Supporting Information for "Forecasting eruptions at poorly known volcanoes using analogues and multivariate renewal processes"

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Introduction

The supplementary material contains figures of the data and the results from some of the data analysis (Figures S1 to S7), and a sensitivity analysis exploring the influence of missing data from the candidate volcano and analogues on the forecasts (Text S1). R code used

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to carry out the data analysis and plot the results and some instructions on running the R code can be found via `https://github.com/athenatingwang/AnalogVolcanoForecast`.

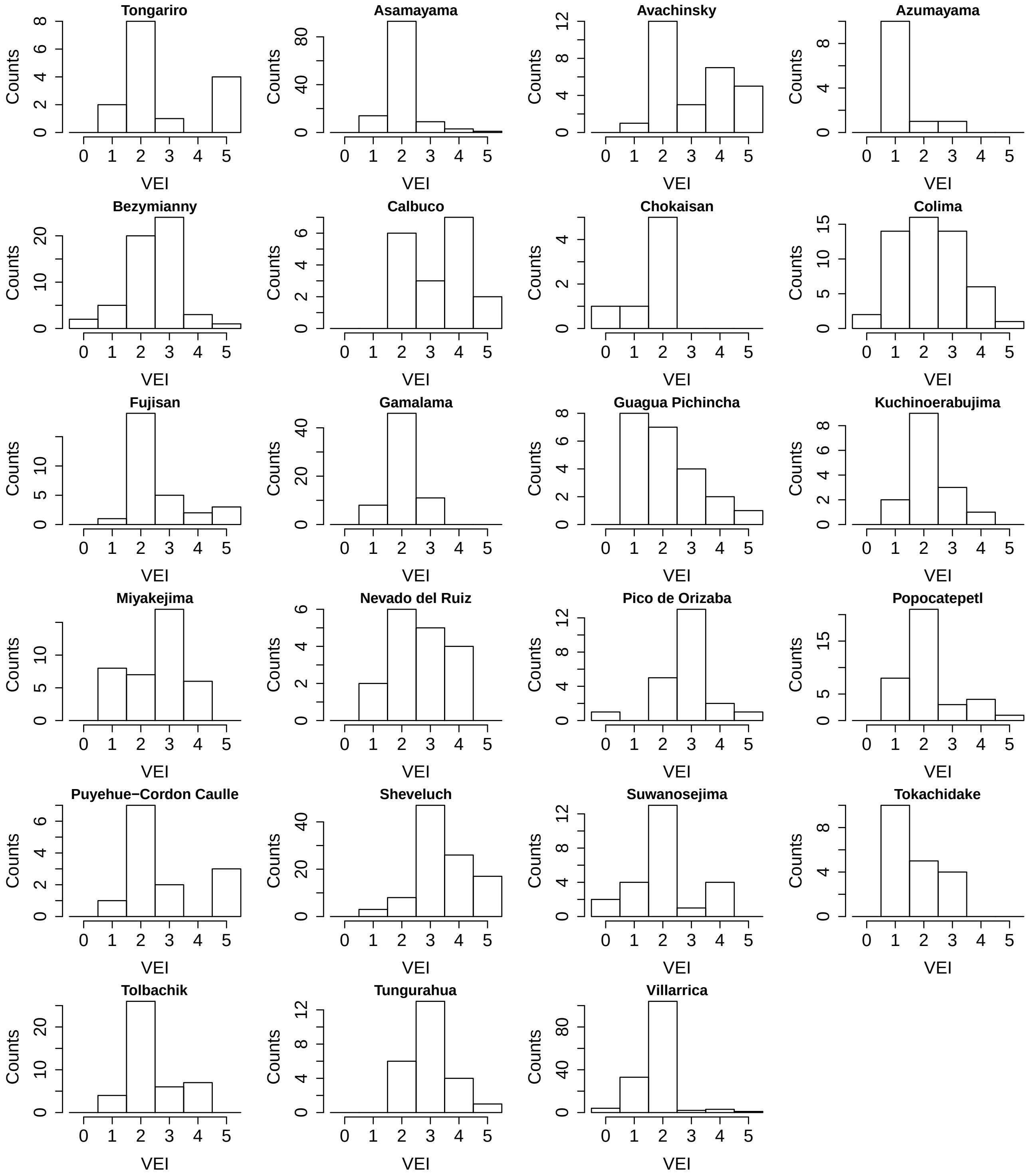

Figure S1. VEI frequency plot for GVP records of the empirical analogue volcanoes.

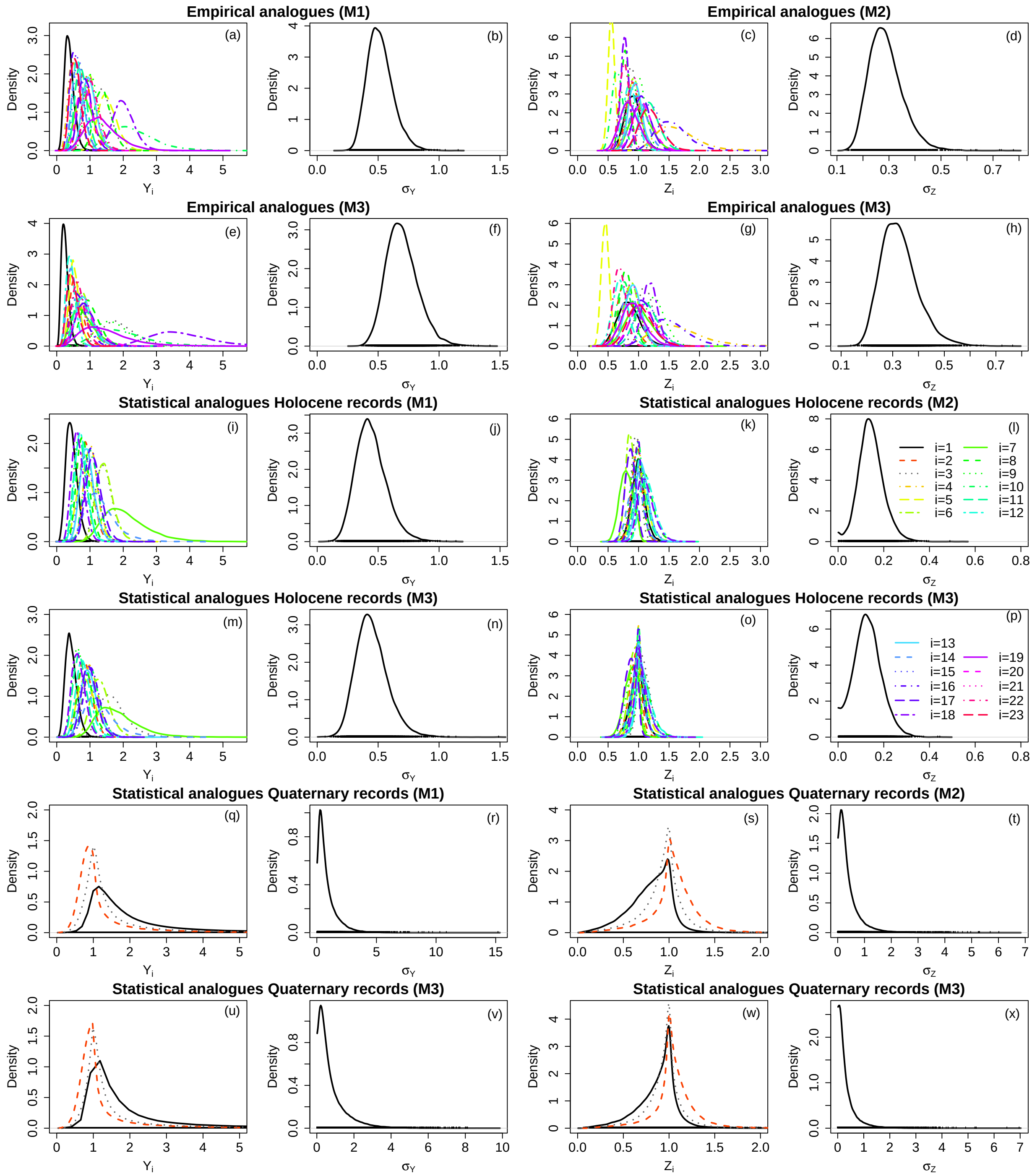

Figure S2. The posterior distributions of Y_i, σ_Y, Z_i and σ_Z. (a,b) Model M1, (c,d) Model M2, and (e,f,g,h) Model M3 fitted to the GVP records of VEI≥3 eruptions from Tongariro and the 22 empirical analogues. (i,j) Model M1, (k,l) Model M2, and (m,n,o,p) Model M3 fitted to the GVP records of VEI≥3 eruptions from Tongariro and the 17 statistical analogues. (q,r) Model M1, (s,t) Model M2, and (u,v,w,x) Model M3 fitted to Tongariro, Three Sisters, and Puyehue-Cordón Caulle eruptions with minimum VEI 3.

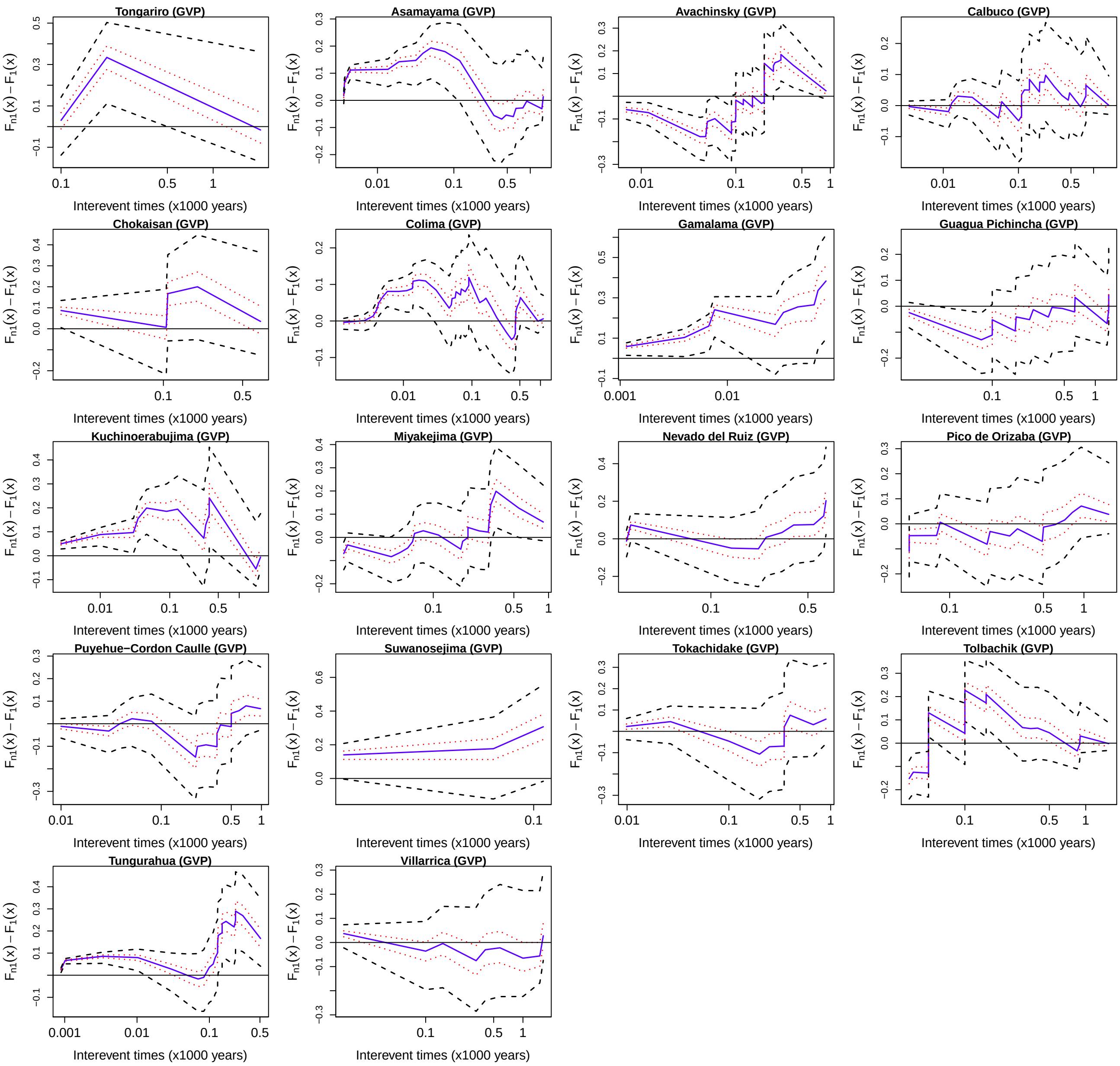

Figure S3. The posterior predictive distributions of $\bar{F}(x_{ij}) - \hat{F}(x_{ij})$ $(i = 1, \cdots, 18; \quad j = 1, 2, \cdots, N_{3,i})$ for model M1 fitted to the GVP records of VEI≥3 eruptions from Tongariro and the 17 statistical analogues. The dotted lines indicate the 25% and 75% quantiles, and the dashed lines indicate the 0.5% and 99.5% quantiles.

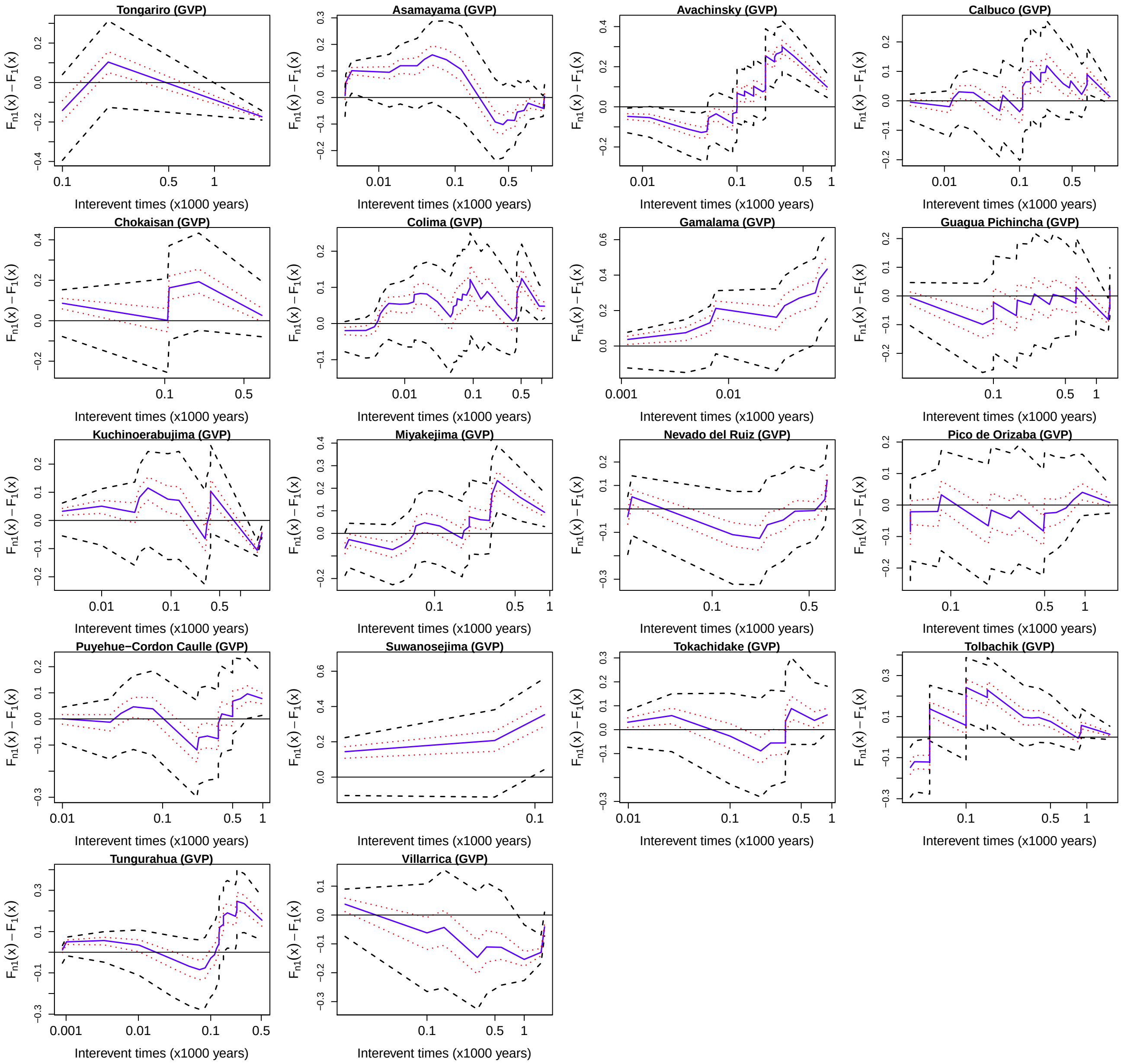

Figure S4. The posterior predictive distributions of $\bar{F}(x_{ij}) - \hat{F}(x_{ij})$ ($i = 1, \cdots, 18$; $j = 1, 2, \cdots, N_{3,i}$) for model M2 fitted to the GVP records of VEI≥3 eruptions from Tongariro and the 17 statistical analogues. The dotted lines indicate the 25% and 75% quantiles, and the dashed lines indicate the 0.5% and 99.5% quantiles.

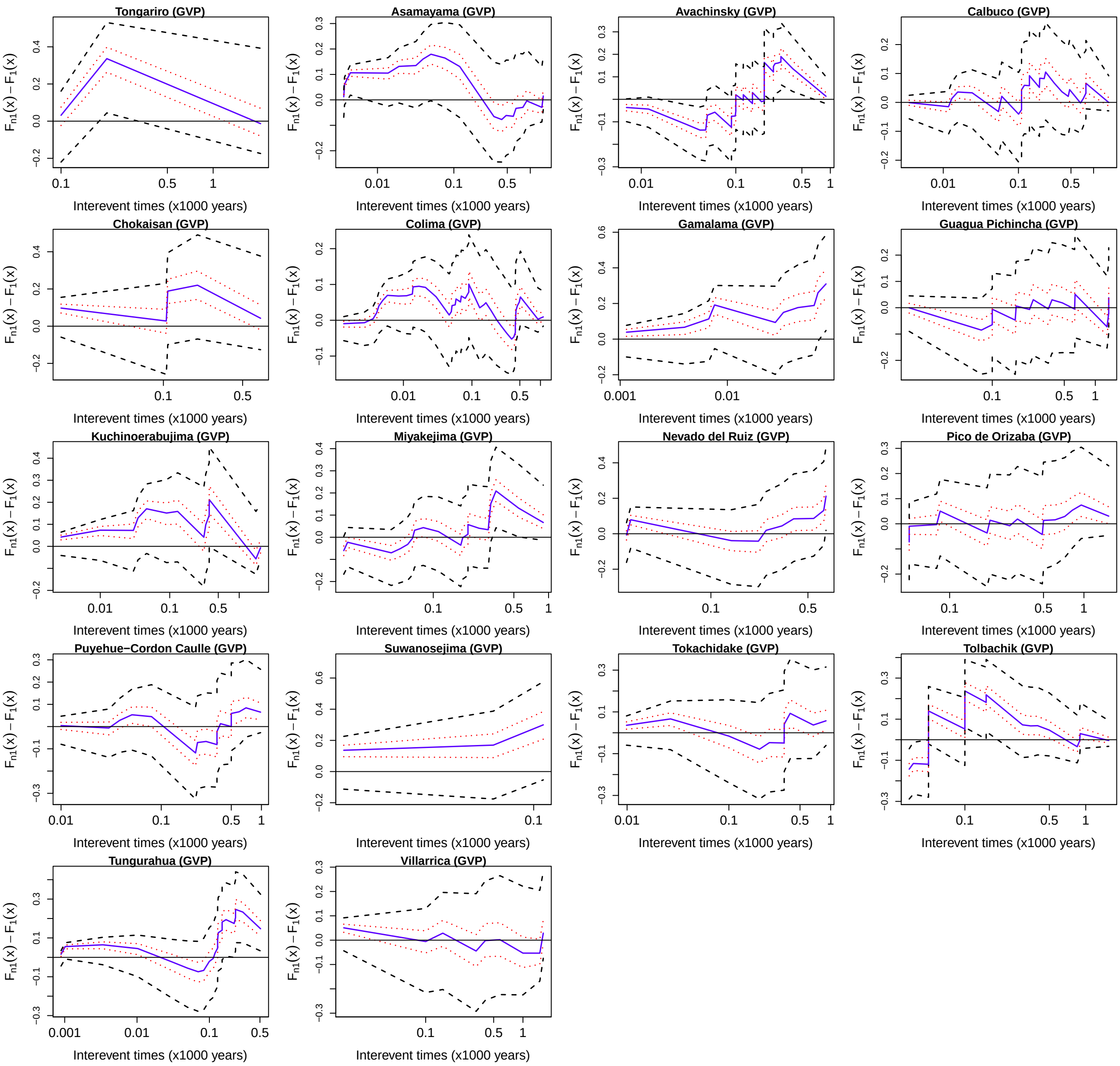

Figure S5. The posterior predictive distributions of $\bar{F}(x_{ij}) - \hat{F}(x_{ij})$ $(i = 1, \cdots, 18; \quad j = 1, 2, \cdots, N_{3,i})$ for model M3 fitted to the GVP records of VEI≥3 eruptions from Tongariro and the 17 statistical analogues. The dotted lines indicate the 25% and 75% quantiles, and the dashed lines indicate the 0.5% and 99.5% quantiles.

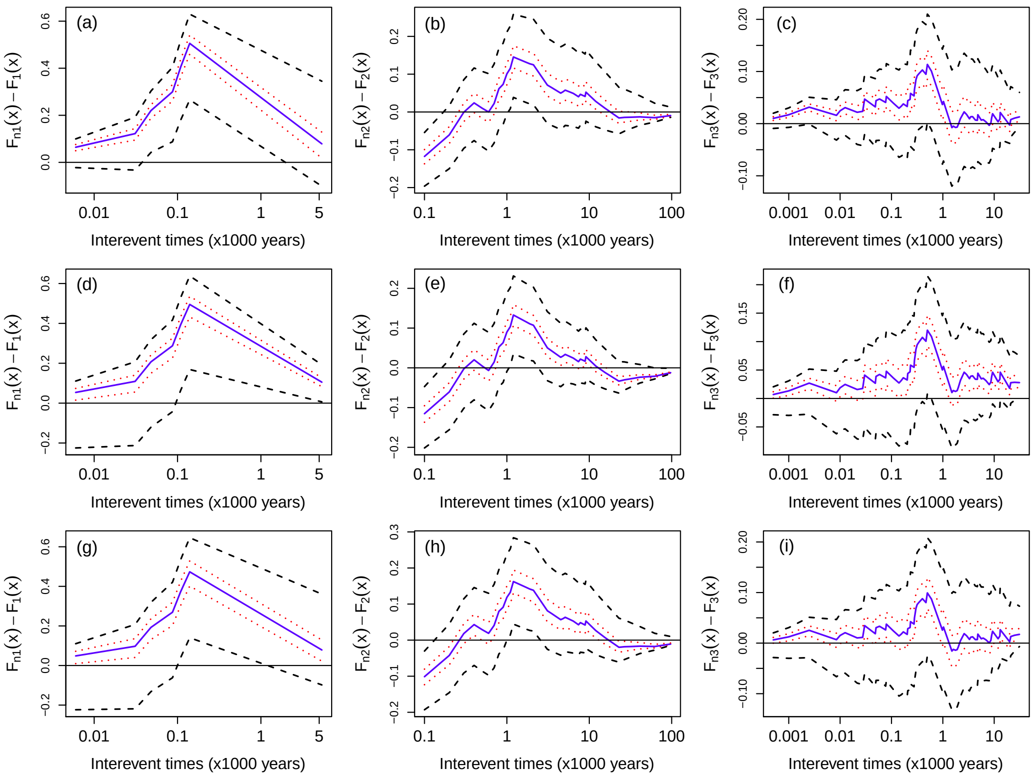

Figure S6. The posterior predictive distributions of $\bar{F}(x_{ij}) - \hat{F}(x_{ij})$ ($i = 1, 2, 3$; $j = 1, 2, \cdots, N_{4,i}$) for the three models fitted to the eruptions with minimum VEI 4 from the geological analogues, (a,d,g): Tongariro, (b,e,h): Three Sisters, and (c,f,i): Puyehue-Cordón Caulle. (a,b,c): Model M1, (d,e,f): Model M2, (g,h,i): Model M3. The dotted lines indicate the 25% and 75% quantiles, and the dashed lines indicate the 0.5% and 99.5% quantiles.

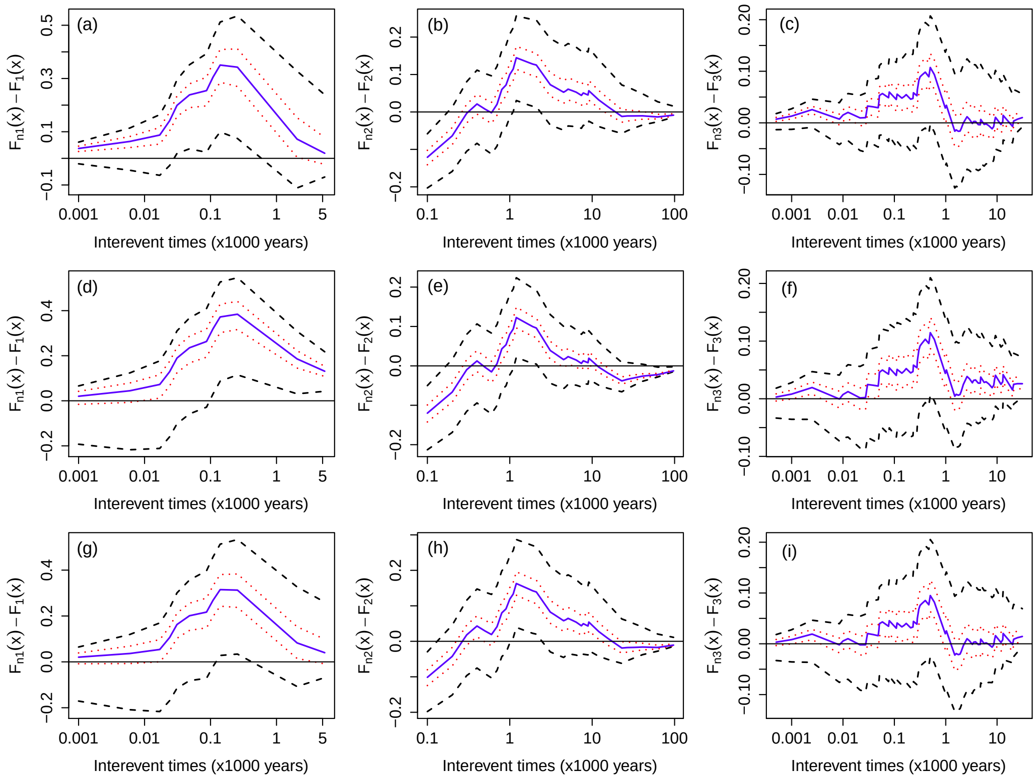

Figure S7. The posterior predictive distributions of $\bar{F}(x_{ij}) - \hat{F}(x_{ij})$ ($i = 1, 2, 3$; $j = 1, 2, \cdots, N_{3,i}$) for the three models fitted to the eruptions with minimum VEI 3 from the geological analogues, (a,d,g): Tongariro, (b,e,h): Three Sisters, and (c,f,i): Puyehue-Cordón Caulle. (a,b,c): Model M1, (d,e,f): Model M2, (g,h,i): Model M3. The dotted lines indicate the 25% and 75% quantiles, and the dashed lines indicate the 0.5% and 99.5% quantiles. Model M3 performed the best for all data, and model M2 performed the worst based on the residual analysis. For model M3, the horizontal line at zero is mostly within the 99% credible intervals in Figures S5, S6, and S7, except possibly for medium interevent times of Tongariro and short interevent times of Three Sisters. This is understandable for Tongariro, because it has only 14 eruptions with minimum VEI 3, and 9 eruptions with minimum VEI 4 in the geological record. The record of Three Sisters doesn't include any very short interevent times (<100 years), which may contribute to the slight deviation at short interevent times. This provides some assurance that the model is not overfitting the data. Based on the discussion above, we would adopt the forecast from model M3.

Text S1. Sensitivity analysis

This section includes a sensitivity analysis exploring the influence of missing data from the candidate volcano and analogues on the forecasts. In this analysis, we explored two types of missingness. The first assumed that the probability of missing an eruption is uniform over time. The second assumed that the probability of missing an eruption is more likely for earlier events as there is evidence that earlier eruptions are more likely to be missing. For the simulation study, data was removed to see if missing data affects forecasts for the next occurrence time. This was done in two ways: 1) simulating 50% more inter-event times than observed in the catalogue for each volcano in the statistical analogues, then randomly removing 10%, 20%, 30% and 40% of the data; and 2) simulating 10%, 20%, 30% and 40% more data than observed for each volcano and then remove the same amount that was added randomly so each time we end up with the same number of events as listed in Table 1 in the manuscript.

The simulation study suggests that the hierarchical model greatly reduces the influence of missing events on hazard forecasts with up to 40% missing data from the records of all the volcanoes (both candidate and analogue volcanoes). Below we give the details of the simulation study and the results.

S1 1. Parameter estimates from model M3

The statistical analogues identified in the main manuscript and the posterior mean values of Y_i and Z_i for the best model M3 are listed in Table S1. The posterior mean values of the parameters α and β for model M3 are $\hat{\mu_\alpha} = 0.752$ and $\hat{\mu_\beta} = 0.332$.

Table S1. Posterior means for Y_i, Z_i, and number of VEI$\geq$ 3 eruptions (n_i) for Tongariro and the 17 statistical analogue volcanoes.

Name	$\hat{\mu_{Y_i}}$	$\hat{\mu_{Z_i}}$	n_i
Tongariro	0.434	0.975	6
Asamayama	0.857	0.943	21
Avachinsky	1.732	1.095	42
Calbuco	1.048	0.996	30
Chokaisan	0.919	1.021	7
Colima	1.323	0.928	46
Gamalama	1.860	0.876	12
Guagua Pichincha	0.880	1.098	18
Kuchinoerabujima	0.667	0.904	15
Miyakejima	1.065	1.008	24
Nevado del Ruiz	0.708	0.991	12
Pico de Orizaba	0.816	1.102	17
Puyehue-Cordon Caulle	0.994	1.063	18
Suwanosejima	1.315	0.972	5
Tokachidake	0.969	1.055	12
Tolbachik	1.069	0.996	28
Tungurahua	0.990	0.870	25
Villarrica	0.658	1.052	11

S1 2. Sensitivity analysis for model M3

Using R, inter-event times for VEI$\geq$ 3 eruptions were simulated for Tongariro and the 17 statistical analogue volcanoes. This was done by using the rweibull function to simulate random inter-event times from a Weibull distribution. The posterior means of $\hat{\mu}_{Y_i}$, $\hat{\mu}_{Z_i}$ (Table S1) and $\hat{\mu_\alpha}$ and $\hat{\mu_\beta}$ obtained by fitting model M3 to the data from Tongariro and the 17 statistical analogues were used as the true parameters of the Weibull renewal process. R uses the pdf

$$f(x) = \frac{a_i}{b_i}\left(\frac{x}{b_i}\right)^{a_i-1} e^{-\left(\frac{x}{b_i}\right)^{a_i}}$$

where

$$a_i = \hat{\mu}_{Z_i}\hat{\mu_\alpha}$$
$$b_i = \hat{\mu}_{Z_i}^{1/\hat{\mu}_{Z_i}\hat{\mu}_\alpha}\hat{\mu}_{Y_i}^{-1/\hat{\mu}_{Z_i}\hat{\mu}_\alpha}\hat{\mu}_\beta^{1/\hat{\mu}_{Z_i}}$$

for the ith volcano. Using these parameters, we can simulate inter-event times of VEI$\geq$ 3 eruptions from the ith volcano. The last inter-event time of each volcano was replaced with a random value from a uniform distribution between 0 and the simulated value to censor the data. This represents the time since the last eruption.

S1 2.1. First simulation study with missing data

Data was removed to see if missing data affects forecasts for the next occurrence time. This was done by simulating 50% more inter-event times than observed (n_i in Table S1) in the catalogue for each volcano, then randomly removing some of the data. Two methods were used to remove data. In the first method it was assumed that the probability of missing an eruption is uniform. The second method assumed that the probability of missing an eruption is more likely for earlier events as there is evidence that earlier eruptions are more likely to be missing (Mead and Magill,

2014) [see reference list in main manuscript]. This was carried out using an inverse logit function in the software R to convert the occurrence times to probabilities. The function `invlogit` was used from the `LaplacesDemon` package in R, which takes a vector of real numbers (x) and transforms them into probabilities (p) in the range $[0, 1]$ using the function

$$p = \frac{1}{1 + e^{-x}}.$$

For both removal methods, if the occurrence time was within the last 500 years it would not be removed as studies have shown that eruption records for the last 500 years are most likely to be complete (Mead and Magill, 2014).

Each method was carried out for 10%, 20%, 30% and 40% missing data. Model M3 was fitted to the five datasets (no missing, 10%, 20%, 30% and 40% missing) and forecasts for the next eruptions were obtained. This was repeated 100 times to obtain the uncertainties of the forecast time for the next eruption with 0%, 10%, 20%, 30% and 40% missing data. This was done using both of the data removal methods. We also use only the eruption record of Tongariro and a Weibull renewal process to forecast the next eruption (that is, without any analogues). Figure S8 shows the plots of the 95% credible interval for the 0.5%, 50% and 99.5% quantiles of the forecast time of the next eruption are shown for uniform probability of missing data and non-uniform probability of missing data for using Tongariro only and using analogue volcanoes.

The credible intervals for the 0.5%, 50%, 99.5% quantiles of the forecast times in Figure S8 tend to get wider as the percentage of missing data increases. The medians of the 0.5%, 50% and 99.5% quantiles of the forecast times tend to increase as the percentage of missing data increases for both of the removal methods. However the credible intervals for each quantile overlap for different percentage of missing data when using analogue volcanoes. This suggests

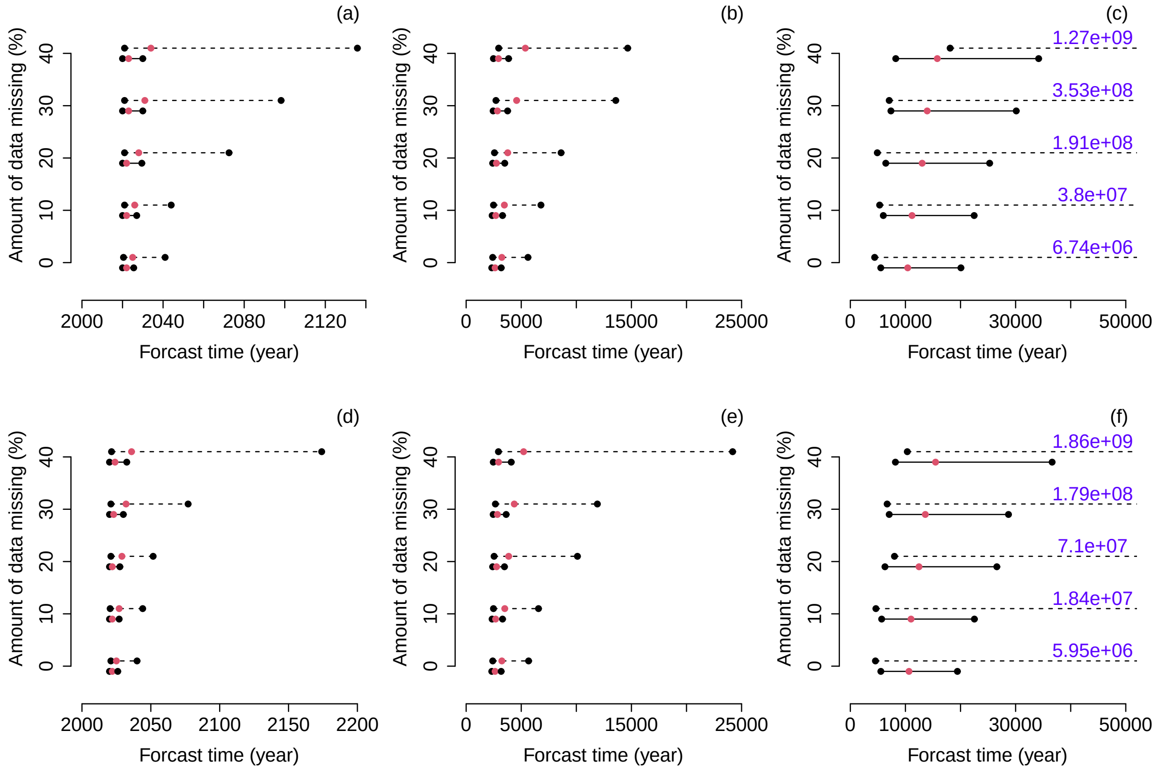

Figure S8. (a,b,c): 95% credible intervals for the 0.5%, 50% and 99.5% quantiles respectively of the forecast time for the next eruption, using uniform removal method with 0%, 10%, 20%, 30% and 40% missing data. (d,e,f): 95% credible intervals for the 0.5%, 50% and 99.5% quantiles respectively of the forecast time for the next eruption, using non-uniform removal method with 0%, 10%, 20%, 30% and 40% missing data. Dashed lines are forecasts from the model using Tongariro only, and solid lines are forecasts from model M3 using analogue volcanoes. Red dots show the medians and black dots show the upper and lower limits of the 95% credible interval. Values for the upper limit of the 99.5% quantile for Tongariro only are shown in blue as they are too large to represent in the plot.

that the hierarchical structure of the model and borrowing strength from analogue volcanoes help reduce the effects of missing data on the forecasts for future eruptions. Moreover, the forecasts from model M3 using analogue volcanoes produce much narrower credible intervals than that from the Weibull renewal process using only Tongariro. The widths of the credible intervals of the forecasts from the Weibull renewal process using only Tongariro are much more than twice the widths of that from model M3 using analogue volcanoes. This further demonstrates the robustness of the hierarchical model using analogues in forecasting future eruptions.

S1 2.2. Second simulation study with missing data

Another method was used to obtain these credible intervals. This involved simulating 10%, 20%, 30% and 40% more data for each volcano and fitting model M3 to these datasets to obtain forecasts for the next eruption. The same amount of data that was added then got removed randomly if the occurrence time was not within the last 500 years using both removal methods (uniform or time dependent). Model M3 was then fitted to these incomplete datasets and forecasts were obtained. This was repeated 100 times saving the 0.5%, 50% and 99.5% quantiles each time to obtain 95% credible intervals. Plots of the 95% credible intervals for the forecasts from the Weibull renewal process using Tongariro only and for the forecasts from model M3 using analogues are shown in Figure S9 for uniform probability of missing data and Figure S10 for non-uniform probability of missing data. The results appear to be similar to that from the first simulation.

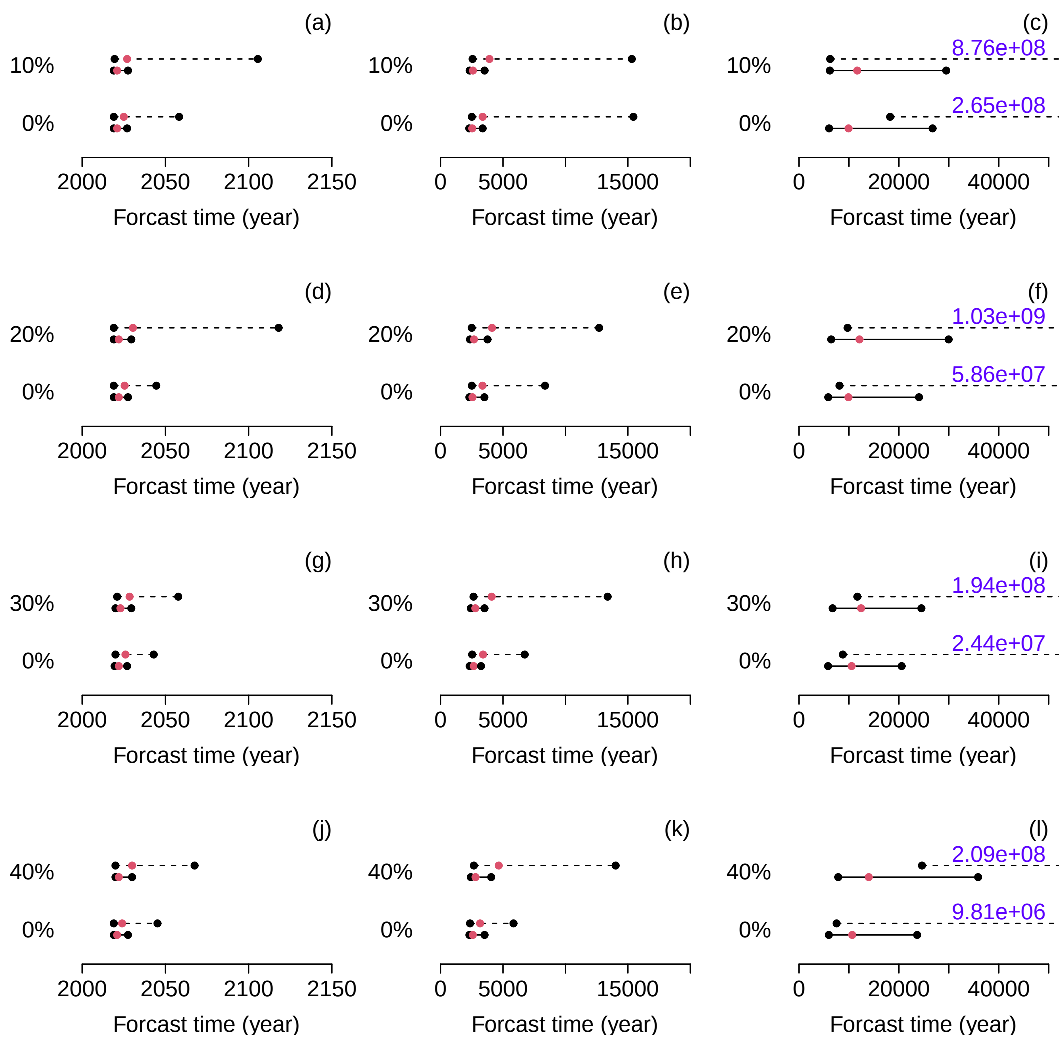

Figure S9. Credible intervals (95%) for the forecast time of the next eruption using uniform method of removing data. (a,b,c) 0.5%, 50% and 99.5% quantiles respectively with 10% missing data. (d,e,f) 0.5%, 50% and 99.5% quantiles respectively with 20% missing data. (g,h,i) 0.5%, 50% and 99.5% quantiles respectively with 30% missing data. (j,k,l) 0.5%, 50% and 99.5% quantiles respectively with 40% missing data. Dashed lines are forecasts from the model using Tongariro only, and solid lines are forecasts from model M3 using analogue volcanoes. Red dots show the median and black dots show the upper and lower limits of the 95% quantile. Upper limit values are shown in blue where they are too large to represent in the plot.

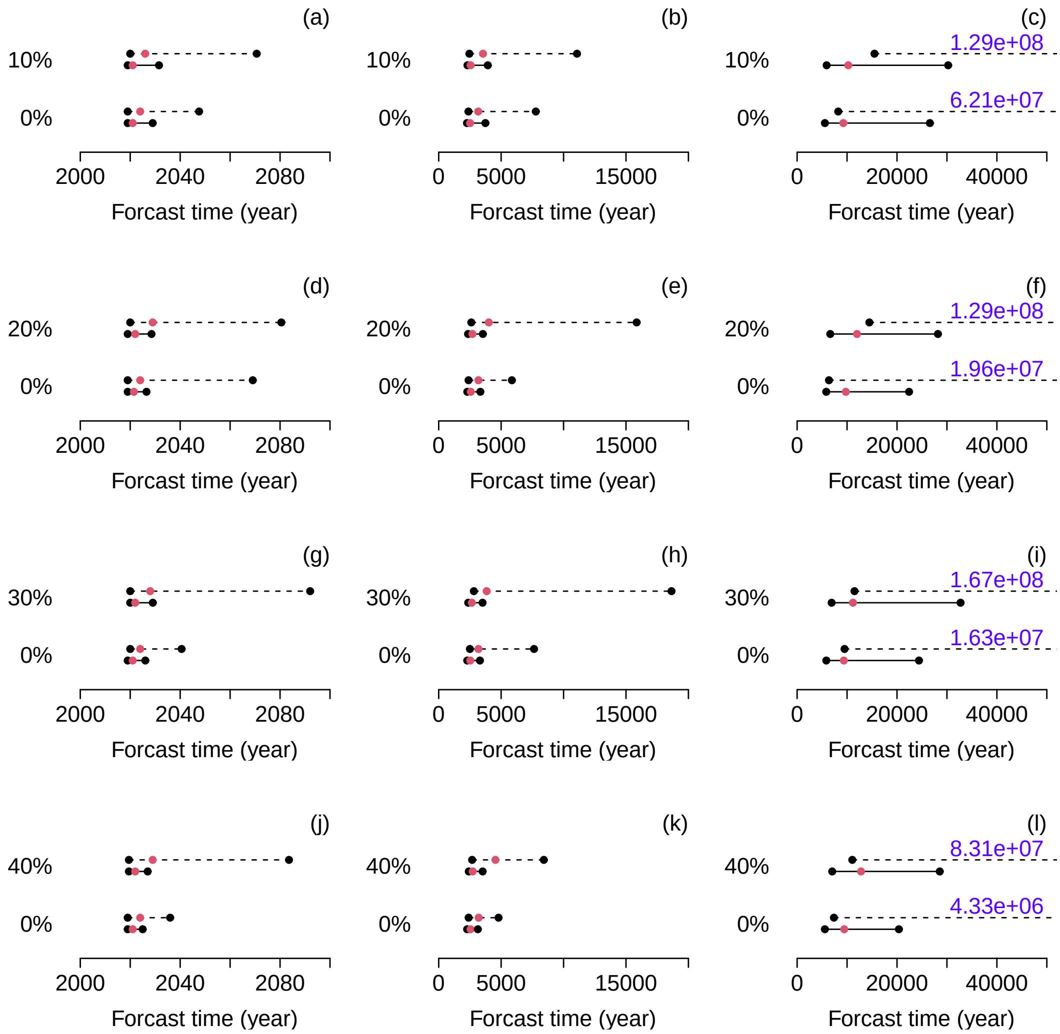

Figure S10. Credible intervals (95%) for the forecast time of the next eruption using non-uniform method of removing data. (a,b,c) 0.5%, 50% and 99.5% quantiles respectively with 10% missing data. (d,e,f) 0.5%, 50% and 99.5% quantiles respectively with 20% missing data. (g,h,i) 0.5%, 50% and 99.5% quantiles respectively with 30% missing data. (j,k,l) 0.5%, 50% and 99.5% quantiles respectively with 40% missing data. Dashed lines are forecasts from the model using Tongariro only, and solid lines are forecasts from model M3 using analogue volcanoes. Red dots show the median and black dots show the upper and lower limits of the 95% quantile. Upper limit values are shown in blue where they are too large to represent in the plot.