Pattern Formation in a Neural Field Model

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Amanda Jane Elvin

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Note: This thesis differs slightly from that submitted for examination in that it includes four additional references and some minor spelling and grammatical changes.

Abstract

In this thesis I study the effects of gap junctions on pattern formation in a neural field model for working memory. I review known results for the base model (the "Amari model"), then see how the results change for the "gap junction model".

I find steady states of both models analytically and numerically, using lateral inhibition with a step firing rate function, and a decaying oscillatory coupling function with a smooth firing rate function. Steady states are homoclinic orbits to the fixed point at the origin. I also use a method of piecewise construction of solutions by deriving an ordinary differential equation from the partial integro-differential formulation of the model. Solutions are found numerically using AUTO and my own continuation code in MATLAB. Given an appropriate level of threshold, as the firing rate function steepens, the solution curve becomes discontinuous and stable homoclinic orbits no longer exist in a region of parameter space. These results have not been described previously in the literature.

Taking a phase space approach, the Amari model is written as a four-dimensional, reversible Hamiltonian system. I develop a numerical technique for finding both symmetric and asymmetric homoclinic orbits. I discover a small separate solution curve that causes the main curve to break as the firing rate function steepens and show there is a global bifurcation. The small curve and the global bifurcation have not been reported previously in the literature. Through the use of travelling fronts and construction of an Evans function, I show the existence of stable heteroclinic orbits.

I also find asymmetric steady state solutions using other numerical techniques. Various methods of determining the stability of solutions are presented, including a method of eigenvalue analysis that I develop. I then find both stable and transient Turing structures in one and two spatial dimensions, as well as a Type-I intermittency. To my knowledge, this is the first time transient Turing structures have been found in a neural field model. In the Appendix, I outline numerical integration schemes, the pseudo-arclength continuation method, and introduce the software package AUTO used throughout the thesis.

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