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**OPTIMAL ROBUST CONTROL SYSTEMS DESIGN
AND ANALYSIS
BY STATE SPACE APPROACHES**

A THESIS PRESENTED IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
TECHNOLOGY AT MASSEY UNIVERSITY.

HAO WEI

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To my mother for her love

仅以此论文献给我最亲爱的母亲

ABSTRACT

This thesis provides a fundamental investigation of robust control, both the issues of robust controller design and robustness analysis of control systems are addressed. The techniques presented evolve from time domain descriptions of linear systems and employ state space approaches. A comprehensive review of the field is given and several significant advances are presented. These include some new design and analysis techniques and some new perspectives on existing techniques. The thesis is fundamental in nature, systematically developing and criticising algorithms and methodologies. Some numerical examples are employed to illustrate the results.

Robust control addresses problems caused by discrepancies between nominal system models used for conventional controller design and analysis, and actual 'real' systems. Much of the classical work in the field assumed no knowledge of possible (or even probable) uncertainties and considered system tolerance to some general, imprecise classes of discrepancy. This tended to lead to conservative designs which degraded system performance to an unnecessary extent.

The modern trend is to provide a 'precise' prediction of possible (probable) uncertainties, described by an uncertainty model. This aims to avoid the consideration of unfeasible discrepancies which often caused the conservatism and will tend to minimise performance degradation. However, tolerance to further (hopefully small) unpredicted uncertainties should still be considered as such residual discrepancies will always exist. This modern trend is supported in this thesis and one of the main potential benefits of the new methodologies will be less conservative designs.

The principle contributions include: systematic methods for the design of cost-optimal robust controllers for both full state feedback and output feedback systems. These explicitly consider a nominal system model and an admissible domain of uncertainties and also provide some inherent robustness to residual uncertainties. Furthermore, a new method for the analysis of the robustness of given full state feedback controllers is presented. For an admissible domain of uncertainty of given structure, the maximal magnitude is determined such that stability and performance criteria are upheld.

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