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***APPLICATION OF MARKOV CHAIN MODEL
IN STREAMFLOW FORECASTING***

A thesis
submitted in partial fulfilment
of the requirements for the degree
of
Master of Science in Geography
at
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by
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Abstract

This thesis presents an approach to streamflow forecasting based on a Markov chain model to estimate conditioned probabilities that a one time-step ahead streamflow forecast will be within a certain streamflow range. In this application a set of "states of flow" defined over streamflow ranges (intervals) forms a finite state space of a Markov chain. Flood forecasting is made by focusing on a preselected state of flow as a flood state.

A multi-objective (two criteria) function for the quantification of the model performance is introduced. Specifically designed for a flood forecasting and warning system the two criteria are the probability of issuing a false alarm and the probability of failing to forecast a flood event. The goal is to minimize both criteria simultaneously together with a preference of accepting more false alarms than misses.

The model has two options of making a forecast: (1) a Threshold Forecast (ThF) approach in which a forecast is based on the probability of making a one-step transition from any state into the flood state; (2) the Most Probable Event (MPE) forecast approach selects the state of flow where the next streamflow is most likely to occur.

Forecasts being probabilistic, there are several options for deciding on when it is appropriate to issue a flood warning in the probabilistic framework. A search for the appropriate probability p_0 is made on interval $[0,1]$ through evaluation of the objective function at each p_0 , using data sets from three North Island catchments (Akitio River, Makakahi River and Kiwitea Stream).

The model applying the option of threshold forecasts performed generally well depending on the relative costs assigned to false alarms and misses. The model performed better on the Akitio River which has strongly fluctuating streamflows than on the Makakahi River and Kiwitea Stream which have relatively modest variations in flows.

When the Model applied the option of the most probable event forecasts did not perform well as the probabilities of false alarms were found to be too high for the model to be accepted.

The outcome of this study suggests a simple short-term flood forecasting procedure especially for rivers with strongly fluctuating flows.

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Chapter 1

INTRODUCTION

Streamflow forecasting is an important component in a variety of routine water management activities. The forecasting problem deals with the development of models which can foretell the future state of streamflow. Its application in this study, however, is aimed at flows associated only with flooding. Flood forecasting is one of the most practicable options in attempts to minimize effects of floods.

1.1 Rationale of the study

Like many places in the world, much of New Zealand's developed land, both urban and agricultural, lies in river basins susceptible to flooding. The interaction between the physical flood event and human use of the floodplain results in flood hazard. The level of loss depends on both the characteristics of the flood and the affected human use systems such as the magnitude of the flood and its speed of onset, and the types of land use affected.

Property damage associated with floods ranges from physical destruction of structures caused by water flow to damage due to wetting. Losses from floods can be grouped into three somewhat overlapping types: property damage, social disruption and human casualties.

Any physical material adversely affected by flood water is a property loss, e.g. transport infrastructure, buildings and industrial machinery. In rural areas damage includes loss of stock and crops as a result of siltation and/ or inundation.

Social disruption losses range from re-scheduling to termination of activities in preparation for, and/or as a result of flooding. For example, a damaged factory may

cease production, a damaged transport system will frustrate the distribution of goods and services to and from the community, or disease may breakout due to unsafe water.

While in most cases injuries and loss of lives are minimal, the toll of human suffering and mental anguish as a result of property losses and social dislocation may be very considerable. For example a very disruptive and long term displacement of many families in the Invercargill flood created so much stress that in some cases family breakup took place (Ericksen, 1986).

In general, losses by flood events can really be significant. For the small community of only 3702 people in Paeroa, property loss during the 1981 floods exceeded \$7 million and social disruption was estimated at \$4 million. A review of flood losses in New Zealand (Ericksen, 1986) suggests that since 1968 the overall flood-loss bill to the nation may be as high as \$1.5 billion or \$90 million per year, and the losses are continuing to rise.

In order to alleviate the effects of flooding there are several options available, classified into the following three groups:-

(1). Modifying flood events through catchment treatment and river control works. Measures of this type aim to adjust floods to the convenience of people and this may be accomplished in two ways: to alter runoff through catchment treatment measures, and to alter stream flows by use of flood protection structures. Both seek to change parts of the local hydrologic cycle and so reduce the incidence of flooding.

(2). Modification of flood-loss susceptibility through land use management, flood warning, and community preparedness. This approach seeks to alter human use and occupance on floodplains to forms that are more compatible with the flood risk. This may be accomplished through land use management, flood-proofing buildings, emergency programmes or flood forecasting and warning. No attempt is made to modify the magnitude of the flood; rather to lessen flood losses.

Land use management seeks to guide the type of land uses and level of development within flood-prone areas to keep the flood hazard minimal, yet to maximize net benefits from its continued use (Ericksen, 1986). Specific structural design measures that make normal buildings less susceptible to flooding are the usual flood-proofing measures. For emergency procedures, a set of interrelated measures beginning with a flood forecasting and warning system through to individual and collective emergency actions is used to reduce flood-loss susceptibility. Flood forecasting, which provides information on flood levels, is also important for other adjustments, like flood-proofing to work satisfactorily.

(3) Modification of the flood-loss burden is intended to enable affected individuals to share the burden of loss with others, particularly in the unaffected areas. This may be done through pre-event insurance where the individuals pay a premium against losses from flooding they might experience. In effect, the insurance spreads the burden of loss in time and space among insurees. Another approach of burden sharing are the various post-event means which include government grants, subsidies, loans, public appeals for material or funds to help in the relief of flood victims and for rehabilitating affected areas.

The focus in this study is on the flood forecasting and warning option. At a macroeconomic level, the damage that potentially can be prevented by action taken in response to flood forecasts is much greater than the cost of providing forecasts (Krzysztofowicz R. and Davis D. R., 1983). Development of a flood forecasting option as a one of the measures used to mitigate damage from flooding, remains an important area of study in hydrology.

In New Zealand, regional councils are responsible, following the Resource management Act of 1991 for operating flood forecasting systems and for day-to-day monitoring of river flows. Most forecasting methods are classified as "manual", in that they depend on a substantial amount of judgement on the part of the forecaster. The assessments of conditions which could result in flooding are based on the forecaster's previous experience and local knowledge. Rainfall-runoff models and flow-to-flow routing models

are being applied for only a few rivers (Pearson C.P. and Jordan R.S., 1991).

1.2 Criteria for acceptable model forecasts

The best possible forecast is that which completely and identically describes a process which will occur in future. This is, however, difficult to attain since all forecasting techniques contain an element of uncertainty. The acceptable model forecast, therefore, should at least be with minimum variance of forecast errors and it becomes necessary to specify the form and accuracy of the forecast.

In this study a stochastic approach employing a simplified "Markov-chain flow model" is applied to three catchments. The methodology is built around (1) a "Markov-chain flow model" and (2) a model performance quantification objective function. The first element provides a mechanism of short-term streamflow forecasting by estimating the probabilities that the one-step ahead streamflow forecast will be within specified flow ranges. With this approach, flood forecasting is possible by focusing on a pre-selected state of streamflows. The second element establishes the model performance measures and expresses them in terms of a two-criteria objective function to assess the ability of the model to forecast whether the flood threshold probability will be exceeded. The criteria are the probabilities of issuing false alarms and misses prior to flood events. With a desire to simultaneously minimize false alarms and misses, the quantification is designed to serve as a decision aid to the targeted users such as New Zealand regional councils.

The model was tested on the Akitio River, Makakahi River and Kiwitea Stream, all under the jurisdiction of the Manawatu Wanganui Regional Council (North Island). The results aim to provide relevant information on the suitability of the model depending on the costs associated with false alarms and misses of flood events.