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Unconscious Processing in Children:

Developing Mathematical Concepts
Through Mathematical Puzzles

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of the requirements for the degree
of Master of Arts in Psychology
at Massey University

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Abstract

The degree of occurrence of unconscious versus conscious processing of information in children is unclear. Also unclear is knowledge of mechanisms and reasons for selective transference of unconsciously processed sensory and perceptual information into conceptual, conscious and verbal awareness. Examples of unconscious processing were evidenced in the difficulty some children experienced verbalising processes and naming classifications after solving some mathematical puzzles. Correct solutions indicated children had clear conceptual understanding of the structure of the task. A series of personally designed junior mathematical puzzles utilising environmental materials to aid development of pre-mathematical concepts of classification, patterning, seriation, ideas of conservation and one to one correspondence in five and six year old New Zealand primary school children were used, and extended to incorporate mathematical concepts taught to twelve year old children at New Zealand intermediate school Form 2 level. These senior puzzles incorporated concepts of set theory, probability, matrices, tessellations, and rotational patterning and ordering, with some puzzles developed to adult difficulty levels. Some adults had difficulty with some junior puzzles, and found senior puzzles as difficult as the twelve year olds they were designed for. Mathematically able six year old children solved some senior puzzles successfully. A hypothesis developed that children could master mathematical concepts considered beyond their age ability defined by the current school curriculum, provided concepts were presented in manipulable and visual form. This was supported in the present study in 1985 - 1986 where 92 six to ten year old junior and senior children with the highest and lowest mathematical ability or special learning difficulties in three primary schools researched the puzzles. Schools selected senior children from national age normed Progressive Achievement Tests (P.A.T.) mathematics results. Junior children were teacher assessed. Unexpected findings included unconscious processing of some unfamiliar concepts with difficulties verbalising unfamiliar and familiar concepts, contrasting

conscious deliberation required for multiple concepts, transfer of learning and use of strategy, indicating ability, especially in conjunction with speed, novel approach, use of symmetry and a younger age of child. Puzzles were diagnostic in detecting and in remediating mathematical understanding of single concepts. The formal mathematics of some remedial and extension children improved, suggesting unconscious transfer of concepts, and some children who previously disliked mathematics or school in general developed a liking for both. No gender performance differences emerged. P.A.T. performance did not correlate with puzzle performance, emphasising differences between P.A.T. formal verbal mathematics and nonverbal visual spatial logic puzzle mathematics, or predominantly left versus right brain mathematical processing respectively, possibly explaining difficulties children had verbalising nonverbal actions. Two P.A.T. average children included with extension children performed above the highest P.A.T. children. Lack of P.A.T. correlation indicates formal mathematics alone may be inadequate to identify all mathematically able children, to remediate all having difficulties, or to extend those needing lateral enrichment. As pre-mathematical concepts incorporated into junior puzzles are prerequisites for formal mathematics, mathematical concepts incorporated into senior puzzles may aid unconscious transfer into formal mathematics through conscious awareness from verbal introspection, providing useful remediation and enrichment if embedded within future curricula.

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Preface and Overview

'Unconscious Processing in Children,' presents experiences indicating information transmits both unconsciously and consciously, intrapsychically within the person, interpsychically between people, and between the person and the environment, with information processed, interpreted, integrated and applied both unconsciously and consciously. Little is known of how sensations translate into perceptions, and perceptions move into concepts. Little is also known of how concepts are available to operable consciousness and action awareness, or of how action awareness transfers into consciously expressible language. The operation of multiple concepts, transfer of learning and strategic application indicate conscious awareness of structure, and applied conscious choice.

The differentiation between what is consciously learnt and what is unconsciously absorbed and processed is unclear. Active awareness and focus of attention may express through sensory and motor systems as action, or verbally through verbal thought and speech. Individuals 'know' more than conscious content. Mechanisms selecting incoming and processed information into conscious awareness are not established, nor is there a unified definition of consciousness or unconsciousness. Why do some items and not others become conscious? Is consciousness to be defined in terms of what can be verbally described, when actions describe knowledge awareness where language can not? Why some items come to conscious awareness and some do not is unknown, but consciousness manifestations must be selected by unconscious mechanisms via sensory awareness into perceptual awareness, and have central meaning to the individual, showing exercise of free will operates strongly at both unconscious and conscious levels.

The degree of conscious scanning in which a person participates is dependent on the degree of sensory and selective perceptual and conceptual awareness coming into conscious attention. This focus of attention is also dependent on individual consciousness states which may alter the nature of the potential for conscious processing at any given

time. Variables such as sleeping, waking, degree of consciousness, alertness, tiredness, physique, health, emotions, aloneness, togetherness, age, sensory and perceptual acuity, cognitive awareness and integration, motivation, and cultural biases and language will affect what comes into conscious awareness. Unconscious scanning and processing appear fully present and constantly aware through all consciousness variables, with overall awareness of experiences and learnings.

The present study looks at how specific intrapsychic nonverbal and possibly unconscious information transmission processes contrast with specific intrapsychic verbal and conscious information transmission processes involved in the solving of mathematical puzzles designed for children and adults, and the interacting effect external judicious questioning has in extending these processes. The focus on unconscious processing emerged from the present study, in the part unconscious processes played in supporting the original hypothesis that children are able to understand mathematical concepts at much higher levels than is generally believed or taught within the educational system, and from unexpected findings that emerged. Unconscious processing is the theme underlying each major finding in the present study, for which there are no definitive interpretations. They include:

- Performing a correct answer without conscious awareness of the process, and difficulty verbalising classifications.
- The ability of children to perform above what is considered usual for their age.
- The breaking of Piaget's linear steps of conceptual development, with mixed conceptual performance in children.
- High ability untapped and unknown in two children considered average. What are the P.A.T.'s and the curriculum missing? What is the unidentified ability?

- The incidence of upper and lower ranges of children's formal mathematics improving using the puzzles, particularly in remedial junior and senior children.
- The biggest question - how manipulating things is translated into, and the forerunner of formal mathematical work.

The present study was not intended as a scientific experiment in the traditional sense, but was an exercise in teaching in progress. Its purpose was an exercise in discovery and extension for individual children, with puzzles and questioning techniques aimed to facilitate maximum self discovery of learning and minimum content teaching. Individual work in small groups facilitated 'catching the moment' with judicious questioning helping children take the discovery as far as possible. This encompassed diagnostic, remediation and lateral extension and enrichment at every level for every age of child. Children worked at their own pace and level. Emphasis was always on building onto what children could already do and achieve by themselves, and how far this could be extended. It was essential to work with children in this manner as good teaching practice, and for the purposes of the present study to find as full a range of possibilities and benefits and uses for the puzzles as possible.

The non experimental approach was selected as non intrusive, non disruptive and low key, emotionally and educationally appropriate, and to simultaneously maximise individual educational effects. It was also necessary to simulate the way the puzzles might be used in normal classroom use to determine their impact as classroom materials. No attempt was made to experimentally have each child attempt the same puzzles, or to attempt them in the same order as any other child, for reasons of varying age groups and ability levels within mathematical groups in addition to individual differences and needs. Questioning was always responsive to the actions of individual children reflecting these differences. Maximising educational advantage meant tailoring to individually assessed needs.

The present study also sits in limbo between a quantitative and a qualitative approach. One quantitative aspect is the use of national age normed P.A.T. mathematics results of quantitative formal mathematical understanding used to group senior children for the present study. Although many puzzles had correct solutions, and some puzzles had more than one arrangement or creative interpretation for a correct answer, all puzzles allowed for individual perceptual and conceptual awareness to manifest in levels of performance. Performance levels typically ranged from too difficult through to exceptional or novel performance, with various questioning and non questioning speed and strategic performance levels exhibited within these extremes. Performance levels provided ordinal data in the present study for the qualitative nature of the puzzles themselves. Non parametric analyses were used, with Spearman's Rho correlations measuring levels of puzzle performance of children against their formal school mathematical groups, ages and class levels respectively, and Mann-Whitney U-Tests measuring gender differences in levels of performance with the puzzles.

There are three parts to the present study:

Part 1 - Introduction

Part 2 - The Puzzles

Part 3 - Implications

Part 1

Introduction

Chapter 1

Unconscious Processing

Chapter 1 looks at current possible theoretical explanations and explorations of unconscious cognitive processing, presenting some definitions of consciousness indicating lack of consensus amongst theorists as to what consciousness is. Conscious versus unconscious processes are discussed. The degree to which these occur and can be introspected also divide the theorists. Theories on modes of learning continue the debate over whether people process information consciously or unconsciously, and in which

order. Issues of introspection and protocol procedures with children look at advantages and disadvantages of the talking aloud method and the clinical interview, and the part language plays in this introspection. Consciousness in the form of unconscious awareness of concepts through actions, through to conscious awareness of concepts through language expression is discussed, along with the acquisition of language as a means of bringing the concept into conscious awareness. Nonverbal versus verbal conscious awareness is discussed. Consciousness is presented as processed integrated awareness, with cerebral evidence of unconscious processing and left and right hemispheric processing leading into a discussion of possible mechanisms for the movement of sensations into perceptions and concepts, and into conscious awareness.

Chapter 2

The Present Study

Chapter 2 introduces the present study, firstly looking at aims of the present study with prior assumptions and hypotheses underlying these aims. A rationale for the use of the mathematical puzzles in the teaching of mathematics and pre-mathematics to school children incorporates Piaget's stages of cognitive development. Pre-mathematical concepts embedded in the puzzles, mathematical readiness in children, and senior puzzle concepts are described. A method section describes the participants and their selection, followed by the rationale and use of the New Zealand Progressive Achievement Tests in mathematical group selection as the quantitative measure incorporated in the present study. Materials used in the puzzles and the nature of the puzzles themselves and how children process them are the qualitative measure. Procedures and types of questioning used with the children are presented. The qualitative aspects of the data collection and analysis follow.

Part 2

The Puzzles

Chapters 3 - 8 highlight and integrate some facets of unconscious and conscious information processing observed in children attempting the mathematical puzzles.

Chapter 3

Children Unable to Verbalise Classifications

Chapter 3 discusses unconscious information transmission process events where children were able to solve some puzzles correctly which could not be solved without conceptual understanding, yet these children were unable to verbalise or name their chosen classifications and processes without extensive questioning. Puzzles incorporating process of elimination, rote performance, and confabulation follow, showing how none of these elements were involved in this unconscious processing. Chance factors are unlikely.

Chapter 4

Perceptual Priorities

Chapter 4 looks at perceptual priorities children exhibited while solving the puzzles, where unexpected concepts were likely to be processed unconsciously or to create perceptual and conceptual difficulties. Puzzles involving expected or familiar elements versus unexpected or unfamiliar elements within perceptual dimensions such as colour, shape, size, pattern, reversals and rotations are presented. Child profiles showing how children progressed through perceptual and conceptual difficulties are presented.

Chapter 5

Multiple Concepts

Chapter 5 elaborates on perceptual difficulties occurring more frequently in puzzles where in addition to perceptual dimensions, multiple concepts such as ordering, alternating or patterning required for the solution of a puzzle necessitated a conscious focus of attention.

Chapter 6

Transfer of Learning and Use of Strategy

Chapter 6 shows how some children manipulating multiple concepts exhibited a conscious transfer of learning, while other children applied a conscious use of strategy, increasing the speed of the solution of the puzzle, and indicating ability in the child concerned, especially where these approaches manifested in a younger child.

Chapter 7

Diagnostic Progressions

Chapter 7 looks more closely at several puzzles giving progressive diagnostic information on a child's level of perceptual and conceptual awareness as indicated by the levels of performance and specific difficulties encountered. They encompass all of the previously presented difficulties in previous chapters. These fall into two groups, of puzzles not involving enumeration and puzzles involving enumeration.

Chapter 8

Range in Levels of Performance

Chapter 8 presents some puzzles where children exhibited a mixed range of levels of performance within and across mathematical group categories, emphasising the general lack of correlation between mathematical group categories and levels of performance. Some puzzles present children exhibiting a mixed range of levels of performance within a particular mathematical group, similarly showing a lack of correlation between mathematical group category and levels of performance. Also presented are two children from an average mathematical group category exhibiting a high level of performance, supporting the general lack of correlation between mathematical group categories and levels of performance in this study.

Part 3

Implications

Chapter 9

Correlation Results

Chapter 9 presents correlation results from comparisons of levels of performance across mathematical group categories, school classes, and ages of children using nonparametric analyses of Spearman's Rho correlations, and Mann-Whitney U-Tests for measuring gender performance differences. These are looked at in conjunction with puzzle difficulty level.

Chapter 10

Implications in the Teaching of Mathematics

Chapter 10 looks at ways which support the hypothesis that children are able to understand mathematical concepts at much higher levels than is generally believed or taught within the educational system, and provides possible explanations for unconscious processing, with implications for the teaching of mathematics. Previously presented findings in the present study, previously unrepresented findings, and findings relating specifically to unconscious processing are listed. Some possible reasons for the general lack of correlation between P.A.T. mathematics scores and puzzle performance are examined in the discussion, with an exploration of P.A.T. mathematics and the mathematical puzzles and what they each measure, and the nature and degree of these different mathematical assessments. It includes discussion on verbal and nonverbal processing and the validity of both P.A.T. mathematics and the mathematical puzzles in determining the acquisition of mathematical concepts in children. Left and right brain function, consciousness and the brain, and aspects of quantum consciousness lead into the movement of percepts to concepts and the function of language in concept formation. Thoughts on Piaget's stages of cognitive development, and the place of intuitive, creative and divergent thinking are followed by definitions of mathematical ability and able children. Acceleration and enrichment programmes are discussed, with concluding implications for the teaching of mathematics, limitations of the present study and possibilities for future research followed by a conclusion.

Appendices include junior mathematical puzzles, the Eileen Churchill Number Readiness Test, three manufactured adult puzzles, and puzzles in approximate order of difficulty.