the bottom so that it ‘floats’ a few metres below the surface. The transducer is mounted on this buoy. However, the effects of currents and tides may move the buoy laterally.
- A submarine or subsurface buoy equipped with a motion-sensing system and an upward-looking echo sounder may be workable.

2.4.1.2 <u>Surface Following</u>

2.4.1.2.1 Buoys

The standard technique is to measure the motion of a buoy on the water surface. The buoy may be a small sphere or platform. Spheres that float just below the surface follow the motion of water particles in a Lagrangian manner. A discus buoy will follow the water surface in an Eulerian manner.

In shallow water, a buoy or boat may use an echo sounder to measure water depth.

Typically, however, an accelerometer is mounted in the buoy in such a way that it remains vertical. By measuring the vertical acceleration, then integrating twice, the surface position can be calculated. Buoys do not follow the wave surface exactly – their response varies with wave frequency. It is necessary to compensate for the response of large buoys [Steele 1993; Tucker 1989; Gnanadesikan 1993].

Buoys may be tethered to the ocean floor to prevent them drifting away. In this case, the buoy’s response will also depend on the mooring constraints, current, tides, and wind speed. Alternatively, buoys may be allowed to drift. In this case, the buoy may tend to ride around wave crests rather than over them.

Various brands and models of data buoys are in use. However, the market is dominated by the Waverider, manufactured by Datawell bv, The Netherlands.

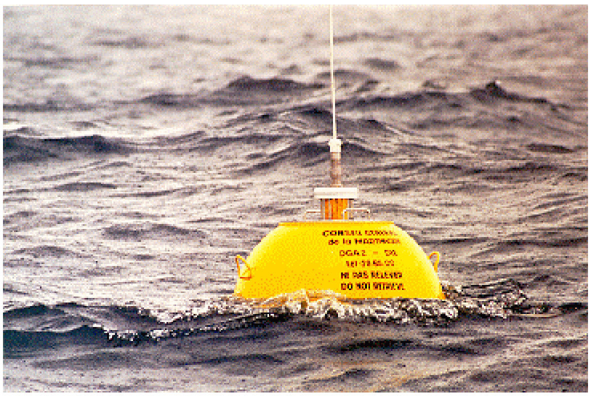

Figure 2-5: Datawell Waverider (Photo Meteo-France)

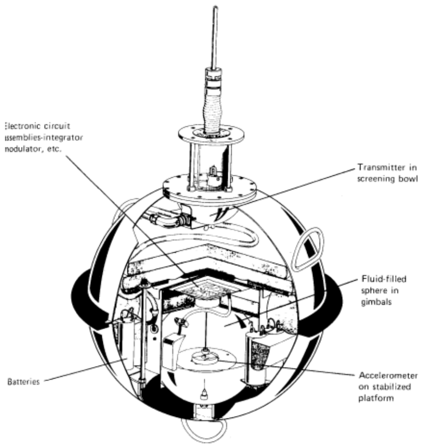

Figure 2-6: Waverider Internals [Tucker 1991]

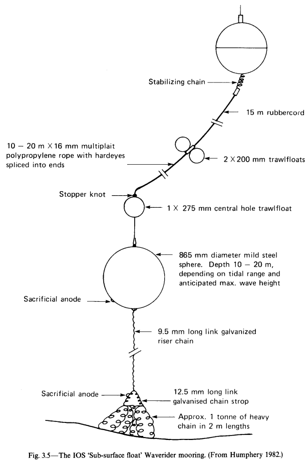

Figure 2-7: Waverider Mooring [Tucker 1991]

This is a buoy of 0.7 or 0.9m diameter. It is tethered to a flexible rubber cord 15m in length. To ensure that the accelerometer remains vertical, the Waverider uses a fluid filled sphere mounted on gimbals. The accelerometer itself is mounted on a horizontal disk, suspended on wires inside the sphere. This arrangement gives a compound pendulum with a natural period of approximately 120 seconds, so that at the 0.04-0.2Hz frequencies (5-25 second periods) typical of ocean waves the accelerometer remains approximately vertical. The accelerometer mounting mechanism is somewhat fragile, being easily damaged by mishandling or steep breaking waves.

Despite having 900N of buoyancy, the Waverider underestimates the highest waves by being dragged through the crests or floating around them [Allender 1989]. [Longuet-Higgins 1986] concluded that it results in slight underestimation of water surface elevation. The buoy's response varies with wavelength – for periods below 1.8 secs (wavelength below 5m) it does not follow the wave's surface. [Tucker 1991] concluded that the motion of the buoy reduces the magnitude of higher frequencies. The response is flat for wave periods of 5-10 secs, and there is some attenuation between 10-25 secs. The buoy samples the accelerometer at 0.3906 second intervals, meaning that the maximum frequency that can be sampled accurately is 1.2801Hz. Maximum wave height is 20m, and the maximum current for deployment is 1m/s. [Datawell Web]

A hybrid approach combining surface-following and 'fixed' sensors may be applied. For example, a pressure sensor may be mounted below the water on a buoy or ship which is relative unresponsive to waves. This may be a large ship, or a buoy that supports a massive damping disk hanging below it [Bascom 1980, p172]. An example described in [Bascom 1980, p150] is FLIP (Floating Laboratory Instrument Platform), which has a heave period of 28 seconds. In a storm with seas up to eighty feet, the platform moved about six inches vertically. To compensate for movement of the platform, its own motions may be measured and combined with the sensor measurements to provide more accurate readings.

2.4.1.2.2 Shipborne Recording

These are somewhat inaccurate by modern standards, but have been widely used. They are a combination of a vertical accelerometer and a pressure sensor, mounted so that even in the steepest waves they will remain below water level. The accelerometers measure the ship's motion, and the water pressure is added to these measurements to calculate wave height. The vessel must be stationary while wave recording, or Doppler shifts cause problems with calculating wave frequency. The response of the vessel needs to be factored into the wave calculations.

Using a small ship itself as a wave measurement device is considered in [Tucker 1991, section 3.5.2]. He suggests using a triaxial accelerometer to measure the ship's motion, from which vertical acceleration can be derived mathematically and used to calculate vertical displacement. He suggests that this might be successful up to about 0.2Hz.

2.4.2 Remote Sensing

Satellites and aircraft have been used extensively to remotely sense wave information. Both have disadvantages over in situ methods in that they give low-resolution answers; i.e. they give an average reading over a large area (perhaps hundreds of square metres). In addition, a typical satellite orbits once per 100 minutes. Thus events of less than 100 minutes duration may not be recorded at all. A satellite may only overfly the same location once every 17 days.

A radar altimeter is used to remotely sense wave height from a satellite. A radar pulse is beamed vertically downwards, and the shape of the return pulse gives wave height. This technique is relatively inaccurate (± 0.5m or 10%), and low resolution (approximately 10km diameter). This gives wave height only; no period or direction information can be derived.

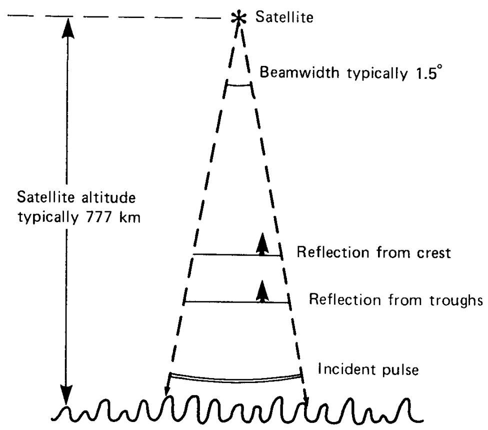

Figure 2-8: Satellite radar altimetry

In a similar approach to the satellite radar altimeter, an aircraft equipped with a laser altimeter can collect wave information. Whilst there may be some low-frequency drift due to the aircraft's movements, this can be removed during data analysis.

2.4.3 Directional Measurement

Measurement of the directional spectrum of a wave field relies on measuring several properties of the waves in that field. Properties that have been used include

- Elevation
- Water velocity
- Surface slope
- Dynamic pressure

Another approach to estimate direction is to employ multiple spatially distributed sensors. Numerous researchers [Barber 1963, Davis 1977, Howell 1993, Krylov 1966, Panicker 1970, Borgman 1977, Krogstad 1988, Nwogu 1989, Young 1994] have used spatial arrays made up of multiple surface piercing or bottom-mounted transducers to make directional measurements of waves. In general, a polygonal arrangement of at least 3 sensors is needed to measure waves in an omnidirectional spectrum. The number and arrangement of sensor elements defines the performance of the array. [Carvalho 2000]

Combined pressure/water velocity gauges may also be used. These instruments are typically bottom-mounted (eg the Woods Hole directional current meter), and measure the water pressure to determine wave height and measure the velocities of water particles in two orthogonal directions [Simpson 1969, Kobune 1986].

In deep water, buoys are used to measure wave directional data. The buoy measures heave (vertical displacement) to determine wave height, and measures pitch and roll to determine the direction of the wave's motion. Whilst there are several manufacturers, by far the most popular heave-pitch-roll buoy is the Datawell Directional Waverider. This buoy uses the same technique as the Waverider to measure vertical motion of the buoy. To determine wave direction, the Directional Waverider is equipped with a magnetic compass and two accelerometers, mounted horizontally and at right angles to each other. From these sensors, it is possible to determine the instantaneous pitch and roll of the buoy, and therefore the slopes of the wave surface in the X and Y directions. [Longuet-Higgins 1962, Ewing 1987, Tucker 1989, Brisette 1994]

Another in-situ approach, which has not had great success, was to use a cloverleaf buoy (Figure 2-9). A cloverleaf buoy consists of three buoys, connected together mechanically such that they can move independently of each other. The relative movement of the parts is then measured and can be used to determine the wave's slope. Effectively a cloverleaf buoy produces the same data as a heave-pitch-roll buoy. [Mitsuyasu 1975]

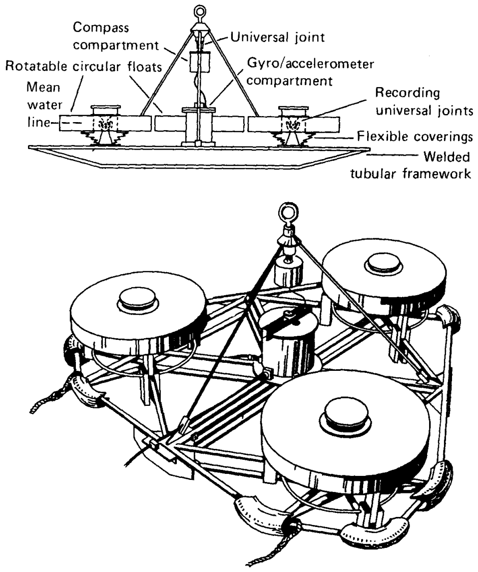

Figure 2-9: Cloverleaf buoy

Various remote-sensing techniques have been used to determine the directional wave spectrum:

- Stereophotogrammetry [Simpson 1969]
- Marine Radar [Young 1985]
- HF Radar [Tyler 1974, Trizna 1977, Trizna 1980]
- Side-looking airborne radar [McLeish 1980, Bascom, p175]
  This has been reported to allow measurement of wavelength (10-50m resolution) and wave direction (2°)
- Synthetic Aperture Radar (SAR) [Beal 1986, Rufenach 1991, Alpers 1981]
  SAR is computationally difficult, but resolution is about 20m, and a 100km wide swath is scanned. In addition, it is possible to derive information about the 2D spectrum from SAR data.

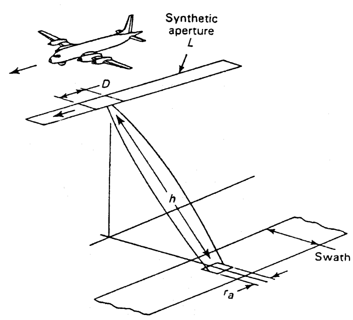

Figure 2-10: Aircraft Synthetic Aperture Radar

2.5 Analysis

Having gathered a time series of wave height and slope information, there are several ways to analyse the data [Massel 1996, p408]:

- Fourier Expansion Method: This assumes a linear wave model (i.e. sinusoidal motion). Fourier series measurement can also be used to represent trochoidal waves, which are another example of periodic motion.
- Maximum Likelihood Method: This is marginally better than the FEM
- Maximum Entropy Method: Autoregressive/FEM based on a probability density distribution. A combined MLM/MEM approach gives better results than MLM
- Bayesian Directional Method: This method is difficult to implement, but very accurate. It is preferred when at least 4 wave properties have been recorded. [Long 1979]