TEACHERS DEVELOPING COMMUNITIES OF MATHEMATICAL INQUIRY

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ABSTRACT

This study explores how teachers develop communities of mathematical inquiry which facilitate student access to, and use of, proficient mathematical practices as reasoned collective activity. Under consideration are the pathways teachers take to change classroom communication and participation patterns and the mathematical practices which emerge and evolve, as a result.

Sociocultural theories of learning underpin the focus of the study. A synthesis of the literature reveals the importance of considering the social and cultural nature of students' learning and doing mathematics in intellectual learning communities—communities in which shared intellectual space creates many potential learning situations.

A collaborative classroom-based qualitative approach—design research—falls naturally from the sociocultural frame taken in the study. The design approach supported construction of a communication and participation framework used to map out pathways to constitute inquiry communities. Study group meetings, participant and video observations, interviews, and teacher recorded reflections in three phases over one year supported data collection. Retrospective data analysis used a grounded approach and sociocultural activity theory to present the results as two teacher case studies.

Managing the complexities and challenges inherent in constituting communication and participation patterns each teacher in this study successfully developed communities of mathematical inquiry within their own classrooms. Important tools that the teachers used to mediate gradual transformation of classroom communication and participation patterns from those of conventional learning situations included the communication and participation framework and the questions and prompts framework.

Significant changes were revealed as the teachers enacted progressive shifts in the sociocultural and mathematical norms which validated collective inquiry and argumentation as learning tools. Higher levels of student involvement in mathematical dialogue resulted in increased intellectual agency and verbalised reasoning. Mathematical practices were shown to be interrelated social practices which evolved within reasoned discourse.

The research findings provide insights into ways teachers can be assisted to develop a range of pedagogical practices which support the constitution of inquiry communities. For New Zealand teachers, in particular, models for ways teachers can draw on and use their Maori and Pasifika students' ethnic socialisation to constitute mathematical inquiry communities are represented in the case study exemplars.

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E rima te'arapaki, te aro'a, te ko'uko'u te utuutu, 'iaku nei.

Under the protection of caring hands there's feeling of love and affection.

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CHAPTER ONE

INTRODUCTION TO THE STUDY

Engagement in practice-in its unfolding, multidimensional complexity-is both the stage and the object-the road and the destination. (Wenger, 1998, p. 95)

1.1   INTRODUCTION

Important changes have occurred in how mathematics classrooms are conceptualised over recent years. These changes are in response to the need to consider how mathematics education might best be able to meet the needs of students in the 21st century. A central hallmark of the changes is a vision of students and teachers actively engaged in shared mathematical dialogue in classrooms which resemble learning communities (Manouchehri & St John, 2006). The shared dialogue includes use of effective mathematical practices, the specific things successful learners and users of mathematics do when engaged in reasoned discourse during mathematical activity (RAND, 2003). These changes propose the creation of teaching and learning experiences which are radically different from those which have been traditionally offered in New Zealand primary school mathematics classrooms. It is against this background that this study, *Teachers developing communities of mathematical inquiry,* was conducted. The main purpose of this study was to investigate and tell the story of the journey two teachers take as they work at developing intellectual learning communities in primary classroom settings which support student use of effective mathematical practices.

This chapter identifies the central aim of this study. The background context outlines how the current reforms in mathematics education in New Zealand are shaped by both political goals and the direction taken by the wider international mathematics community. The significance of considering the social and cultural nature of learning in mathematics classrooms is discussed within the context of the current numeracy initiatives within the primary sector.

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1.2 RESEARCH AIM

This study aims to explore how teachers develop communities of mathematical inquiry that supports student use of effective mathematical practices. The focus of the exploration is on the different pathways teachers take as they work at developing communities of mathematical inquiry, their different pedagogical actions, and the effect these have on development of a mathematical discourse community. The study also investigates how changes in the mathematical discourse reveal themselves as changes in how participants participate in and use mathematical practices. That is, the research also seeks to understand how the changes in participation and communication patterns in a mathematical classroom support how students learn and use effective mathematical practices.

1.3 BACKGROUND CONTEXT OF THE STUDY

During the latter stages of the 20th century New Zealand society experienced rapid and widespread technological changes and an economic climate of competitive complex overseas markets. Knowledge was viewed as increasingly immediate, complicated, and constantly changing. This initiated a political response which involved the establishment of ambitious social and cultural goals—goals which aimed to transform New Zealand to a knowledge-based economy and society. Education was positioned as a "transformational tool which enriches the lives of individuals, communities and societies" (Clark, 2007, p. 2); an important vehicle to ensure future participation in employment and citizenship for all New Zealanders.

High levels of numerical literacy were seen to be integral to full participation in a knowledge-based, technologically-oriented society (Maharey, 2006). This fuelled increased political attention towards mathematics education, already a source of concern as a result of the relatively poor performance of New Zealand students in the 1995 Third International Mathematics and Science Study (Young-Loveridge, 2006). A key consequence of those results was the development of 'The Numeracy Development Project' (NDP), an on-going

nation-wide initiative designed to increase student proficiency through strengthening teachers' professional capability in mathematics teaching (Young-Loveridge, 2005).

The NDP built on more than a decade of national and international efforts (e.g., Department for Education and Employment, 1999; Ministry of Education, 1992; National Council of Teachers of Mathematics, 1989, 2000; New South Wales Department of Education and Training, 1999) to reform mathematics teaching and learning. Concerns with poor levels of achievement of many students in Western society and the disaffection of groups of people towards mathematics supported an impetus for reform focused on achieving success for *all* students as successful mathematical thinkers and users.

The teaching model advocated in the NDP requires that teachers reconceptualise their teaching and learning practices from a previous predominant focus on rote learning of computational rules and procedures. Inherent in the shift is the repositioning of all members of the classroom community as active participants because teachers are required to "challenge students to think by explaining, listening, and problem solving; encourage purposeful discussion, in whole groups, in small groups, and with individual students" (Ministry of Education, 2005, p.1). Students are cast as active communicators as they collaboratively analyse and validate the mathematical reasoning within solution strategies offered by their peers.

The focus on students' interactions and communicating of mathematical reasoning within the NDP resonates with goals previously established in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992). This document maintains that "learning to communicate about and through mathematics is part of learning to become a mathematical problem solver and learning to think mathematically" (p. 11). This theme is reiterated in the current policy initiative. *The New Zealand Curriculum (draft)* (Ministry of Education, 2006a) states that "students learn as they engage in shared activities and conversations with other people. Teachers can facilitate this process by designing learning environments that foster learning conversations and learning partnerships, and where challenges, support, and feedback are readily available" (p. 24). Similarly, international policy documents (e.g.,

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Australian Education Council, 1991; Department for Education and Employment, 1999; National Council of Teachers of Mathematics, 2000) exhort the importance of teachers developing mathematics classroom communities which support student communication, problem solving and reasoning capacity. Collectively, these documents affirm the significance of classroom interactions and the nature of the discourse for the development of the kinds of mathematical thinking and learning envisaged within the mathematics reform strategies.

Influenced by contemporary sociocultural learning theories (e.g., Forman, 2003; Lave & Wenger, 1991; Sfard, 1998; Wells, 2001a; Wenger, 1998) the current mathematics reform efforts attend to the essentially social and cultural nature of cognition. Within this perspective, "mathematics teaching and learning are viewed as social and communicative activities that require the formation of a classroom community of practice where students progressively appropriate and enact the epistemological values and communicative conventions of the wider mathematical community" (Goos, 2004, p. 259). The current curriculum document (Ministry of Education, 2006a) and NDP both advocate pedagogical changes that increasingly expect and direct teachers to reconceptualise and restructure their teaching and learning practices to move towards developing communities of mathematical inquiry in which students are given opportunities to learn, and use, effective mathematical practices.

Under consideration in the context of this study are the mathematical practices which emerge and evolve as teachers reconstitute the classroom communication and participation patterns. As discussed in the following chapter, learning mathematics is more than acquiring mathematical knowledge. To become competent learners and users of mathematics students also need to learn how to "approach, think about, and work with mathematical tools and ideas" (RAND, 2003, p. 32). Evidence (e.g., Ball, 2001; Goos, 2004; Lampert, 2001; Wood, Williams, & McNeal, 2006) shows that students are capable of powerful mathematical reasoning when provided with opportunities to engage in reasoned collective mathematical discourse. Reasoned discourse in this form extends past explaining mathematical ideas, to include interrelated practices of justification, verification

and validation of mathematical conclusions, coupled with an efficient use of mathematical language and inscriptions.

However, learning and using reasoned discourse requires a classroom culture which actively supports collective inquiry. The role the teacher has in developing a culture of mathematical inquiry is central to the learning practices which develop in it (Goos, 2004). Therefore, it is timely to explore exactly what pedagogical practices teachers use, the communication and participation patterns which result, and how these support students learning and use of effective mathematical practices.

1.4   RATIONALE FOR THE STUDY

Wells (2001b) reminds us that in trying to understand as well as to adjust and develop one's practice, theory emerges from the practice and supports sense-making of the practice. However as Wells notes, theory is "only valuable when it shapes and is shaped by action" (p. 171). The sociocultural learning perspectives evident in the policy initiatives spanning the past decade, provide ample justification for directing specific attention to examining how teachers establish effective learning partnerships in classroom communities in which the use of reasoned collective mathematical discourse is the norm. Whilst we have international studies (e.g., Borasi, 1992; Brown, & Renshaw, 2000; Goos, 2004; Goos et al., 2004; Lampert, 2001) which illustrate how the key sociocultural ideas have been understood and applied within mathematics classroom contexts, we have limited research directed towards exploring and examining how New Zealand teachers might develop such mathematical classroom learning communities, the pedagogical practices they might use to develop reasoned discourse, the roles they might take in them, and the reasoning practices which emerge and evolve.

A review of the NDP (Ministry of Education, 2006b) professional development material reveals comprehensive guidance for how teachers might teach the mathematical knowledge and strategies. But how teachers might organise student participation in communal mathematical discourse that extends beyond the teacher leading strategy reporting and

questioning of strategy solutions sessions to independent cooperative reasoning is not described (Irwin & Woodward, 2006). In reading the many New Zealand NDP evaluation reports we learn about enhanced teacher capacity and capability to teach numeracy, and we read of student increase in knowledge and strategy levels. However, what is not revealed is how these gains might equate to student growth in the use of the communicative reasoning processes, and to mathematical practices from which theorists (Boaler, 2003a; RAND, 2003; Van Oers, 2001) suggest deeper mathematical reasoning and conceptual understandings develop. A focus on how teachers extend communal exploration of mathematical explanations towards reasoning for justification and an investigation into the mathematical practices that emerge and are used is appropriate at this time.

Central to mathematical proficiency is conceptual understanding of mathematical ideas; conceptual understanding means not only knowing a mathematical concept but also being able to reason its truth. Although New Zealand policy initiatives over the past decade have promoted the need for teachers to develop classroom cultures which foster student participation in learning dialogue—including the presentation of arguments, and challenge and feedback on mathematical reasoning (Ministry of Education, 1992, 2006)—these are ambitious goals for change (Huferd-Ackles, Fuson, & Sherin, 2004; Silver & Smith, 1996). Gaps exist in our understanding about the challenges New Zealand teachers might encounter as they change the classroom interaction norms towards inquiry and argumentation. We need to know how teachers themselves might best be supported to learn to use reasoned mathematical discourse as they establish its use within their classroom culture.

International studies illustrate that in primary classrooms where the use of discourse extends to justification and argumentation both the cognitive demand and student participation in conceptual reasoning increases (Forman, Larreamendy-Joerns, & Brown, 1998; Wood et al., 2006). However, reasoning at complex cognitive levels and working through a communal argumentative process is not something many younger students can achieve easily without explicit adult mediation (Mercer, 2002). In New Zealand there appears to be little research which has examined the explicit pedagogical practices used to

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mediate justification and argumentation with young students. A direct focus is needed to consider the specific adaptations teachers make in pedagogical practices which best match the social and cultural context of their students.

Through the collaboration with teachers, this study advances professional development beyond the stage of immediate involvement in the NDP. Exploring from the perspectives of teachers how they construct communal learning communities within a sociocultural learning frame, provides possible models other teachers might use. The research partnership used in this study provides space to understand from the teachers' perspective what they find important and significant as they explore and examine possible changes in classroom practices consistent with sociocultural theory but also appropriate to the complex situations of their classroom contexts. Importantly, this focus supports the development of better understanding of how the central theoretical ideas inherent in the sociocultural learning perspective can be enacted within the real-life messy context of mathematics primary classrooms.

1.5   OVERVIEW OF THE THESIS

The thesis is divided into eight chapters. This chapter has set the scene and provided a rationale for the study. Chapter Two reviews sociocultural learning theories and establishes its links to classroom discourse, communities of mathematical inquiry and mathematical practices. Chapter Three discusses the background research on the teaching and learning of mathematical practices in classroom inquiry communities. Chapter Four provides descriptions of different mathematical practices which emerge in the social and mathematical norms of classroom communities. Chapter Five outlines the use of a qualitative design approach and discusses the methods used to collect and analyse data. Chapters Six and Seven present the results of teachers' developing classroom communities of mathematical inquiry. The teachers' pedagogical practices and the mathematical practices which emerge and evolve as a result are discussed in relation to the literature. Chapter Eight completes this thesis. The journey of the two teachers is discussed. The conclusions and implications and recommendations for future research are presented.

CHAPTER TWO

THE BACKGROUND TO THE STUDY

Learning mathematics involves learning ways of thinking. It involves learning powerful mathematical ideas rather than a collection of disconnected procedures for carrying out calculations. But it also entails learning how to generate those ideas, how to express them using words and symbols, and how to justify to oneself and to others that those ideas are true. Elementary school children are capable of learning how to engage in this type of mathematical thinking, but often they are not given the opportunity to do so. (Carpenter, Franke, & Levi, 2003, p. 1)

2.1   INTRODUCTION

An important observation made in Chapter One was that students learn to use effective mathematical practices in 'learning communities' which include articulation of reasoned mathematical thinking. For this reason, close examination will be given in this chapter to sociocultural theories of learning that underpin this study. The defining characteristic of this theoretical frame shifts the focus from individuals to activity settings—the learning environments and communities structured by teachers.

Section 2.2 provides a description of mathematical practices. It explains why they are considered reasoned performative and conversational actions (Van Oers, 2001) used in collective activity. Explanations are offered as to how students gain membership and identity through engaging in the discourse of different activity settings.

Section 2.3 describes how mathematical practices evolve through socially constructed interactive discourse. It reveals that interactive dialogue is central to how mathematical practices are developed and used collectively in classroom communities. Theoretical justification is provided for the focus on communication and participation patterns. Contemporary views of the zone of proximal development (zpd) are drawn on to show how

dialogue supports growth and use of mathematical practices and the development of a collective view.

Section 2.4 defines communities of mathematical inquiry and the role taken by teachers within them. Mathematical inquiry and argumentation are discussed. Reasons are given for the importance of this form of discourse. Explanations are offered concerning why discursive interaction poses many difficulties for both teachers and students in classroom communities. In addition, the concepts of socio-cultural[1] and mathematical norms are defined.

2.2   MATHEMATICAL PRACTICES

This study drew on sociocultural perspectives of learning as a theoretical rationale to explain links between mathematical instructional practices and the mathematical practices which students construct and use. Theorising from this perspective provides opportunities to describe how teachers structure learning environments so that students learn to participate in and use mathematical practices as reasoning acts. These acts are embedded within the social and communicative context of classroom communities of practice (Lave & Wenger, 1991). Mathematical practices are specific to and encapsulated within the practice of mathematics (Ball & Bass, 2003). Theorising using a sociocultural perspective offers ways to understand how opportunities that students have to construct and use proficient mathematical practices are dependent on the mathematical activity structured in the cultural and social life of classroom communities.

Van Oers (2001) defines mathematical activity as the "abstract way of referring to those ways of acting that human beings have developed for dealing with the quantitative and spatial relationships of their cultural and physical environment" (p. 71). He refers to mathematical practices as the mathematical activity

> with the values, rules and tools adopted in a specific cultural community we tend to speak of a *mathematical practice*. Any practice contains *performative actions* and operations that just

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[1] Socio-cultural norms refer to the social and cultural norms of the mathematics classroom (See p. 25).

carry out certain tasks which have mathematical meaning within that community (like performing long division). On the other hand, practices also comprise *conversational actions* that intend to communicate about the mathematical operations or even about the mathematical utterances themselves (p. 71).

Understanding mathematical practices as comprised of not only performative but also conversational actions is in accord with the increased curricular attention in the past decade to communication in mathematics classrooms (Ministry of Education, 1992; 2004a; National Council of Teachers of Mathematics, 1989, 2000). Many researchers have promoted the need for the development of classroom mathematics communities in which inquiry and validation of reasoning are considered central (e.g., Brown & Renshaw, 2006; Goos, 2004; National Council of Teachers of Mathematics, 2000; Wood, et al., 2006).

The use of the term 'practice' within the current study is taken from the definition provided by Scribner (1997). Scribner used the term 'practice' during her research of adults who were engaged in everyday work tasks, "guided by a practice approach to cognition" (p. 299). Scribner clarified that by a practice she was referring to

> a socially-constructed activity organised around some common objects. A practice involves bounded knowledge domains and determinate terminologies, including symbol systems. A practice is comprised of recurrent and interrelated goal-directed actions. Participants in a practice master its knowledge and technologies and acquire the mental and manual skills needed to apply them to the accomplishment of action goals. (p. 299)

Mathematical practices in this frame are constructed and used within situated, social and cultural activity settings. This theoretical position presents a way to understand how teachers, as more knowledgeable members of mathematical communities, socialise students to participate in learning and to explore how students learn to use the communicative reasoning processes—the mathematical practices—from which deeper mathematical thinking and understanding develops (Forman & Ansell, 2002; Van Oers, 2001; Wood et al., 2006).

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2.2.1 MATHEMATICAL PRACTICES AS REASONED COLLECTIVE ACTIVITY

The notion that mathematics teaching should enable all students to participate in the construction and communication of powerful, connected, and well-reasoned mathematical understanding is a common theme in current mathematical research literature (e.g., Alrø & Skovsmose, 2002; Ball & Bass, 2003; Boaler, 2003a; Carpenter et al., 2003; Carpenter, Blanton, Cobb, Franke, Kaput, & McClain, 2004a; Lampert & Cobb, 2003; National Council of Teachers of Mathematics, 2000; Romberg, Carpenter, & Kwako, 2005; Sfard, 2003). In keeping with this notion, mathematical learning is mediated through participation in reasoned collective discourse. As Ball and Bass (2003) explain this reasoned collective discourse involves more than individual sense-making.

> Making sense refers to making mathematical ideas sensible, or perceptible, and allows for understanding based only on personal conviction. Reasoning as we use it comprises a set of practices and norms that are collective not merely individual or idiosyncratic, and rooted in the discipline. Making mathematics reasonable entails making it subject to, and the result of, such reasoning. (p. 29)

In Chapter 3, a review of relevant research illustrates how teachers' actions in these communities can ensure that students are provided with opportunities to progressively engage in and appropriate a set of reasoning practices. Through these practices the students learn to think, act, and use the mathematical practices of expert mathematical problem solvers.

Mathematical practices are not fixed to specific groups of mathematical users, nor are they only invoked in schools, or academic mathematical settings (Civil, 2002). They are used by successful mathematics users of all ages to structure and accomplish mathematical tasks. They encompass the mathematical know-how beyond content knowledge which constitutes expertise in learning and using mathematics. Put simply, the term 'practices' refers to the particular things that proficient mathematics learners and users do. Examples of mathematical practices include "justifying claims, using symbolic notation efficiently, defining terms precisely, and making generalizations [or] the way in which skilled

mathematics users are able to model a situation to make it easier to understand and to solve problems related to it" (RAND, 2003, p. xviii). The proposal that mathematical practices entail more than what is usually thought of as mathematical knowledge enlarges our view of mathematical learning. To develop robust mathematical thinking and reasoning processes, students need opportunities not only to construct a broad base of conceptual knowledge; they also require ways to build their understanding of mathematical practices; these "ways in which people approach, think about, and work with mathematical tools and ideas" (RAND, p. xviii).

Until the last decade the predominant research focus, derived from psychological models, has more generally centred on student construction of cognitive structures. Only recently has research explicitly examined how teachers structure mathematical activity to support student construction and use of mathematical practices. Viewing mathematical learning as embedded within reasoned mathematical discourse offers an alternative way to consider student outcomes. From this position students' opportunities to construct rich mathematical understandings might well be related to the quality or types of classroom discourse and interactions in which they participate. This provides a different explanation from one that focuses on individual capabilities or the presentation skills of teachers (Lerman, 2001). Theorising that mathematical learning occurs as a result of sustained participation in reasoned mathematical discourse is consistent with contemporary sociocultural learning perspectives (Forman, 2003; Mercer & Wegerif, 1999b).

2.2.2   THE DISCOURSE OF COMMUNITIES OF MATHEMATICAL INQUIRY

The prominence accorded to discourse in the context of the current study "reflects an enterprise and the perspective of a community of practice" (Wenger, 1998, p. 289). In this form, discourse is "a social phenomenon" (Bakhtin, 1981, p. 259) in which the past, present and the future are encapsulated within the contextualised and situated nature of utterances of specific genres. Individuals participate with varying levels of expertise across a range of speech genres but using an individual voice (Bakhtin, 1994). The individual voice, however, is a social voice with dialogic overtones of others, given that "thought itself…is born and shaped in the process of interaction and struggle with others' thoughts, and this

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cannot but be reflected in the forms that verbally express our thought as well" (Bakhtin, 1994, p. 86).

The dialogic nature of discourse provides a useful tool for the current study to explore how teachers scaffold students in the use of the language of inquiry and argumentation—the language which supports mathematical practices. Students learn to question, argue, explain, justify and generalise through the models provided by teachers and other participants in the dialogue. These models are appropriated and explored as the students learn how to use mathematical talk. They try them out, expand and extend them and transform them into their own words or thoughts for future use.

Discourse in this context is not a practice in itself; rather it is a resource used in the context of a variety of practices which overlap but are often distinctly different (Wenger, 1998). Membership is provided to the students through their ability to participate in, and use, the discourse of different practices. Identities are constructed in relation to a situation, that is, "different identities or social positions we enact and recognise in different settings" (Gee, 1999, p. 13). Thus, when students engage in mathematical practices within a community which uses inquiry and argumentation, they not only use a social language, or speech genre, they also display the particular identity which is appropriate to the mathematical situation. Gee and Clinton (2000) outline how

> each different social language gives us different verbal identities. But language never does it by itself. It is always something part and parcel of something bigger, what we will call *Discourse*, with a capital D. Discourses are ways of talking, listening, reading and writing— that is using social languages—together with ways of acting, interacting, believing, valuing, and using tools and objects, in particular settings at specific times, so as to display and recognise particular socially situated identities. (p. 118)

For the purposes of the current study the term Discourse with a capital D is not specifically used, however the meaning attributed to it by Gee and his colleague will be drawn on. Framing discourse in this manner provides ways to consider how students may construct distinctly different identities as mathematical learners, as a result of access to participation

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in different classroom activity settings. How participants engage in the discourse may be affected by the values and beliefs they hold toward their role, and the role of others, in using the discipline specific forms of mathematical inquiry and reasoning. Theorising in this way offers explanation for how the mathematical practices of participants in mathematical activity may be constrained or enabled depending on how teachers have structured students' access to, and use of the discourse. To develop this point further, I will draw on sociocultural learning perspectives and activity theory.

2.3 SOCIOCULTURAL LEARNING PERSPECTIVES AND ACTIVITY THEORY

Sociocultural learning theories offer ways to view mathematical learning as contextualised, which is to view learning-in-activity. Within this theoretical frame, the social, cultural, and institutional contexts are not considered merely as factors which may aid or impede learning. Rather, these social organisational processes are *integral* features of the learning itself (Forman, 1996).

The sociocultural perspective on learning is an appropriate means to explain how mathematical practices are learnt and used in classroom communities. This is particularly so because "a goal of the sociocultural approach is to explicate the relationships between human action, on the one hand, and the cultural, institutional, and historical situations in which this action occurs, on the other" (Wertsch, del Rio, & Alvarez, 1995, p. 11). The sociocultural learning theory described in the next section draws on three themes that Wertsch (1985) developed from Vygotsky's work in the early 19th century.

2.3.1 COMMUNICATIVE INTERACTION AND THE MEDIATION OF MATHEMATICAL PRACTICES

To understand the development of a phenomenon Vygotsky (1978) maintained that it should be studied in the process of change, rather than at an end-point in its development. Central to this stance is the claim that modes of thinking and reasoning used in mathematical practices are not composed of already formed concepts which can be

transmitted; rather they are always in a state of construction and reconstruction (Mercer; 2000; Sfard, 2001; Wells, 1999). Mathematical practices can therefore be thought of as a part of a dynamic and evolving transformative process. That is, they are both part of the process of developing rich mathematical understanding, and they are also its product.

The idea that mathematical practices emerge and evolve within a community of users is commensurate with Vygotskian thinking. According to Vygotsky, the social origins of higher mental functions—thinking, logical reasoning and voluntary attention are created by social processes. They appear first between people on a social plane and then within individuals on a psychological plane (Wertsch, 1985). Language and communication that is used initially in the social context is described by Vygotsky (1978) as fundamental to learning. He maintained that language served two functions, "as a communicative or *cultural tool* we use for sharing and jointly developing the knowledge—the 'culture'— which enables organised human social life to exist and continue...and as a *psychological tool* for organising our individual thoughts, for reasoning, planning and reviewing our actions" (Mercer, 2000, p. 10).

The intertwined concepts of mediational means and mediational action are at the core of sociocultural thinking (Lerman, 2001; Wertsch, 1985). All activity, including the mental processes of thinking and reasoning, is mediated by tools and signs (Vygotsky, 1978). In mathematics classrooms tools include the material and symbolic resources, communicative patterns, spoken words, written text, representations, symbols, number systems and participation structures. These mediational means are "embedded in a sociocultural milieu and are reproduced across generations in the form of collective practices" (Wertsch, Tulviste, & Hagstrom, 1993, p. 344). Thus, shaped in sociocultural specific ways, mathematical practices evolve within mathematical inquiry communities in response to the communication and participation patterns teachers structure within the activity settings (Tharp & Gallimore, 1988).

In the current study, in order to explain how the construct of activity settings relates to the construction of mathematical practices, theories of Vygotsky's contemporary, Leontiev

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(1981), are drawn on. Activity as it is proposed by Leontiev, provides a way to account for the transformation of mathematical practices within the complex and goal-directed activity enacted by teachers in classroom communities. Leontiev, like Vygotsky, defined individuals' thoughts and actions in relationship to their goal-directed social and culturally specific activities. The key elements of activity Leontiev identified as always "material and significant" (p. 13), "primarily social" (p. 14) and with a "systematic structure" (p. 14).

Contexts in which collaborative interaction, intersubjectivity and teaching occur are termed activity settings (Gallimore & Goldenberg, 1993; Tharp, 1993). There are many diverse forms of activity settings ranging from informal home based groups to the more formal school settings. Activity settings "are a construct that unites (1) objective features of the setting and the environment and (2) objective features of the motoric and verbal actions of the participants with (3) subjective features of the participants' experience, intention, and meaning" (Tharp, 1993, p. 275).

According to Tharp and Gallimore (1988), activity settings, participants in them, and their goal-directed activity are not arbitrary; rather they result from "the pressures and resources of the larger social system" (p. 73). The membership of the activity setting is determined by the goals and the setting. In turn, the activity is "performed only when the time is congruent with the character of the operations and the nature of the personnel" (p. 73). The meaning, or motivation, for the activity is goal dependent although "in the emergent intersubjectivity of group performance in its time and place, meaning continues to develop, to emerge, to explain, and to perpetuate" (p. 73).

The construct of 'activity settings' provides a useful explanatory tool to view how different participants transform their beliefs and values of mathematical practices as they strive to gain alignment with others in their zpd. Within the everyday contexts of the classroom mathematics community there are important variables to consider "(1) the personnel present during an activity; (2) salient cultural values; (3) the operations and task demands of the activity itself; (4) the scripts for conduct that govern the participants' actions; (5) the purposes or motives of the participants" (Gallimore & Goldenberg, 1993, p. 316). These

variables present ways to explain connections between participants, the constraints and supports provided by others in the activity setting, and the cultural patterns, norms and values enacted. In turn, changes in the activity settings can be mapped to shifts in the communication and participation patterns which teachers enact. Shifts in these structures alter the demands on the participants and thus prompt change in the interactional scripts, participation roles and their beliefs and values.

2.3.3   ZONES OF PROXIMAL DEVELOPMENT

According to Vygotsky (1986) conceptual reasoning is a result of interaction between everyday spontaneous concepts and scientific concepts. Scientific concepts involve higher order thinking which is used as students engage in more proficient mathematical practices. Vygotsky maintained that "the process of acquiring scientific concepts reaches far beyond the immediate experience of the child" (p. 161). Although his research was not centred on schooling he suggested that school was the cultural medium, and dialogue the tool that mediated transformation of everyday spontaneous concepts to scientific concepts.

Articulated reasoning, inquiry and argumentation in the construction of reasoned mathematical thought is the focus of the current study. Whilst the exact nature of how external articulation becomes thought has been extensively debated (Sawyer, 2006a), sociocultural theorists are united in their belief that collaboration and conversation is the key to the transformation of external communication to internal thought. This occurs as students interact in zpds they construct together. The zpd has been widely interpreted as a region of achievement between what can be realised by individuals acting alone and what can be realised in partnership with others (Goos, Galbraith, & Renshaw, 1999; Tharp & Gallimore, 1988). Traditional applications of the zpd were used primarily to consider and explain how novices were scaffolded by experts in problem solving activity (Forman, 2003). Often learning was described as occurring through the expert demonstrating, modeling, guiding or explicitly explaining (Forman & McPhail, 1993). In this form, scaffolding was interpreted as teachers or more expert others "assisting the child in identifying, sequencing, and practising sub-goals for eventual guided assembly…the

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asymmetrical structuring of the passive child through a process of breaking down the task" (Stone, 1993, p. 180).

More contemporary interpretations of the zpd provide ways to consider learning which occurs when levels of competence are more evenly distributed across members in the zpd. Learning in this form for individuals, groups and whole classroom communities occurs during mutual engagement in collective reasoning activity (e.g., Brown & Renshaw, 2004; Enyedy, 2003; Goos, 2004; Goos et al., 1999; Lampert, 1991; Mercer, 2000; Wells, 1999; Yackel, Cobb, & Wood, 1991; Zack & Graves, 2001). Lerman (2001) describes the participation in the mathematical discourse and reasoning practices as pulling participants forward into their zpds. The zpd is thus defined

> as a symbolic space involving individuals, their practices and the circumstances of their activity. This view takes the zpd to be an ever-emergent phenomenon triggered, where it happens, by the participants catching each other's activity. It is often fragile and where it is sustained, a process of semiotic mediation and interaction emerges. (Lerman, 2001, p. 103)

Defining the zpd as a symbolic space provides a useful means for the current study to explain how participants in activity settings mutually appropriate each others' actions and goals (Newman, Griffin, & Cole, 1989). In doing so, teachers and students are required to actively engage and work to understand the perspectives taken by other participants. Teachers pulled into the zpd work with their students' understandings and attitudes. In turn, the students identify with the attitudes and values of the teacher who represents the social and cultural practices of the wider mathematical community (Goos, 2004).

As described earlier, the most recognised use of the zpd pertains to the scaffolding metaphor used by Bruner (1990, 1996). In this form, scaffolding supports the learner to achieve their goals 'of the moment' (Sawyer, 2006a). However, the learner is not a passive recipient, rather, the negotiation of goals is co-constructed within the activity setting. For example, Mercer (2000) described how students were scaffolded by teachers to participate in reasoned mathematical inquiry co-constructed in the active contributions of all

participants. He termed student inquiry of each other's reasoning in the zpd "interthinking" (p. 141).

Mercer (2000) described interthinking in shared communicative space as creating an intermental development zone founded in the shared knowledge and goals of all community members. Connected to the variable contributions of all participants the quality of the communicative space, therefore, was "dependent on the existing knowledge, capabilities and motivations of both the learner and the teacher" (p. 141). The variable contributions create the need to continually renegotiate and reconstitute the zpd. Mathematical meaning is generated through collective inquiry as participants talk, think, and reflectively consider, what is being claimed. If the shared dialogue succeeds then the students are extended beyond their usual capabilities but if the dialogue fails to sustain alignment of all members then the zpd collapses and collective mathematical reasoning fails.

The construct of interthinking, pulling participants into a shared communicative space, extends a view of the zpd beyond scaffolding. It supports consideration of the learning potential for pairs or groups of students working together with others of similar levels of expertise in egalitarian relationships (Brown & Renshaw, 2004; Goos, 2004; Goos et al., 1999; Renshaw & Brown, 1997). The partial mathematical knowledge and skills that members contribute support collective understanding. Through joint activity, opportunities are made available for the group to encounter mathematical situations which involve erroneous thinking, doubt, confusion and uncertainty (Goos et al., 1999). Negotiation requires participants to engage in exploration and speculation of mathematical reasoning. This is an activity which approximates the actual mathematical practices used by mathematicians.

Within a group the development of a collective view is dependent on all members sharing goals and values which support mutual engagement in mathematical practices. Wenger (1998) outlines that

> mutual engagement involves not only our competence, but the competence of others. It draws on what we do and what we know, as well as on our ability to connect meaningfully

to what we don't do and what we don't know—that is, to the contributions and knowledge of others. In this sense, mutual engagement is inherently partial; yet, in the context of a shared practice, this partiality is as much a resource as it is a limitation. This is rather obvious when participants have different roles…where mutual engagement involves *complementary* contributions. But it is also true…[for those]…who have largely *overlapping* forms of competence. Because they belong to a community of practice where people help each other, it is more important to know how to give and receive help than to try to know everything yourself. (p. 76)

Collaboration and construction of a collective view is not always premised on immediate consensus. Azmitia and Crowley (2001) explain that "dissension can also serve as a catalyst for progress either during or following the collaborative session" (p. 58). They maintain that the essential ingredient is mutual engagement in transactive dialogue. Transactive dialogues

> are conversations in which partners critique, refine, extend, and paraphrase each other's actions and ideas or create syntheses that integrate each other's perspectives, have been linked to shifts in…scientific reasoning. These transactive dialogues may be the epitome of collaborative theory construction because in many cases, individuals walk away with a joint product for which they are no longer certain (and may not care) who gets credit for particular ideas. (Azmitia & Crowley, 2001, p. 58)

Thus, the dialogue functions "as a thinking device…formed as a system of heterogeneous semiotic spaces…in which languages interact, interfere, and organise themselves" (Lotman, 1988, p. 36-37). In classrooms where teacher structuring of such activity is the norm, opportunities are provided for students to be inducted into more disciplined reasoning practices. The "lived culture of the classroom becomes, in itself, a challenge for students to move beyond their established competencies" (Goos et al., 1999, p. 97) to become more autonomous participants in classroom communities of mathematical inquiry. As summarised by Burton (2002)

> coming to know mathematics depends upon active participation in the enterprises so valued in that community of mathematics practice that they are accepted as knowing in that community…the important words to stress here are 'active participation'. The identity of the

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learner is as an enquirer and the valued enterprises reflect not only the knowledge of mathematical objects and skills but also the knowledge of how mathematical enquiry is pursued. (p. 158)

Thus, coming to know and do mathematics is described as the outcome of active participation in collective inquiry. Within an activity setting the mathematical practices that students come to know and use are directly linked to the forms of social and cultural practices they have access to, within their mathematical classroom community. The use of the term 'inquiry' in the current study portrays a view of students collaboratively participating in mathematical practices within explicitly constructed classroom communities. In the next section an explanation of communities of mathematical inquiry and the discourse of inquiry and argumentation is developed further. The role taken by teachers as they implicitly and explicitly facilitate the communication and participation patterns and interactional strategies which position students to engage in the discourse is also described.

2.4   COMMUNITIES OF MATHEMATICAL INQUIRY

Within a contemporary sociocultural learning perspective the development and use of mathematical practices is matched by the students "increasing participation in communities of practice" (Lave & Wenger, 1991, p. 49). Describing mathematics classrooms as communities of practice points to a group united through common purpose and joint social activity. All participants are considered legitimate; although some members have more power and knowledge of valued skills (for example the teacher as the old-timer), while other members (often the students as the new-comers) are more peripheral. In this context, the constructs of the zpd and intersubjectivity supports the more peripheral member's development and use of effective and appropriate mathematical practices. Thus "learning as legitimate peripheral participation means that learning is not merely a condition for membership, but is itself an evolving form of membership" (Lave & Wenger, p. 53).

The transformation of mathematical practices is part of a dynamic participatory process—a process that is aligned with appropriation of the norms and values of the specific

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community of practice. Because the students make individual meaning through varied forms of participation in particular goals, social systems, and discursive practices mathematical practices can be viewed as the outcome of the accepted and codified forms of communication and participation practices—the discourse practices (Goos, 2004).

Van Oers (2001) outlined how a "community committed to a particular style of conversational actions with regard to a special category of objects can be named a community of discourse" (p. 71). Van Oers differentiated between communities of mathematical practice and communities of mathematical discourse. In the first form, the users of mathematics can be making calculations in supermarkets and shops and often in idiosyncratic ways. In contrast, communities of mathematical discourse "mainly includes persons interested in reflectively understanding mathematical actions" (p. 71). It is communities of mathematical discourse which receive particular attention in the current study.

Likewise, based on observations across classrooms Cobb, Wood, and Yackel (1993) report differences in the discourse practices involving "talking about talking about mathematics" (p. 99). For example, the practices and beliefs of students in classrooms which use memorisation and rote procedures contrast with those which have an inquiry discourse approach (Goos, 2004). Goos suggested that the practices and beliefs constructed in these classrooms

> frame learning as participation in a community of practice characterized by *inquiry mathematics*—where students learn to speak and act mathematically by participating in mathematical discussion and solving new or unfamiliar problems. Such classrooms could be described as *communities of mathematical inquiry.* (p. 259)

Other researchers have described these classroom communities variously as in an inquiry mathematics tradition (Cobb, Wood, Yackel, & McNeal, 1992); an inquiry co-operation model (Alrø & Skovmose, 2002); and communities of inquiry (Goos et al, 1999; Wells, 1999; Zack & Graves, 2001). In the current study 'communities of mathematical inquiry' is

the term used to consider the communal context in which meaning is mutually constituted as students participate discursively in reasoned actions and dialogue.

2.4.1 THE COMMUNICATION AND PARTICIPATION PATTERNS OF COMMUNITIES OF MATHEMATICAL INQUIRY

Within communities of mathematical inquiry mathematical reasoning is set within a social world, where all participants are bound together in the dialogicality of "mutual regulation" and "self regulation" (Sfard, 2001, p. 27). In communities premised on inquiry, value is placed on "practices such as discussion and collaboration…valued in building a climate of intellectual challenge" (Goos, 2004, p. 259). Engagement is founded on

> a communicative relationship between equals that requires *participation, commitment, and reciprocity. Participation* means there are opportunities in the dialogue for the individuals to become engaged, question others, try out new ideas, and hear diverse points of view. *Commitment* implies that the participants will be open to hearing the positions of other speakers. *Reciprocity* means a willingness to engage in an equilateral exchange with others. In this mode, the *structure* of discourse is multidirectional and responsive. The *content* of the dialogues is dynamic, connected and unscripted. The *purpose* of the dialogue is to participate and engage others in deep inquiry into the meaning of things. (Manouchehri & St John, 2006, p. 545)

The open and responsive dialogue and the participants' partially overlapping zones of mathematical understandings and skills are inclusive of multiple levels of knowledge and expertise (Goos, 2004). This enables the orchestration of many productive interactions. The increase and range of activity settings offers students additional ways to assume varying participatory roles (Forman, 1996). Changes are also evident in the way members of communities relate to each other, to the classroom power and authority base, and to the discipline of mathematics itself (Cornelius & Herrenkohl, 2004).

Many researchers have noted that the roles and scripts of such activity settings are challenging for both teachers and students (e.g., Ball & Lampert, 1999; Mercer, 2000; Rojas-Drummond & Zapata, 2004; Wells, 1999). The role of the teacher as the

unquestionable authority is challenged. Wider diversity in the roles, task demands and interactional scripts are also demanded of the students (Forman, 1996). Assuming responsibility "to propose and defend mathematical ideas and conjectures and to respond thoughtfully to the mathematical arguments of their peers" (Goos, 2004, p. 259) is often not familiar to, nor used automatically by students. Moreover, many students on entry into inquiry classroom communities may hold contrary beliefs about argumentation, considering it either unnecessary or impolite (Forman et al., 1998; Rittenhouse, 1998; Van Oers, 1996).

We now have considerable evidence of the beneficial effects of students articulating their mathematical reasoning, and inquiring and challenging the reasoning of others (e.g., Ball, 1991; Cobb et al., 1993; Enyedy, 2003; Lampert, 2001; McCrone, 2005; Sherin, Mendez, & Louis, 2000; Wood, 2002; Wood et al., 2006). It forms a "central role in thinking and scientific language" during mathematical activity (Rojas-Drummond & Zapata, 2004, p. 543). These researchers claim that argumentation is a powerful reasoning tool in that it "allows individuals to deny, criticise and justify concepts and facts, as well as find opposing views and generate a new perspective in social interaction or in self-deliberation" (p. 543). It is through engaging in structured mathematical arguments as a precursor to argumentation that "students are able to experience mathematics as a discipline that relies on reasoning for the validation of ideas" (Wood, 1999, p. 189).

Argumentation is defined as the act of presenting grounds for taking a particular position, view or conclusion; or to confront the positions, views, or conclusions taken by others (Billig, 1996). The diversity of views and disagreement are considered important elements in sustaining active engagement of students in inquiry (Wells, 1999). However, being able to engage in dialogue appropriate to specific social situations is a learnt skill which requires explicit scaffolding (Bauersfeld, 1995; O'Connor & Michaels, 1996).

The teacher's role is of prime importance in structuring the activity settings in which students are held "accountable to disciplinary standards of inquiry and to fellow students' contributions and ideas" (Cornelius & Herrenkohl, 2004, p. 477). Research studies have illustrated how enacted participation structures and the use of specific interactional

strategies can enable all classroom members to have a voice in the intellectual activity. O'Connor and Michaels (1993, 1996) described an interactional strategy used by teachers which they termed revoicing. They drew on the seminal work of Goffman (1974; 1981) to explain how speakers characterised and positioned themselves or animated others through words and talk in social situations. In their study of a 3rd and 4th grade teacher and a 6th grade teacher revoicing was used in subtle ways to rephrase and expand student talk in order to clarify, extend, or move the discussion in different directions. The teachers positioned all members in each classroom community to mutually engage in intellectual activity. As a result, the students learnt to take various intellectual roles and to use reasoned inquiry and argumentation. Through such means, students become enculturated into mathematical dialogue embedded within discursive interaction. The interactional strategy of revoicing offers a way for the current study to explore how teachers can 'socialise' students in mathematical situations.

2.4.2 SOCIO-CULTURAL AND MATHEMATICAL NORMS

Sullivan, Zevenbergen, and Mousley (2002) describe the socio-cultural norms of a mathematics classroom as stable patterns of behaviour or practices, organisational routines and forms of communication which "impact on approaches to learning, types of response valued, views about legitimacy of knowledge produced, and responsibility of individual learners" (p. 650). These socio-cultural "norms are inferred by discerning patterns or regularities in the ongoing interactions of members of a community...patterns in collective activity within a community" (Cobb, 2002, p. 189-190). All classroom mathematical communities have their own norms and these differ significantly from one classroom to another (Boaler, 2003a). These interaction patterns encapsulate the values and beliefs with which the students align in the classroom community (McClain & Cobb, 2001). They define the expectations and obligations in classroom participation structures and are often the "hidden regularities" in classrooms (Wood, 1998, p. 170).

The mathematical norms, in contrast, relate specifically to mathematics (McClain & Cobb, 2001, Sullivan et al., 2002). Drawing on Wood's (2001) description of the interplay of the social nature of student learning, the students' developing cognition and the structure that

underlies mathematics, Sullivan and his colleagues define mathematical norms as the "principles, generalisations, processes and products that form the basis of the mathematics curriculum and serve as the tools for the teaching and learning of mathematics itself" (p. 650). Alternatively, McClain and Cobb use the term sociomathematical norms—a term which has its roots in social psychology—to describe the mathematical norms. Kazemi and Stipek (2001) explain that they are the negotiated variables constituted within discursive interaction

> For example whereas explaining one's thinking is a social norm, what counts as a mathematical explanation is a sociomathematical norm...discussing different strategies is a social norm, comparing the mathematical concepts underlying different strategies is a sociomathematical norm. Finally, working on tasks in small groups is a social norm; requiring students to achieve consensus using mathematical arguments is a sociomathematical norm. (p. 60)

These normative understandings evolve within mathematical activity and support higher level cognitive activity. They are important elements in the development of a mathematical disposition and intellectual agency (McClain & Cobb, 2001). Theorising that mathematical practices and mathematical norms are interrelated offers us a way to explain how mathematical practices are transformed as they are negotiated. At the same time, explanations can be made of how subtle differences in communication and participation classroom structures affect student opportunities to engage in and appropriate increasingly proficient mathematical practices.

2.5   SUMMARY

This review has provided a theoretical background to the current study. It began with an examination of mathematical practices. I have conceptualised mathematics learning as learning ways of thinking and reasoning (Carpenter et al., 2003) and I have framed that conceptualisation within sociocultural perspectives. Within the school setting, opportunities to develop powerful reasoning require the establishment of effective mathematical learning communities. It was noted that inquiry communities provide intellectual space to support

the emergence and evolution of increasingly proficient mathematical practices. A gap in the literature was identified which related to understanding how teachers structure mathematical activity so that their students come to know and use mathematical practices. This study aims to examine the action pathways teachers take which facilitate student access to, and use of, proficient mathematical practices in inquiry communities.

The construction of inquiry communities presents significant challenges for teachers. Teachers are required to rethink their roles and responsibilities and those of their students within the discourse patterns they structure in such classroom communities. Both teacher and student views are challenged in regard to the place of inquiry and argumentation in mathematics classrooms. Chapter 3 will outline relevant research studies which relate to the pedagogical actions teachers have used to structure communication and participation patterns—structures which engaged students in collective mathematical practices. The various forms of mathematical discourse will be examined. The importance of collective reasoning will be explained to show its influences on student beliefs and mathematical goals.

CHAPTER THREE

THE BACKGROUND RESEARCH ON TEACHING AND LEARNING OF MATHEMATICAL   PRACTICES IN COMMUNITIES OF MATHEMATICAL INQUIRY

> For students and teachers, the development of understanding is an ongoing and continuous process…the development of understanding takes time and requires effort by both teachers and students. Learning with understanding will occur on a widespread basis only when it becomes the ongoing focus of instruction, when students are given time to develop relationships and learn to use their knowledge, when students reflect about their own thinking and articulate their own ideas, and when students make mathematical knowledge their own. (Carpenter & Lehrer, 1999, p. 32)

3.1   INTRODUCTION

Attention was drawn in Chapter Two to the importance of student engagement in reasoned mathematical discourse within learning communities. Particular significance was given to the discourse of inquiry and argumentation and its importance in student development and use of mathematical practices. The connective thread through this chapter is placed on the interactional strategies and the communication and participation structures which teachers use to engage students in collective inquiry and argumentation.

Section 3.2 outlines how the structuring of learning environments which involve inquiry and argumentation is a complex and challenging task for teachers. For that reason, this review commences with three studies which outline the roles taken by exemplary teachers, in classrooms in which proficient mathematical practices were developed and used. Further studies are then drawn on to illustrate the differential outcomes for student reasoning related to the different classroom interactional patterns. The section concludes with a discussion of socio-cultural norms and mathematical norms and their relationship to student agency. Relevant literature is drawn on to illustrate the importance of teachers developing intellectual partnerships within classroom communities.

Section 3.3 outlines the forms of discourse commonly used in mathematics classrooms. Appropriate literature is used to outline the difficulties teachers have in restructuring the discourse patterns towards inquiry. This literature also illustrates the problems students encounter when engaging in communal inquiry and the subsequent effects on student values, goals and mathematical identity. To conclude, specific participation structures which teachers have used to socialise students into collective mathematical discourse in zones of proximal development are described.

3.2 STRUCTURING COMMUNITIES OF MATHEMATICAL INQUIRY

In Chapter Two, sociocultural and situative learning theories were used to explain and justify the focus of the current study. Specifically the focus is on the development of classroom communities of mathematical inquiry and participation and communication patterns that support construction and use of proficient mathematical practices. Providing students with space to engage in disciplined ways of reasoning and inquiry presents considerable challenge (Ball & Lampert, 1999; Franke & Kazemi, 2001). Teachers are not only required to change what they teach, but also how it is taught (Sherin, 2002b). Many teachers lack experience of learning in such environments (Huferd-Ackles, et al., 2004; Silver & Smith, 1996) and as a consequence, their fundamental beliefs about learning and teaching are challenged (Goos, Galbraith, & Renshaw, 2004; Rousseau, 2004; Weiss, Knapp, Hollweg, Burrill, 2002). Moreover, traditional beliefs held by both teachers and students about the non-contentious nature of mathematics and about mathematical discourse as non-adversarial are put up for challenge (Weingrad, 1998). As a result, teachers are often required to learn as they teach (Davies & Walker, 2005).

To effect change, teachers constructing such communities need models to draw upon. They need to know the pedagogical actions that are appropriate, and have an understanding of the mathematical practices which may emerge. Within the literature only a limited number of studies have documented the formation of such communities. These have predominantly documented more senior level classrooms (e.g., Borasi, 1992; Brown, 2001; Goos, 2004; Goos et al., 2004; Lampert, 1990b, 2001) than those classrooms that are under examination

in the current study. There also appear to be no classroom studies with this focus available in New Zealand.

The previous chapter highlighted the importance of the teacher's role in developing inquiry cultures and associated proficient mathematical practices. Constructing less hierarchical, more interactive learning communities requires explicit teacher actions to ensure that all students can access the discourse (Goos et al., 2004; Lampert, 2001; Van Oers, 2001).

Examination of the research literature reveals a range of studies of teachers who used specific pedagogical actions associated with various mathematical practices (e.g., Ball, 1991, 1993; Bowers et al., 1999; Brown & Renshaw, 2004; Cobb et al., 2001; Enyedy, 2003; Kazemi & Stipek, 2001; Mercer, & Wegerif, 1999b; Moschkovich, 2002b, 2004; O'Connor, 2002; Sherin, 2002a; Sherin, Mendez, & Louis, 2004; Saxe, 2002; Van Oers, 2001; Wood et al., 2006; Yackel, Rasmussen, & King, 2001; Zack & Graves, 2001). But the focus of these studies was not placed on the pedagogical actions the teachers took to scaffold student participation in construction and use of mathematical practices. Therefore this section begins with three studies in which the actions and beliefs of the teachers were closely documented as they established and worked in communities of mathematical inquiry. These studies provide a cohesive view of the teachers' actions, the roles they took, their beliefs and the mathematical practices which developed within the communities.

3.2.1 MODELS OF TEACHERS MEDIATING MATHEMATICAL INQUIRY CULTURES

The seminal work of Lampert (1990a; 1990b; 1991; 2001) has exemplified the key role teachers have in inducting Grade 5 students into what Lampert (2001) termed 'studying'. By this is meant learning how to be proficient users of mathematical practices. Lampert described studying as including "activities like inquiring, discussing, thinking, reading carefully, and examining closely" (p. 365). Central to Lampert's notion of studying was discursive interaction and the development of productive discourse. She provided compelling evidence that she, as the teacher, took a significant role in orchestrating a respectful exchange of ideas, within extended conversations involving inquiry and

challenge. Lampert, as teacher-researcher, structured her lessons to resemble mathematical arguments in which all participants engaged in disciplined public construction and evaluation of reasoning. An initial focus was placed on structuring the discourse norms. These norms were continually revised and reconceptualised so that over time the press for explanatory justification and generalisation increased.

Within Lampert's (2001) classroom, developing student autonomy and competence in mathematics included ensuring that students gained access to both the mathematical content discussions and the social discourse process (i.e., when and how to explain, question, agree, disagree or challenge). Lampert's role as a skilful listener was evident as she used instruction that shifted back and forth between mathematical content discussions and talking about the social discourse process. Lampert explicitly taught 'politeness' strategies, the essential qualities or norms for ways students might disagree. By affirming the importance of disagreement and inducting students into ways to question and disagree, students were then able to articulate their reasoning, accept opposing views, and negotiate at cognitively advanced levels. In short, through her shaping of discursive interaction the students in the community learnt "the practice of mathematics" (p. 51).

Goos (2004) examined another classroom mathematical community where through specific pedagogical actions the students were inducted into the use of inquiry practices. Goos examined a Year 11/12 classroom community of inquiry over 2 years. She observed that it was the teacher who provided predictable patterns for how the students were to engage in inquiry discourse. He "scaffolded the students' thinking by providing a predictable structure for inquiry through which he enacted his expectations regarding sense-making, ownership, self-monitoring, and justification" (p. 283). However, as the year progressed Goos reported that he "gradually withdrew his support to pull students forward into more independent engagement with mathematical ideas" (p. 283). To facilitate student participation in mathematical inquiry the teacher demonstrated a commitment to personal sense-making and willingness

> to deal with more abstract ideas concerning conjecture, justification and proof. Typically he
> modeled this process of inquiry by presenting students with a significant problem designed

to engage them with a new mathematical concept, eliciting their initial conjectures about the concept, withholding his own judgement to maintain an authentic state of uncertainty regarding the validity of these conjectures, and orchestrating discussion or presenting further problems that would assist students to test their conjectures and justify their thinking to others. (p. 283)

As students engaged interactively in communicative activity in the classroom they learnt the mathematical practices associated with inquiry.

Based on the continuing study of senior classrooms Goos and her colleagues (2004) offered "five assumptions about doing and learning mathematics that appear[s] to be crucial to creating the culture of the community of mathematical inquiry" (p. 100). These assumptions (see Table 1) underpinning teacher and student actions were derived from classroom observations and interviews from a 2-year study of four Year 11-12 mathematics classes. Although construction and use of mathematical practices was not a key focus of the research report, evidence is provided that the mathematical practices are subsumed within the doing and learning of mathematics.

Table 1

Assumptions about doing and learning mathematics implicit in teacher-student interactions.

Assumptions	Teacher actions	Student actions
Mathematical thinking is an act of sense-making, and rests on the processes of specializing, generalising, conjecturing and convincing	The teacher models mathematical thinking using a dialogic format to invite students to participate. The teacher invites students to take responsibility for the lesson content by providing intermediate or final steps in solutions or arguments initiated by the teacher. The teacher withholds judgment on students' suggestions while inviting comment or critique from other students.	Students begin to offer conjectures and justifications without the teacher's prompting. During whole class discussion students initiate argumentation between themselves, without teacher mediation.
The process of mathematical inquiry are accompanied by habits of individual reflection and self-monitoring	The teacher asks questions that encourage students to question their assumptions and locate their errors. The teacher presents 'what if' scenarios.	Students begin to point out and correct their own and each other's errors, and those made by the teacher. Students ask their own 'what if' questions.
Mathematical thinking develops through teacher scaffolding of the processes of inquiry	The teacher calls on students to clarify, elaborate, critique and justify their assertions. The teacher structures students' thinking by asking questions that lead them through strategic steps.	Students spontaneously provide clarification, elaboration, critiques, and justifications. Students take increasing responsibility for suggesting strategic steps.
Mathematical thinking can be generated and tested by students through participation in equal status peer partnerships	The teacher structures social interactions between students, by asking them to explain and justify ideas and strategies to each other.	Students form informal groups to monitor their progress, seek feedback on ideas, and explain ideas to each other.
Interweaving of familiar and formal knowledge helps students to adopt the conventions of mathematical communication	The teacher makes explicit reference to mathematical language, conventions and symbolism, labelling conventions as traditions that permit communication. The teacher links technical terms to commonsense meanings, and uses multiple representations of new terms and concepts.	Students begin to debate the appropriateness and relative advantages of different symbol conventions.

(From Goos et al., 2004, p. 99)

In another paper which described the senior classrooms Goos and colleagues (2002) provided important evidence of what happened when the teacher structured student participation in a joint construction zone. They described how peers of comparable expertise worked together in a context in which challenge was a common element. They outlined how the interplay between transactive dialogue and self-regulated decisions were significant in the creation of collaborative zones. Transactive reasoning was characterised as clarification, elaboration, justification and critique of one's own thinking and the thinking of others. Goos and her colleagues noted that the lack of transactive challenge led to student failure to construct a collaborative zone.

These important studies illustrate that teachers in inquiry based classrooms fulfilled significant roles. They modeled the practice of inquiry, facilitated classroom communities of inquirers and were co-participants in the mathematical practices. Initially they scaffolded student thinking toward development and use of proficient engagement in mathematical practices and later they facilitated student authoring of their own reasoning within the established intellectual partnerships. These research studies have implication for the current study. Together, they suggest possibilities of what communities of mathematical inquiry and student participation in mathematical practices might look like. They provide a useful background with which to view teacher actions as they develop student participation in mathematical practices.

3.2.2   VARIATIONS IN PRACTICES OF CLASSROOM INQUIRY COMMUNITIES

Whilst it is widely documented that differences in the teaching and learning practices exist between more conventional classrooms and those of inquiry communities, studies also report differences within inquiry communities themselves (see Kazemi & Stipek, 2001). The differences, attributed to the varying demands placed on expectations and obligations within the structured interaction patterns, have important implications for the mathematical practices which result.

Boaler (2003a) outlined how the varying interaction patterns students experienced in mathematics classrooms resulted in different students' reasoning patterns. She examined the practices of three inquiry classroom teachers who used similar curricular. The discernible differences were directly related to how the students were held accountable to their own learning and the learning of the collective within structured communication and participation patterns. For example, one teacher over-structured the problem solving activity so that all cognitive challenge was removed. A second teacher under-structured activity so that the students were left frustrated and without resources to proceed independently. However, the third teacher carefully structured both the problem solving activity and her responses to student questions. She implicitly directed the students to engage in and use mathematical practices to resolve their difficulties. Boaler explained that the teacher deflected authority back to the students to validate their own reasoning using mathematical practices and the discipline of mathematics itself. Through these actions she established a classroom culture in which all members had agency. Within this structure, higher levels of intellectual reasoning were achieved as the students increasingly assumed responsibility to validate their own mathematical thinking and that of others.

Similarly, Wood and her colleagues (2006) provided persuasive evidence that higher levels of complexity in students' articulated mathematical reasoning were "closely related to the types of interaction patterns that differentiated class discussions among…4 classroom cultures" (p. 222). Their analysis of 42 lessons in five classes of 7 and 8 year olds revealed qualitative differences between students in conventional and reform-oriented classrooms. These differences were manifest in both the nature of their interactions and what they articulated. In accord with Boaler's (2003a) findings, significant differences were directly attributed to characteristics of the cultures. Those which required greater involvement from participants resulted in higher intellectual levels of verbalised thinking.

Building on her earlier studies (e.g., Wood, 1994, 2002; Wood & Turner-Vorbeck, 2001; Wood & McNeal, 2003) Wood's (Wood et al., 2006) recently reported study clarifies how student reasoning is extended to higher intellectual levels in inquiry and argumentative cultures. In a previous study Wood (2002) reported on the research of six Year 2/3 classes

over a period of 2 years. Using analysis of the generalised "patterns of interactive and communicative exchanges" (p. 64), Wood constructed a theoretical framework. Key dimensions involved the students' 'responsibility for thinking' and 'responsibility for participation'. Four classroom cultures were delineated: traditional, strategy reporting, inquiry and argument. Wood and McNeal (2003) subsequently reconceptualised these as conventional, strategy reporting, and inquiry/argument. They also added a thinking dimension.

In Wood and McNeal's (2003) classroom research they found that teacher use of questioning and prompts were significant factors in shifting student reasoning toward more complex intellectual levels. When comparing the questioning behaviours between traditional and reform classrooms they noted that

> questioning in conventional teaching was directed at prompting children to give teacher expected information, while strategy reporting and inquiry/argument emphasised student exploration of methods and justification of student ideas. Comparison of strategy reporting and inquiry/argument revealed teachers differed in the frequency of prompting for mathematical thinking during inquiry interactions and situations involving proof and justification. The resolution of differences in students' answers was dealt with differently by teachers in reform cultures. Teachers in strategy reporting class cultures emphasised proof of a correct answer through the use of concrete objects, while teachers in inquiry/argument relied on children's explanations and justification to resolve differences in reasoning. (p. 439-440)

The researchers reported that those classrooms which supported an argument culture had an added element of challenge and debate. Dispute or challenge from the teacher or students initiated further debate and prompted need for justification to support reasoning. Responsibility for sense-making was most evident in inquiry and argument cultures. Interactions between teachers and students increased as the discussion cultures shifted from traditional, to report, to inquiry and argument. This was matched with an increase in frequency and complexity of teacher questioning and prompts.

These significant studies illustrate the consequences of teacher structuring of the interaction structures—the participation and communication patterns. Collectively, the studies draw attention to the differential outcomes in both the quality of student mathematical understandings and their interactions which resulted from the different interactive structures. They also provided evidence that only in inquiry/argument cultures were students provided with opportunities to construct a shared perspective. In an inquiry culture student thinking

> was extended to include whether a method or result is reasonable...the pulling together of ideas for making a judgement...and identifying flaws...and strengthening arguments by considering the mathematics from a different perspective...—all as a process for establishing shared mathematical meaning. (Wood et al., 2006, p. 248)

Thus, Wood and her colleagues provided evidence that, through discursive interaction, students appropriated the knowledge and skills of when and how to engage in mathematical inquiry and argumentation—key aspects of coming to know and use mathematical practices.

These various research studies have implications for the current study. They illustrate the importance of teachers developing mathematical learning cultures in which students are able to mutually engage in the discourse of inquiry and argumentation. They also revealed the significance of the structured interaction patterns. Collectively these research studies provide convincing evidence that if students are to engage in inquiry and reasoned mathematical talk, then teachers need to explicitly frame classroom interaction patterns.

3.2.3 THE SOCIO-CULTURAL AND MATHEMATICAL NORMS OF COMMUNITIES OF MATHEMATICAL INQUIRY

Recent studies have outlined how differences between classroom mathematics environments are directly linked to the enacted social-cultural and mathematical norms of classroom communities (Cobb et al., 1993; Kazemi, 1998; Sullivan et al., 2002; Wood, 2001; Yackel & Cobb, 1996). Both norms are important to the way in which students consider and use mathematical practices. Socio-cultural norms affect how students

construct a mathematical disposition, whereas the mathematical norms, also referred to as sociomathematical norms, affect the development of mathematical agency and autonomy.

Cobb and his colleagues (1993) report on a year-long teaching experiment in a second-grade classroom. They describe how the teacher's explicit scaffolding led to the students knowing when and how to talk about 'talking about mathematics'. They outline how other interaction practices were renegotiated as required. For example, the students learnt that the focus of discussion was on solution strategy, not just provision of correct answers. They also learnt that all contributions (including errors) had value. The teacher used social situations to discuss explicitly her expectations and obligations of the participants in joint mathematical activity. Higgins (2005), in a New Zealand study, also illustrated the importance of explicit teacher scaffolding of interaction norms. Higgins outlined the actions a Numeracy teacher of predominantly Maori students took to create collective responsibility for mathematical reasoning. The teacher encouraged the students to risk-take in reasoning, while at the same time ensuring that their mana, or feeling of self-worth, was protected.

The previous studies, and those described in other studies (e.g., Blunk, 1998; Cobb et al., 2001; Huferd-Ackles et al., 2004; Kazemi, 1998; Kazemi & Stipek, 2001; White, 2003; Wood, 1999; Yackel & Cobb, 1996; Zack & Graves, 2001), identify specific conditions which supported rich mathematical activity in inquiry environments. These include risk-taking, safe learning environments, polite and respectful exchange of ideas, provision of thinking time, valuing all contributions, collective analysis of errors and a need for agreement and disagreement during discursive exchanges. In all of these studies teacher enactment of these socio-cultural norms affected the values and beliefs students held towards mathematics—their mathematical disposition.

Mathematical agency or intellectual autonomy however, results from enacted mathematical norms. McClain and Cobb (2001) supplied persuasive evidence of how 1st Grade students gained intellectual autonomy as they negotiated and renegotiated sociomathematical norms (mathematical norms) concerning appropriate mathematical explanation or justification.

Kazemi and Stipek (2001) compared the teaching practices used by four 4th and 5th grade teachers. They illustrated that although the social norms (socio-cultural norms) in each classroom were similar, the sociomathematical norms (mathematical norms) of one classroom were markedly different. As a result, mathematical explanations provided by students in that classroom were comprised of argumentation, justification and verification of reasoning. The students in this classroom analysed and validated their reasoning, and the reasoning of others. Collectively, these studies and many others (e.g., Bowers et al., 1999; Cobb et al., 2001; McClain, Cobb & Gravemeijer, 2000; Yackel, 1995; Zack & Graves, 2001) provide strong evidence that autonomous thinkers emerge when teachers engage their students in negotiation and renegotiation of socio-cultural and mathematical norms. Autonomy and agency to validate one's own reasoning and the reasoning of others' are central to the mathematical practices used by successful mathematical learners and users.

3.2.4 INTELLECTUAL PARTNERSHIPS IN THE MATHEMATICAL DISCOURSE OF INQUIRY

The development of intellectual partnerships is fundamental to student engagement in mathematical practices at proficient levels of expertise. Intellectual partnerships evolve when students are required to use their own autonomy or authority in critical analysis and validation of their own and others' mathematical reasoning. Within these partnerships all participants have a voice and shared authority in mathematical inquiry and the validation of reasoning (Amit & Fried, 2005; Povey, Burton, Angier, & Boylan, 2004).

Stigler and Hiebert's (1999) large scale comparative study of Japanese and American teachers demonstrated that many of the American teachers in their study perceived themselves, or were perceived by their students as the main source of authority. As external authority figures they had the responsibility to ensure that their students learnt certain mathematical rules and procedures. They also assumed that it was their responsibility to ask all the questions and validate answers. Hamm and Perry (2002) investigated the practices of six American 1st Grade teachers and reported the same findings. The teachers, despite their reported intent to require students to validate their reasoning, seldom stepped away from their authoritarian role. These studies and others (e.g., Skott, 2004; Stipek, Givven, Salmon,

MacGyvers, & Valanne, 2001; Weiss et al., 2002) illustrate the way in which the beliefs held by the teachers and students affect how authority was viewed in the classrooms.

Intellectual partnerships are grounded in collaborative dialogue (Amit & Fried, 2005). Amit and Fried explained that when specific students or the teacher are considered the authority in discussion, collaborative dialogue is difficult. They used two case studies of 8th Grade classrooms to illustrate the effects of externally attributed authority. When teachers and more expert peers were given immense and unquestioned authority this resulted in a lack of reflective analysis on their part or challenge to students' sense-making. These researchers argue that authority should be dynamic and fluid and that teachers must work "to make their students into colleagues who finally will completely share authority with them" (p. 165). Researchers have noted however, that construction of intellectual partnerships is a lengthy process which requires explicit teacher actions (Boaler, 2003a; Goos et al., 2004; McCrone, 2005; Povey et al., 2004)

In examining inquiry environments that provide space for intellectual partnerships to develop, research studies highlight the importance of rich challenging mathematical activity. Researchers argue that teachers need to make mathematical activity problematic in a form which represents real disciplinary specific inquiry (Cornelius & Herrenkohl, 2004; Engle & Conant, 2002; Forman, et al., 1998; Lampert, 1990b; 2001; O'Connor, 2002; Peressini & Knuth, 2000; Smith & Stein, 1998; Stein & Smith, 1998). Stein (2001) maintained that rich problematic mathematical tasks are ones which promote argumentation. These problems, which are often open-ended and elicit multiple strategies, force students to examine the generality of their reasoning, and often modify their reasoning. In a study of middle school students' engagement in rich problematic tasks Stein illustrated students' use of justification including initiating and backing claims appropriately, critically evaluating their own and others' arguments and validating their reasoning during extended dialogue.

Lampert (1990b, 1991) and O'Connor (2002) both used position driven discussions to elicit multiple strategies and solution pathways. Lampert (1991) illustrated how through

discursive dialogue the students learnt disciplinary inquiry and explanatory justification. Position driven discussions involve a teacher facilitating discussion which centres on a framing question. For example, in O'Connor's study in a 5th Grade classroom the teacher posed the question 'can all fractions be turned into a decimal'? The teacher skilfully managed the tension between pursuit of collective reasoning and the diverse individual contributions using the "divergences to create a discussion with some of the properties of real mathematical discovery" (p. 180). The discussion concluded with a shared understanding which had been validated by the collective community.

Shared ownership and active engagement in the discourse is central to students' developing reflective validation of their own reasoning and the reasoning of others. The work of Boaler (2003) and Wood and her colleagues (2006) (see section 3.2.2) illustrated that a press towards inquiry and argumentation created a community of validators. Likewise, Huferd-Ackles et al. (2004) outlined how an elementary teacher shifted classroom discourse patterns and created a community of validators. The teacher explicitly scaffolded the discourse toward reasoned collaborative talk. Shifts in the discourse were matched with increased levels of student investment in their own and collective reasoning. As the teacher progressively shifted toward a more facilitative position, student thinking became more central and the students learnt to talk with authoritative mathematical understanding. Teacher researcher Ball (1991, 1993) also illustrated in an elementary classroom how the teacher's structuring of social interactions provided the students with increased opportunities to collectively engage in and learn the reasoning discourse of mathematical practices. Ball noted that the students' growth in use of the discourse was matched with their sense of agency in the classroom.

Cobb et al. (2001) also linked teacher scaffolding of shared ownership of inquiry discourse with student perceptions of authority and agency. Students in their study progressively shifted from peripheral to more substantial participatory positions as they participated in the reasoning discourse related to measure activities. The students assumed responsibility to validate their own mathematical thinking, rather than using the teacher, or text, as sources of external authority. Hunter (2006) and Young-Loveridge (2005a) in two recent New

Zealand studies also provide convincing evidence that students in inquiry focused cultures viewed both mathematics and their role as mathematical learners differently. In both studies involving students in middle school, the students recognised that it was their responsibility to validate their own reasoning and the reasoning of others. They indicated that they took communal responsibility for sense-making and described a proficient use of mathematical practices to do so.

Together, these studies of inquiry communities illustrate that both intellectual autonomy and shared understanding are supported when there is a press toward inquiry. Mathematical communities develop when classroom cultures are premised on the reciprocal responsibility of all participants to engage actively in collective mathematical reasoning. Implications for the current study include the need to consider how teachers and students negotiate the socio-cultural and mathematical norms and how these affect the mathematical practices. Likewise, consideration is needed of the actions teachers take to develop intellectual partnerships in which students become successively more proficient in the use of mathematical practices. There appears to be limited research studies available in New Zealand related to these factors.

3.3   FORMS OF DISCOURSE USED IN MATHEMATICS CLASSROOMS

The forms of talk that students participate in shape the mathematical practices they come to know and use. They also affect the students' beliefs and influence their goals during mathematical activity.

3.3.1 UNIVOCAL AND DIALOGIC DISCOURSE

How teachers socialise students' use of the discipline-specific mathematical discourse is of prime importance. The function of discourse in more traditional forms of mathematical talk focuses on transmitting meaning (univocal discourse) framed around what the teacher wants to hear. Mehan's (1979) examination of traditional pedagogies noted the typicality of the univocal form in which the teacher initiated the discourse, selected students to respond,

evaluated responses and provided feedback or questions until the answers sought were provided. In contrast, inquiry discourse is focussed on generating meaning (dialogic discourse). It requires a more balanced partnership in the dialogue (Knuth & Peressini, 2001; Lotman, 1988; Manouchehri & St. John, 2006; Wertsch, 1991).

Knuth and Peressini (2001) provide vignettes from a 4 year professional development project aimed to support teachers to foster meaningful mathematical discourse. The first vignette illustrates how a teacher structured univocal discourse through unintentionally shaping the discourse so that it aligned with her own perspective. The same teacher in the second vignette structured dialogic discourse so that multiple layers of meaning were generated. The vignettes illustrated the complexities and challenges for teachers to orchestrate inquiry community interactional patterns. This is of particular interest because both forms of discourse are always on a continuum—univocal discourse ceases at the point where listeners have received an intended message and dialogic discourse commences within the structure of "give-and-take communication" (Knuth & Peressini, p. 321).

Restructuring discourse patterns is an activity fraught with many pitfalls. This is particularly so because the more conventional forms of classroom discourse in which teacher talk has dominated is likely to be the most common form of talk both students and teachers have experienced in their former mathematics classrooms (Lampert & Cobb, 2003). Nathan and Knuth (2003) analysed data from a 2 year study of a 6th Grade classroom teacher as she reconstructed classroom interactional strategies to support dialogic discourse. In the first year, although she appeared to scaffold more open communication norms, her actions indicated that she retained a central position and dominated the flow of discourse. In the second year, she facilitated increased mathematical communication but removed herself from scaffolding the participants' mathematical reasoning. This resulted in discourse which lacked rigorous mathematical analysis and argumentation.

These studies and others (e.g., Kazemi & Franke, 2004; Sherin, 2002a, 2002b; Sherin et al., 2004; Steinberg, Empson, & Carpenter, 2004) illustrate the multiple issues teachers face in managing the discourse of inquiry and argumentation. One challenge to teachers is the

messy dialogue which evolves when student thinking is central. Teachers also face conflict when structuring learning opportunities which may initially give a sense of being unsuccessful (e.g., Schwan Smith, 2000). Other studies identified problematic situations are related to teachers' need to finely balance their role in the dialogue (Chazan & Ball, 1999; Lobato, Clarke, & Ellis, 2005). For example, when and how should teachers insert questions and challenge? When should they insert their mathematical ideas and explanations to ensure that student mathematical understandings are being analysed and advanced? All these issues have implications for teachers in the current study as they orchestrate the discourse of inquiry and argumentation.

3.3.2   INQUIRY AND ARGUMENTATION

Framed in a sociocultural learning perspective, proficient mathematical practices evolve through collective participation in discursive interaction which has as its focus the emergence of shared meaning (Forman, 2003; Mercer, 2000; Wells, 1999). In this frame, student access to and valuing of the discourse of inquiry and argumentation is recognised as key to their participation in mathematical practices.

In separate studies involving New Zealand students, Meaney (2002) and Bicknell (1999) both reported the ambivalent beliefs some students held toward the value of communicating mathematical explanations, or the importance these had for learning mathematics. Similarly, Young-Loveridge (2005b) and her colleagues (Young-Loveridge, Taylor, & Hawera, 2004) reported considerable variation in student views towards the value of explaining their strategies and listening to others. Interview data of the views of 180 nine to eleven year-olds included students who had been in discussion-intensive *New Zealand Numeracy Development Project* classrooms. Although the students stated that they liked to explain their reasoning, many did not recognise the value of explaining their reasoning and discounted the value of attending to the thinking of others. Nor did they realise the importance of discursive exchange as a means to advance their own thinking or in the construction of a collective view. Young-Loveridge suggested that the responses of the majority of interviewees indicated that "they continued to regard mathematics from an individualistic perspective as being a private activity of little or no concern to others in their

class" (p. 28). Their identity as mathematical learners remained focused on a model of mathematics learning as *acquiring* knowledge rather than *participating* in the construction of understandings through interactions in communities of learners (Cobb & Lampert, 2003; Sfard, 1998, 2001, 2003).

Numerous other studies have illustrated the difficulties students encounter when engaging in communal discussions (e.g., Gee & Clinton, 2000; Lampert, Rittenhouse, & Crumbaugh, 1996; McCrone, 2005; Mercer, Dawes, Wegerif, & Sams, 2004; Rojas-Drummond & Mercer, 2003; Rojas-Drummond, Perez, Velez, Gomez, & Mendoza, 2003; Rowe & Bicknell, 2004; Wegerif & Mercer, 2000; Wegerif, Mercer, & Rojas-Drummond, 1999). Irwin and Woodward (2006), in a New Zealand study, examined the talk that Year 5-6 students used when working in small cooperative problem solving groups independent of the teacher. Whilst the teacher consistently modeled inquiry strategies during large group discussions Irwin and her colleague observed that when the students worked together beyond the teacher's close scrutiny they exhibited difficulties using inquiry talk. The teacher verbally instructed them to work cooperatively but did not discuss why nor provide explicit scaffolding. At times individual students appeared to adopt aspects of inquiry talk but predominantly competitive talk and the use of procedural rules prevailed. As a result, the use of unproductive forms of talk prevented the construction of communal views.

A lack of explicit teacher guidance on student use of shared language may explain why students do not realise the benefits which accrue through listening closely to the reasoning of others, or developing a collective view. For example, Sfard and Kieran (2001) illustrated the difficulties two 13 year old students encountered during a 2 month teaching sequence. Sfard and Kieran examined their mathematical discussion, within the context of the immediate mathematical content, to explore the effectiveness of their communication. Although, both students' achievement results significantly improved, close data analysis illustrated that this enhanced performance could not be attributed to their conversations. Analysis showed that the students had placed little value on the other's contributions during discussion.

Mercer (1995, 2000) and his colleagues (e.g., Wegerif, Mercer, & Dawes, 1999; Wegerif et al., 1999) examined the forms of mathematical talk students used when working with their peers in small groups. In their many studies in English and Mexican schools they have consistently noted that students use three different forms of talk, and these in turn use different degrees of engaging constructively with each others' reasoning. The most productive form of talk the students engage in, they termed exploratory talk. Exploratory talk they characterised as language which included such phrases as 'because', 'if', 'why', 'I think' and 'agree'. However, without explicit teacher intervention and scaffolding they noted that the students consistently used the two other forms of unproductive talk the researchers termed disputational and cumulative talk.

Disputational talk, Mercer (2000) has described as discussion characterised by cyclic forms of assertions and counter-assertions which remain unexamined by the participants. Short utterances and lack of explicit reasoning are used as the participants primarily focus on self-defence. Rather than trying to reach joint agreement individuals struggle to hold control. Although actions are not overtly uncooperative Mercer noted participants' perspectives "compete with rather than complement each other" (p. 97).

Cumulative talk, Mercer (2000) described as lacking the element of confrontation, that is questions and argument are avoided. During this form of talk the participants build on each others' thinking adding their own ideas and "in a mutually supportive, uncritical way construct together a body of shared knowledge and understanding" (p. 97). The construction of a collective view is accomplished but not evaluated by the students.

Exploratory talk involves students' mutual investment in developing collective reasoning and occurs as a result of reasoned debate. Exploratory talk supports students engaging in each other's reasoning in mutually constructed zones of proximal development.

> Exploratory talk is that in which partners engage critically but constructively with each other's ideas. Statements and suggestions are sought and offered for joint consideration. These may be challenged and counter-challenged, but challenges are justified and

alternative hypotheses are offered. In exploratory talk, knowledge is made publicly accountable and reasoning is visible in the talk. (Mercer & Wegerif, 1999b, p. 96-7)

Teacher actions to ensure that students engage at high levels of inquiry make certain that mathematical reasoning is visible and therefore available for challenge and debate. At the same time, the accessibility of the reasoning means that a collective view can be constructed (Mercer, 2002; Wells, 1999).

As previously discussed, the forms of talk that students use in mathematical activity influence their goals and attitudes and the mathematical practices they use. Whilst recent New Zealand policy documents (e.g., Ministry of Education, 1992, 2006) have promoted notions of students working interactively, making explanations, justification and generalisations of their mathematical reasoning, there is a gap in the research literature in New Zealand on the forms of talk and actions teachers should take which best support these mathematical practices.

3.3.3 INTERACTIONAL STRATEGIES USED BY TEACHERS TO ENGAGE STUDENTS IN THE DISCOURSE

The previous section outlined the complexities involved in teacher orchestration of learning partnerships with students in the discourse. Beyond ensuring that the students understand the rights and responsibilities enacted in classrooms, teachers are also required to "align students with each other and with her, as proponents of a particular hypothesis or position" to ensure collective intellectual reasoning (O'Connor & Michaels, 1996, p. 66).

O'Connor and Michaels (1996) illustrated that higher levels of reasoning were fostered when teachers scaffolded students to participate in collective dialogue. Forman and her colleagues (1998) illustrated 'teacher revoicing' in a middle school classroom investigation of area measure. The teacher revoiced and thus aligned and realigned the students to engage in collective mathematical argumentation. The study provided clear evidence that the teacher use of revoicing positioned the students to use mathematical practices to examine

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and explore the generality of mathematical concepts—rather than rely on mathematical rules and algorithms.

Teachers have a central role in inducting their students into the construction of persuasive mathematical arguments. Drawing on episodes from a middle school mathematics lesson Strom, Kemeny, Lehrer, and Forman (2001) illustrated how the teacher enculturated the students into the mathematical practices. The models of teacher revoicing showed the complexities involved in the positioning of students as intellectual owners of their reasoning. In the extended dialogue, all forms of reasoning were seriously considered, mathematical terms were clarified, and the contexts of arguments were integrated with previously introduced ideas and terms. Through the teacher's choice of specific words and indirect speech, students were animated and positioned to intellectually engage in collective argumentation.

Similarly, Empson (2003) illustrated how the situated identities of two low performing Grade One students were enhanced through explicit teacher revoicing, and animating activity. Revoicing was a powerful tool the teacher used to ensure students were considered mathematically competent by their peers. This resulted, in them contributing productive mathematical ideas within a variety of roles. These included the roles of problem solvers and claim makers. Empson's study highlights a key equity issue of relevance to the current study—that teachers ensure that *all* students are able to engage authoritatively in mathematical activity and gain intellectual agency.

Collectively these studies and many more like them (e.g., Cobb et al., 2001; Forman & Ansell, 2002; Huferd-Ackles et al., Kazemi & Stipek, 2001; Yackel, 2002; Zack & Graves, 2001) report positive outcomes for students when their teachers aligned them to take specific roles in the use of inquiry and argumentation in the development of collective thinking. The studies demonstrated that through explicit teacher positioning the students gained a sense of authorship and authority toward their mathematical activity. Positive mathematical dispositions were developed in which they saw themselves as users and doers of mathematics.

3.3.4 THE MATHEMATICAL DISCOURSE AND THE DEVELOPMENT OF SITUATED IDENTITIES

In recent years there has been considerable discussion and debate as to how diverse student groups are positioned into taking—or not taking—appropriate intellectual roles in mathematics classrooms. Explanations have varied. One theme in the literature of relevance to the current study, relates to the patterns of interaction and how these mediate identities for diverse students (e.g., Black, 2004; Baxter, Woodward, Voorhies, & Olson, 2002; Boaler, 2006a, 2006b; Civil & Planas, 2004; Cobb & Hodge, 2002; Empson, 2003; Khisty & Chval, 2002; Lubienski, 2000a, 2000b; Martin, Pourdavood, & Carignan, 2005; Moschkovich, 2002b; Planas & Civil, 2002; Planas & Gorgorió, 2004; Pourdavood, Svec, & Cowen, 2005; Varenne & McDermott, 1998; White, 2003).

Planas and Gorgorió (2004) illustrated how a teacher's actions influenced how students perceive themselves and others as valid contributors to mathematical dialogue. Space to engage in mathematical activity provided differentially by the teacher resulted in different groups of students constructing different mathematical meaning and identity. These researchers revealed how a teacher unintentionally influenced how and when the local and immigrant students participated in mathematical activity. The teacher positioned the local students to discuss and argue their reasoning through to sense-making. The immigrant students were required to enumerate numbers rather than engage in explanation or argumentation. The differential treatment meant that immigrant students' were unable to contribute to the discourse; their contributions were perceived as invalid by local students. Unlike the local students the immigrant students were not socialised into how to participate in collective argumentation and construct communal reasoning. This resulted in their construction of different identities as mathematicians.

Pourdavood et al. (2005) maintain that teachers must ensure that all students are explicitly scaffolded to have a 'voice' and agency in the discourse. For example, White (2003) showed how 3rd Grade students were positioned as agents responsible for validation of their own thinking and the thinking of others. Pivotal to their agency were norms which placed

value on sharing and respecting each others' contribution within mathematical dialogue. The actions of two teachers in White's study emphasised to the classroom community that all ideas would be seriously considered and constructively examined and explored.

Martin et al. (2005) also illustrated how a pedagogical focus on engaging students in collective reasoned inquiry discourse resulted in the development of a positive mathematical disposition and sense of agency. These researchers examined the mathematical experiences of two low socioeconomic status African American children. They drew on classroom observations, interviews with the students and their parents and teachers. They also observed the two students during mathematical activity paired with students from more conventional schools. Significantly, their observations revealed that the two students maintained their sense of 'voice' independently of their teacher as they drew the 'outsider students' into working collaboratively.

Collectively, these studies illustrated the different identities students construct as an outcome of their participation in collective mathematical reasoning. Persuasive evidence was provided that the salient features of teachers' actions affected the forms of discourse the students participated in. Likewise, it affected how they considered the outcome of discursive interaction on their knowledge construction.

Ensuring that all students are socialised into the discourse of mathematical inquiry is a key equity issue (Boaler, 2006a, 2006b; Martin, et al., 2005; Gutierrez, 2002; MacFarlane, 2004; Moschkovich, 2002b). In addition, specific to the New Zealand context of this study MacFarlane (2004) promotes the need for teachers to be "culturally responsive" (p. 27) to Maori and other diverse students. Culturally responsive teaching provides space "which is culturally, as well as academically and socially, responsive" (p. 61). This is of particular relevance to the current study because the school has a wide range of diverse learners in the classrooms.

3.3.5 EXAMPLES OF FRAMEWORKS USED TO STRUCTURE COLLECTIVE REASONING DURING INQUIRY AND ARGUMENTATION

This chapter has focused on the close connections between student participation in inquiry and argumentation and the development of mathematical practices. How teachers scaffold student participation in mathematical practices and the communication and participation structures they use are significant. The complexities involved in achieving this and the many issues involved have been identified. This final section reviews a number of international studies that illustrate the positive outcomes for individual and communal learning when specific frameworks are used which scaffold student inquiry and argumentation in zones of proximal development. These studies were influential in informing the implementation of the current study.

The first framework is provided by Mercer (2000) and his colleagues. They constructed a programme they termed 'Talk Lessons'. The ground rules teachers used specified how the students interacted. They included the following criteria

> (1) all relevant information is shared; (2) the group seeks to reach agreement; (3) the group takes responsibility for decisions; (4) reasons are expected; (5) challenges are accepted; (6) alternatives are discussed before a decision is taken; and (7) all in the group are encouraged to speak by other group members. (Mercer & Wegerif, 1999a, p.99)

In the planned intervention of nine lessons students were explicitly scaffolded to engage productively in collective argumentation. These lessons focused on developing a communal view. In order to achieve interthinking lessons began with "integrated teacher-led whole class dialogue and group activity, so that children could be expected to begin their activity and discussion with a shared conception of relevant knowledge and of how they should talk and think together effectively" (Rojas-Drummond & Mercer, 2003, p. 103).

Rojas-Drummond and Mercer (2003) reported that the students in both the target and control classes were assessed using the Ravens Progressive Matrices psychological test prior to being trained in the use of 'talk lessons'. Post assessment used the problem solving section of the Ravens Progressive Matrices. Completed individually and in small problem solving groups, the results illustrated significant differences. The target class students used more exploratory talk, and through using exploratory talk were more successful at solving

the Raven's puzzles. What was more significant was that they were also more successful at solving problems alone. Their improved relative performance indicated that they had appropriated the grounds rules of the exploratory talk, and so were "able to carry on a kind of silent rational dialogue with themselves" (p. 105).

Similar results were attained in a larger study, which used the alternative British official Standard Attainment Task Mathematics and Science assessments (Rojas-Drummond & Mercer, 2003). Studies using the 'Talk-Lesson' format have also been replicated in Mexico with comparable results. In the large Mexican study Rojas-Drummond and Zapata (2004) outlined how explicit teacher scaffolding of students in the use of exploratory talk resulted in progressive shifts toward increased levels of exploratory talk and argumentation. Through the use of talk-lessons the students negotiated more frequently, constructed more arguments and provided a variety of perspectives for consideration. Elaborated reasoning was articulated in that they

> tended to present their arguments in a more explicit way, and to provide more supports to sustain their opinions, making their reasoning more explicit in their language. They also used a greater number and variety of links to mark the logical organisation between different components of arguments, such as the assertions, the supports and the conclusions, thus making their interventions more understandable to others. (p. 554)

Another international study which reported on the use of a structure which scaffolded students to participate in mutual inquiry of open-ended problems is described by Alrø and Skovmose (2002). Teachers applied their Inquiry-Cooperation Model across age groups. The students were specifically taught to actively listen, and identify each others' perspectives. Claims were proposed as the tentative positions and thinking aloud made reasoning visible and available for scrutiny. Ideas were clarified through teacher or student reformulation and then subjected to challenge. These researchers maintained that the participation structure scaffolded the students' development of "mathemacy" (p. 136). Their understanding shifted beyond a focus on numbers and rules to also exhibit a reflective autonomy toward considering the validity of their reasoning.

Brown (2001, 2005) and his colleague (Brown & Renshaw, 1996, 1999, 2000, 2004; Renshaw & Brown, 1997) reported on collective argumentation, a framework that they used to structure collective reasoning. This framework was designed for teachers to scaffold student participation in inquiry and argumentation in zones of proximal development. The collective argumentation was organised around the following key strategies that require students to

> *represent* the task or problem alone, *compare* representations within a small group of peers, *explain* and *justify* the various representations to each other in the small group, and finally *present* the group's ideas and representations to the class to test their acceptance by the wider community of peers and teacher. (Brown & Renshaw, 2004, p. 136)

Brown (2005) reported on the outcome of a teacher's use of collective argumentation in a Year 7 longitudinal classroom study. He outlined how the students engaged in collaborative interactions using the structured format and as a result constructed and reconstructed dynamic and generative zpds. The zpd as an intellectual or social space made the reasoning visible and supported critical examination and evaluation of key mathematical concepts. The learning situations and the joint partnerships created were not dependent on 'expert-novice' partnerships but rather were often comprised of individuals with incomplete but relatively equal knowledge. In that situation the interactions created multi-directional zpds. For example, Brown and Renshaw (2004) outlined how the teacher used collective argumentation to scaffold students to "adopt different speaking positions or voices" (p. 12). They described how students began from 'my voice', shifted to 'your voice' and finally developed 'our voice'. Thus, ownership of explanations were shared and defended within intellectual partnerships. The reciprocal interaction required that participants explained and justified their reasoning. In turn, they expected clarification and justification of reasoning from others. Challenge through disagreement and conflict was as important in this process as agreement and consensus (Brown & Renshaw, 2004).

The previous studies reported on structures used extensively with large numbers of students and for lengthy periods of time. Rowe (2003) however, illustrated positive learning outcomes in a short 4-week New Zealand study. The intervention was designed to develop

verbal interactions at higher cognitive levels. It was framed around students working with these key points

> Wait and give individuals time to think for themselves; Be specific with feedback and encouragement; Give help when asked in the form of a specific strategy, idea or question rather than an answer; and support agreement or disagreement with evidence. (Rowe & Bicknell, 2004, p. 496)

Collectively, these studies provide compelling evidence that students can and do learn to participate in inquiry and argumentation through explicit scaffolding. Structuring discursive interaction so that all participants are able to access a shared intellectual space creates many potential learning situations for the participants. The partnerships and learning situations are conducive to the students learning and using mathematical practices. Although none of these studies focused directly on mathematical practices and how these were transformed, the intellectual climate created a context for their use.

3.4   SUMMARY

This review recognises the complexities and challenges teachers encounter in constructing inquiry learning communities. The review began with specific studies which depicted particular teachers and their actions and beliefs as they developed and worked within inquiry communities. Important literature illustrated how variations in the interactional patterns of inquiry communities affected how the students in them participated in inquiry and argumentation. In the second section, literature related to the forms of talk used in classrooms including the discourse of inquiry and argumentation was reviewed.

Woven through the review of literature were descriptions of specific interactional strategies and communication and participation structures used by teachers to induct their students into intellectual climates. Studies drawn on in this review provided convincing evidence that teachers can establish such intellectual climates through a range of pedagogical practices. There is however a gap in the literature which describes more specifically how mathematical practices as interrelated reasoned verbal and performative actions are taught

and learnt within intellectual communities. This finding is consistent with that of The RAND Mathematics Study Panel (2003) who noted how little attention had been given to the understanding of mathematical practices "such as problem solving, reasoning, proof, representation…" (p. 33) as a coordinated group which are used skilfully and flexibly by expert users of mathematics. Some of these various studies will be drawn on in the following chapter to outline background research for the mathematical practices. How these evolve and are transformed through teacher guidance will be described. These are presented as single practices in the following review because there appear to be few studies available which consider them as integrated practices.

CHAPTER FOUR

THE MATHEMATICAL PRACTICES OF COMMUNITIES OF MATHEMATICAL INQUIRY

> School mathematics should be viewed as human activity that reflects the work of mathematicians—finding out why given techniques work, inventing new techniques, justifying assertions, and so forth. It should also reflect how users of mathematics investigate a problem situation, decide on variables, decide on ways to quantify and relate the variables, carry out calculations, make predictions, and verify the utility of the predictions. (Romberg & Kaput, 1999, p. 5)

4.1 MATHEMATICAL PRACTICES

The observations made in the previous chapters draw attention to the fact that mathematical practices are grounded within collective practices. These practices involve reasoned performative and conversational actions and occur in social and cultural activity systems and amongst multiple participants (Saxe, 2002; Van Oers, 2001). Mathematical practices evolve and are transformed within the community they are developed. For that reason the literature reviewed in this chapter draws on studies which correlate with the age of the primary school students (aged 7 -12) in the current study, and the mathematical practices students within this age group learn and use during mathematical activity.

Previously I noted that student learning and use of proficient mathematical practices were both dependent on how teachers structured classroom participation and communication patterns. Likewise, in this chapter recognition is given to how inquiry and challenge supports both emergence and change in the mathematical practices students use. Relevant literature is drawn on to discuss and describe what mathematical practices are and how these are used by successful mathematical learners to develop collective reasoning. Links are made to the actions teachers take to engage student participation in those practices and how participation mediates development and use of mathematical practices.

4.2 MATHEMATICAL EXPLANATIONS

Mathematical explanations are statements which commence from well reasoned conjectures (Whitenack & Yackel, 2002). These conjectures, although provisional, are statements which are used to present a mathematical position the explainer is taking. They make visible and available for clarification, or challenge, aspects in the reasoning which may not be obvious to listeners (Carpenter et al., 2003).

An extensive body of research undertaken with primary aged students has outlined criteria for what is accepted as well structured mathematical explanations (e.g., Bowers et al., 1999; Carpenter et al., 2003; Cobb, Boufi, McClain & Whitenack, 1997; Cobb et al., 2001; Cobb et al., 1992; Forman & Larreamendy-Joerns, 1998; Kazemi & Stipek, 2001; Perkins, Crismond, Simmons and Unger, 1995; Reid, 2001; Whitenack, Knipping, & Novinger., 2001; Whitenack & Yackel, 2002; Yackel, 1995; Yackel & Cobb, 1996). The criteria include the need for explainers to make explanations as explicit as required by the audience, relevant to the situation, and experientially real for the audience. Explainers also have to supply sufficient evidence to support the claims. This may require that the explainer provides further elaboration or re-presentation of the explanation in multiple and rich relational ways. Concrete material or graphical, numerical, or verbal contextual support may also be needed.

Teachers have a key role to play in the construction and use of mathematical explanations within classroom communities. The important studies of Wood and her colleagues (2006) discussed in the previous chapter, illustrated that is the teacher who establishes how students participate in developing, using and analysing mathematical explanations. Moreover, these researchers showed how the questions and prompts teachers use shape the explanations students make. Other studies undertaken within Grade 1 to Grade 5 classes (e.g., Bowers et al., 1999; Kazemi & Stipek, 2001; McClain & Cobb, 2001; Whitenack et al., 2001) have illustrated important pedagogical actions. Effective teachers pressed for acceptable explanations. These were differentiated between the different, more sophisticated, more efficient and easily understood explanations. The teachers also ensured

that explanations were accessible and understood by the community. They revoiced explanations to ensure that conceptual explanations were maintained. Other actions included ensuring that their questions and those of their students' were framed so that solution strategies were directed toward specific clarification of mathematical explanations.

For example, the following vignette from McClain and Cobb (2001) illustrates how the teacher facilitated an efficient explanation. Clearly evident is the students' implicit recognition that acceptable explanations should be easily understood by all participants and that they should be of a conceptual not calculational nature. The teacher's mediational actions supported the students to autonomously validate their own reasoning.

Pressing the explanatory reasoning towards easy and efficient solution strategies

Towards the end of a 6 month teaching experiment in which norms for what made an explanation different and notions of easy and hard had been established in the community the following task was presented: *There is fourteen cents in the purse. You spend seven cents. How much is left?* Initially a student Kitty used an arithmetic rack and made two rows of seven beads. Then she moved four beads on one rod and three on the other leaving groups of three beads and four beads respectively. When she finished another student interjected:

Teri	I think I know a way that might be a little easier for Kitty.
Teacher	You think so?
Teri	[comes to the board] We know that seven plus seven equals fourteen because we have seven on the top and seven on the bottom…It might just be easier if we just moved one of the sevens on the top or the bottom (points to each group separately).
Teacher	You mean move a whole group of seven altogether?

Kitty nods in agreement as she looks at Teri. The term 'easy' used in this context referred to a thinking strategy or grouping solution (structuring the 14 as a composite of two sevens) that Teri considered easily accessible for the classroom community.
From McClain and Cobb (2001)

Learning to construct knowledge of what constitutes an explanation which is conceptually not procedurally based and matching it appropriately to an audience may initially pose considerable challenges for some students. The various studies of Cobb and his colleagues and other researchers (e.g., Cobb, 1995; Cobb et al., 1992; Cobb et al., 1993; Kazemi, 1998; Kazemi & Stipek, 2001; Yackel, 1995; Yackel et al., 1991) documented the

difficulties students had in elementary grade classrooms as they developed the ability to construct appropriate explanations. For example, Yackel and Cobb (1996) outlined how a group of 2nd Grade students in their study were unable to assess what needed to be said and what could be assumed as 'taken-as-shared' in their community. The researcher also noted the difficulties some students had in maintaining focus on mathematical reasoning. They reported that these students changed their explanations in response to peer or teacher reaction because they interpreted the social situation as more important.

A group of New Zealand researchers (e.g., Anthony & Walshaw 2002; Meaney, 2002, 2005) also drew attention to the difficulties younger students have in applying appropriate evaluative criteria regarding sound explanations. At the secondary level Bicknell (1999) described how many students lacked an understanding of what was required in writing explanations. Despite their use of "varying modes of representations for an explanation from purely symbolic to a combination of symbols, diagrams, and words" (p. 81) they lacked confidence to assess the adequacy of their explanations. These studies have important implications for the current study. Given the recent focus on students explaining their reasoning in the New Zealand Numeracy Development Project (Ministry of Education, 2006b) ensuring that students can make and evaluate mathematical explanations is a key task for teachers. Providing explanatory reasoning is an important precursor for explanatory justification and argumentation (Cobb et al., 1997).

4.3   MATHEMATICAL JUSTIFICATION

Explanations become explanatory justification when explainers are required to provide further evidence in order to address disagreement or challenge to their reasoning (Whitenack & Yackel, 2002). The following vignette illustrates the subtle differences between mathematical explanation and justification. The vignette illustrates how the teacher scaffolded the shift between two mathematical practices. Initially she prompted the students to examine the explanation. She then facilitated the use of it as a scaffold for other participants to develop arguments—justification that supported, refined or refuted the ideas in the explanation. As part of the socially interactive process of argumentation the questions

used by both the teacher and the students shifted from asking 'what did you do' to those which encompassed challenge like 'but why would you'.

Providing explanatory reasoning and justification of the reasoning

Ms Jones has reconvened the Grade-2 class to listen to an explanation for the following problem: *Aunt Mary has 31 pieces of candy on the counter and Uncle Johnny eats 15 pieces of candy. Show how much candy Aunt Mary has on the counter now.* Casey has recorded on the board $30 - 15 = 15$; $15 + 1 = 16$. He then explains:

Casey   Um I took that away from the 31. And plus 1 equals 16. If you take that 1 and add it onto the 30 to make 31, it's just when you minus you just have 1 higher number, and if you take the 1 off the 30, add onto the 15, you get 16. And that's where I got the 16.

The teacher then opened the discussion by asking for comments or questions.

Shari   That was really cool. Because sometimes people know that 15 plus 15 equals 30. So if you know that it might be easier.

Ms Jones confirmed with Casey that he had used the doubles fact then continued:

Teacher   So that was something Casey knew and he worked with that first. Does anybody else have a comment or question?

Teri   Why would you add it [the 1] onto 30 and put it onto the 16? The 1 to the 16?

Casey   I added it onto the 15 to get the 16.

Teri   Huh?

Ms Jones   Does that make sense Teri?

Teri   But why would you add it to the 30 first and then the 15?

Ms Jones   Can you show us your original thinking? What did you have up first? I think that might help her to see what you did.

From Whitenack and Yackel (2002)

In this vignette, the teacher supported continued questioning until the explainer had persuaded the listeners of the legitimacy of his mathematical actions. At the same time, she directed him to re-evaluate and re-present his original reasoning. Through these actions the teacher ensured that justification was also used to ascertain or convince the explainer himself of the validity of what he was arguing.

Krummheuer (1995) described justification as reasoned and logical argumentation which consists of a combination of conjectures or claims, and one or more pieces of supporting

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evidence. Included in the claims are supporting evidence which might be comprised of analogies, models, examples and possibly beliefs and a conclusion. Krummheuer drew on the theories of Toulmin (1958) to examine the reasoning process of Grade Two students' analysis and verification of their own and their peers' reasoning. He presented schematically how two students together constructed reasoned argumentation. The initial statement (data) provided the foundation for an assertion (conclusion). A further statement in the form of an inference (warrant) was made which verified reliability of the assertion. Further warrants (backing) were then provided. These strengthened the conclusion and provided specific evidence which illustrated how the data led to the specific claim. Cobb (2002) also analysed a middle school statistics class drawing on Toulmin's work. He illustrated that it was the teacher's expectation for additional warrants and backing that led to the use of conceptual rather than calculational discourse.

Many studies have shown that explanatory justification is constructed and reconstructed when teachers have pressed their students to take specific positions to make reasoned claims (e.g., Enyedy, 2003; Manouchehri & St. John, 2006; Martin et al., 2005; Saxe, 2002; Sherin, 2002a; Wood et al., 1993). The previous chapter discussed the importance of teacher use of position statements and problems which engage students in disciplinary specific inquiry. O'Connor (2002) outlined how a teacher used a position driven statement related to rational numbers with her 5th Grade class. The interactional strategies the teacher used supported individual students to make reasoned claims. O'Connor described the discursive conversational turns the dialogue took. Positions would be maintained or changed from turn to turn as the students rethought their claims in light of counterexamples. Thus, the teacher created an intellectual climate in which the students had space to evaluate all perspectives used in the arguments.

Constructing classroom cultures which provide participants in the dialogue with space for extended thinking is important for the development of justified claims. Explanations are extended to justification when the students are required to suspend judgment to examine the perspectives taken by others and construct supporting evidence for either agreement or disagreement. For example, Lampert (1990b) in her classroom studies outlined how she

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used strategy solutions as "the site of mathematical argument" (p. 40). The strategies were considered hypotheses and used to facilitate student questioning which took the "form of a logical refutation rather than judgment" (p. 40). Kazemi and Stipek (2001) also illustrated how a 5th Grade teacher required that students suspend judgment (agreement or disagreement with solutions) until they had constructed justification for their stance.

The following vignette taken from Sherin et al. (2004) illustrates the teacher's orchestration of open dialogue which required the middle school students to use extended thinking time to evaluate the positions taken in a claim. The students were asked to discuss various mathematical explanations for six graphs. Each graph represented a different way a flag might be hoisted to the top of the flagpole. The teacher engaged the students in each others' perspectives and pressed them to validate their positions. He withheld his own evaluative judgement but required that they validate either agreement or disagreement. In the extended discourse, he allowed the students to control the flow but also slowed it to provide close examination of claims. The provision of time enabled the students to not only resolve their differing views but also to work from confusion to sense-making.

Justifying agreement or disagreement	
Joey	I agree.
Mr Louis	Why?
Joey	Because it shows like that he, he waited a while, then it went straight up and it didn't take time.
Mr Louis	Ben?
Ben	Well I agree because it would be really difficult to do that because he's really small like me and so probably the only way he could do it would have to be like stay on the top of the flagpole and then jump down like holding the rope and…
Sam	It's possible if you have a real long flag.
Lisa	It still takes time!
Mr Louis	That's an interesting point. Are you guys with this conversation? Did you hear what Sam said? Sam, do you want to make that point again?
Sam	If you have a real long flag and it's the length of the pole, it's in both places at one time.
Mr Louis	What do people think about that idea? Can someone rephrase what Sam said in a different way that might help clarify it for people?
From Sherin et al. (2004)	

In the above vignette, the teacher's request for Sam to revoice the claim in an alternative form indicated that what was considered acceptable justification for a claim had still not been established. This illustrated the role teachers have in scaffolding what is accepted in the community as valid mathematical justification.

Specific classroom social norms may also demand that mathematical explanations be extended to justification. These include norms which place value on the individual's right to disagree until convinced. For example, White (2003) described how a teacher in her study of 3rd Grade learners pressed the community towards collective thinking. Through extended discussion she positioned and repositioned the students to accept or reject answers freely and work towards mathematical clarity and consensus. White, like a number of other researchers (e.g., Blunk, 1998; Rittenhouse, 1998; Weingrad, 1998) illustrated the importance of teachers positioning students to engage 'politely' with each others' thinking whether agreeing or disagreeing.

Other studies have illustrated the importance of classroom cultures where the students are confident that they can express their lack of understanding or inquire about their own erroneous thinking (e.g., Kazemi. 1998; Simon & Blume, 1996; Whitenack & Knipping, 2002; Yackel, 2001). Situations which challenge reasoning are often caused by the introduction of higher levels, or new ways of thinking. To gain agreement requires extended and open discussion and debate. For example, Whitenack and Knipping illustrated how Grade 2 students accommodated the reasoning provided by various participants. Introduction of notation by one student shifted the reasoning to a more advanced level which challenged many participants' thinking. Consequently, in the extended dialogue which followed, justification was required in many forms and models (concrete material, pictures and number sentences) before the claim gained explanatory relevance for all participants.

Enyedy (2003) showed how 7th Grade students' claims in the context of computer simulated probability problems were structured and explained logically with each assertion building from those previously understood. However, challenge to the validity of one pair

of students' public prediction led to social conflict which resulted in the need for "justification and evidence" (p. 385). Enyedy maintained that the communal nature of the argument "encouraged the students to make their reasoning explicit and public so that it could be challenged, tested and modified" (p. 391). Likewise, Kazemi (1998) showed how the teacher used open dialogue, discussion and debate to reconstruct erroneous reasoning in a Grade 4-5 classroom, when a student explained and notated that $1/2 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8$ equaled both 1 and 1/8 and the teacher engaged the entire class in analysis of the reasoning and scaffolded the development of justification using fraction equivalence.

Teacher construction of norms which promote an expectation of mathematical sense-making and development of a collective view have been described as important in pressing students towards constructing justifications (Mercer, 2000; Rojas-Drummond & Zapata, 2004; Simon & Blume, 1996; Yackel, 2001). For example, Inagaki, Hatano and Morita (1998) provided persuasive evidence of how the press toward a shared perspective supports development of justification. In their study of 11 Grade 4-5 classes the students were asked to develop collective consensus during a session of whole class argumentation in each classroom. Individual students were asked to select and provide persuasive mathematical justification for the most appropriate solution strategy for addition of fractions with different denominators. In their findings the researchers outlined how different students considered and reconsidered their choices. They also incorporated other students' utterances into their subsequent justification for their own change in choice of solution strategy. Following each classroom session specific students were interviewed including some who had remained silent throughout the extended discussion. All students were able to nominate individual students who were proponents of particularly plausible arguments. They also provided clear reasons as to why these students' justification had either convinced them or caused them to reconsider their own selected solution strategy. Inagaki and his colleagues suggested that the silent participants had selected 'agents' who 'spoke' for them and even when they were unable to align their view with an 'agent' they responded to "proponents' and opponents' arguments in their mind" (p. 523).

The study of Inagaki et al. (1998) illustrated that in the act of developing a collective view all participants were required to take into consideration each other's perspective. In identifying with all participants' perspectives, the listeners and interlocutors were bound in communality in the views they held. Not only were they required to accommodate the opinions of others they were also required to accommodate their own views. Their personal perspective already coloured by the dialogic overtones of others, acted as a thinking tool which they used to accommodate, or negotiate alternative views, on their path to construction of collective reasoning.

4.4   MATHEMATICAL GENERALISATIONS

The previous sections have illustrated that teacher actions which scaffolded student participation in reasoned dialogue were of significance in promoting a shift from mathematical explanation to justification. The need to validate claims, in turn, scaffolds the development of generalised models of mathematical reasoning.

Inherent in generalised models are the processes, procedures, patterns, structures and relationships. New cases are not created; rather existing concepts are further developed and extended through reflective and evaluative reasoning (Lobato, Ellis, & Muñoz, 2003; Mitchelmore, 1999; Mitchelmore & White, 2000; Romberg & Kaput, 1999; Skemp, 1986). For example, Enyedy (2003) illustrated how a teacher used computer simulations with 7th Grade students to construct probabilistic reasoning. In the search for reasoned justification the students shifted from "local *models of* a situation" (p. 375) to "*models for*" (p. 375) the generalised situation. Similarly, Gravemeijer's (1999) Grade One classroom study showed how a ruler was the initial model. In the discursive dialogue the teacher scaffolded the ruler's gradual transformation "from measuring to reasoning about the results of the measuring" (p. 168). Gravemeijer explained that in using the ruler as a model for "reasoning about flexible mental-arithmetic strategies for numbers up to 100" (p. 168) the teacher enabled the students to generalise from a model of the numbers as referents, to a model for numbers as mathematical entities in their own right.

Skemp (1986) explained that engaging in generalising is a

> sophisticated and powerful activity. Sophisticated, because it involves reflecting on the *form* of the method while temporarily ignoring its content. Powerful, because it makes possible conscious, controlled and accurate reconstruction of one's existing schema—not only in response to the demands for assimilation of new situations as they are encountered *but ahead of these demands*, seeking or creating new examples to fit the enlarged concept. (p. 58)

Teachers are required to support their students into assuming a 'mindful' approach to recognising patterns, combining processes, and making connections at an elevated level of awareness (Fuchs, Fuchs, Hamlett, & Appleton, 2002). The RAND Mathematics Study Panel (2003) describe this as using "intellectual tools and mental habits" (p. 38) to pattern seek and confirm. Generalisations evolve through cycles of reflective pattern finding as theories are publicly proposed, tested, evaluated, justified, and revised (Carpenter et al., 2004b). Although the forerunner to generalisations may have begun as an intuitive leap (as many generalisations do) they have then undergone critical reflective analysis to verify their validity (Skemp, 1986).

The following vignette from Lanin (2005) in a 6th Grade classroom offers an example of how a teacher used patterning tasks to scaffold the students to construct a numeric generalised scheme. In the social milieu of the classroom the teacher provided time and space for the students to construct and reconstruct their thinking as they searched for and reflectively tested patterns. In the class discussion the teacher facilitated wide ranging discussion of possibilities. This made each contribution a platform for other participants to build from. As a result, the community resolved the problem situation using a well reasoned mathematical generalisation.

Constructing and justifying a generalisation

The students had been given the following problem to solve using a computer spreadsheet: *The Jog Phone Company is currently offering a calling plan that charges 10 cents per minute for the first 5 minutes for any phone call. Any additional minutes cost only 6 cents*

per minute. The students were required to determine a general rule for phone calls that were 5-minutes or longer. Dirk made links with previous problems and focused on number sequences generated in the problem situation and using a rate-adjust strategy derived the rule $C = 6n + 20$ (C is the cost of the phone call in cents and n is the number of minutes). When questioned he explained:

Dirk	I did 6 because it [costs 6 cents] for 6 minutes after and then, like the 20 was the number that I needed to get to 56.
Questioner	Oh, I see so you adjusted. So when you took 6 times 6 you got 36 and then you said, "it needs to be 56 cents", so you add 20 back on. Did you assume it was right or did you try some other numbers?
Dirk	No, I tried other numbers like 7 and if it went up 6 then I was right, and if it didn't then it was wrong.

In the following whole class discussion a range of generalisations were presented which linked back to the context of the situation through the use of generic examples. Dirk presented the final generalisation further describing his rate-adjust strategy:

| Dirk | Well I just did times 6 cents because it's 6 cents every minute and when I put in 6 it said 6 times 6 equals 36 and then I just added 20 to get 56 because I knew 50 plus 6 was 56. And then I wanted to make sure that it didn't work for only 6. So I did 7 and it went up 6, and it's supposed to do that and then I did 8 and it went up 6 and it's supposed to do that. |

Questioned further Dirk could not relate how his rule related to the context of the phone cost problem. However another student provided further justification for the generalisation.

| Teacher | Where is that plus 20 coming from? |
| Student | Because on the first 5 minutes he added 6 instead of 10 and there are 4 cents left over [for each minute] and then you need to add 4 for every minute and 4 times 5 for 5 minutes, 4 extra cents is 20. And that's where the 20 extra cents comes from. |

This student had illustrated conceptual understanding of the relationship between adding 6 repeatedly and multiplying by 6.
From Lanin (2005)

A number of international studies have reported the difficulties many students encounter when asked to construct and justify generalisations (e.g., Falkner, Levi & Carpenter, 1999; Smith & Phillips, 2000; Warren & Cooper, 2003). For example, Saenz-Ludlow and Walgamuth (1998) and Carpenter, Levi, Berman and Pligge (2005) report difficulties which include a lack of knowledge of the meaning of operations, relationships between operations, the equivalence relationship and equals sign. Anthony and Walshaw (2002) in a New Zealand study described similar findings. These researchers outlined the difficulties

the groups of Year 4 and 8 students in their study had when asked to produce generalised statements related to number properties and relationships. Although many of the students drew on the basic properties of addition most did not exhibit explicit awareness of commutativity; neither its mathematical structure, nor its mathematical properties. These researchers argued the need for teachers to provide students with opportunities to make "explicit their understanding of why number properties such as commutativity and identity 'hold good'" (p. 46).

Numerous studies have shown that explicit focusing of student discussion on the relationships between numbers properties and operations resulted in their powerful construction of generalisations (e.g. Blanton & Kaput, 2002, 2003, 2005; Carpenter & Levi, 2000; Carpenter et al., 2005; Kaput & Blanton, 2005; Schifter, 1999; Vance, 1998). For example, Carpenter and Levi (2000) illustrated how a Grade 2 teacher directed student attention toward number properties and operations including the properties of zero. As a result they constructed a number of robust generalisations. Blanton and Kaput (2005) also outlined how a 3rd Grade teacher explicitly integrated algebraic reasoning into her classroom practice. As a result the students

> generalized about sums and products of even and odd numbers; generalized about properties such as the result of subtracting a number from itself, expressed as the formulization $a - a = 0$; decomposed whole numbers into possible sums and examined the structure of those sums; and generalised about place value properties (p. 420).

Other studies in early and middle primary classrooms (e.g., Carpenter & Levi, 2000; Carpenter et al., 2003; Carpenter et al., 2005; Falkner et al., 1999; Kaput & Blanton, 2005; Lampert, 1990; 2001; Saenz-Ludlow & Walgamuth, 1998) have illustrated teacher use of mathematical tasks which focus on relational reasoning (e.g., relational statements which use the equals sign). This provoked rich classroom dialogue and provided teachers with insight into students' reasoning.

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In order for students to develop generalised understandings Watson and Mason (2005) promote teacher use of 'example' spaces. In these intellectual spaces, the students are able to suspend judgement as they search for and identify plausible patterns or counterexamples to challenge and disprove their hypothesis. These researchers maintain that the ability to generalise quickly and broadly through a cyclic search for mathematical patterns using conceptual reasoning is a hallmark of what proficient mathematics problem solvers do. But to make this happen students need rich conceptual understanding of the underlying structures and properties of mathematical procedures (Kaput & Blanton, 2005; Skemp, 1986). Some researchers (e.g., Blanton & Kaput, 2003; Watson & Mason, 2005) argue that teachers should transform existing instructional material from problems with single numerical responses, to ones which lead to students providing a range of conjectures. They explain that variations on problem parameters (often by size) decrease chances of the students using simple modeling or computing of answers. Rather, they are nudged in the direction of thinking about the problem in general.

Teacher use of open-ended mathematical problems within a class of problems is promoted in many studies (e.g., Kaput, 1999; Kaput & Blanton, 2005; Krebs, 2003; Lanin, 2005; Lesh & Yoon, 2004; Mitchelmore, 1999; Sadovsky & Sessa, 2005; Smith, 2003; Sriraman, 2004). These studies outlined how use of open-ended problems, not easily solved empirically, drew sustained effort and supported rich dialogue. Krebs (2003) illustrated how 8th Grade students were structured to work within 'example' spaces. She outlined how they persevered with challenging problems, deliberated at length, explored and tested a number of hypotheses before constructing their generalisations. In doing so, they also made connections both between the representations (tables, symbols, graphs) and to previous problems and tasks.

Teacher use of such problems increases opportunities for students to engage in inquiry and argumentation and to develop powerful models of their reasoning (Carpenter & Levi, 2000; Carpenter et al., 2005; Kaput, 1999; Warren & Cooper, 2003). For example, in the following vignette from Blanton and Kaput the teacher increased the numbers beyond the students' immediate arithmetic capacity. This action eliminated their capacity to compute

and so they were pressed to examine the properties of the numbers. As a result, the student showed that she knew that she needed to only consider the last digits of the two numbers to validate the generalisation.

Generalising the properties of odd and even numbers
The teacher asked the students whether the sum of 5 + 7 was even or odd:

Teacher	How did you get that?
Tory	I added 5 and 7, and then I looked over there [indicating a list of even and odd numbers on the wall] and I saw it was even.
Teacher	What about 45,678 plus 856,317, odd or even?
Mary	Odd.
Teacher	Why?
Mary	Because 8 and 7 is even and odd, and even and odd is odd.
From Blanton and Kaput (2003)	

Also illustrated in the vignette was how the teacher's use of the question 'why' supported further explanation of the generalisation.

Blanton and Kaput (2003) maintain that teachers should use specific questions like "Tell me what you were thinking? Did you solve this a different way? How do you know this is true? Does this always work?" to focus student attention and draw justification about the properties and relationships of numbers (p. 72). For example, Strom et al. (2001) revealed the importance of teacher questioning. Within a Grade Two classroom episode of dialogic talk the students' everyday measurement and area experiences were progressively mathematised as the teacher pressed the students toward generalisation and certainty. The teacher's use of questions like 'why'; 'does it work for all cases'; 'can you know for sure' nudged the students to search for patterns and to consider underlying generalised mathematical structures. Similarly, Zack (1997, 1999) in an elementary class showed that challenge through teacher and student questioning acted as a catalyst which promoted a search for patterns and need for generalised reasoning. In the student initiated dialogue, patterns of conjecture and refutation were constructed as the participants worked towards collective agreement. The logical connectives (because, but, if…then) used in response to the specific questions supported the construction of generalised explanations.

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4.5 MATHEMATICAL REPRESENTATIONS AND INSCRIPTIONS

Mathematical representations and inscriptions are important social tools which are used to mediate individual and collective reasoning within classroom communities (Carpenter & Lehrer, 1999; Cobb, 2002; Forman & Ansell, 2002; Greeno, 2006; Saxe, 2002). In the current study the term inscriptions is used to include a range of representational forms which proficient users of mathematics employ and understand. These include symbolic and invented notation and other "signs that are materially embodied in some medium...such as graphs, tables, lists, photographs, diagrams, spreadsheets and equations" (Roth & McGinn, 1998, p. 37).

Concrete material and problem situations grounded in informal and real world contexts potentially provide a starting point to develop multiple forms of representation (Smith, 2003). These serve as reference points which are then mathematised—progressed toward abstraction and generalisation. For example, McClain and Cobb (1998) in a Grade One class described an instructional sequence which began as "experientially real" (p. 61). This initial context served as a means to both "fold back" (p. 59) and "drop back" (p. 67) when needed, to maintain conceptual understanding. They described how the students abstracted and generalised a form of quantitative reasoning which they represented symbolically on a number line.

Heuser (2005) depicted a study in which 1st and 2nd Grade students developed fluid use of symbolic representation for computation procedures. Heuser described how hands-on-activity embedded in problem situations laid the foundations. Reflective discussion advanced the students from modeling problems through drawings to development of numerical procedures to explain their invented strategies. Heuser outlined how many students generalised their invented numerical computational strategies to solve other more complex and non-routine problems. Many of the invented notation schemes closely resembled more standard procedures but the students retained a rich sense of place value.

When students are engaging in inquiry and argumentation effective teachers draw on the different representational forms individuals use to make the reasoning public and accessible for community exploration and use (Sawyer, 2006a). Likewise, the obligation to make available multiple explanations of reasoning influences how students learn and use ways of representing or inscribing their current reasoning (Cobb, 2002; Lehrer & Schauble, 2005; McClain, 2002; Roth & McGinn, 1998; Saxe, 2002). For example, Whitenack and Knipping (2002) illustrated the interdependent relationship of inquiry and challenge that led to more advanced models for conceptual reasoning being developed. The 2nd Grade students devised invented notational inscriptions for two digit addition and subtraction. However, the need for further clarification and justification led to the claims being recast flexibly as pictures and invented symbolic schemes. These resulted in generalised symbolic schemes which served as explanatory tools which clarified, explained and interpreted and substantiated the claims.

Strom et al. (2001) outlined how discursive dialogue was used to transform students' informal knowledge depicted as invented representations into notation schemes which included formal algorithms. Teacher actions were integral to ways in which the students' representational models supported explanation, generalisation, and development of collective certainty. The teacher press led to inscriptions being used to flexibly "re-present" (Smith, 2003, p. 263), re-explain and re-construct the concepts embodied in the models. In the extended discussion the students learnt ways to examine, explore, and debate the notation schemes and assess their adequacy. Thus learning to make use of, and comprehend, representational forms was achieved through social interactive activity (Greeno, 2006). The representational models had also been used as reflective tools which facilitated student thinking about their learning and the learning process (Sawyer, 2006a).

4.6   USING MATHEMATICAL LANGUAGE AND DEFINITIONS

Negotiating mathematical meaning is dependent on students' access to a mathematical discourse and register appropriate to the classroom community. Students who display mathematical literacy are able to use the language of mathematics to maintain meaning

within the context of its construction, in its form or mode of argumentation and matched to audience needs (Moschkovich, 2002b; Thompson & Rubenstein, 2000). Forman (1996) proposes that these include the differing use of mathematical terms and ways of defining these terms. Included also are use of a particular syntax, use of brevity and precision to present oral and representational forms of the discourse or to respond to challenge (Mercer, 1995).

Gaining fluency and accuracy in mathematical talk requires a shift from an informal use of terms and concepts to a more narrow and precise register (Meaney & Irwin, 2003). Meaney and Irwin maintain that "if students are not encouraged to use mathematical language (both terms and grammatical constructions) then eventually their mathematical learning will be restricted" (p. 1). Drawing on a study of Year 4 and 8 students in New Zealand these researchers showed the students more readily supplied answers which conformed to a social not mathematical register. Likewise, in Australia Warren (2003) documented 87 eight year olds' responses to explaining and connecting meanings for words commonly used in addition and subtraction. Warren noted that students neither comprehended the many nuances of mathematical language nor made connections with how words overlapped in meaning. In a 2005 study Warren (2005) reported results of a longitudinal study of 76 Year 3-5 students. The study investigated the development of student understanding of words commonly linked to equivalent and non-equivalent situations. She indicated that student construction of meaning for terms like 'more' or 'less' remained tied to their meanings in arithmetic operations and 'equal' denoted the answer. Warren illustrated that the narrow use of the concepts was established by Year 3 and remained firmly embedded in the Year 5 students' thinking.

Different social groups and communities make use of varying discourse and communication patterns specific to their situated context (Gee, 1999; Moschkovich, 2003; Nasir, Rosebery, Warren, & Lee, 2006). For example, Irwin and Woodward (2005), in a New Zealand study, noted the prevalence of "colloquial terms and conversational conventions" (p. 73). The teacher promoted mathematical explanations of reasoning in the predominantly Pasifika Year 5/6 discussion intensive Numeracy classroom. However, the

researchers concluded that the lack of emphasis on "the use of the mathematics register, both the terms and the discourse of premise and consequence" (p. 73) affected how the students talked together mathematically. Latu (2005), in another New Zealand study, showed the difficulties encountered by a group of secondary Pasifika students. The group used mathematical terms but these were restricted to exact contexts in which they were learnt. Relational statements in word problems were least understood. However, Latu demonstrated that those Pasifika students able to code switch between a first language and the language of mathematics (in English), performed better than those who had only restricted forms of English as their first language.

Other studies have illustrated that diverse students are able to gain fluency in mathematical discourse when teachers focus specifically on the rich use of mathematical language and terms. For example, Khisty and Chval (2002) outlined how a teacher engineered her 5th Grade Latino student's learning environment. Her engagement of students in lengthy periods of collaborative problem solving included an expectation that they explain and justify their mathematical reasoning. She explicitly modeled the use of mathematically rich language and complete mathematical statements. As a result, the students were inducted into fluent use of the specialised discourse of mathematics.

Moschkovich (1999) used a classroom episode in a 3rd grade classroom of English second language learners. The teacher "did not focus primarily on vocabulary development but instead on mathematical content and arguments as he interpreted, clarified and rephrased what students were saying" (p. 18). The teacher carefully listened to the students, probing and revoicing what they said to maintain focus on the mathematical content of their contributions. As a result, they gradually appropriated both the use of the mathematical register and knowledge of how to participate in mathematical dialogue.

4.7   SUMMARY

This literature review of mathematical practices used by primary aged students illustrated a range of mathematical practices and described how each evolved and was transformed

within the social and cultural context of classrooms. Discursive interaction and its role in the development of communal sense-making wove across the mathematical practices. In the discursive dialogue, explaining mathematical reasoning was extended to justifying it. In turn, the need for multiple forms to validate the reasoning led to generalising. Representing or inscribing reasoning and using mathematical language appropriately were also embedded in the discursive activity of collective reasoning. Learning and doing mathematics was outlined as an integrative social process. Each mathematical practice engaged in, within classroom communities is intrinsically interconnected and related reciprocally to the other mathematical practices.

Gaps in the literature correlated with those identified by the RAND Mathematics Study Panel (2003). This group of researchers noted the paucity of literature which outlined how mathematical practices, as a collection of connected and well-coordinated reasoning acts, were used skilfully and flexibly by expert users of mathematics. There also appeared to be a scarcity of New Zealand literature which explicitly related to mathematical practices or understanding how these developed within classroom communities. The current study moves away from a focus on single mathematical practices to consider how mathematical practices are used as integrated practices. How the teacher structures the environment which provides students with cognitive space to participate in the mathematical practices will be examined.

The following chapter describes the methodology of design research used in the current study. Design research falls naturally from the sociocultural frame taken in the current study.

CHAPTER 5

METHODOLOGY

> What teachers do is strongly influenced by what they see in given teaching and learning situations. For example, as teachers develop, they tend to notice new things...these new observations often create new needs and opportunities that, in turn, require teachers to develop further. Thus the teaching and learning situations that teachers encounter are not given in nature: they are, in large part, created by teachers themselves based on their current conceptions of mathematics, teaching, learning, and problem solving. (Lesh, 2002, p. 36)

5.1   INTRODUCTION

This study aims to investigate how teachers develop mathematical inquiry communities in which the students participate in proficient mathematical practices. A classroom-based research approach—design research—was selected. This approach followed naturally from the theoretical stance taken by the researcher and used in this study.

Section 5.2 states the research question. In section 5.3 an interpretive qualitative research paradigm is examined. Illustrations are provided to show how a qualitative paradigm supports the aims of this research. Section 5.4 provides an explanation of design research as it is used in this study. Explanation is given of how design research is congruent with collaborative partnerships between teachers and researchers within the situated context of schools and classrooms. How the framework of communicative and participatory actions was designed is outlined. In Section 5.5 ethical considerations are discussed in relation to school based collaborative research.

Section 5.6 outlines how participation by the school and teachers in the design research was established. Section 5.7 describes data collection in the classrooms and the process of data analysis. The different research methods used in the study are outlined and an explanation is given of how these were used to maintain the voice of the teachers and capture their perspectives of the developing discourse and mathematical practices. Section 5.8 describes

the approach taken to analyse the data. Finally, Section 5.9 outlines how the findings are presented.

5.2   RESEARCH QUESTION

The study addresses one key question:

How do teachers develop a mathematical community of inquiry that supports student use of effective mathematical practices?

The focus of exploration is on the different pathways teachers take as they develop mathematical communities of inquiry; their different pedagogical actions, and the effect these have on development of a mathematical discourse community.

Implicit in the question is how changes in the mathematical discourse reveal themselves as changes in how teachers and students participate in and use mathematical practices. That is, the research also seeks to understand how the changes in participation and communication patterns in a mathematical classroom support how students learn and use proficient mathematical practices.

5.3   THE QUALITATIVE RESEARCH PARADIGM

This research is guided by a qualitative interpretive research paradigm (Bassey, 1995) and draws on a sociocultural perspective. The interpretive and sociocultural approaches adopted in this study are grounded in "situated activity that locates the observer in the world" (Denzin & Lincoln, 2003, p. 3). They share the notion that reality is a social construct and that a 'more-or-less' agreed interpretation of lived experiences needs to be understood from the view of the observed (Merriam, 1998; Scott & Usher, 1999). Recognition is given to descriptions based within social meanings which are always subject to change during social interactions (Bassey, 1995). Such a view enables one to report on the 'lived' experiences of the teachers and their students, and use their 'voice' to interpret the multiple realities of the construction of mathematical inquiry communities and their complementary mathematical practices.

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Qualitative researchers acknowledge that the 'distance' between the researcher and researched is minimised as each individual interacts and shapes the other (Creswell, 1998; Denzin & Lincoln, 2003). A report of mathematical practices in inquiry communities needs to be understood as an interpretation of the researcher and the teachers involved in the study. A collaborative and participatory approach was appropriate given that the purpose was "to describe and interpret the phenomena of the world in attempts to get shared meanings…deep perspectives on particular events and for theoretical insights" (Bassey, 1995, p. 14). Moreover, the qualitative design research methods provided "a formal, clear and structured place for the expertise of teachers to be incorporated within the production of artefacts and interventions designed for use in the classroom" (Gorard, Roberts, & Taylor, 2004, p. 580).

5.4   DESIGN RESEARCH

Design research has been gaining momentum as a classroom-based research approach which supports the creation and extension of understandings about developing, enacting, and maintaining innovative learning environments (Design Based Research Collective (DBRC), 2003). Design research is commonly associated with the important work of Brown (1992) and Collins (1992) who situated their research and design within the real-life, messy, complex situations of classrooms. However, the foundations for it span the past century. It has its roots within social constructivist and sociocultural dimensions and the theories of Piaget, Vygotsky, and Dewey who shared a common view of classroom life "not as deterministic, but as complex and conditional" (Confrey, 2006, p. 139).

Design research provides a useful and flexible approach to examine and explore innovations in teaching and learning in the naturalistic context of real world settings (Barab, 2006; Collins, Joseph, & Bielaczyc, 2004). Its methods emphasise the design and investigation of an entire range of "innovations: artefacts as well as less concrete aspects such as activity structures, institutions, scaffolds and curricula" (DBRC, 2003, p. 6). Moreover, these methods offer ways to create learning conditions which current theory

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promotes as productive but which may not be commonly practised nor completely understood. This has relevance for this study which sought to explore how teachers develop mathematical inquiry communities which support student use of proficient mathematical practices.

Design research is an approach which links theoretical research and educational practice. There are many varied forms and terminology for design research accepted within education, each with its own different goals, methods, and measures (Kelly, 2006). Variants include classroom teaching experiment—multi-tiered and transformative; one-to-one teaching experiment; teacher and pre-service teacher development experiment; and school and district restructuring experiments. Irrespective of the form design research is described as

> *experimental,* but not an experiment. It is hypothesis *generating* and *cultivating*, rather than *testing*; it is motivated by emerging conjectures. It involves blueprinting, creation, intervention, trouble-shooting, patching, repair; reflection, retrospection, reformulation, and reintervention. (Kelly, p. 114)

Within design research the "central goals of designing learning environments and developing theories or 'prototheories' of learning are intertwined" (DBRC, 2003, p. 5). This sits easily with the current study given that a key aim is to explore and broaden knowledge of how teachers enact participation and communication patterns in mathematical communities that support student use of efficient mathematical practices. Although this research study was conducted in a single setting the intent is to add to broader theory through offering insights about the process. This is attempted through the provision of rich accounts of the pedagogical actions teachers take to develop inquiry communities and the corresponding mathematical practices the participants engage in. Barab (2006) maintains that through providing "methodological precision and rich accounts…others can judge the value of the contribution, as well as make connections to their own contexts of innovation" (p. 154).

A central feature of design research is that it leads "to shareable theories that help communicate relevant implications to practitioners and other educational developers"

(DBRC, 2003, p. 5). Of importance to the current study is how design research is able to "radically increase the relevance of research to practice often by involving practitioners in the identification and formulation of the problems to be addressed, and in the interpretation of results" (Lesh, 2002, p. 30). The collaborative and ongoing partnership between the teachers and myself as a researcher was a significant feature of this study. Engaging in partnership with the teachers as a study group and with each individual teacher in his or her classroom setting, supported the wider evaluation, reflection and re-defining of specific aspects of the intervention. It also required the use of methods commonly used in design research to develop accounts that documented and connected the processes of enactment to the outcomes under investigation.

An important aspect of the design research approach is the account provided of how the designed intervention interacts with the complexities within authentic educational settings (Cobb et al., 2003; Confrey, 2006). Due to the many dependent variables within the educational setting many regard design research as messier than other forms of research. Gorard and his colleagues (2004) justify the messiness of design research explaining that it evolves through its characterisation of contexts. It does this by revising the procedures at will, which "allow[s] participants to interact, develop profiles rather than hypotheses, involve users and practitioners in the design, and generate copious amounts of data of various sorts" (p. 580).

5.4.1 TESTING CONJECTURES: COMMUNICATION AND PARTICIPATION FRAMEWORK

Design research is an interventionist and iterative approach which uses on-going monitoring both in the field (and out of the field) of the success or failure of a designed artefact to gain immediate (and accumulating) feedback of the viability of the theory it is grounded in (Collins, 1992; Kelly, 2006). A collaborative relationship between the teachers and researcher and the trialling of artefacts in the field within iterative cycles has similarities to the approach used in action research. An important difference is that it is the researcher who introduces the problem and takes a central position in the collaborative

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design, trialling, systematic documentation, and the reflection and evaluation of an intervention artefact as it is implemented within an authentic context. In the current study, the designed artefact took the form of a communication and participation framework (see Table 2, Section 5.4.2). The framework outlines a conjectured set of communicative actions (verbal and performative) and a conjectured set of participatory actions that progressively constitute effective mathematical practices in an inquiry classroom. The intent was that the framework of communicative and participatory actions would guide teachers as they scaffolded students to engage in collective reasoning activity within communities of mathematical inquiry.

According to Confrey and Lachance (2000) conjectures are inferences "based on inconclusive or incomplete evidence" (p. 234-5) which provide ways to reconceptualise either mathematical content or pedagogy. In the current study, these conjectures related to possible communication and participation patterns, which if teachers pressed their students to use, could result in them learning and using proficient mathematical practices. Steffe and Thompson (2000) maintain that generating and testing conjectures "on the fly" (p. 277) is an important element of design methodology. The process of designing tools such as frameworks or trajectories and then subjecting them to on-going critical appraisal is a crosscutting feature of design research. Cobb and his colleagues (2003) explain that design research "creates the conditions for developing theories yet must place these theories in harm's way" (p. 10) in order to extend them and benefit from other potential pathways which may emerge as the research unfolds.

The beginning point for the design of the communication and participation framework used in the current study drew on a theoretical framework proposed by Wood and McNeal (2003). Their two-dimensional hierarchical structure (discussed previously in Chapter Three, Section 3.2.2) illustrated that student responsibility for thinking and participating in collective activity deepened in a shift from conventional to strategy reporting discussion contexts, deepening again in the further shift to inquiry and argument contexts. Their descriptions of the communicative and participatory actions teachers prompt students to use in each discussion context, which link to different levels of student engagement in

collective reasoning practices, provided an initial basis for the communication and participation framework. It was then extended through the use of many studies previously discussed in Chapter Three and Four. In these studies, the specific communication and participation patterns teachers scaffolded their students to use, which gradually inducted them from peripheral to more central positions in the collective discourse, were identified. These were added to the communication and participation framework as conjectures of possible actions teachers could scaffold students to use, to provide them with opportunities to learn and use mathematical practices within inquiry cultures.

The structure of the communication and participation framework was comprised of two separate components—communication patterns and participation patterns. Vertically, the framework outlined a set of collective reasoning practices matched with conjectures relating to the communicative and performative actions teachers might require of their students, to scaffold their participation in learning and using mathematical practices. Likewise, conjectures of a set of participatory actions teachers may expect of their students to promote their individual and collaborative responsibility in the collective activity were added. The horizontal flow over three phases sketched out a possible sequence of communicative (and performative) actions, and participatory actions, teachers could scaffold their students to use, to gradually deepen their learning and use of proficient mathematical practices within reasoned inquiry and argumentation. The design of the communication and participation framework recognised the possibility that the participating teachers might be novices within inquiry environments and not have had many opportunities to engage in collective reasoned discourse. Therefore, clear and descriptive details of communicative and participatory actions teachers could press students to use were provided.

The communication and participation framework was designed and used by the teachers as a flexible and adaptive tool to map out and reflectively evaluate their trajectory or pathway. They drew on the communicative and participatory actions outlined in the framework, to plan individual pathways of pedagogical actions to use, to guide the development of collective reasoned discourse in inquiry classroom communities. Although the teachers

reached similar endpoints their individual pathway they mapped out using the framework was unique. The flexible design of the framework responsively accommodated each teacher's need for less or more time to unpack and manage the changing participation and communication patterns requirements at different points. While the framework was sufficient in itself as a guide some teachers required clarification and further elaboration and more detailed explanation of the communication and participation actions they might press the students to use (see Section 7.3.2 for how the framework was used as a tool to support specific teacher needs and Appendix K for a more detailed explanation and expansion of one section of it).

An additional tool which was developed in conjunction with the communication and participation framework during the current study took the form of a framework of questions and prompts (see Appendix E). The framework of questions and prompts was designed in collaboration with the teachers in the study group setting and complemented the communication and participation framework. The need for and use of this tool emerged and evolved through collaborative reflective examination of classroom communication and participation patterns and the communicative and participatory actions outlined in the communication and participation framework. The basis for the framework of questions and prompts also drew on the important work of Wood and McNeal (2003). It evolved and was extended during the current study as collaborative observations were made of how the use of specific questions and prompts influenced how the students participated in communicating their mathematical reasoning. Conjectures were made that teacher modeling of the questions and prompts as well as a press on students to use them would deepen student agency in the mathematical discourse.

Further discussion of how the framework of questions and prompts developed collaboratively in the study group setting, how it was used in classrooms, and how it was used as an additional evaluative tool by the teachers is provided in this chapter (see Section 5.6.5). The communication and participation framework used by the teachers is presented on the following page.

Table 2

The communication and participation framework: An outline of the communicative and participatory actions teachers facilitate students to engage in to scaffold the use of reasoned collective discourse.

		Phase One	Phase Two	Phase Three
Communicative actions	Making conceptual explanations	Use problem context to make explanation experientially real	Provide alternative ways to explain solution strategies.	Revise, extend, or elaborate on sections of explanations.
	Making explanatory justification	Indicate agreement or disagreement with an explanation.	Provide mathematical reasons for agreeing or disagreeing with solution strategy. Justify using other explanations.	Validate reasoning using own means. Resolve disagreement by discussing viability of various solution strategies.
	Making generalisations	Look for patterns and connections. Compare and contrast own reasoning with that used by others.	Make comparisons and explain the differences and similarities between solution strategies. Explain number properties, relationships.	Analyse and make comparisons between explanations that are different, efficient, sophisticated. Provide further examples for number patterns, number relations and number properties.
	Using representations and inscriptions	Discuss and use a range of representations or inscriptions to support an explanation.	Describe inscriptions used, to explain and justify conceptually as actions on quantities, not manipulation of symbols.	Interpret inscriptions used by others and contrast with own. Translate across representations to clarify and justify reasoning.
	Using mathematical language and definitions	Use mathematical words to describe actions.	Use correct mathematical terms. Ask questions to clarify terms and actions.	Use mathematical words to describe actions (strategies). Reword or re-explain mathematical terms and solution strategies. Use other examples to illustrate.

Participatory Actions		Active listening and questioning for more information. Collaborative support and responsibility for reasoning of all group members. Discuss, interpret and reinterpret problems. Agree on the construction of one solution strategy that all members can explain. Indicate need to question during large group sharing. Use questions which clarify specific sections of explanations or gain more information about an explanation.	Prepare a group explanation and justification collaboratively. Prepare ways to re-explain or justify the selected group explanation. Provide support for group members when explaining and justifying to the large group or when responding to questions and challenges. Use wait-time as a think-time before answering or asking questions. Indicate need to question and challenge. Use questions which challenge an explanation mathematically and which draw justification. Ask clarifying questions if representation and inscriptions or mathematical terms are not clear.	Indicate need to question during and after explanations. Ask a range of questions including those which draw justification and generalised models of problem situations, number patterns and properties. Work together collaboratively in small groups examining and exploring all group members reasoning. Compare and contrast and select most proficient (that all members can understand, explain and justify).

5.5 ETHICAL CONSIDERATIONS

The ethics of social research focuses on the need to protect all participants from possibility of harm (Babbie, 2007; Berg, 2007). Of central importance in entering school and classroom communities for research purposes is the need to consider potential harm to teachers and their students. Berg outlines the many difficulties social researchers encounter when engaging with issues of "harm, consent, privacy and confidentiality" (p. 53). Mindful of these difficulties, this section examines and discusses the principles and practices which underwrite this classroom study.

5.5.1 INFORMED CONSENT

Integral to ethical research is the fundamental principle of informed consent (Bogdan & Biklen, 1992). Participants need to be "fully informed about what the research is about and what participation will involve, and that they make the decision to participate without any formal or informal coercion" (Habibis, 2006, p. 62). In this study, it was particularly important that the teachers had complete understanding of the research process, for many reasons. These included the collaborative nature of the research, the extended length of the research project and the extensive time commitment required for discussion and review of pedagogical practices, study group activity and viewing video-captured observations.

The teachers' informed decision to participate was integral to the study. Robinson and Lai (2006) maintain that in school situations the "power differentials for gaining free and informed consent needs to be carefully considered" (p. 68). In this study, individual consent was gained through comprehensive discussions. Anticipation of issues which posed the possibility of harm were carefully considered and sensitively discussed. These included the possibility of embarrassment on being observed and videoed in classrooms and talking about and viewing video-captured observations with me and other study group members. The benefits of participating in the study were explored so that the teachers could better balance risks with possible outcomes. The teachers were also clear about their right to withdraw at anytime. This was particularly important given the length of the design-based research project.

Informed consent is also a key issue when working with children. Important factors include full cognisance of what participation in the research implies for them (Habibis, 2006). In this study, the students first discussed the project in their classrooms and were provided with student and parent information sheets which provided additional information about the research.

5.5.2   ANONYMITY AND CONFIDENTIALITY

Anonymity and confidentiality are key ethical issues (Habibis, 2006). Although all participants in this study were allocated pseudonyms, assuring anonymity remained problematic given that the teachers in the school community were aware who the participants (teachers and students) in the study were. Also the involvement of the school in a larger project[1] placed the research within scrutiny of a wider audience.

Habibis (2006) outlines how confidentiality overlaps with anonymity—both are concerned with maintaining the privacy of respondents.

> However, while anonymity is concerned with the identification of individual respondents, confidentiality is concerned with ensuring that the information they provide cannot be linked to them. Even if the respondents in a study are identified, the principle of confidentiality means that their specific contribution cannot be identified. (p. 67)

In this research, although the participants in the study discussed and viewed selected video excerpts, beyond this immediate group the specific contributions of individuals could not be linked back to individual participants. In addition, interviews and further discussions of lessons occurred on an individual basis so that each person's confidentiality in relation to others in the research was protected.

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[1] This project was one of four nested projects of a larger Teaching Learning Research Initiative study.

5.6   THE RESEARCH SETTING

This research began with my approach to Tumeke School in the final term of the year before the research was planned. I was invited to meet with the senior management team to discuss further the possibilities of the staff participating in the collaborative research project. At a staff meeting to outline the proposed research involvement for the teachers and students I responded to the teachers' many questions and confirmed that it was each teacher's right to choose to participate in the study. Following this exploratory staff meeting the Principal confirmed that in subsequent smaller team meetings the staff had independently confirmed their wish to participate in the research. The Principal explained that the teachers at Tumeke School regarded their involvement as professional development. They considered that their involvement would provide them with opportunities to extend their understanding of numeracy teaching beyond the NDP in which they had all recently participated. The Principal also notified the Board of Trustees and verified their support.

5.6.1   DESCRIPTION OF THE SCHOOL

Tumeke School[2] is a small New Zealand suburban full-primary[3] school situated on the city boundary. The school's decile ranking[4] of 3 reflects the low socioeconomic status of individual families within the community. The school roll fluctuates between 170 and 250 students. Students of New Zealand Maori and Pacific Nations backgrounds comprise 70% of the school roll. The remainder include 21% New Zealand European students and 9% Thai, Indian, and Chinese students. The majority of students from the Pacific Nations and other backgrounds are New Zealand born, bilingual and with English spoken as a second language in their home. The school has a high pattern of transience; approximately 30% of students in each class enter and leave during the school year.

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[2] Tumeke is a code name for the school
[3] A school which includes both primary and intermediate aged students
[4] Each state and integrated school is ranked into deciles, low to high on the basis of an indicator. The indicator used measures the socio-economic level of the school community. The lowest decile ranking is a decile 1.

For organisation and team planning purposes, Tumeke School is organised in three clusters: Junior, Middle, and Senior teams. Altogether, there are nine composite[5] classes, each with students within age bands of two or three years.

5.6.2 THE PARTICIPANTS AND THE BEGINNING OF THE RESEARCH

At the beginning of the school year seven teachers expressed interest in participating in the study (three teachers from the senior school level and four teachers from the middle school level). The teachers were provided with information sheets (see Appendix A) and we met to discuss in more depth the research aims, the nature of the study, and their role as co-researchers. In discussing ethical issues I explained that anonymity was not possible because other teachers at the school knew their identity. Issues of confidentiality were discussed and I outlined the need for "an environment of trust" (Robinson & Lai, 2006, p. 204), especially in relation to expected collaborative critiquing of classroom excerpts. I also reinforced that participation in the research was voluntary and that the teachers could withdraw from the study at anytime.

In this first meeting other issues were discussed and explored. The teachers expressed concern about the time they might be expected to commit to the research. They also discussed how to ensure that the focus of the research on mathematics did not detract from their professional growth in other curriculum areas. They were also concerned about how we could manage the participation of all their students, including those who did not have parental consent. Having another adult in the classroom and taking part in video recorded observations posed concerns for some members of the group. These teachers expressed personal fear about what the researcher and video recorded observations might reveal about both their mathematical understandings and their pedagogical expertise. The outcome of this wide ranging discussion was a shared understanding that the ultimate purpose of the research was a collaborative examination of ways to enhance student engagement in mathematical discourse which supported student achievement of higher levels of mathematical understandings. The openness of the discussion provided the beginnings of a

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[5] Classes comprised of more than one age group of students

joint partnership in the research and ensured that the teachers saw that they were in a position in which they had "power over knowledge" (Babbie, 2006, p. 301).

In the first term of the school year I was invited to present at two staff meetings. In the first meeting I provided an overview of the Numeracy Development Project (Ministry of Education, 2004a) including content focused on the strategy and knowledge stages. In the second staff meeting I facilitated discussion and exploration of the participation and communication patterns of mathematical inquiry classrooms. I also provided the staff with a number of articles which described elements of inquiry classrooms. These research articles[6] were selected with the intention of seeding ideas of what inquiry classrooms and the use of different collective reasoning practices in them might look like. Subsequently, the articles supported on-going discussion among the team and were useful in the eventual implementation of the previously described communication and participation framework (see Table 2).

During Term One I was in Tumeke School two mornings a week. This provided opportunities for the teachers to further discuss the study with me on an individual and informal basis. In response to their requests I worked collaboratively alongside the teachers

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[6] The research articles included:

Fraivillig, J., Murphy, L., & Fuson, K. (1999). Advancing children's mathematical thinking in everyday classrooms. *Journal for Research in Mathematics Education, 30*(2), 148-170.

Huferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81-116.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal, 102*(1), 59-79

Rittenhouse, P. S. (1998). The teacher's role in mathematical conversation: Stepping in and stepping out. In M. Lampert, & M. L. Blunk (Eds.), *Talking Mathematics in School* (pp. 163-189). Cambridge: University Press.

Stein, M. (2001). Mathematical argumentation: Putting umph into classroom discussions. *Mathematics Teaching in the Middle School, 7*(2), 110-113.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education, 30*(2), 171-191.

Wood, T., & McNeal, B. (2003). Complexity in teaching and children's mathematical thinking. *Proceedings of the 2003 Joint Meeting of PME and PMENA* Honolulu, HI: CRDG, College of education, University of Hawaii.

Mercer, N., & Wegerif, R. (1999b). Children's talk and the development of reasoning in the classroom. *British Educational Research Journal, 25*(1), 95-112.

White, D. Y. (2003). Promoting productive mathematical classroom discourse with diverse students. *The Journal of Mathematical Behaviour, 22*(1), 37-53.

during their numeracy lessons. At times the teachers asked me to observe them teaching numeracy lessons and give them feedback. My on-going involvement in the school supported my "access to the site under investigation" (Scott & Usher, 1999, p. 100). My acceptance by the teachers and students as an 'adopted' member of the staff established the foundations for our subsequent collaborative role. As Bogdan and Biklen (2007) explain, "qualitative researchers try to interact with their subjects in a natural, unobtrusive and nonthreatening manner" (p. 39). My presence in the school throughout the first term had established me as an accepted person around the school and in all the classrooms, with staff members and the students.

Towards the end of the first term, I reconfirmed with the seven teachers that they wanted to participate in the research. The teachers distributed information sheets and consent forms (see Appendix B and Appendix D) to their students and the students' parents or caregivers. These were returned to a collection box in the school office.

5.6.3   PARTICIPANTS IN THE STUDY GROUPS

The seven teachers who participated in the study were a diverse group. This group included two Maori teachers, one teacher who was part Cook Islands and part European, two New Zealand European teachers and two Indian teachers and their teaching experience ranged from 3 years to 20 years. Early in the study, three of the teachers withdrew due to medical and family reasons leaving four participants in the study, two of whom are included as detailed case studies in this research report. These two teachers were selected to reveal the distinctly different ways in which they transformed their classrooms into communities of mathematical inquiry. Whilst these two teachers had engaged in similar activities in the study group setting and while they appeared to follow a similar path guided by the communication and participation framework, close examination of their goals and motives, and those of their students, led to important differences in the pedagogical practices the teachers took to construct discourse communities. These two case study teachers and their class composition are described in the following section.

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5.6.4   THE CASE STUDY TEACHERS AND THEIR STUDENTS

Ava described herself as part New Zealand Maori-part New Zealand European. She was in her ninth year of teaching. The students in her Year 4-5 class were ethnically diverse: 47% were New Zealand Maori, 25% were from the Pacific Nations, 24% were New Zealand Europeans and 4% were other groupings. Based on the results from the New Zealand Numeracy Development Project assessment tool (Ministry of Education, 2004c), 24% of the students were achieving at significantly lower numeracy levels than that of comparable students of a similar age grouping in New Zealand schools; 55% were one level below and 21% were working at the level appropriate for their age group. No students were working at a level above the level attained by their age group nationally.

Moana described herself as a New Zealand Maori married to a Cook Islander and socially involved in both cultural groups. She was in her fifth year of teaching. The students in her Year 4-5-6 class were predominantly New Zealand Maori or from the Pacific Nations. 40% of the students were New Zealand Maori, 58% were Pacific Nations and 2 % were of New Zealand European ethnicity. In Moana's class 31% of the students were achieving at significantly low numeracy levels; 59% were one level below and 10% were at the level average for their age group. Like Ava's students, none of the students were achieving at a level above the average for their age group.

5.6.5   STUDY GROUP MEETINGS

Study group meetings with participating teachers continued at regular intervals throughout the duration of the research. In this setting, I had many roles including being a participant, and acting as a resource for the teachers within a collaborative sharing of knowledge environment.

Study group activities included examining research articles (see Appendix E) for descriptions of the communication and participation patterns of inquiry classrooms, looking at how teachers changed the forms of mathematical talk towards that of inquiry and argumentation, and reading about mathematical practices students used in inquiry

environments. A DVD 'Powerful practices in mathematics and science' (Carpenter et al., 2004b) from an international research study stimulated an examination of inquiry practices. The teachers also re-examined the materials and learning activities in the NDP (Ministry of Education, 2004b) and discussed which learning tasks best supported their goals to change the classroom discourse patterns. A joint decision was made that the group would adapt this material for use within the study.

The video classroom records (see section 5.7.4) were used extensively in the study group context. Sections were selected to view and examine and through repeated viewing critical incidents were able to be identified; so too were the antecedents and consequences of these events. During viewing the discussion centred on the communication and participation patterns students used and how these supported student engagement in mathematical practices, the use of the different mathematical practices, and the pedagogical actions associated with the students' engagement in increasingly proficient mathematical practices. Other activities included collaborative review (with a peer or the group) of selections of their classroom video data and transcripts in order to 'tell its story'. As part of this activity the teachers recorded explanations on their matching transcripts (see Appendix F). These included their intentions at the outset of the lesson and what they noticed as the students engaged in mathematical activity. It also included how as teachers they adjusted their actions or facilitated discussion to match what the students were saying and doing as they engaged in mathematical practices.

In the third study group meeting (Phase 1, Term 2, Week 7) the teachers used the video observations to support the design of a framework (see Appendix E) of the questions and prompts to support the development of specific mathematical practices. As described previously the questions and prompts suggested by Wood and McNeal (2003) provided the foundations for the framework. Examination of the teachers' classroom video records supported identification of further questions and prompts. This framework became a useful tool for the teachers to help model the types of questions and prompts they wanted their students to use.

5.7   DATA COLLECTION

This section describes how the design experiment methodology and data collection was used in this research. Table 3 traces the study research programme over the research year from entry to the school to the point of withdrawal from the school and outlines the schedule of observation phases.

Table 3

A time-line of data collection

2003 Term 4	
Oct-Nov.	Initial contact with Tumeke School and meeting with Senior Management. Staff meeting to outline and discuss the proposed research.
2004 Term 1	
Week 2-10	Researcher present in the school two mornings a week working collaboratively with 7 teachers to provide in-class and out-of-class support.
Week 2	Meeting with 7 teachers to discuss the aims of the research. Information sheets and consent forms given to teachers.
Week 4	Staff meeting: A summary of the NZNDP strategy and knowledge stages.
Week 7	Staff meeting: Discussion and exploration of the discourse patterns of inquiry communities. Research articles provided for background reading. Teacher participation reconfirmed. Information sheets and consent forms provided for students and caregivers.
Week 9	Initial study group meeting to negotiate a tentative communication and participation framework.
Week 10	Unstructured interviews with the teachers.
Term 2	
Week 1-4	Data collection in the senior school in Years 6-8.
Week 3	Senior school study group meeting.
Week 5	Middle and senior school study group meeting.
Week 5-10	Data collection in the middle school in Years 3-6.
Week 7	Study group meeting with the case study teachers.
Week 10	Study group meeting with case study teachers. Unstructured interviews.
Term 3	
Week 2-5	Data collection with the case study teachers in Years 3-8.
Week 4	Study group meeting with case study teachers.
Week 7-10	Data collection with the case study teachers in Years 3-8.
Week 7	Study group meeting with case study teachers.
Week 10	Study group meeting with case study teachers.
Term 4	
Week 3-6	Data collection with the case study teachers in Years 3-8.
Week 5	Study group meeting with case study teachers.
Week 7	Unstructured interviews with case study teachers.

5.7.1 DATA COLLECTION IN THE CLASSROOMS

Data collection began at the start of the second school term in the senior level of the school. In the first 4 weeks two to three video observations of hour-long mathematics lessons were made in each senior classroom. In week 5 of the school term data collection began in three middle school classrooms and continued for the final five weeks of the school term. In the third term of the school year data collection occurred in the classrooms of the four case studies during weeks 2-5 and weeks 7-10. In the final term data collection began in week 3 and was completed in week 6.

5.7.2 PARTICIPANT OBSERVATION

In any classroom study observations occur on a continuum. They may consist of less structured descriptive data which attends to minute detail or they may be tightly structured to focus on specific and selected events or persons. The degree to which the researcher participates in observations is also on a continuum with participation at its highest level when observations are least structured and the researcher is a participant observer (Creswell, 1998).

A sociocultural perspective in which the "world is understood as consisting of individuals and collections of individuals interacting with each other and negotiating meanings in the course of their daily activities" (Scott & Usher, 1999, p. 99) fits well with participant observation techniques. Participant observation is grounded in establishing "considerable rapport between the researcher and the host community and requiring the long term immersion of the researcher in the everyday life of that community" (Angrosino, 2006, p. 732). Developing a balanced relationship between all participants was an essential feature of the current study and so I took the role of participant observer, shifting back and forth from total immersion in the learning context to complete detachment. This role is consistent with that taken by many researchers who have collaborated with teachers in design-based research (e.g., McClain & Cobb, 2001; Whitenack & Knipping, 2002).

5.7.3 VIDEO-RECORDED OBSERVATIONS

The use of video-recording as a tool which flexibly captures and then supports study of the moment-to-moment unfolding of complex classroom interactions has made it a "powerful and widespread tool in the mathematics education research community" (Powell, Francisco, & Maher, 2003, p. 406). Within design based research it has become widely used to collect and archive large amounts of both visual and aural data within the naturalistic contexts of classrooms (Roschelle, 2000; Sawyer, 2006b).

Video-data capture was used extensively in this research primarily due to two features which Bottorff (1994) identifies as its strengths: density and permanence. The notion of density refers to the way in which it is able to capture in depth observations of simultaneous events and provide audio and visual data in real time. This included capturing antecedents and consequences of events as well as nuances of tone and body language of participants.

Each mathematics session was video recorded using one camera. Within each lesson video recorded observations targeted specific lesson events. The first section of each lesson was recorded as the teacher discussed and outlined the mathematical activity with the students. Then the camera was positioned to focus on one small group (selected by the teacher) to capture the students' interactions as they problem solved. Finally, the concluding large group sharing session was video recorded.

The video-recorded data became a permanent record readily available for subsequent review and iterative analysis. In this research, it was used extensively by all members of the team to examine multiple factors on both an immediate and longitudinal basis. Together we discussed and analysed many aspects in the emergence of each classroom mathematical inquiry community.

Although video is a valuable methodological tool for gathering data, its use is not problem free (Bottorf, 1994; Hall, 2000; Roschelle, 2000). Roschelle explains that the introduction of the video camera in a naturalistic context causes change in the ways participants interact and behave. To minimise unwanted effects caused through use of the video, prior practice

runs, explaining and discussing the purpose of taping, and the teacher modeling of everyday behaviour took place in each classroom.

Hall (2000) and Roschelle (2000) also caution that video data is neither a complete account, nor theory free. "These realities both constrain and shape later analyses and presentation of results" (Powell et al., 2003, p. 408). This research drew on multiple sources of data to supplement and triangulate the video observations in recognition that video-recording can be mechanical and theory loaded. These included field-notes of aspects of the classroom context including the physical setting, the organisation of groups, the activities and discussion, the interactions and dialogue of the teacher with the students, with me, and of student to student and documents.

5.7.4   DOCUMENTS

Documents are an important source of evidence to augment the audit trail (Bogdan & Biklen, 1992). In this study the documents included the mathematics problems the teachers used with the students, student written work samples, the charts recorded within large group explanations and those the teachers used to support the communication and participation patterns. Also included were the informant diaries completed by the teachers. Some teachers regularly completed diaries; others were constrained by lack of time and pressure in their daily life as teachers. Although the teacher recorded reflections reduced as the study progressed, discussion with me became more frequent. Similarly, their conversations with their peers became more frequent. They discussed interesting or puzzling aspects of student reasoning, student participation in mathematical practices, or the outcome of their use of specific problems or activities.

Teacher reflections about their videoed lessons were another important source of data. The video observations were transcribed as soon as possible after each observation and made available for the teachers to read. The teachers added reflective comments to the transcripts as they read them (see Appendix F). Additionally, they would watch sections of the video observations which had attracted their attention in the transcriptions and add comments. The video excerpts and notes would be discussed in the study group context. Such activity

supported richer development of taken-as-shared interpretations of what was evolving in the classroom communities. It enabled me to gain "better access to the meanings of the participants in the research" (Scott & Usher, 1999, p. 100). Maintaining the voice of all participants and capturing the teachers' perspectives about emerging inquiry discourse patterns and their corresponding mathematical practices was a central aspect in this study.

5.7.5   INTERVIEWS WITH TEACHERS

Where possible, following each lesson observation, the teacher and I briefly discussed the learning activities, and explored our shared view of the communicative interactions and emerging mathematical practices. On a micro-level these short discussions resulted in small modifications of the learning activities and enactments in the communication and participation framework.

Three scheduled interviews were held with the case study teachers. In the first interview the teachers described their students and the achievement levels of their classes. They also outlined their attitudes towards mathematics and the approaches they used when teaching mathematics. The second interview took place with the four case study teachers as a group at the end of the second term. This interview explored how the shifts in the interaction patterns were challenging the teacher and students beliefs about their roles in the classroom. The teachers also made direct comparisons with the previous mathematical classroom cultures they had enacted, the classroom communication and participation patterns and how they had organised mathematical activity. At the conclusion of the study the four case study teachers participated in a semi-structured interview to capture their reflections on their involvement in the research study.

Interviewing offers flexibility for the researcher to probe deeper for alternative or richer meanings. Kvale (1996) suggests that the interviewer should be a "miner" (p. 3) or "wanderer" (p. 5) and engage in interviews which resemble conversations which lead participants to recount their "stories of their lived world" (p. 5). However, whilst more informal interviews can act as 'conversations' Babbie (2007) notes the need for interviewers to be aware that "they are not having a normal conversation" (p. 307). Rather

they need to listen closely and promote participant talk. This was an important consideration in this research. The interviews supported my intention of understanding, from the perspective of the teachers, the actions they took to construct an inquiry community and how they perceived the effect of these actions on the mathematical practices the students used. On one hand, I was required to be a design research partner and co-share knowledge of the developing inquiry cultures and mathematical practices. On the other hand, I needed to balance this with acting as a "socially acceptable incompetent" (Lofland & Lofland, 1995, p. 56-57). This required setting aside a personal knowledge of teaching mathematics in an inquiry classroom and offering myself in the role as "watcher and asker of questions" (p. 56).

5.7.6   EXIT FROM THE FIELD

Withdrawing from the close relationship of the year-long collaborative design based research was an important process. In addition to the exit interviews, the teachers met with my research supervisors to debrief. The following year I participated in a team planning meeting and provided support for the teachers as they mapped out ways to develop mathematical inquiry communities with their new classes. Then at regular intervals I continued to informally visit and provide on-going support. The teachers were also provided with conference papers related to the research project to read and confirm. They have become members of a local mathematical professional group and have co-presented workshops at local professional development days and one co-presented a workshop with me at a national conference.

5.8   DATA ANALYSIS

Consistent with the use of a design approach, data collection and analysis held complementary roles in that one informed the other in an iterative and cyclic manner.

5.8.1   DATA ANALYSIS IN THE FIELD

Data analysis began concurrently with data collection in an intensive on-going process to identify tentative patterns and relationships in the data. Memos supported the contextualisation of the data, the development of theoretical insights, and noted connections between the immediate situation and broader theory. Initial coding themes were used to feed-back to shape subsequent decisions in the design research. Of particular importance in the iterative and on-going analysis in the field was the review of the individual teachers' progress in enacting aspects of the communication and participation framework—confirmation of aspects of its utility or need for extension, expansion, or revision of particular teacher actions and supports were noted. This iterative and recursive process is consistent with design research in which the researcher conducts "on-going data analyses in relation to theory" (Barab, 2006, p. 167).

5.8.2   DATA ANALYSIS OUT OF THE FIELD

At completion of data collecting I began the daunting task of analysis of the data set in its entirety—a key element of design research methodology used to maintain trustworthiness of the research findings (Cobb, 2000; McClain, 2002b). Qualitative data analysis is described as an art form which includes an important "creative element" (Willis, 2006, p. 260). It is a complex, time-consuming and intuitive process which requires a constant spiralling forward and backward from concrete chunks of data toward larger and more abstract levels (Creswell, 1998). In this study, a search for explanatory patterns drew on constant comparative methods. Using the general approach of Glaser and Strauss (1967) to manage a volume of data and build theory inductively from it is consistent with other qualitative and design-based research studies (e.g., Bowers et al., 1999; Cobb, 2000; Cobb & Whitenack, 1996; McClain, 2002b; Moschkovich & Brenner, 2000).

Glaser and Strauss (1967) contend that the development of theoretical ideas should occur alongside the collection and analysis of data. These scholars used the term 'grounded' to indicate that the theoretical constructs are embedded in the data collection and analysis process. Important to this qualitative approach is the iterative use of coding and memo-

writing for the generation of larger and more abstract concepts (Willis, 2006). Willis outlines coding as a process in which the qualitative researcher attributes meaning toward chunks of text, identifies the key concepts in the text and labels those which are most salient. In this process, incidents in the data are coded and constantly compared in order to develop and define their properties. Then, so that the codes and their properties can be integrated, the relationships among the concepts are closely examined. Eventually as patterns of relationships in the concepts are clarified, some which were initially noted are discarded as irrelevant to the emerging theoretical constructs (Glaser & Strauss).

In this research, coding began with the data collected in one classroom. To confirm or refute initial codes and memos recorded in the field, transcripts and field notes were read and reread and video observations repeatedly viewed. Hunches grounded in the data were formulated using a process Bogdan and Biklen (1992) term "speculating" (p. 157). Patterns in communication and participation and incidents and events which occurred and reoccurred during classroom interactions were noted. The use of codes and memos to maintain a way of developing the emerging 'hunches' into a more coherent form continued. These were added to field notes, teacher reflections, the transcribed video observations and the classroom artefacts. Follow up questions and words or incidents circled to be considered further were added.

In order to bring meaning and structure to the volume of data from the four case study classrooms I needed to use a data reduction process (Scott & Usher, 1999). Data reduction involved sampling from the data set by including what was relevant and excluding what was irrelevant to the research question. The analysis of one classroom was used to create an initial set of codes, simple descriptions which sought to give meaning to the mathematical discourse; the participatory and communicative acts taking place in the mathematics lessons. The codes covered both single lines of words and action or chunks of text (see Table 4).

Table 4

Examples of the codes

Teacher tells students to ask for an explanation which is different.
Teacher asks group members to support each other in explanation.
Teacher asks for collective support when student states lack of understanding.
Teacher asks students to predict questions related to strategy solution.
Student expects support when stating lack of understanding or asking for help.
Student adds ideas or questions that advance collective thinking.
Student tells group members that group work involves team work.
Student makes sure that everyone else understands strategy solution.

After coding all the data I then began the task of manually sorting the codes into coherent patterns and themes to use to guide analysis of the other three case study classrooms. At the conclusion of this first iteration twelve themes for the teachers' actions and ten themes for students' actions had been constructed (see Table 5).

Table 5

Examples of the themes for teacher and student actions

Teacher actions	Student actions
The teacher asks for collective responsibility to the mathematics community.	The students use strategies which support collective responsibility to the mathematics community.
The teacher provides models for how students are to question, clarify, and argue to draw explanatory justification of mathematical thinking.	The students use questioning, clarification and argumentation to support sense making and the development of explanatory justification.
The teacher positions specific students so they are able to engage equitably in mathematical activity.	The students position themselves and others to engage in mathematical activity and validate their reasoning.
The teacher asks the students to make clear conceptual explanations of their thinking.	The students make clear conceptual explanations of their mathematical thinking.
The teacher emphasises the need for sense-making and scaffolds students to ask questions to make of a mathematical explanation.	The students use specific questions which support sense-making of mathematical explanations.

Following the first level of coding I engaged in refinement of the codes, testing and retesting these, discarding some, and verifying or adding others as they emerged in the data.

The evidence of teacher actions and student actions which supported each code was closely examined, added to, redefined or deleted. Evidence which supported each theme was developed and used to examine the data of each setting. (See Table 6)

Table 6

Examples of the evidence for one theme

The theme: The teacher emphasises the need for sense-making and scaffolds students to ask questions to make sense of a mathematical explanation.
1. The teacher asks students to track each step of an explanation.
2. The teacher explicitly affirms students for following step by step.
3. The teacher asks for explanation to be re-explained in a different way.
4. The teacher models questions which lead to clear conceptual explanation (what, where, is that, can you show us, explain what you did).
5. The teacher asks for use of materials to back up explanation.
6. The teacher asks listeners to clarify thinking behind steps in an explanation.

In the final iteration the underlying patterns and themes were used to develop conceptual understanding of how the teacher's pedagogical practices guided by the communication and participation framework, initiated students into the use of effective mathematical practices within inquiry communities. Data analysis tables of each classroom activity setting supported the exploration and sense-making of the themes in early drafts of the findings.

Gabriel (2006) maintains that embedding the writing of sections of the findings as part of the data analysis process is an important tool which supports theorising and sense-making of the data. In this study, writing based on the preliminary analysis developed in the field began immediately. Through writing conference presentations (Hunter, 2005, 2006, 2007a, 2007b) and chapter revisions, a richer sense of the relationships involved in changing communication and participation patterns in classroom communities emerged.

5.8.3 SOCIOCULTURAL ACTIVITY THEORY DATA ANALYSIS

As illustrated in the previous section a grounded approach provided a useful tool to code the data in an ordered and systematic way. In turn, sociocultural activity theory provided an additional conceptual means to examine and analyse the interaction between various elements of the activity system and reveal significant differences in each of the teacher's

development of a discourse community. Of particular relevance were the subjects—the teachers and students—and their "expanding involvement—social as well as intellectual" (Russell, 2002, p. 69) in communicating and participating in proficient mathematical practices in inquiry mathematics classroom communities. Also under consideration was the object or focus of activity—the overall direction of activity and its shared purpose or motive. Russell notes that the "direction or motive of an activity system and its object may be understood differently or even contested, as participants bring many motives to a collective interaction and as conditions change" (p. 69). As the communicative and participatory patterns changed in classroom communities, of interest was evidence of the resistance, discontinuities and deep contradictions produced in the activity system. These related particularly to the actions the teachers took in enacting the communication and participation patterns and how they interpreted these actions in their interviews when they reviewed and discussed the lessons and video observations.

5.9 DATA PRESENTATION

The findings are reported in the form of case studies of two classroom teachers and the pedagogical actions they took to construct communities of mathematical inquiry and their corresponding mathematical practices. Direct quotations are used in the findings, drawn from the interview data, discussions with teaching colleagues and me, and the teachers' own verbal or written statements as they examined video observations and transcripts or reviewed and reflected on lessons. The teachers' voice provides a way to gain deeper understanding of the teachers' goals and motives as they reconstructed the communication and participation patterns towards inquiry in their classrooms.

The findings are reported in three distinct phases. These phases relate to the three school terms in which data collection took place, although there is some overlap which corresponds with changes in communication and participation patterns in the classrooms and their related mathematical practices. Vignettes of mathematical activity are provided to illustrate the actions of the teachers and students. They provide detail of the mathematical practices as these emerged and were refined.

5.9.1 TRUSTWORTHINESS, GENERALISABILITY AND ECOLOGICAL VALIDITY

Although objectivity, reliability and validity are factors which make design research a "scientifically sound enterprise" (DBRC, 2003, p. 7) these attributes are managed differently within this form of research. Gravemeijer and Cobb (2006) argue that a central aim of the design research approach is to achieve ecological validity—that is that descriptions of the results should offer the starting point for adaptation to other situations. These researchers explain that an aim of the approach is to construct local instructional theory which can act as a frame of reference (not direct replication) for teachers who want to adapt it to their own classroom situations and personal objectives. To achieve this requires thick description of all details including the participants, the learning and teaching context, and what happened in the design research supported by analysis of how the different elements may have influenced the process. In the current study thick description is provided of all elements in the research. The study's ecological validity is further strengthened through use of teacher input into how best to adjust their individual pathways towards enacting the communication and participation patterns of inquiry while also accommodating their particular circumstances.

In the design research approach the conventionally accepted factors of validity and reliability are replaced by need to establish credibility and trustworthiness (Lincoln & Guba, 1985). Trustworthiness of the research centres on the need for the credibility of the analysis (Cobb, 2000; Cobb & Whitenack, 1996; McClain, 2002b). McClain outlines the need for a "systematic analytical approach, in which provisional claims and conjectures are continually open to modification" (p. 108). Documentation in all the phases should include "the refining and refuting of initial conjectures. Final claims and assertions can then be justified by backtracking through the various levels of the analysis" (p. 108). Other criterion which increases credibility and trustworthiness of analysis include prolonged engagement with participants in the field by the researcher (Lincoln & Guba, 1985) and peer critiquing of analysis. This research was driven by the need for a systematic, thorough, and auditable approach to the analysis of the large sets of video records, observations and

transcriptions generated during its life. Prolonged engagement with the teachers as co-researchers, and their students in classroom communities was a strong feature of the study. The on-going collaborative teacher-researcher partnership provided peer critique and important insights into understanding and interpreting activity from the teachers' personal perspectives.

In design research the issue of generalisability is addressed by viewing events as "paradigmatic of broader phenomena" (Cobb, 2000, p. 327). Cobb explains that considering "activities and events as exemplars or prototypes…gives rise to generalisability" (p. 327). However, Cobb clarifies his position outlining that this is not generalising in a traditional view where particulars of individual situations "are either ignored or treated as interchangeable" (p. 327) with similar situations. Instead, he argues that the post hoc theoretical analysis which occurs after data collection is completed is relevant for interpretation of cases across wider situations. Generalisability was supported in this research through the use of retrospective analysis, the careful detailing of the setting and participants, and the use of multiple classrooms and teachers as cases.

5.10   SUMMARY

This chapter began by outlining the broad research question used in this study. The selection of the research paradigm and use of design research followed naturally from the sociocultural perspective of this study. The key characteristics of design research methodology were explored in relation to the collaborative relationship between the teachers and myself.

Descriptions were provided of the data collection methods and how these captured the teachers' perspective on the emergence of a discourse community and its corresponding mathematical practices. Consistent with the use of design methodology, data collection and analysis held complementary roles in that one informed the other in an iterative and cyclic manner. Data analysis occurred concurrently with data collection, then after data collection

had concluded, of the complete data set. The use of sociocultural activity theory provided a way to establish important differences between the case study teachers.

In the following chapters I present the findings of two case study teachers. The literature signalled the many complex situations which teachers may encounter as they develop mathematical discourse communities. These chapters illustrate the different pathways teachers take as they engage with the complexities of developing classroom communities of mathematical inquiry.

CHAPTER SIX

LEARNING AND USING MATHEMATICAL PRACTICES IN A COMMUNITY OF MATHEMATICAL INQUIRY: AVA

One needs an identity of participation in order to learn, yet needs to learn in order to acquire an identity of participation. (Wenger, 1998, p. 277)

6.1   INTRODUCTION

The literature chapters drew attention to the need for student engagement in reasoned mathematical discourse in learning communities if they are to learn and use proficient mathematical practices. Persuasive evidence was provided that teachers can establish such intellectual climates through the application of a range of pedagogical actions. In this chapter and the next, each case study is organised in three distinct phases which directly relate to the three school terms in which data collection occurred. Each section reports on the pedagogical actions of the two teachers (Ava and Moana), and then the transformative changes these caused in the mathematical practices used in the classroom community. A commentary that links the changes in mathematical practices to the literature accompanies each section.

For Ava's case study Section 6.2 describes Ava's beliefs about doing and using mathematics and how these beliefs shaped the initial classroom learning context. Section 6.3 outlines the different tools Ava used to mediate the foundations of an inquiry classroom culture. Descriptions are provided of how the safe collaborative classroom culture supported student participation in questioning and explaining mathematical reasoning.

Section 6.4 describes the actions Ava took to engage the students in inquiry and argumentation. A close relationship is illustrated between changes in the participation and communication patterns and increased student engagement in collaborative argumentation

(Andriessen, 2006). Evidence of increased use of exploratory talk (Mercer, 2000) and more efficient use of mathematical practices is provided.

Section 6.5 outlines how Ava's actions to further transform the participation and communication patterns resulted in an intellectual climate where students mutually engaged in proficient mathematical practices. How learning partnerships within multi-dimensional zones of proximal development emerged is described.

6.2   TEACHER CASE STUDY: AVA

In an interview at the beginning of the study Ava outlined how she had always liked teaching mathematics. However, she reported that her earlier participation in the New Zealand Numeracy Development Project (NDP) (Ministry of Education, 2004a) had caused a loss of confidence in her ability to meet her students' mathematical needs. She outlined how the new instructional materials and strategies in the NDP, and its focus on students explaining their mathematical thinking, conflicted with how she had previously taught mathematics. Formerly she believed that it was her responsibility to explain the rules and procedures and that she had felt confident about her ability to do so. The concerns Ava voiced are consistent with those other researchers (e.g., Rousseau, 2004; Sherin, 2002b; Silver & Smith, 1996; Weiss et al., 2002) have identified as teachers change their pedagogical practices to implement inquiry based learning. As an experienced teacher Ava had been secure in her routines and in her understanding of the nature of mathematics. But her involvement in the professional development had led to dissonance in her previously held beliefs about both teaching mathematics and what she was being asked to implement.

Despite the reservations Ava had toward the NDP she had implemented aspects of what she had learnt in the professional development. She reported that she used or adapted the NDP lesson outlines to teach the students solution strategies. She had also encouraged the students to generate and explain their strategy solutions to her and a group of listeners. However, initial observations at the start of the study showed that these sharing sessions took a format in which a student would explain a strategy and the other students would sit

listening in silence. If questions were asked, it was Ava who more often asked them. Ava had done what Sherin (2002b) reports many teachers do when given new curricula designed to change the content of instruction—she had *transformed* the material to fit with her familiar routines. As a result, the classroom context was consistent with a 'conventional culture' (Wood, 2002). Although Ava had constructed a classroom climate in which the students talked more in the mathematics lessons, the teacher-led questioning elicited teacher-expected answers. Ava had retained a central position in the classroom as the main source of mathematical authority.

6.3   ESTABLISHING MATHEMATICAL PRACTICES IN A COMMUNITY OF MATHEMATICAL INQUIRY

Through discussion with me and the study group Ava established the immediate actions she wanted to take. To do this she drew on the communication and participation framework (see Figure 1, section 5.4.1) and the research articles described previously in Chapter 5. The articles offered her ways to consider how she might establish an inquiry environment. They also provided models for her to consider of students engaging in mathematical practices. The communication and participation framework provided Ava with a tentative pathway to structure her students' participation in reasoned mathematical activity. In the first instance, the mathematical practice she aimed to extend focused on students' participation in mathematical explanations. This participation included how they constructed and explained their arguments. It also included how the other students as listeners actively engaged with the reasoning being used. Ava indicated that to achieve this she would need to build a more collaborative classroom environment—a classroom culture where student reasoning was a key focus.

6.3.1   CONSTITUTING SHARED OWNERSHIP OF THE MATHEMATICAL TALK

At the beginning of the study Ava and her students held distinctly different roles within the classroom discourse community. The communicative rights and responsibilities were what Knuth and Peressini (2001) described as asymmetrical. Ava was the dominant voice and

she used univocal discourse as the most prevalent form of communication. To enact changes to the discourse patterns Ava directly addressed the new 'rules' for talk. She made explicit the changes being enacted by commencing each mathematics session with a discussion of expectations. She outlined and examined with the students how they were required to work together to build a mathematical community. She emphasised that working together involved an increase in collaborative participation in mathematical dialogue involving them both as listeners and as talkers. She repositioned herself from the central position of 'mathematical authority' to that of 'participant in the dialogue'. She modeled the shift explicitly by placing emphasis on the use of the words 'we' and 'us' as she participated in discussions. For example, during a mathematical discussion at the end of the first week of the study she asked: *Can you show **us** with your red pen what would happen? **We** want to know*. Through consistent use of similar statements she indicated to her students that she, too, was a member of the classroom community.

Changing the rules and roles in the community

The classroom interactions had previously been shaped by asymmetrical patterns of discourse and by the mathematical authority which Ava implicitly held. To re-mediate how they all participated in communicating mathematical reasoning required the reconstitution of the interactional norms. When we reviewed the video observations Ava explained her reasons for the explicit discussions. In the first instance, I interpreted that her overarching purpose was directed toward developing active student engagement in increased collaborative interaction and dialogue within classroom community. She wanted the students to be a key part of the changes and so her discussions were designed to motivate the students and help them to understand and adopt the changes. In an informal discussion during the first month of the study she commented that many of the students were finding the changes challenging. She related these specifically to the expectations they had of her role as 'mathematical authority'. The gradual repositioning of herself from the role of authority to that of participant indicated her shift toward a more "dynamic and fluid" (Amit & Fried, 2005, p. 164) view of authority in the classroom. Ava had taken the first step toward the development of an intellectual partnership in the classroom community based on shared ownership of the classroom talk.

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6.3.2   CONSTITUTING A SAFE LEARNING ENVIRONMENT

Ava, in her discussions with me, noted a need to establish a safe learning environment. She considered that in a safe environment her students would be willing to engage in mathematical dialogue more readily. She anticipated that the increased focus on their need to explain and justify their reasoning might cause a temporary reduction in her students' confidence. Therefore in these beginning stages of the study, in order to provide support for the students, she made many statements to them which affirmed her belief in their ability to cope with the changes. She confirmed to them that she understood their need for time to learn how to engage in mathematical dialogue. She regularly expressed belief in their mathematical ability and used this to explain why she believed in their ability to participate in mathematical conversations. For example, she told them: *Sometimes you share that mathematics magic really quietly with someone next to you or near you... I know and you know that it is simmering, that things are happening and that's all right.*

At the same time, she explicitly outlined how explaining or questioning reasoning required the students to take both social risks and intellectual risks. She also indicated that the students needed to take risks in their own reasoning when making sense of other's explanations and justification.

The following vignette illustrates the explicit attention Ava gave to the development of a classroom climate as she engaged the students in discussion of their responsibilities within their changing roles.

Intellectual risk taking
Before the students began their mathematical activity Ava discussed a need to risk-take.
Ava   Remember how yesterday we talked about in maths learning how you go almost to the edge? So therefore I am going to move you out of your comfort zone. It's lovely being in a comfortable cosy place. Even as an adult we love to be there too. But if you are already there then it's time to move

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	on, out a little bit…so you go out there…maybe a bit more…a bit further next time and come back in again…
Sandra	And when you are out there you will make that your comfort zone. Then move on and make that your comfort zone.
Ava	So your comfort zone as Sandra said will move. You may have been here. It will go out a little bit. You will get used to this over here and you will think oh that's cool, I am quite comfortable here. I don't feel threatened. I am not stressed. I can do this. Hey so if I can do this and I was here before as Sandra said I will go over here.
(Term 2 Week 5)	

Changing beliefs, affective relationships

Early in the study, examination of the data reveals that Ava was very aware of tensions and contradictions between the interactional norms she wanted to establish and the previous norms which still shaped the community's current learning behaviour and beliefs. However, if student beliefs about 'doing and using mathematics' were to be re-mediated the tensions and contradictions were expected and indeed necessary components to activate changes in the current learning environment. Russell (2002) explains that activity systems constructed by humans are continually subject to change; change which is driven by contradictions within their various elements. Ava recognised that affective needs were an important consideration to this change. The actions Ava took in attending to the students' affective needs when establishing collaborative learning are explained by a sociocultural learning perspective. During collaborative interaction and co-construction of meaning, intellect and affect are interdependent elements "fused in a unified whole" (Vygotsky, 1986, p. 373).

Ava's actions in placing direct focus on how the students would be assisted to make changes within a safe supportive environment aimed to provide them with affective support to take the required social and intellectual risks. The establishment of a safe, supportive, positive learning environment is reported as an essential pedagogical component for engendering learning competence (Alton-Lee, 2003; Cobb, Perlwitz, Underwood-Gregg, 1998; Povey et al., 2004; Wells, 1999). Her carefully crafted care and support incorporated the key elements Mahn and John-Steiner (2002) identify as components of emotional

scaffolding including the "gift of confidence, the sharing of risks in the presentation of new ideas, constructive criticism and the creation of a safety zone" (p. 52).

6.3.3 COLLABORATIVE CONSTRUCTION OF MATHEMATICAL EXPLANATIONS IN SMALL GROUPS

Ava stated that in line with the NDP she regularly used small activity groups in her mathematics lessons. These groups of 3-4 students were required to construct solution strategies for explaining at a larger sharing session. However, in reviewing of videotapes with me Ava recognised that the students working in their small groups predominantly engaged in use of either cumulative or disputational talk (Mercer, 2000). Cumulative and disputational talk, as we saw in Chapter 3, has been characterised as an unproductive form of talk which limits how group members explore each other's mathematical reasoning. Ava acknowledged that her students needed more specific guidance on how to work collaboratively.

In the first instance, Ava focused on how the students participated together in small group activity. She outlined to the students her requirement that they actively engage in listening, discussing, and making sense of the reasoning used by others. To develop their skills to work collectively she stressed that all group members needed to engage in construction of mathematical explanations and be able to explain them to a wider audience.

In accord with the pathway she had mapped out, Ava initiated an immediate shift towards establishing that the mathematical explanations the students constructed should be well reasoned, conceptually clear, and logical. She explicitly scaffolded how they were to provide an explanation: *Talk about what you are doing…so whatever number you have chosen don't just write them. You say I am going to work with…or I have chosen this and this because…and this is what I am going to do.* She outlined not only how these explanations needed to make sense for a listening audience but also how listeners needed to make sense of the explanations offered by others. To develop their skill in the examination and analysis of explanations she provided opportunities for the small group members to construct, explain, and, in turn, question and clarify explanations step-by-step. The

following vignettes from three separate observations early in the study illustrate how Ava inducted the students into public construction and evaluation of conceptual explanations within collaborative zones of proximal development (Vygotsky, 1978).

Collaborative interaction and sense-making

The students had individual time to think about a solution strategy then Ava said:

Ava — You are going to explain how you are going to work it out to your group. They are going to listen. I want you to think about and explain what steps you are doing, each step you are doing, what maths thinking you are using. The others in the group need to listen carefully and stop you and question any time or at any point where they can't track what you are saying.

(Term 2 Week5)

The students were asked to construct individual conjectures then Ava directed them to examine and explore each solution strategy:

Ava — They might say I think it is 59. That's cool but they have to back it up, explain how they came up with it. They have to say why. So I want you before you even begin to go around in your group and actually talk about it. Someone in your group may ask you a question. For example, that's an interesting solution. Why do you think that? Could you show us how you got it?

(Term 2 Week 6)

Ava explained and explored the group roles then directed them:

Ava — Argue your maths. Explore what other people say. Listen carefully bit by bit and make sense of each bit. Don't just agree. Check it all out first. Ask a lot of questions. Make sure you can make sense that you understand. What's another important thing in working in a group?

Alan — Share your ideas. Don't just say I can do it myself that adds on to teamwork.

Ava — That's right. We do need to use each other's thinking…because we are very supportive and that's the only way everyone will learn. So we have to be discussing, talking, questioning, and asking for clarification. Whatever it takes to clarify what you understand in your mind.

(Term 2 Week 5)

Interaction scripts, peer collaboration, making and clarifying mathematical explanations

Ava had assumed that the students would construct appropriate knowledge because they were asked to interact cooperatively in a small group. This is a common misconception (Mercer, 2000). The initial examination of lessons in Ava's classroom confirmed what

Mercer and other researchers report (e.g., Irwin & Woodward, 2006; Rojas-Drummond et al., 2003; Wegerif et al., 1999): the students had many difficulties engaging in and using mathematical talk in the small group situation

To transform how the students interacted in mathematical activity in their small groups required explicit renegotiation of both what the task required and the scripts for conduct— the ground rules which shape the interaction (Gallimore & Goldenberg, 1993). Ava's use of clear directives to outline her expectations gave the students a working knowledge of what their obligations were. It also provided them with understanding of the learning potential of small group interactions, offering them a motive to participate appropriately and develop ownership of the ground rules. She laid the foundations for peer collaboration (Forman & McPhail, 1993) as she structured the group activity. The students were introduced to and practised skills for explaining their reasoning and examining the reasoning of others. Many of these actions parallel the pedagogical actions taken by Lampert (2001) when she instituted small group collaboration. These included: establishing with the students recognition of themselves as valuable sources of knowledge; emphasising mutual responsibility for sense-making; and the requirement of individual responsibility for understanding, thus removing any possibility that a lack of understanding could be attributed to others.

6.3.4 MAKING MATHEMATICAL EXPLANATIONS TO THE LARGE GROUP

Each mathematics lesson included small group activity and a larger sharing session. In the sharing session groups of students provided the mathematical explanations they had constructed together. In these sessions Ava took a key role. She questioned and provided prompts to ensure that the explanations were tied to their problem form. When required, she specifically asked questions to make the explanations experientially real for the listeners. To promote clarification of explanatory reasoning Ava discussed and modeled the use of specific question starters (e.g., what, where, is that, can you show us and explain what you did) and monitored how they questioned each other. She would ask them to rephrase a question if she judged that it had not elicited the information required to make sense of an explanation. She directly focused their attention on paradigmatic models of students using

questioning effectively to make sense of an explanation. For example, Ava asked: *Did you see that? For example, as we are saying add two, that's what Ruru was saying, okay so we've got these two here. Then Alan asked a very good question, why aren't we adding on three each time?*

Through analysing and discussing video observation excerpts with me and the study group, Ava became increasingly aware of the potential learning opportunities student contributions to discussions offered. In class she began to listen more closely to the reasoning used in small groups in order to structure who presented explanations to the larger group sessions. For example, the following vignette shows how Ava specifically selected a group to share 'faulty' thinking. During the presentation she directly interceded at regular intervals to facilitate space for listeners to think about, question, and clarify each section. But, she herself did not evaluate it.

Providing space to question and clarify mathematical explanations
Ava selected a group to explain their solution strategy to the large group. She stated:
Ava   I know for some of you, it pushed you out to the edge. I can see some semi and that means partly confused looks and that's okay... you will need to question anything you don't understand. We are going to listen step by step and when you finish one step put the lid on the pen and see if anyone has got any questions... Remember you need to talk about what your group did.
Sandra   [records 52 on a sheet] The rule was timesing it by two and it goes up.
Ava   Right. Can you stop there and wait for questions.
Pania   How did you get that?
Ava   [prompts] How did you get what?
Pania   How did you work out it was 52 sticks?
Sandra   We went 25 and 25 equals 50. No. 12 and 12 equals 24 plus 2 would equal 25.
Ava   You people need to be really listening to what is being said here. You have got to ask yourself does it make sense to you.
Pania   I don't think it will make sense because 24 plus 2 would equal 26.
Ava   Well tell you what. You have done really well. There are some adults who would not even know how to work through that sort of problem so well done.
Ava then selected another group to provide the solution strategy for the problem used in the vignette in section 6.3.6. *(Term 2 Week 7)*

After the teaching session she explained her selection to me. She said that she knew that this group's explanation would challenge the reasoning of others and require that the listeners make sense of it, section by section. She saw it as a tool which provided practice for the other students to question and clarify sections as needed.

Changing roles and rules, mediating sense-making and the norms for mathematical explanations

The observational data shows that Ava's expectations of both how mathematical explanations were given and the student's role in listening to them, ran counter to the students' previous experiences. As discussed previously, Ava had voiced the potential conflicts she knew the students would have between their perception of her 'teacher' role and the more facilitative role she planned to develop. Hence, she structured the large group interactions directly. Her requirement that explanations be explained in sections, with each section supported by space to 'think', made the reasoning open and visible and available for clarification and challenge. The models of how to make an explanation and the questions she explicitly modeled and used during the sessions provided the students with tools to sense-make. Through these actions Ava began to pull forward all participants in the dialogue into what Lerman (2001) termed a symbolic space—a zone of proximal development in which she was scaffolding sense-making.

Kazemi and Franke (2004) noted the difficulties many teachers experience when attending to the different reasoning their students use. The discussions Ava had with me and her colleagues, of the ways her students reasoned, were the tools which mediated her growth in understanding and her value of them. Ava's use of student contributions and erroneous reasoning shifted thinking past a focus on correct answers to look at and explore the solution strategies. Thus the students were 'pressed' (Kazemi & Stipek, 2001) to assume their new role, to analyse the reasoning and be accountable for their own sense-making. In addition, Ava's requirement that explanations be conceptually based and her emphasis on construction, explanation and clarification of them led to the constitution of mathematical norms for what constituted clear and logical explanations in the classroom community.

6.3.5 LEARNING HOW TO AGREE AND DISAGREE TO JUSTIFY REASONING

In the early stages of the study the participation structure that Ava made available to students operated as a scaffold for the development of argumentation. Although mathematical argumentation was not a strong feature of how the students interacted Ava initiated and maintained discussion about the need for both agreement and disagreement in the construction of reasoned explanations. For example, when a student stated that working as a group required agreement she responded by asking: *Yes you could be agreeing with what the person says…but are you always agreeing, do you think?* She carefully structured ways in which the students could approach disagreement and challenge. As illustrated in the following vignette, Ava scaffolded ways in which they could disagree with an explanation so that justification became necessary.

Scaffolding ways to disagree
As the students worked together she reminded them:
Ava — Arguing is not a bad word…sometimes I know that you people think to argue is…I am talking about arguing in a good way. Please feel free to say if you do not agree with what someone else has said. You can say that as long as you say it in an okay sort of way. If you don't agree then a suggestion could be that you might say I don't actually agree with you. Could you show that to me? Could you perhaps write it in numbers? Could you draw something to show that idea to me? That's fine because sometimes when you go over and you do that again you think…oh maybe that wasn't quite right and that's fine. That's okay.
(Term 2 Week 6)

Beliefs about disagreement and argumentation

Inducting students into the use of justification and collaborative argumentation (Andriessen, 2006) was begun through discussions of the need for not only agreement but also disagreeing and arguing. As Mercer (2000) and his colleagues (Rojas-Drummond, & Zapata, 2004; Wegerif, & Mercer, 2000) have shown, when students maintain constant agreement often cumulative talk results—a less productive form of mathematical reasoning.

Therefore, Ava had to ensure that the students understood that even when working collaboratively agreeing with the reasoning of others is not always productive mathematically. Her regular references of the need for both agreement and disagreement, and her specific scaffolding of ways to challenge, provided assistance to the students to shift the discourse patterns from cumulative or disputational forms of talk, toward use of exploratory patterns (Mercer, 2000).

Although Ava had initiated discussion of the need for disagreement she noted the tensions these expectations placed on the students. During analysis of a video excerpt Ava stated: *Disagreeing is so hard for these students so I am supporting them and ensuring that they're okay with the concept of agreement and disagreement, also how to approach each other when voicing their opinions.* Her statement reveals her recognition of the contradictions between the new rules for talk, those used previously, and those which governed the forms of talk used in their wider community beyond school. As Gallimore and Goldenberg (1993) explain, children "are shaped and sustained by ecological and cultural features of the family niche" (p. 315). Ava's statements and careful structuring of ways to argue illustrated that she was aware that for many of her students from non-dominant groups[1] (Nasir, et al., 2006) mathematical argumentation as a specific speech genre (Gee, 1999) was not necessarily within their current repertoire of cultural practices (Gee, 1992). In addition, she recognised that the form many of her students might have participated in previously was an oppositional or aggressive form of argument (Andriessen, 2006) which she thought would have shaped negative beliefs about 'arguing'. As a result, she knew that transforming their beliefs and constituting mathematical argumentation as a situated discourse practice in the classroom might initially be problematic. She knew that to shift to an inquiry environment required that the students learn the discourse of inquiry and challenge and therefore she purposely continued to lay the foundations for its development.

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[1] Non-dominant groups include students who speak national or language variations other than standard English and students from low income communities.

6.3.6 GENERALISING MATHEMATICAL REASONING

In conjunction with a focus to shift the students towards generalising Ava chose to use a series of problems which required the students to engage in active and extended discussion of their reasoning. These problems[2] were intentionally devised in the teacher's study group to support the use of early algebraic reasoning. They required exploration of patterns and relationships and the construction of a rule as an unexecuted number sentence to describe the relationship. The problems began as single tasks but collectively they provided a means to scaffold the students to construct and voice generalisations. During task enactment Ava emphasised the physical representation of the problems to illustrate the recursive geometric patterns. However, discussion with me led to Ava varying the task parameters so that the students were also encouraged to use analytical approaches. As the problem contexts were extended Ava pressed the students to look beyond the concrete constraints of the immediate situation. She directed them to make and test their conjectures on a range of numbers and explore whether they worked on all numbers. She began teaching sessions with discussion and direct modeling of the use of diagrams, ordered pairs and tables of data. These operated as a structure in which the students were able to explore and develop systematic strategies to test out the patterns that produced the data.

As I described in Section 6.3.4 Ava carefully selected groups to provide explanations to the larger group. In the following vignette she had observed functional reasoning in one group. She used this group's explanation as a way to shift the wider groups' thinking towards generalising. Continuing from the vignette in section 6.3.4 Ava takes a different role, recognising that this group are presenting an argument which many of the listeners may have difficulty accessing. The explainers had established and recorded a functional relationship between the number of sticks needed to develop a triangular pattern and the number of triangles. She facilitates a prolonged discussion of their model and how they came to develop it. She participates in questioning and directs attention to their notations. Her questioning prompted the listeners to reflectively analyse their reasoning. Her questions drew further explanation of how the group had reflectively constructed the

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[2] For examples of the problems see Appendix G

generalisation together through persistent pattern seeking and examining and re-examining the reasoning they were working with.

Generalising a functional relationship	
Rachel	[recording] We started off with a table and we stuck a picture of a little triangle and we wrote sticks. We know that one triangle has three sticks but from then on it is two. So two triangles have five sticks…We put a sign that said plus one here then we went from one to ten plus one.
Ava	Put the lid on the pen for now. Anybody got any questions?
Sandra	Why did you do that?
Rachel	Well it helps us with our rule. We put a sign up here that says times two.
Rachel explains and records x 2 and + 1, records from 1 to 10 and records the value of each number x 2 +1 underneath	
Ava	Ruru can you follow what Rachel's group have done so far? Can you explain what you see happening here? Wang what about you?
Wang	The first one is the one triangle makes up to three because they are adding two on.
Ava	[points at the ordered pairs] Why though have Rachel and their group… done this? Why have they put these numbers down here as well as these two numbers? You are right in that they are adding two on. But why have they done it like this? Why? Maybe somebody else can help?
Sandra	To help them know what their strategy is.
Ava	[asking the explaining group] You will accept that?
Sandra	[interjecting] Most of us haven't done the two times and the one plus.
Ava facilitated further discussion and questioning related to the use of the systematic recording of the table and ordered pairs. Then she asked the explainers to continue.	
Aorangi	Well six times two equals twelve…plus one equals thirteen. Then Rachel when she was thinking of this, that's when she saw Pania writing it. She thought of this [points at the recorded rule]. She thought six times two plus one equals thirteen…Then what we did, we did brackets around the six and the two. We started thinking it was times two inside and plus one outside.
Ava	Can you stop there just for a moment. Think about this. All of you think about your own explanations. Wang said they are adding two on…is this group adding two…where are the two? We need to think about what they are doing?
Alan	Instead of plus two, plus two…they are timesing by two. So they didn't have to go long…it was six times two plus one. Or it could be one hundred times two plus one.
(Term 2 Week 8)	

Through construction and exploration of problems in the study group, Ava was introduced to possible ways she could mediate her students' early development of algebraic reasoning. However, it was the informal discussions after teaching sessions which shifted Ava's understanding that the students' attention on use (and enjoyment) of manipulatives can in some instances potentially hinder their development of underlying mathematical understandings. Moyer (2001) identified the tension many teachers have when using manipulatives as one of balancing student confidence and enjoyment with maintaining a press towards more generalised mathematical understandings. In the lesson the shift in emphasis from the use of manipulatives and the concrete situation extended the problems so that they became more open-ended. Ava provided the students with example spaces (Watson & Mason, 2005); these became intellectual spaces in which they then had opportunities to search for and test examples and counterexamples of numerical patterns and relationships.

Using student explanations in which there was evidence of a 'mindful' approach (Fuchs et al., 2002) to reflective pattern seeking provided a foundation for inducting them into the use of more "intellectual tools and mental habits" (RAND Mathematics Study Panel, p. 38). The reasoning contributed by the explainers provided a platform for other participants to make connections and to build from. The use Ava made of student explanation, her modeling of careful listening, and the positioning of other students to access the reasoning, are important actions which influenced student beliefs about themselves as mathematical doers and users.

6.3.7   USING AND CLARIFYING MATHEMATICAL LANGUAGE

Ava placed direct attention on scaffolding how the students used talk to describe and question explanations and provide appropriate responses. The following vignette illustrates how Ava accepted the students' use of short utterances and informal terms in the mathematical discussions but then facilitated further discussion which focused directly on

how the language of mathematics was being used. Ava revoiced, rephrased, or elaborated on sections of explanations to clarify the mathematical meanings of words

Revoicing to clarify and define mathematical terms	
After a group had modeled an explanation using equipment and described the action of repeatedly adding three sticks as equivalent to squares times three they are challenged.	
Jo	Isn't that just plusing three sticks not timesing it? You are not timesing you're adding.
Pania	Well what she sort of means it is like it is going up.
Alan	Is that timesing going up?
Ava	When we talk about timesing what do we actually mean?
Jo	We mean multiplying not adding. Adding is plus [indicates a + with fingers] that sign.
Sandra	You mean when you add two more squares on, that is multiplying?
Ava	Rachel was saying she is adding three, adding another three, so that's three plus three plus three. So if you keep adding three all the time what is another way of doing it?
Alan	You can just times instead of adding. It won't take as long and it is more efficient.
Ava	Yes you are right. Did you all hear that? Alan said that you can just times it, multiply by three because that is the same as adding on three each time. What word do we use instead of timesing?
Alan	Multiplication, multiplying.
(Term 2 Week 7)	

Multi-levels of language

Examination of the data reveals that Ava regularly attended not only to how the students used mathematical talk to explain their reasoning, but also to their use of the mathematical language. Ava accepted the students' use of colloquial terms but revoiced (O'Connor & Michaels, 1993) and rephrased what they said, using rich multi-levels of word meanings. Her actions in providing multiple levels of words and revoicing and extending the students' mathematical statements paralleled the actions of a teacher Khisty and Chval (2002) described. Ava, like that teacher, ensured through her revoicing and provision of multiple layers of meaning that her students could access the specialised discourse of mathematics.

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6.3.8 SUMMARY OF THE FIRST PHASE OF THE STUDY

In this section I have outlined how Ava maintained a focus on constructing a safe collaborative classroom environment where the students gradually gained confidence to participate in explaining, questioning and clarifying mathematical reasoning. The trajectory Ava mapped out using the communication and participation framework guided shifts in the classroom interaction patterns and establishment of the foundations for an inquiry classroom culture (Wood & McNeal, 2003). Ava's immediate focus was placed on scaffolding the mathematical discourse. She mediated the establishment of classroom norms concerning how students participated in the mathematical talk and what mathematical thinking was discussed.

This initial focus is consistent with what a number of other researchers have noted (e.g., Rittenhouse, 1998; Silver & Smith, 1996; Wood, 1999); teachers when constituting inquiry communities first establish the norms for discourse, and then they turn their attention to the content. In this environment the students began to learn to use more proficient forms of mathematical practices as they learnt to talk about 'talking about mathematics' (Cobb et al., 1993).

6.4   Extending the mathematical practices in a mathematical inquiry community

Our reflections on the success of the initial establishment of the interaction patterns prompted us to move forward drawing on further steps on the communication and participation framework. Ava stated that the students were ready: *to move from the 'what' to the 'why.* She wanted the students *to engage in meaty mathematical arguments using various methods to convince others or to justify why they agree or disagree with a solution.* The communication and participation framework was used to plan shifts toward increased student use of inquiry and challenge.

6.4.1 PROVIDING AN ENVIRONMENT FOR FURTHER INTELLECTUAL GROWTH

As Ava and the students repositioned themselves in the mathematical discussions the students gradually assumed more agency over when and how they engaged in the discourse. In accord with the communication and participation framework, Ava introduced the use of a 'thinking time', a pause in mathematical dialogue designed to support all participants' active cognitive engagement. Its purpose was to offer opportunities for participants to analyse arguments, frame questions, or structure their own reasoning and explanations. Ava explicitly outlined to her students how 'think time' was a sense-making tool which provided them with an opportunity to reconsider and restructure their arguments. She regularly reinforced its use, halting discussions to outline how she observed its use: *I am pleased to hear the way you are all exploring each others' thinking. Stopping and thinking…having a think time. It's good to hear things like "I thought it was that one but now I think it is this one and this is the reason why". Or "I thought that but now I think this because we"…and explaining it and using drawings to back up what you are saying.*

Learning to participate in mathematical dialogue continued to be a risk-taking act for some. 'Think time' was used to scaffold those students who were initially diffident or unconfident about publicly explaining or responding to questions and challenge. Rather than accepting silence from a student as reason to bring questioning or explaining to an end, Ava would suggest a 'think time'. But she would clearly indicate that they were still responsible to explain: *You have a think about it while I pop over to this group so we can hear their thinking then we will come back to you.* Examination of the data reveals that through balancing 'think time' with an expectation of accountability the students became more willing to express partial understandings or outline difficulties they had understanding sections of an argument.

Earlier in the study, the vignette in section 6.3.4 illustrated how Ava used an explanation based on faulty thinking to provide the students with practice in questioning and sense-making. However, often the reasons for the misconceptions remained unexamined. Through reflective discussion with me, Ava's awareness of the potential learning

opportunities in examining and analysing erroneous reasoning increased. In class she began to use problems which required students to examine and justify their selection of a solution strategy. These included solution strategies which had common misconceptions students of their age consistently make and which appear reasonable when seen from the perspective of the learner (see Appendix I). Then in sharing sessions Ava would purposefully select a group who had an argument which had the potential to progress all their thinking. This is illustrated in the following vignette when Ava selects a group who are arguing that an erroneous solution strategy is correct. Despite two previous groups' clear and well argued explanations Ava hears the on-going questions and challenges. Her response is to facilitate close examination of the reasoning using student challenge to explore the many partial understandings they all hold.

Making the reasoning clear, visible, and open to challenge

For the problem[3] under discussion Ava selected a group to explain that she knew had developed an incorrect explanation.

Sandra	[records 1/2, 1/5, 3/10] My argument is Colin is correct on his one and I am going to show you how it goes...I think that Colin took all the first numbers from the top numbers and he has added them together which is five. Then he's got all the bottom numbers and he's picked one which is the five to get to his whole.
Ava	Stop Sandra. You have a mountain of questions.
Pania	Why did you choose the bottom one the five?
Sandra	Because that's how he probably got a whole. Because if he went five and [points at the 3/10] ten he would get a half and if he went five and [points at the 1/2] two he would get two wholes and a half. But he said he had a whole.
Rachel	In other words you are saying that you are adding to five and using the other five [points at the denominator of 5] to get a whole?
Ava	So can you just use one from there? Can you show us using pictures to convince us?

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[3] Colin and Junior are having an argument about their homework. In the problem the clown eats $^1/_5$ of one fruit square, ½ of another fruit square and $^3/_{10}$ of the last fruit square. The clown then puts the remainder of the fruit squares on one plate in the fridge to eat later. Their teacher has asked them to say how much fruit square as a fraction, the clown ate altogether. Colin says that the clown altogether ate 1 fruit square but Junior says that the clown ate $^5/_{17}$. Who is correct and why? Can you work out what each of them has done and what they were thinking? Work together and discuss how you would explain the correct answer to them. You will need to work out a number of different ways to explain so that they are convinced. Think about using pictures as well as words and numbers to convince them

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Sandra	[draws a rectangle and colours in each slice as she explains] They basically ate one fifth, another fifth and then three fifths. So that made them into fifths instead of them [points at 3/10]. So that would make five and five [at the bottom] would equal one whole.
Rangi	How did you make them into fifths?
Sandra	[points at the denominators in 1/2 and 3/10] Just pretended those were not there.
	Further discussion takes place and Ava directs the students to reread the problem.
Rangi	[points at the numerators then denominators] Can I show you something? Junior was wrong in the problem because first he added those together which was five. Then he added that together and it was seventeen.
Rachel	What Sandra is doing is adding the top three which equals five…and taken a five from the bottom to make a whole.
Ava	[records a list of different fractions] Can you do that? Does that make sense? Would it still work the same if for example you have different numerators which will not add up to any of your denominators? I am going to leave that thought with you. Would it still work? If you think it won't explain it to us. If you think yes you prove it either way.
(Term 3 Week 4)	

Thinking space, increased independence, errors as a resource, mathematical analysis, learning in the act of teaching

Evident in the data is a shift in positioning of Ava and her students as they grew in what McCrone (2005) terms communicative competence—knowing when and how to initiate and participate in interactions using appropriate mathematical talk. Introducing a 'think time' as a tool to use during social interaction provided the students with a thinking device—a space to reconceptualise the reasoning under consideration. In this space, their explanations and the explanations of others became reflective tools (Cobb et al., 1997) which contributed to developing richer levels of mathematical discourse.

Providing a gap in the interactions also acted as a form of social nurturing (Anthony & Walshaw, 2007) for those less confident members of the community. In addition to the previously described affective support that Ava provided in the first phase, provision of a space to think and the added press to participate indicated that she "cared for" (Hackenberg, 2005, p. 45) her students and their engagement in the mathematical discourse. But the 'caring' meant that a hesitant response, or no response at all, did not signal a dependent

relationship, nor remove responsibility for participation. Rather, it led to increased independence indicated in the expectation that they take ownership of their explanation or identify aspects of reasoning they were unsure of.

The classroom observations revealed that problems which required extended dialogue and negotiation of sense-making, including those that examined misconceptions, were important tools which mediated how the students participated in the mathematical discourse. The errors became "resources for learning" (Alrø & Skovsmose, 2002, p. 22). As Brodie (2005) explains, many misconceptions are reasonable errors which "make sense when understood in relation to the current conceptual system of the learner, which is usually a more limited version of a mature conceptual system" (p. 179). The errors in this situation served as a point of continuity for the community to discuss and grapple with complex ideas. Building on errors as an entry point for further discussion is a feature of many studies located in inquiry communities (e.g., Fraivillig, 2001; Fraivillig, Murphy, & Fuson, 1999; Kazemi, & Stipek, 2001; White, 2003).

Ava analysed and noted her own increased pedagogical skill to 'notice' (Sherin, 2002b) student thinking and learning when she reviewed the video excerpt: *I saw here that they wanted to continue asking questions and Sandra was really starting to rethink and I think that there would have been more than Sandra in the rethinking. I am giving them food for thought trying to challenge their thinking to see if they can understand and discuss why and why not adding different denominators. It doesn't hurt to leave them hanging. What a shift for us all...before it was about the right answer and now...* In collaborative discussion with me she voiced how she was learning to facilitate productive discourse which maintained a fine balance between providing knowledge and allowing the group to struggle through to co-construction of arguments. Ava was "learning in the act of teaching" (Davies & Walker, 2005, p. 273), drawing together her knowledge of the mathematical content, the learning task, and knowledge of how student reasoning could be used to progress or deepen understanding.

6.4.2 COLLECTIVELY CONSTRUCTING AND MAKING MATHEMATICAL EXPLANATIONS

Ava continued to monitor how the students constructed and explained their reasoning. To facilitate the group processes Ava introduced the use of one large sheet of paper and one pen to assist the recording of a strategy solution to explain to the larger group. Individual group members were each required to explain a solution strategy then after close examination select the most efficient one. The students also knew that during large group sessions Ava would halt an explainer and require that other members of the small group continue or clarify the explanation. This encouraged members to closely examine and explore their reasoning, ask each other questions, and search for alternative ways to explain and engage in reasoned debate about their thinking.

The following vignette illustrates how Ava constructed collaborative group processes to include students' responsibilities to each other and their listening audience. In turn, the students extended their responsibility for each other's sense-making but also anticipated how the explanation would be understood by the larger group. The attention participants in small groups placed on their analysis of their explanations scaffolded support for each other when explaining and justifying their explanations in the larger sessions.

Constructing a collaborative explanation and justification

Before the small groups began to work Ava described their group responsibilities.

Ava	You need to make sure that everyone in your group all work together to make sense of it. Together you need to know what you're…saying and what you are doing. You may need to use your fractions pieces and lots of different ways to make it make sense to all of you in the group…When it comes to the sharing time you need to be able to explain and justify what you are saying in lots of different ways. We are all going to need to be able to see what you are saying, see your reasons behind your explanations. I am going to ask anybody in the group to explain. So you have to make sure that everybody in the group can explain anything you are asked.

After this group of students have explored three different solution strategies for the problem[4] they discuss which one to provide to the larger group.

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[4] See problem on p. 20

Rachel	About this one, it's a bit hard to understand because it was so fast.
Tipani	Okay. The truth is this is the most efficient way. [Points] That's a good way. That's a good way. But that's the most efficient.
Rachel	[points] Yeah but that one is the most efficient because it's easier to understand. This is more confusing even if it is the fastest. So let's go with the one we know everyone will understand.

When Rangi provides their group's solution strategy in the sharing session he is challenged:

Tipani	Why did you shade in two tenths and call it one fifth? You didn't explain why. You just said there was only one fifth left and you just shaded it in?
Rangi	The clown ate one whole and it says he ate half. Then he ate three tenths and then there was only one fifth. So I had to shade in two because that was the only squares left.
Josefina	[steps in to support her group member] I can explain why. An easier way…because when you divide tenths into fifths there is two of the tenths resembles one of the fifths. So that's why she shaded in two because two tenths equals one fifth.

(Term 3 Week 4)

Communal discourse, mathematical explanations, objects of reflection

Both the communication and participation framework and our discussions were tools which mediated the continued press toward constituting an inquiry classroom participation structure where collaborative interaction in the discourse was central. As a communal activity, classroom discourse involves both individual and shared accountability (McClain & Cobb, 1998). Collins (2006) explains that the key idea "is to advance the collective knowledge of the community…to help individual students learn" (p. 55). Multiple zones of proximal development were constructed as participation patterns shifted from single explainers representing groups, to all members assuming responsibility to track what their explainer outlined and when required, intervene to respond to inquiry and challenge.

I previously described in Chapter 4 how many researchers reported that teacher expectations were vital to the quality and types of explanations constructed. In this study it was Ava's expectations which scaffolded the development of the classroom community's criteria for an acceptable mathematical explanation. Evident in the communal discourse was the requirement that explanations be experientially real and conceptually clear and

provide as much information as the listening audience required. In the act of comparing and analysing explanations to find those which were different, more sophisticated, or efficient, the explanations became explicit objects of reflection (Cobb & Yackel, 1996).

6.4.3 ENGAGING IN EXPLANATORY JUSTICATION AND MATHEMATICAL ARGUMENTATION

The requirement for conceptual explanations provided the foundations with which to build mathematical argumentation. We noted during evaluative discussion of the classroom video excerpts that when the students provided explanations they often drew on more than one form to validate their explanatory reasoning. To extend this practice Ava required that all the groups construct multiple ways to validate their thinking. Problems (see Appendix H) which required multiple ways to convince others were developed in the study group and used by Ava. In class, Ava explicitly discussed 'maths arguing' and how mathematicians use it as a tool to make mathematical reasoning clear: *Mathematicians engage in arguing. They do it all the time. It is good to have a healthy mathematical argument. You actually learn a lot more from arguing about your maths than not arguing about your maths. It just opens it all up...all the thinking.*

In the study group, the teachers and I had examined the video records of each classroom and recorded the questions and prompts the teachers and students used during mathematical activity. Building on the questions and prompts Wood and McNeal (2003) used to delineate classroom cultures, we developed a framework (see Appendix E) based on questions and prompts which had emerged in the classroom talk. As I explained in Chapter Five (see section 5.4.1) these were the questions and prompts used by participants in the discussion. These questions and prompts were identified as tools which mediated further examination and extension of the mathematical reasoning and the use of different mathematical practices. In class Ava used the models to guide how she scaffolded the students to use specific questions and prompts so that they engaged in the mathematical practices. Separate wall charts were constructed relating to the different types of questions to use with mathematical explanations and mathematical justification. In subsequent lessons Ava added questions which arose during the dialogue—questions which asked why and prompted for

justification and validation of reasoning. Ava also assisted the students by asking them to prepare group responses to questions they might be asked in the larger situation: *Think about the questions that you might be asked. Practise using some of those questions like why does that work or how can you know that is true. Try to see what happens when you say if I do that... then that will happen.*

Ava's emphasis on the need for justification and validation of reasoning was accepted and modeled by the students. They recognised that it supported possibilities for confirmation or reconstruction of their reasoning. They regularly appropriated Ava's words to prompt argumentation from their fellow students. For example, one student observed another student frown and directed: *Argue against them if you need to, if you disagree with their maths thinking. So they can think about it again and maybe change. Or you might agree in the end with their stuff.*

The following vignette illustrates how the press toward justification and validation of reasoning led to student engagement in exploratory talk (Mercer, 2000). Each step in the argument was closely examined and rehearsed. Then, for clarity, or ease of sense-making, the sections were reworked, reformulated and re-presented.

Using multiple means to justify and validate an argument

After sustained discussion of drawings and possible solution strategies for a problem[5] one member of a small group provides a verbal explanation:	
Tipani	I know what to do. It might be a little complicated and we might have to do something else to show that, to prove it. Half of the fruit square pretend that the half is in tenths. So one half is equal to five tenths plus three tenths [points at the 3/10 on the problem sheet] which equals eight tenths plus one fifth which is two tenths, so that means Colin was correct, one fifth which is two tenths which equals one whole which is one fruit square.
Ava	Are you convinced Pania?
Pania	It was a little fast.
Ava	Perhaps recording it Pania will mean that you get to see the bits that were too fast. But remember to keep asking those why questions. You need to be convinced one hundred percent.

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[5] See problem p. 20

Tipani records the fraction symbols and draws a rectangle which she divides into 10 segments. She colours each segment and points at the symbol as she explains:

Rachel   Pretend I don't understand so explain it to me.
Tipani   [colours in the segments and points at symbols] So this is one half, this one. Then I got three tenths and I am pretending that instead of it being just like that, the one half I am doing it in tenths. So because of that I am going three tenths and five tenths which is...
Rachel/Pania   Eight tenths.
Tipani   Then I know that one fifth is equivalent to two tenths. So I shaded that in, in my head and that's how I got one whole.
Rachel   So Tipani what about the fifth? Shouldn't we use that too?
Tipani   [pointing at the segments] No because eight, that's the first mark, that's the second mark because that's eight and then one fifth is equivalent to two tenths so that's it.
Rachel   So what you've done is you made one fifth...
Tipani   Into two tenths.
Rachel   So you are changing the one fifth into two tenths because that is the equivalent fraction and then using the equivalent fraction? [She records 1/5 = 2/10 on a sheet].
Pania   Yeah. One fifth equals two tenths.
Rachel   Why...yeah she has taken one half...and three tenths and she is using one fifth as an equivalent fraction. So it is one whole.
Pania   Ten tenths.
Rachel   [records 10/10 = 1] One whole.
Tipani   Because that's the equivalent fraction.
(Term 3 Week 4)

Accessing a social language, interthinking, maths arguing

An analysis of classroom observations suggests the framework of questions and prompts were tools which mediated further change in the classroom discourse patterns. Directly focusing the students on their need to use specific forms of questions to gain particular mathematical information had the effect of shaping the form and content of the discourse. These actions assisted the students to access what Wood (1998) described as the form, "knowing how to talk" and content, "knowing what to say" (p. 170) in the mathematical discourse. This provided the students with access to what Bakhtin (1994) terms a social language or speech genre and a specific identity.

Reviewing the video taped lessons provided insight into how the students appropriated and used the various models of ways to question and challenge. To begin, we observed that the students appeared to do what Ava described as "*parroting questions*" as they tried out ways to inquire and challenge. But later students were observed to rephrase and reformulate the questions, expand and extend them and make them 'their own'.

Ava's actions, in providing students with explicit ways to frame close examination and exploration of the reasoning of others, resonates with many other studies which have used specific structures to frame the classroom interactions (e.g., Brown, 2005; Mercer, 2000; Rojas-Drummond & Zapata, 2004; Rowe & Bicknell, 2004). Like the students in these studies, the students in Ava's room used the social language to engage in joint activity, to do what Mercer (2000) described as interthinking. The press toward a shared perspective shifted explanations to justification and validation of reasoning, particularly when other participants in the discussion either did not understand what was being explained, or disagreed with the reasoning. In these situations, the students were required to negotiate a shared perspective using what Ava and her students called 'maths arguing'. They listened closely to the reasoning of others, questioned then re-explained, re-modeled and rehearsed arguments until they became integrated as 'our' voice.

Although the students became more skilled at 'maths arguing' Ava continued to carefully structure the affective components of the classroom culture. She outlined to me toward the end of the second phase how she consistently aimed: *to establish a supportive environment in which it is okay and mathematically sound practice to disagree with someone else's thinking. This puts the onus on the explainer to clearly demonstrate how they worked through a solution. It also puts the responsibility of seeking clarification and asking analytical questions onto the other group members.* However, she recognised the inherent on-going difficulties this held for some of the students, commenting: *I feel a bit like a cracked record going over all these norms all the time but we always have new students joining. Also I am aware that these students are growing into this behaviour now but disagreeing can be so hard for these students so I find that I have to keep almost giving*

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them permission to disagree or argue. Her reasons for the tensions were discussed previously in section 6.3.5.

6.4.4   PROBLEM SOLVING AND INSCRIBING MATHEMATICAL REASONING

When the study began, the study group discussed the need for the students to engage in, interpret, and develop different ways to solve problems independently. On the communication and participation framework it was also planned that the students would develop their own ways to record their solution strategies. But two terms into the study Ava continued to closely structure what they did; reading the problems to them then leading discussion on possible strategies and outlining these step by step. Her detailed directives limited the students' opportunities to engage in pattern seeking and testing of conjectures. She placed little focus on the use of notational schemes other than to require that students match their verbal explanations with some form of representation which varied between informal notations, concrete material, or prepared representational recording forms.

At the end of the first phase our analysis of video excerpts led Ava to reconsider how the students could be scaffolded to engage in problem solving more autonomously. In accord with the communication and participation framework, she began to give the problems to the groups to read and work with independently. She closely monitored how they worked with the problem contexts and how they planned to model their solution strategies. She would halt group work briefly to examine possible conjectures and different pathways. At the same time, she began to structure how she wanted the students to notate. In the larger session she initially recorded: *I am just going to record…someone in your group can explain it and I will record it here and I will check with you that that's what you mean, so I understand where you are coming from because I don't want to be putting down an idea which isn't yours. Okay?* Thus she modeled how she wanted the students to record their reasoning collectively. A pattern of recording names of groups or individuals on the drafts to denote ownership of the reasoning was also established. Draft work was placed on display where they were accessible for later use.

Initially, the inscriptions were used as exploratory tools to explore and explain reasoning but in conjunction with shifts in the pathway Ava had mapped out they became tools used to justify and further validate mathematical thinking. For example, the following vignette shows how a solution strategy is outlined using an informal notation scheme. In the explanation the terms are revoiced and expanded on. In response to questioning, explanation is provided of how the group explored the application of their solution beyond the immediate problem situation to problem situations drawing on a generalisation to verify their own thinking.

An inscription used as a public reasoning tool to generalise
Jo presents an explanation in the large sharing session as Josefina notates:
Jo	What Josefina did was instead of using times five she timesed it by ten. Remember when I was talking about levels and how I timesed the place value? Well instead of 756 it went up to 7560. What she has done now because it is ten times ten now she is halving it because you have to times that by five and half of ten is five. So she halved it and 7000 in half equals 3500 and 500 in half equals 250 and 60 in half equals 30. Then what she did was added 3500 and 250 so she had 3750 then 30 so she got 3780. The strategy she is using is division.
Tipani	It was multiplication and division.
Rachel	Halving. She halved it, divided it by two. The thing is would that work all the time that strategy?
Josefina	We did talk about that. We knew it works really well with five because ten is easier to multiply because as Jo says it goes up in levels like in place value.
Jo	But I thought we could use other numbers like six and do five and then add on one more lot of marbles.
(Term 3 Week 9)

Inscriptions used as tools to validate reasoning publicly, over-structuring, author/ity,

Ava's initial notating provided a model that inscriptions were tools which publicly complemented and validated reasoning. Her explicit attention to verifying that she recorded what was explained emphasised the need for accuracy in recording communal reasoning. The classroom observations clearly illustrated that the manipulatives and problem situations were a beginning point from which many different inscriptions were developed and re-developed in the discursive interaction. Research literature (e.g., Lehrer & Schauble, 2005;

Roth & McGinn, 1998; Sawyer, 2006) suggests that the requirement to make available multiple ways to validate reasoning influences how inscriptions are developed and used in classroom communities. This was evident in this study.

Reflecting on her role as teacher Ava noted the tensions and contradictions she encountered when we analysed a video excerpt: *Did I need to read the problem? Here I was focusing on what and how I wanted the students to problem solve...I think that giving them the sheet with the four bags meant that they were already being shaped.* Over-structuring of activity is consistent with what many teachers do when they restructure the discourse towards inquiry. Ball (2001) and Schifter (2001) outline that often teachers focus on their own perspective of how a problem should be solved or they focus on pedagogical issues rather than the students' mathematical reasoning. In this situation, Ava expressed doubts about the students' ability to read and make sense of the problems together.

Close listening to student reasoning and 'noticing' (Sherin, 2002b) the students' collaborative problem solving skills when analysing the video excerpts with me, mediated the shift in her beliefs. When she gave a problem to a group without discussion she stated: *This is the struggle I have been working through...I thought it was my role to do the 'explaining the problem bit'. In actual fact I may have been doing the students a disservice unintentionally by removing the ownership of the problem from them.* Previously Ava had continued to be what Povey et al. (2004) describe as an "external authority" (p. 44). Her trusting that the students could autonomously read and problem solve together provided them with space in a relationship of shared "author/ity" (Povey, 1995, cited in Povey et al. p. 45).

6.4.5   JUSTIFYING AND GENERALISING MATHEMATICAL REASONING

Generalisations students often use implicitly had been discussed in the study group. Ava knew that these required explicit exploration with the students. However, her growth in recognising and using opportunities which emerged in the classroom dialogue developed slowly. The requirement that the students develop multiple ways to justify their reasoning resulted in them often spontaneously drawing on prior understandings which related to the

underlying structures and properties of mathematical numerical relationships. Through examination of student generated numerical generalisations within video excerpts Ava became more attuned to hearing them in class. She began to use them to explicitly develop and refine the thinking. For example, when a student talked about odd and even numbers she initiated further exploration of numbers beyond the students' immediate number capacity. Thus she eliminated the possibility that they could compute to verify their reasoning. In the following vignette Ava has intervened to use the explanation to examine the commutative property of multiplication applied in a fractional number context.

Generalising using the commutative property of multiplication

After extended discussion a group explain a solution strategy for the problem[6].	
Tipani	[draws a rectangle she divides into 5 sections] I have divided the cake into fifths and she took two fifths of the cake so I am going to shade in two fifths. But she could only eat half of what she took...given that there are two pieces, two divided by two equals one so she could only eat one.
Hinemoa	One piece of what she took.
Ava	One half of what she took. So how much did she eat?
Tipani	I think that they both ate the same amount because Alex ate one fifth which is equivalent to two tenths and Danielle ate...yeah they both ate one fifth or two tenths and so they are both equivalent fractions. So they both ate the same amount.
Ava	Using multiplication how could you word that...if you just looked at the numbers?
Adam	I have got an explanation.
Hone	It is because each one is using the same fraction and they have just turned it around.
Ava	Just turned it around?
Hone	Just like yesterday. It was the same for yesterday. So they are just turned around.
Ava	Think about the numbers and think about multiplication. There are two numbers there. What are you actually doing with the half and the two fifths?

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[6] Faith took $^7/_8$ of a cake but could only eat $^1/_3$ of what she took. Danielle took $^1/_3$ of a cake the same size as Faith's cake but could only eat $^7/_8$ of what she took. Which of them ate more? Explain why the answer works out as it does. Can you work out a number of ways to convince everybody else? Be ready to answer any questions other people will ask you.

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Hinemoa	You are multiplying them because it is just what Hone explained, they are just turned around. The numbers are flipped around but it's the same like we still end up with the same answer.
(Term 3 Week 7)	

Using student voiced generalisations, learning in the act of teaching

Ava's earlier lack of attention to numerical generalisations confirmed Carpenter et al.'s (2003) contention that many teachers despite extensive experience learning and teaching number have not abstracted the fundamental structures of arithmetic. The analysis reveals that it was my nudging and the explicit exploration of video excerpts of student voiced generalisations that mediated Ava's "algebra eyes and ears as a new way of both looking at the mathematics they are teaching and listening to students' thinking about it" (Blanton & Kaput, 2005, p. 440). As in Blanton and Kaput's study, this study showed that Ava's ability to spontaneously transform the mathematical talk into one which used algebraic reasoning grew and this was matched with an increased search by the students for wider ways to justify and validate their arguments.

As I described in the previous phase Ava was learning in the act of teaching and teaching in the act of learning. In the midst of lessons she was required to negotiate among the aspects of her own current knowledge and make on the spot decisions about how to adapt and use this to extend her students' thinking. These requirements created pedagogical tensions for her. After she watched the video excerpt of the preceding vignette she said: *I wondered after why I didn't get them to work with whole numbers to explore it. That's a problem I often meet up with. I don't grab the opportunity to do things like this because I am juggling too many balls in the air at once.* These tensions and the dilemma they caused her have similarly been identified by many researchers (e.g., Ball, 1993; Heaton, 2000; Sherin et al., 2004). Ava is responding to the fact that her transformed role within the inquiry environment has different intellectual demands (Hammer & Schifter, 2001).

6.4.6   USING MATHEMATICAL LANGUAGE

The shift evident in the classroom community toward increased use of justifications and generalisations was matched with a more proficient use of the mathematical language. Ava required that the students be specific in describing their actions and solution strategies. To extend their repertoire of mathematical talk she used a range of cues—hesitation, body language and facial expressions. Also, she would revoice, rephrase and extend the description, naming the strategy specifically: *you are plussing as your strategy, adding two each time so using addition as your strategy*. In turn, as illustrated in the following vignette, the students would appropriate Ava's actions. They would scaffold their fellow students when required or re-explain, revoice, and extend the argument.

Using the mathematical talk to scaffold others
In a sharing session Hemi explains a solution strategy for a problem[7]:

Hemi	So Peter was wrong because he had five eighths which equals two quarters [shades in 1/2 and 1/8 of a shape] so Jack was right because he has three quarters [Shades in 3/4 of a second shape] which is six eighths.
Hone	How can you prove it to us?

Hemi's body language indicates discomfit so Pania intercedes:

Pania	I can show them. This group, that's Hemi and his group are saying that Jack is arguing that five eighths is less than three quarters [Pania draws a rectangle and sections it in eighths. Three times she records 2/8 as she shades two sections then she records 6/8. She draws another rectangle and this time shades in 5/8]. They are saying the three quarters is an equivalent fraction to six eighths. Like them I can prove it is bigger because look this is only five eighths, which is smaller than six eighths. Is that right you guys?

(Term 3 Week5)

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[7] Problem: Peter and Jack are having an argument over their homework. They have been asked if 5/8 is bigger than 3/4. Peter is arguing that because the numbers are bigger in 5/8 then 5/8 must be bigger than 3/4. Is he right or wrong? Can you work out some different ways to explain how you would solve their argument? You need to work together to convince either Peter or Jack using a range of ways including drawings, diagrams and symbols. Discuss the mathematics argument you would use and the different challenges Peter or Jack might make to your argument.

According to Forman and McPhail (1993) if students are to participate in peer collaboration they need a shared discourse. Ava's actions made available a shared understanding of how to participate in the more specialised discourse (Gee, 1992; Gee & Clinton, 2000). This involved the students not only becoming more precise in their use of mathematical words, but also more proficient in collaborating in mathematical argumentation. Within the constructed zones of proximal development, developed in the inquiry environment, not only did Ava pull the students forward (Lerman, 2001) but the students pulled each other forward. Through their appropriation of Ava's actions and words they extended and rephrased each other's short utterances and used their mathematical explanations, terms, and definitions as their own.

6.4.7   SUMMARY OF THE SECOND PHASE OF THE STUDY

A close relationship is evident in the data between the communication and participation framework, Ava and her student's changed roles, and transformation of the participation and communication patterns. The emphasis placed on the use of inquiry and argumentation and the intellectual space Ava provided had the effect of scaffolding consistent use of exploratory talk (Mercer, 2000) during collaborative argumentation (Andriessen, 2006). Increasingly the students engaged publicly in collaborative construction and critical analysis of their mathematical reasoning. A community of inquirers had been established in which the students were using many interrelated mathematical practices with increasing proficiency. Closely aligned to the increase in the efficient use of mathematical practices was Ava's focus on scaffolding discourse in which student thinking was central.

6.5   OWNING THE MATHEMATICAL PRACTICES IN A COMMUNITY OF MATHEMATICAL INQUIRY

Our collaborative discussion and verbal and written analysis of classroom observations prompted Ava to plan further shifts in the classroom interaction patterns. Ava voiced her observations that as the students gained confidence to challenge and debate their reasoning

all their roles had changed. She noted that she was increasingly facilitating the dialogue and the students were taking ownership of the discourse. Maintaining a press toward mathematical inquiry and argumentation continued to be an emphasis but Ava wanted to further increase the flow of mathematical discourse driven by student thinking. She wanted to develop ways to: *support students when they want to inject, butt in with another idea which helps all our thinking.* She explained that this remained: *a learning curve for all of us because we were so used to participating by taking turns.*

6.5.1 MAINTAINING INTELLECTUAL PARTNERSHIP IN COLLECTIVE INQUIRY AND ARGUMENTATION

Whilst maintaining the social norms previously constituted Ava now expected the students to actively assume responsibility for their sense-making: *It is not up to me to actually go around and focus on certain people and say ah that one looks confused... It is not my job to focus in on somebody and say you look like you don't understand...You look like you need to be asking a question. It is your job to be listening, to be watching and trying to make sense of what someone is explaining. If it is not clear to you, you need to jump in straight away.* Ava wanted the students to act autonomously in the discourse. She voiced an observation that when she participated in the small and the larger discussions the students looked to her for permission to question or justify the reasoning. She considered that this often interrupted the interactive flow. So, she introduced the concept of 'no hands up' particularly when there were many questions and challenges for an explanation. Although the students were initially hesitant she prompted them with directives: *Hone you wanted to ask a question? Come on just jump in and do it.* The students also appropriated the words "just butt in" and used them to prompt their less confident peers. Illustrated in the following vignette is how this resulted in an interactive flow of conversation in which the students used proficient forms of mathematical practices to closely examine, analyse, and validate their mathematical reasoning as they worked to solve the following problem.

Facilitating the flow of interactive student led discourse

Beau is doing a T.V. contest and this is a question which is asked.
Which of the following fractions best represents the value of ( )?
$5/10$
$5/100$
$1/100$
$5/1000$
He has to give a reason for his answer and then when questioned further explain and justify his responses?
First of all discuss your answers and then be ready to explain and justify them. Think also of questions the group might ask you and other ways of explaining what they ask.

0   ( ) $1/10$

Mahaki and Chanal are explaining in a large sharing session a solution strategy for the previous problem:

Chanal   [records a numberline and records 1/10 at the end]. We think that it is five hundredths. We think that that's the end [points at 1/10] because one tenth can't go anymore back. You can't equivalent it less. So if it was two tenths you could have done it…you will have to go decimal points with your fractions.

Many students indicate they have a question.

Ava   [prompts] Just butt in with your questions.
Sandra   Why do you think you have to do decimal points with the one tenth?
Chanal   Because with one tenth and fractions you can't go half of just one tenth…
Sandra   So you thought it was five hundredths?
Mahaki   [records the notation scheme as he explains using fractions and decimals] Yeah. We thought it was five hundredths because one tenth times ten equals ten hundredths and if you half that ten it will be five, five hundredths.
Jae   If you half the top one you have to half the bottom one?
Chanal   No, because we are not halving the numbers we are halving the fraction.
Rangi   Why did you timesed it?
Mahaki   So we could get the hundredths. So we could half it.

After further questions and challenge to the terms, formal and informal representations and the reasoning Sandra introduces an alternative explanation.

Sandra   [marks above 1/10, 10%] There's also a different type of way you can do this. The five would have to stand for five percent.
Ava   So what Sandra has shown you, is that one tenth or ten percent which she said was another way is not near half, not near five tenths. It is the first mark. So half of that is five one hundredths because as Chanal and Mahaki told you one tenth is equivalent to ten hundredths and half of that segment is five one hundredths. Certainly not half of ten tenths, or it's equivalence of one hundred one hundredths, or one whole as Mahaki explained so well.
Aroha   [asks the two boys] So why do you put it in decimals?

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Sandra	I know, because there's three different ways to basically explain a fraction, the fraction way, a decimal point way and a percentage way. That's why he has picked one of them. Instead of just doing the fraction or percentage, he's picked the decimal point way, because he may think that's actually his easier point of doing the fraction way.
(Term 3 Week 10)	

Through these actions Ava encouraged student agency over when and how they participated in the mathematical discourse. At other times she explicitly intervened to facilitate slower exchanges. This occurred particularly when new mathematical topics were introduced or when she considered the mathematical concepts under consideration particularly challenging. For example, in the following vignette she participated in a group discussion of the problem used in the previous vignette. This time her interventions maintained a slower flow of productive discourse which enabled the students to focus on inquiry and challenge and ensured that they reflectively considered and reconsidered their reasoning.

	Facilitating a slower flow of productive mathematical discourse
Tipani	[draws a numberline, records 0 then 9 marks and then 1/10] Here is 0 and 1/10.
Ava	Now take you time. Think about it. Right any questions?
Pania	Why are you doing those lines?
Tipani	[records 5/100 at the middle mark] Because each of those lines is representing one tenth, I mean ten tenths. I am thinking that this one is meant to be 5/100.
Mahaki	Why?
Tipani	Basically because of what you said Mahaki.
Ava	Which was? Explain it in your own words and see if Mahaki agrees.
Tipani	That if you times… ten by ten…well I am not actually that sure. I just think that it is five one hundredths. I don't think that it is five one thousandths.
Ava	Well what do you think Mahaki, and you other people who heard what Mahaki explained? Let's take a look at these fractions and think about what Tipani and Mahaki were saying. What do you see when you look at these fractions…what other ways can they be represented apart from that?
Chanal	[points at the 5/100 mark] Mahaki said that one tenth can be ten percent because if you times one by ten you get ten, and you times the ten by ten you get one hundred. So that will be one tenth is like ten percent, so in the middle that will be five percent there.

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Ava	You accept that Mahaki? So what you are saying is that that means five parts out of a hundred and the one tenth there means ten out of a hundred. So what is this one? [Points at 5/10]
Chanal	That is fifty percent.
Pania	So how?
Ava	Yes you jump in here Chanal, if you can explain it in a different way.

Chanal leans forward and picks up the problem sheet with the diagram of a numberline of one divided into ten segments as he says:

Chanal	[points at the first segment on the numberline] I know what. If you go back to there and just pretend you shrink that down to there. There's a hundred right? So that half way mark in brackets would be right there [points at the position it would be in if you had a whole number line not just to one tenth and it represented 1/10] and that would be ten percent and if you halved that ten percent it would be five.
Pania	Five what?
Chanal	[records 5% and 5/100] Five percent or five hundredth.
Ava	Are you all convinced? Or do you want to ask some more questions?
Mahaki	It is five hundredth because as Chanal said that thing there would be just like a little piece of this line…But the other way is to go the percent way. You get ten percent and then half that. That's the quickest way to explain it.

(Term 3 Week 10)

Mutual engagement, intellectual agency and collaborative partnerships, scaffolded participation in mathematical argumentation, exploratory talk

Ava had provided the students with a predictable framework for inquiry and argumentation. Within this frame she had established that the students were required to take personal ownership of their sense-making and justification. Through providing the students with ways to circumvent the need to seek permission to speak she positioned the students as more autonomous participants in the mathematical interactions. In turn, the students responded by participating in the dialogue with decreased teacher assistance. They would analyse the mathematical reasoning then often bypass Ava to ask a question or challenge the reasoning. Or they would step in ahead of Ava's prompting to respond to questions and justify or offer alternative ways of reasoning. Evident in the data is student development of intellectual agency within a climate of mutual engagement (Wenger, 1998) and respect. Ava and her students had constituted a mathematical community within an intellectual partnership. As Cobb (2000a) explains, when students view themselves as autonomous

learners in a mathematical community they are able, independently of teacher or text, to validate their own and others' contributions and reach consensus in the argument.

Ava drew on her 'insider' (Bassey, 1995; Jaworski, 2003) knowledge of classroom culture and from moment to moment selected how she positioned herself in the discourse. In this situation, as the more experienced 'knower and user' of mathematics she worked within negotiated zones of proximal development (Vygotsky, 1978) guiding and scaffolding the students in collaborative argumentation. Andriessen (2006) explains that "when students collaborate in argumentation in the classroom, they are arguing to learn", activity that involves "elaboration, reasoning, and reflection" (p. 443). Ava's actions in slowing the discourse provided intellectual space (Watson & Mason, 2005)—a platform for all participants to access and develop reasoning from. In this space the different students were positioned and repositioned to make and justify claims and counterclaims. Arguments were extended through revoicing (O'Connor, 2002), rephrasing, and elaborating on. In the discursive interaction, the words and reasoning used by others were appropriated, explored, reworked and extended before they became a communal 'voice' (Brown & Renshaw, 2000).

The collaborative construction of a shared view was not always premised on immediate consensus. Instead in this community, as in other inquiry communities reported on (e.g., Brown & Renshaw, 2000; Goos, 2004; Mercer & Wegerif, 1999a, 1999b; Rojas-Drummond & Zapata, 2004), a lack of understanding, dissension and disagreement often acted as the catalyst for further examination and evaluation of the reasoning. The students' use of specific words ("because", "I think", "but why", "so if you", and "I agree") to negotiate and reach a common perspective were evident. These words and phrases Mercer and his colleagues identified as those used in exploratory talk. Exploratory talk as I explained in Chapter 3 is a form of talk which supports proficient use of mathematical practices.

6.5.2 TRANSFORMING INFORMAL INSCRIPTIONS TO FORMAL NOTATION SCHEMES

In this final phase of the study, informal inscriptions and formal notation schemes were important tools which supported a range of mathematical practices. Ava continued to place drafts of the inscriptions on the wall and this provided the students with public access to a range of solution strategies. They regularly reviewed and discussed these independently, particularly when arguing about possible solution strategies, or when making comparisons, or seeking more proficient solution strategies. The inscription drafts on the wall were also regularly used to "fold back to" (McClain & Cobb, 1998, p. 59) during argumentation or for reflective analysis by the groups. Evident in the data is how they would be appropriated, tried out, extended and innovated on and used to scaffold more proficient forms of notation.

In group work Ava pressed the students to use mathematical symbols and formal notation schemes alongside their less formal notational schemes. In the sharing sessions Ava would often build on a group's informal notation scheme to develop more formal notation schemes. The students would also begin from informal inscriptions and then, in search of ways to convince others, develop more formal ways to explain. The students became adept at translating across their multiple representations including both formal and informal schemes to provide explanatory justification as illustrated in the following vignette.

Using multiple representations to validate reasoning
The small group have extensively explored a problem and constructed multiple representations of their reasoning including a formal notation scheme. They are asked for justification and one student explains the reasoning used by another in their group:
Rangi   [using diagrams, fraction symbols and gesturing] First she started off with twelve eighths which she added to six eighths which she knew was two wholes plus two eighths, then she plussed ten eighths because she knew that that made three wholes and one half. Then she was adding five eighths which made four wholes and one eighth and then she added seven eighths onto her four wholes and one eighth which made five wholes.
(Term 3 Week 10)

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Abstracting and generalising notation schemes, folding and dropping back, appropriating inscriptions

The press to use more proficient notation schemes supported a shift from the use of literal interpretations embedded in real world contexts toward mathematising—abstracting and generalising and using more formal notation schemes. Concrete situations provided a starting point and in the reflective dialogue the students gradually advanced from modeling informal schemes to the use of more formal symbolic notation. Consistent with the findings of Whitenack and Knipping (2002), inquiry and challenge also increased the press toward formal notation. This occurred either when a group member could not make sense of an inscription, or when a student introduced a more sophisticated form which progressed all the group's thinking. In these situations, Ava would intervene and explore with them formal notating patterns. However, although Ava specifically pressed towards the use of formal notation schemes, she also recognised the value of informal models to make the reasoning visible. She would select a group who had used informal inscriptions or return to an explanation which had been explained using a less proficient notation scheme as a means to fold or drop back to when needed, to clarify the reasoning (McClain & Cobb, 1998). For example, she explained when we analysed a video excerpt: *I picked Rangi first because I knew that they would use their diagram as a model and that would really help the others to see how the pieces combined.*

6.5.3   INCREASING THE PRESS FOR GENERALISING REASONING

In this final phase of study Ava had become attuned to hearing spontaneously voiced generalisations used to justify reasoning and now regularly built on them. She introduced a requirement that students analyse, compare, and justify differences in efficiency and sophistication between explanations. To extend how the students participated in constructing and exploring generalisations Ava introduced a set of questions which potentially supported the students to generalise. She modeled their use, regularly asking questions like 'does it always work' and 'how can you know for sure'. She also constructed a wall chart of similar questions and prompted the students to use them. She used open-ended problems which had been discussed and written in the study group. The following

vignette illustrates her students working on equivalence generalisations. In the rich discussion, Ava's request for further backing shifts the lens from a single model of number combinations towards a more general model for number combinations.

Generalising concepts of equality

Marama, Arohia and Rangi provide a solution strategy for a problem[8] which they have verified using other combinations in response to an expectation that they were required to convince their listeners.

Arohia	What we did is we kept on doing ninety and we kept on making the middle one...
Marama	Going smaller.
Arohia	Going decreasing and we made the last line going up, increasing. We went nine hundred, ninety, and then ten. Then we went nine hundred, eighty and twenty.
Rangi	It's going down in tens and then it's going up in tens.
Rongo/Hemi	Can you prove that...it equals up to the number like one thousand?
Hinemoa	Does it always work?
Arohia	Yeah, to us it always does. But I can do it by using another number.
Ava	That is even better. Arohia what are you saying? Can you tell us?
Rangi	[records 1000 = 900, 95, 5; 900, 85, 15; 900, 80, 20] I can because all you have to do...first I go in fives and I go up in fives and then I go...
Hone	Why do you do that?
Rangi	Well because I know that ninety five plus five equals one hundred and then ninety plus ten equals one hundred.
Hone	So everything adds up to one hundred? So it is all the same as one hundred?
Rangi	Then I write nine hundred all the way down. So if I add them together they all equal one thousand.
Ava	I know what. Just write down B and in the third place write C. Now that could be anything all right? Now what are you going to do to B? Make a conjecture.
Sandra	You could do B take away five and C take away five.
Hemi	No C plus five
Sandra	It's like what we thought. Basically you can put any number there. It does work with any number.
Ava	And?
Sandra	Like B take away nine hundred and C plus nine hundred. You are keeping the same amount.
Ava	As long as you...

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[8] Mrs Hay goes shopping for Christmas presents. She spends $1000 on three presents and she only uses dollars not cents for them but she uses all of her money. Can you explore patterns for the different amounts each present might have cost? Is there any way you could work out all the combinations that are possible without listing every single one?

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Sandra	Are balancing. So if you go B minus seven hundred and then C plus six hundred it would not work out because it's not balanced. You would have one hundred less.
(Term 4 Week 1)	

Powerful questions, abstracting and mathematising, open-ended problems as 'example spaces'

Making comparisons and searching for patterns and connections between the solution strategies were tools Ava used to develop more generalised reasoning patterns with the students. Likewise, her modeling of specific questions and the wall chart supported students' search for patterns and relationships across problem situations and solutions. Evident in the data is students' increased use of logical connectives (because, but, if…then) to generalise. As other studies have shown (e.g., Blanton & Kaput, 2003; Strom et al., 2001; Zack, 1997, 1999) teacher questioning and the use of specific questions were important elements in classrooms in which students participated in argumentation and justified through generalised reasoning. These studies emphasised the key role of the teacher in facilitating the discourse. In this study Ava's facilitation of sharing sessions was the key to how the students mathematised problem situations so that they became models for more abstract and generalised mathematical thinking. She would structure the dialogue so that gradually student reasoning was pressed toward abstraction.

Ava was mindful that for the students to be able to formulate, model and generalise solutions of one problem situation—as models for problem situations—they needed opportunities to make connections in patterns and relations across problem situations. At times she transformed existing material so that the students were required to provide multiple conjectures. These and the use of open-ended problems provided what Watson and Mason (2005) termed 'example spaces'. These problems offered the students many opportunities in their groups to suspend judgement to conceptualise and re-conceptualise, represent and re-present problem solutions in many forms as they searched for plausible patterns or counterexamples.

6.5.4 SUMMARY OF THE THIRD PHASE OF THE STUDY

The participation and communication patterns of inquiry provided the basis to further develop intellectual partnerships within the classroom community. Justification and the use of many inscriptions provided a platform for the development and exploration of generalised reasoning. Ava facilitated increased student autonomy within student discourse. Mutual engagement in learning partnerships evolved within reciprocal interaction which created multi-directional zones of proximal development.

6.6 SUMMARY

This chapter has documented the journey Ava and her students made as a community of mathematical inquiry was constructed. The tools which mediated the transformation of the classroom's learning culture were described. These included the communication and participation framework which guided the pedagogical actions Ava took to engage the students in increasingly proficient use of mathematical practices.

The classroom culture began as what Wood and McNeal (2003) termed a conventional one. Through steady consistent shifts in the communication and participation patterns the classroom culture became strategy reporting and then an inquiry or argument culture. Change was initiated in the mathematical discourse patterns through specific emphasis placed on modeling the processes of mathematical inquiry within a safe learning environment. As changes in the discourse patterns shifted toward a predominant use of inquiry and argumentation, so too were the mathematical practices extended. Both Ava and her students' use of questions and prompts were important factors which pressed reasoning to higher and more complex intellectual levels. The variations in interaction patterns in the different classroom cultures had differential outcomes for student learning. Student autonomy, responsibility for sense-making of reasoning was most evident in the argument culture. Similarly, the mathematical practices the community members participated in and used became interrelated tools used to justify and validate reasoning.

Inducting students into a culture of inquiry and argumentation responded to both the pedagogical actions Ava took and the beliefs she held about doing and using mathematics. The findings in this chapter showed how each shift in the mathematical discourse and mathematical practices matched shifts in Ava's beliefs toward her students engaging in inquiry and argumentation. The following chapter describes a second teacher Moana who held a different set of beliefs about doing and using mathematics to those held by Ava. Description is provided of how these shaped her construction of a mathematical community of inquiry and the mathematical practices which evolved.

CHAPTER SEVEN

LEARNING AND USING MATHEMATICAL PRACTICES IN A COMMUNITY OF MATHEMATICAL INQUIRY: MOANA

> The transformative practice of a learning community offers an ideal context for developing new understandings because the community sustains change as part of an identity of participation. (Wenger, 1998, p. 215)

7.1 INTRODUCTION

The previous chapter described the transformation of participation and communication patterns which resulted in construction of a community of mathematical inquiry. This chapter also documents the transformation of a classroom community but it illustrates how a different set of beliefs about doing and learning mathematics held by the teacher shaped a slower, more circuitous route to developing a community of mathematical inquiry.

Section 7.2 outlines Moana's beliefs about doing and using mathematics at the beginning of the research. This provides an important background to understanding her subsequent actions. Section 7.3 describes Moana's pedagogical actions which mediated student participation in communicating mathematical reasoning. Effects of the previous learning culture are examined and ways in which change was initiated are described. Descriptions are provided of the wide resources Moana drew on when transforming the interaction patterns from conventional to strategy reporting.

Section 7.4 describes the pedagogical actions Moana took to engage her students at higher intellectual levels of inquiry. A close relationship is illustrated between Moana's previous experiences as a mathematical learner and user and how she positioned student participation in the mathematical discourse. Section 7.5 describes how Moana drew on her students' social and cultural identities to scaffold their appropriation of the mathematical discourse as a social language (Bakhtin, 1984; Gee, 1992). Participants' shifting roles in the

community towards shared ownership of the discourse and mutual engagement in a range of reasoning and performative mathematical actions (Van Oers, 2001) are described.

7.2   TEACHER CASE STUDY TWO: MOANA

In an interview at the start of the research study Moana outlined her understanding of the nature of mathematics. Although she had secondary school mathematics qualifications she explained that she disliked mathematics and lacked confidence in the use of mathematics beyond a restricted school situation. She considered that she was not as clever as those individuals who enjoyed mathematics or appeared more confident in their use and knowledge of mathematics. Thus Moana had constructed a situated identity (Gee, 1999) about herself as a mathematician. She drew on this identity to explain both her personal dislike for mathematics and her difficulties in doing and using mathematics, despite her apparent academic success.

Moana stated that for her mathematics was a body of knowledge and a set of rules which were useful tools in a school context for 'school mathematics'. But beyond school she said she was not confident about applying mathematics to real life problem situations. Moana's beliefs about mathematics as a static body of knowledge are consistent with a traditional view which Stipek and her colleagues (2001) report many teachers hold. In this view, mathematics involves "a set of rules and procedures that are applied to yield one answer…without necessarily understanding what they represent" (p. 214).

In this initial interview Moana outlined how she considered mathematics to be: *a valuable commodity in society*. But she added that she considered that there were many: *barriers to access that knowledge* for many of her diverse group of students. To enable them to access the mathematical knowledge she explained that she used particular pedagogy practices which she considered best suited her students' learning needs and learning styles. In her description of her predominantly Maori and Pasifika students she said that they were: *more practical, hands-on visual learners* and so she said that she always used concrete and manipulative material when teaching mathematics. This label Moana had attached to her

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students conforms to one which Anthony and Walshaw (2007) contend is often used to describe Maori and Pasifika students—kinaesthetic learners. However, when teachers attach the label to Maori and Pasifika students they make assumptions based on what they perceive their learning needs to be. Alton-Lee (2003) maintains that this often results in negative outcomes for these specific students.

Moana explained that she combined a physical hands-on approach with: *a lot of routines and structure. Writing on the lines, from left to right, numbers keeping them nice and tidy in their boxes...lots of practice... lots of hands on...then dragging it all back, putting it on the board...practice over and over.* She described how she was the one who did most of the talking in her mathematics lessons because she considered that her role was to instruct students on how to use the various mathematical procedures. She believed that if they listened carefully and then practised the procedures she showed them they should learn what was required. Her description of the classroom context she had constructed is consistent with what Wood and McNeal (2003) describe as a traditional or conventional classroom. The way in which Moana described her use of mathematics talk as a tool which focused on transmitting knowledge, corresponds with descriptions of traditional classrooms where teacher talk predominates (Mehan, 1979; Wood, 1998). Moreover, the stance she took as personally responsible for her students to learn specific rules and procedures corresponds to that taken by many teachers (Stigler & Hiebert, 1999). Stigler and Hiebert describe these teachers as positioning themselves as external authority figures in conventional classrooms.

Moana had recently been a participant in professional development in the New Zealand Numeracy Development Project (Ministry of Education, 2004a). She described how she had adapted the project's knowledge and strategy activities so that they were always at a concrete and manipulative level. She justified this by referring back to her need to maintain a teaching style which best met the learning needs of her students. Although a central tenet of the NDP is to develop student use and explanation of a range of strategies, Moana explained that she had focused on the use of concrete materials. She said her students learnt through 'doing' and that they found explaining difficult. Moana's adaptation of the NDP

material meant that she could continue to teach in a way which conformed to her own beliefs about the nature of mathematics and what she perceived were the learning needs of her students.

7.3   CHANGING THE INTERACTION NORMS TOWARDS A COMMUNITY OF MATHEMATICAL INQUIRY

Moana had been a quiet member of the study group when the communication and participation framework was discussed and explored at the end of Term One. However, at the combined study group of senior and middle school teachers in the second term Moana participated actively in discussions. As she watched the senior classrooms' video records she probed and questioned the teachers about how they had initiated changes to their classroom communication and participation patterns. She questioned the effect the changes were having on their students. Then she used their descriptions of how they had restructured their classrooms to establish changes she wanted to make. In discussion with me she stated that her immediate focus was on developing student ability to construct and present mathematical explanations. In order to achieve this she described a need to change the ways in which the students interacted and talked together in mathematics.

7.3.1   THE INITIAL START TO CHANGE THE COMMUNICATION AND PARTICIPATION PATTERNS

Moana introduced her expectations for collaborative behaviour by using a laminated chart obtained from another teacher. She told the students that they would be working in small groups and then she read the chart to the students as she displayed it prominently on the wall: *so when you are working and you have a person in your group who can't manage and cope, they need to go and they need to look at the solution pathway and the participation norms.* Each lesson in the first week of the research followed a similar pattern. After students were placed in groups or before large group sessions began Moana read aloud sections of the chart. However, she did not explore or discuss what the chart meant with her students. In this initial stage she discussed what listening meant but did not extend

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description beyond the physical act of listening. For example, when she asked the students what they needed to do to listen, a student said: *use taringa [ears]*. To this, she responded: *yes listen* and that ended the discussion.

At the start of the first research phase Moana used the problems which had been devised in the study group (see Appendix G) to support development of early algebraic reasoning. Consistently each lesson began with the students organised into small groups. Then Moana read them the problem, provided them with concrete material and then instructed them to: *work together and find an answer*. Moana walked around the groups and regularly used comments like: *come on participate people, what did I say about active participation?* As the students worked in these groups observational data revealed a predominant use of disputational or cumulative talk—a form of talk Mercer (2000) describes as not conducive to developing shared thinking.

Small group activity was followed by large group sharing sessions. In these sessions the students constantly interjected and made negative comments to each other. Moana, in turn, asked single answer questions and then validated the answers she wanted by recording them on the whiteboard.

Moana's actions illustrate the considerable challenges Ball and Lampert (1999) contend teachers face in constructing inquiry environments. Her behaviour I interpreted as indicating that she had limited vision of what teaching and learning in such an environment meant. Her students were unable to understand the roles they were now being asked to take. As Hufferd-Ackles and her colleagues (2004) describe, many teachers have had no personal knowledge or experience in learning, or teaching, in inquiry environments. Moana showed her novice approach in her use of her colleague's chart. Her inexperience was also evident when she assumed that telling students to listen and participate in groups was sufficient to establish the social interaction norms she required. She had put in place edges of the changes required but her core pedagogical practices remained largely unchanged.

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7.3.2 CONSTITUTING SHARED MATHEMATICAL TALK

In informal discussion during the first week of the study Moana indicated her need to further consider how she could change the interaction patterns. Our collaborative review of events in this first week and examination of classroom video recordings led to a closer examination of the communication and participation framework. Smaller, more incremental steps (see Appendix K) in the communication and participation patterns were planned to guide the change.

In the first instance to remediate how the students interacted, Moana noted the need for an immediate focus on increasing student cooperation in listening and talking. So that she could carefully guide and shape how the students participated in mathematical activity Moana returned to teaching mathematics in a large group. At this stage she had seen how the students quickly lost focus of the discussion and became disruptive in the small groups. Not using small groups was thus an intermediary step; she eventually wanted to develop student ability to work in them.

Her immediate action as she led whole class mathematical activity and discussion was to ask the students to talk together in pairs. For example, in one of these early lessons Moana made an array of two sets of eight counters then directed the students to consider: *If I've got one set of eight there and another set of eight, how many sets of eight have I got? I don't want you to answer that. I want you to talk. I want you to turn and talk to each other.* Following their discussion she asked one member to report what the other had explained. In another instance, she provided a problem and required that together they make a conjecture and discuss their reasoning but she emphasised their need to both be able to report back. At other times, to keep the focus on listening and sense-making she asked one member of the pair to explain their reasoning and then asked the other to outline and model with materials what their partner said.

Through daily discussion and our informal analysis of how different students participated in the mathematical talk Moana began to more confidently outline her requirements for collaborative activity. Initially she had depended on instructions and strategies she saw

modeled by other study group members when she viewed video records of their classrooms but now she tentatively explored and developed her own repertoire of talk. She used this to establish with the students what she meant by active participation and collective engagement in mathematical discourse. Each lesson began with an outline of her requirements for active participation and she closely monitored how the students worked together. In the following vignette examples are provided of how she adapted her instructions specifically to what she saw happening during shared mathematical activity.

Shaping ways to talk and actively listen to mathematical reasoning

Moana observed that the students had begun to record without discussion. She intervened and told them: Moana   Before you pick up a pen or touch the paper you need to discuss it first and I want you to show me how you can work together listening and building on each other's thinking with those questions. *(Term 2 Week 6)*

After Moana had explained a problem and provided concrete material she stopped them and directed: Moana   I want you to talk to each other before you even touch the sticks. Lots of talking and listening and I might ask you what someone said, not you, so you need to discuss things please and make sense of what someone else says. Listen carefully to each other. I want you to discuss what is happening in your patterns of two…and how many you have. *(Term 2 Week 6)*

Conflicts in changing beliefs, reshaping the rules of talk.

The classroom culture and the interactions in it had previously been shaped by the beliefs Moana had about the nature of mathematics and what it meant to learn mathematics. These beliefs remained robust throughout the first part of this study. Researchers (e.g., Boaler, 2002a; Lampert, 2001) argue that students in schools learn particular mathematical dispositions through the 'school mathematics' they experience. Beliefs that mathematics ideas are predetermined and unarguable are fostered when prior learning experiences emphasise scripted construction of roles, rules and procedures (Hamm & Perry, 2002). This had been Moana's prior experiences and she in turn had constructed a culture in which her students were developing similar dispositions and beliefs. Moana illustrated this view in a

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reflection, recorded after a lesson, as to why the students had difficulties engaging in the talk: *I have always told them, not asked them to talk together. I am used to doing the talking and they are used to me talking too.*

Although Moana discussed the changes she wanted to make to classroom communication and participation patterns with me and the other study group members she was ambivalent about them. She was aware of the contradictions she was causing in her students' beliefs about what constituted a mathematics lesson. She noted as she reviewed a video record of a lesson observation at the end of the second week: *I really wanted these children to do this but they really couldn't see any sense in it at all…it was more of the mechanical or cosmetic workings on how I wanted them to go about talking and listening.* Researchers (e.g., Ball & Lampert, 1999; Mercer, 2000; Wells, 1999) report that students may be challenged by the different roles and scripts they are required to take when shifting from conventional to inquiry classrooms. Conflicts and contradictions in the students' beliefs were expected given the former sociocultural norms which had prevailed including the external attribution of authority to the teacher and more able or dominant students

Contradictions existed also because although Moana had taken small incremental steps towards change, she retained the central position as the authority in the classroom. When she viewed a video excerpt of a classroom observation she critically described her pedagogical actions as: *book style participation norms…going through the cosmetics…but not really going into it…not really actually unearthing it…yes you must validate everybody's answers but at the end of the day just give me the answer.* Hamm and Perry (2002) maintain that teachers relinquishing authority to the classroom community, stepping away "from a central role as classroom leader to allow true public discourse about mathematical ideas" (p. 136), comprises a major challenge. Moana illustrated the on-going difficulties she had in adopting a more facilitative approach although she remained critically aware of her intentions to do so.

7.3.3 CONSTRUCTING MORE INCLUSIVE SHARING OF THE TALK IN THE COMMUNITY

In the second week of data collection Moana voiced her concern that the girls were often passive participants in the discussions. She described their lack of confidence to talk and aligned this with her own experiences as a school student. She outlined her observations of the difficulties the Maori and Pasifika girls in particular had when required to speak to, or question, boys. A decision was made to initially put girls and boys in separate pairs rather than in mixed pairs and monitor the outcome. At times Moana was even more specific and paired particular girls, for example Pasifika girls together. She explained that she wanted all students to develop a mathematical voice and identity and so she was exploring ways to increase how specific students engaged in the mathematical tasks.

Moana read and discussed with the study group a research article[1] which described the pedagogical strategies a teacher used with diverse learners and as a result she became more mindful that an environment which supported her students as risk-takers was needed. Engaging in the discourse of inquiry was such a new experience for Moana and her students that many were understandably hesitant. It was not unusual for a less confident student to stop mid-sentence or indicate that they wanted to ask a question, but withdraw when asked to speak and say "I forgot". At other times, a student would make a conjecture but then sit silently when questioned or shake their head when asked by Moana to respond to further questioning. To establish a caring environment Moana talked with the students about inclusion, support, and collegiality. She closely monitored less confident students' participation, actively supporting them to participate appropriately. For example, when a Pasifika girl responded quietly to a question, she interceded and told her: *you don't have to whisper. You can talk because you want to make sure that you are heard.* She participated in discussions and provided specific scaffolding for less confident students, eliciting and extending their mathematical responses. For example, Moana asked Tere: *What are we*

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[1] White, D. Y. (2003). Promoting productive mathematical classroom discourse with diverse students. *The Journal of Mathematical Behaviour, 22*(1), 37-53.

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looking at here? When Tere responded with: o*nes* Moana probed further: o*ne group of...* and thus drew the more extended answer of: o*h... twenty, one group of twenty.*

At the end of the first month Moana observed that the pairs were able to more collaboratively construct and examine their explanations and so she began to vary the number of students working together. She tentatively placed boys and girls together but continued to closely monitor Pasifika and Maori girls' participation. As she gradually increased group sizes she affirmed their skills of working together and introduced and talked about the notion that they were all members of one whanau (family). She drew on their home experiences and together they explored how family members supported each other. She emphasised that in such groupings different levels of expertise exist but tasks are accomplished through the cooperative skills of all members. She described herself as a whanau member who took the lead in some situations and in other situations depended on other whanau member's expertise.

Moana's introduction of the whanau concept indicated that a gradual shift in positioning had occurred in the classroom culture. Moana had begun to move from the position of total authority—the teacher in control of the discourse—to become a participant in, and facilitator of the mathematical discourse. This shift in positioning aligned to changes in her expectations and obligations of the students. Previously her requests for explanation had used 'me' and 'I' with little reference to other participants: *Tell **me** about yours...Speak up Rata. **I** like the way you are thinking but **I** need to hear you.* Now in an observable shift Moana positioned herself as a participant and listener illustrating this in her request: *can you show **us** what four groups of four look like because **we** want to think of other ways than the adding on, the skip counting don't **we**?*

Changing beliefs, changing roles, mediating different forms of participation in the discourse

Cobb and Hodge (2002) explain that often teachers and researchers attribute poor mathematical achievement to specific attributes the students themselves lack. Similarly,

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Moana attributed her own problems with mathematics to qualities she lacked stating in an interview before the research began: *I didn't have the ideas that other kids had so I just never said anything. I just thought they were all brighter than me.* In the same interview she described her students' low achievement levels attributing these to their lack of prior experiences at home, their lack of interest and engagement in mathematics, and their passive approach to learning. In using a "deficit-model approach" (Civil & Planas, 2004, p. 7) that attributed the problems in constructing mathematical understandings, hers included, to individual traits she undervalued the role of the social learning context. Shifting classroom communication patterns and awarding increased focus on the social context was understandably the site of many tensions: *at some point I am thinking is this maths? Is this going to work? Trusting that someone has the best interests of these children in mind with such an emphasis on talking and them hearing what they say and I wonder about if they get confused by what they say to each other.*

Moana's actions in explicitly scaffolding specific groups of students were designed to assist all students to access the shared discourse. Many researchers (e.g., Civil & Planas, 2004; Khisty & Chval, 2002; Planas & Gorgorió, 2004; White, 2003) have illustrated successful outcomes for diverse learners when teachers developed classroom cultures which mediated student participation in the mathematical discourse. The discourse Moana wanted the students to access Bakhtin (1981) and Gee (1992) term a social language. She had begun to recognise that the students needed to learn more than mathematical knowledge and a set of procedures—they were also required to learn ways of talking, listening, acting and interacting appropriately when working with mathematical ideas.

Moana's focus on different forms of grouping aimed to reposition less confident students to participate. She wanted to increase their participation and change how they saw themselves—their roles and identities as mathematical doers and users. Changing the organisational structures of who worked with who caused an immediate shift in all participants' roles. Moana noted the: *changes evident in girl culture. The girls are starting to say something. Just like testing out what they can say.* Who participates and how in mathematics classrooms is influenced by the organisational structures in them and the

memberships these create. As Civil and Planas (2004) explain, the "the internalization of certain roles, derived from these memberships, certainly has many implications for learning" (p. 11). This was evident in this study.

Moana had previously maintained a distance between herself and the students. Within the first month she made an observable shift to include herself in the community. Her actions reveal the value she placed on developing affective relationships. Drawing on the concept of the whanau indicated that she wanted to build reciprocity—mutual respect which empowered all members of the community. According to MacFarlane (2004) whanau "is often defined as the notion of a group sharing an association, based on things such as kinship, common locality, and common interests" (p. 64). Integrated within the whanau concept are specified forms of behaviour. These include care and concern within collaborative support but also "assertive communication" (p. 78) which links to appropriate ways to voice thinking. Moana's careful scaffolding of each student's participation laid the foundations for them to assertively communicate their reasoning during mathematical activity. Moreover, I interpreted her introducing the whanau concept as evidence that she too had begun to develop her own pathway and a more assertive voice in constructing an inquiry community.

7.3.4   LEARNING TO MAKE MATHEMATICAL EXPLANATIONS

As described previously, Moana had adapted the Numeracy Development Project (Ministry of Education, 2004a) to better match the pedagogical beliefs she held. As a result, the students had learnt a different form of mathematics than that intended—one in which the use of concrete materials, and recording and practising rules and procedures prevailed. Initial classroom observations revealed the students' immediate response when given a problem was to compute an answer. Moana wanted to address this behaviour but she was concerned that her students' growing mathematical interest and confidence be maintained. In the first instance she redesigned a set of problems and used a family of television cartoon characters which she considered would better engage student attention. Using these she addressed their persistent answer seeking behaviour, directly outlining her expectations: *it's*

not about the answer. It's about how you solve it. You need to be talking about it. Discuss it, what you are doing and then what you are doing all the way.

Moana was aware that the students had many difficulties explaining their reasoning fully. In accord with a revised section of the communication and participation framework (see Appendix K) Moana explicitly scaffolded ways to extend their explanations. In their pairs she emphasised need for extensive exploration and examination of each of the sequential steps. Through these actions the students began to recognise what needed to be included in an explanation to meet their audience's need. They were also learning how to respond to questioning with clarification of their reasoning.

In sharing sessions Moana closely attended to their verbal explanations and stepped in to scaffold those who needed support and to address students' use of everyday language and short utterances. The following vignette illustrates how Moana revoiced to name solution strategies or to press the students to explain their actions and solution strategies.

Scaffolding mathematical explanations

Aporo uses counters to model how his group solved the problem[2].	
Aporo	Two, four, six, eight, ten, twelve.
Moana	So that's called skip counting because you are skipping across the numbers.
Tere	We kept adding like two more. We counted in twos.
Moana	Counting in twos. Yes that is skip counting.
(Term 2 Week8)	

Faa lays out an array of three rows of three counters to explain a solution for the problem[3].	
Faa	We went three times three equals nine people.
Moana	But there might be people in here who are not sure…
Faa	Yes. Because three plus three equals six and plus another three equals nine.
Moana	Did you see how he solved that and explained it? What he said?
Tui	He said three plus three equals six plus another three equals nine or three times three equals nine.
(Term 2 Week8)	

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[2] Mrs Dotty has baked some muffins. She puts them on the bench to cool in two rows. Each one of the six Dotty children sneaks in and takes two of them so when Mrs Dotty returns they have all gone. How many muffins did Mrs Dotty bake?

[3] Mrs Dotty has a car which has her driving seat and then three rows of three seats in the back. How many Dotty children can she fit in her car?

Teacher dilemmas, changing scripts to focus on the reasoning, teacher revoicing

In inquiry classrooms the teacher role is a complex one with many challenges (Yackel, 1995). Moana described these in her reflections of her lesson video: *as a teacher I am put in a position of 'sculpting' an outcome without the proper tools. The long silences and I feel quite lost on when to jump in and when to let the children struggle. These children are just not used to me letting them struggle.* In informal discussions with me Moana outlined her personal conflict at allowing her students to struggle or be confused. Similarly, she described problems she had scaffolding student development of viable explanations while also making sense of them herself. She also described the challenge of knowing when to question and challenge, when to insert her own ideas, and when to lead discussion back to the mathematical understandings under consideration. Similar dilemmas have been described by other researchers (Chazan & Ball, 1999; Lobato et al., 2005; Schwan Smith, 2000) when teachers shift the communication and participation patterns towards inquiry.

Previously the students had experienced making mathematical explanations through calculating objects arithmetically in an instrumental manner (Skemp, 1986). Constructing conceptual explanations posed immediate and on-going difficulties for many students. Some students continued to interpret explaining as providing procedural steps; others had difficulties knowing what details were required for sense-making and what they could assume as taken-as-shared. These difficulties are similar to those other researchers have described as students learn to explain and justify their reasoning in inquiry classrooms (Cobb, 1995; Cobb et al., 1993; Kazemi & Stipek, 2001; Yackel, 1995; Yackel & Cobb, 1996).

To transform how the students had interacted previously in mathematical activity required that Moana directly address their previous scripts (Gallimore & Goldenberg, 1993). These involved establishing a range of socio-cultural norms identified by many researchers (e.g., Blunk, 1998; Lampert, 2001; Sullivan et al., 2002) as supporting rich mathematical activity and discussion. The small intermediate steps added to the communication and participation framework mediated a gradual shift in the communication and interaction patterns. Moana

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used specific pedagogical practices to establish the foundations for peer collaboration (Forman & McPhail, 1993). These included use of what O'Connor and Michaels (1996) term revoicing—an interactional strategy used to socialise students into mathematical situations. Moana's revoicing subtly repositioned students to extend their explanations.

7.3.5 LEARNING HOW TO QUESTION TO MAKE SENSE OF MATHEMATICAL EXPLANATIONS

Owing to the previous univocal patterns of discourse (Knuth & Peressini, 2001) used in the classroom the students were inexperienced at listening to, and making sense of, each others' explanations. Observing their novice behaviour in video records of classroom observations Moana noted their need to learn to talk and to listen to each other. She stated that she planned to address a former interaction pattern in which interjections were an accepted norm. Observations had provided evidence that the interjections maintained a focus on correct answers, affected self-esteem negatively, and detracted from other students having time or space to actively listen, think, and examine the reasoning.

In the first instance, Moana began to halt mathematical explanations at specific points in the large group discussions. She directed the students to take time to think about what had been explained, and then to ask questions. To focus students on responding appropriately to erroneous reasoning she required that they ask questions and cause the explainer to rethink. For example, when an erroneous explanation was presented she halted discussion, withheld her own evaluation and asked the students to frame a question: *Who has got a really good question they can ask Wiremu to make him rethink. I like the way you are thinking about the question. Would you like some help with your question?* Moana drew on a set of questions (see Appendix E) which had been developed in the study group context and used these as models for the students, of how to question to extend explanatory reasoning. She would listen closely to explanations and regularly halt the explainer to provide space for the other students to ask questions, if none were forthcoming she would often ask questions herself as illustrated in the following vignettes. These show how Moana adopted a number of different roles as she participated in discussions. These included her directly modeling asking questions, scaffolding student questioning and shaping how and what questions

were asked, or acting as an observer and directing attention to student models of active listening and questioning.

Questioning mathematical explanations

Moana as a participant in a discussion listening to an explanation asks a question.
Moana   What have you actually done there Mahine?   Mahine   I have plussed 10 onto 47.   Moana   So you have added 10 onto 47? Are there any questions? Questions like where did you get the ten from?   *(Term 2 Week 7)*

Moana stops an explainer and asks the listeners.
Moana   Do you want to ask a question? You need to start with what.   *(Term 2 Week 8)*

Moana halts an explanation and directs student attention to an example of active listening
Moana   Donald is really listening. He is actually listening and watching what has been taking place. He is not only listening to the person, he is watching when they write things down. His questions are really specific to what the person is doing.   *(Term 2 Week 10)*

Active listening, questioning to provide space for rethinking

In the interaction patterns previously established Moana had assumed that the students were learning through listening. This was evident in a statement she recorded after a lesson when she observed the outcomes of the new interaction patterns: *good to see different thinking coming through, blows me away because before they never had a chance to explore…I just thought that they understood by me talking all the time.* Now she had begun the gradual process of inducting students into an inquiry culture which placed value on active listening and questioning. Within this changed learning climate there was a discernible shift in how the students listened to each other with increased respect.

Evident in the data is how errors had become learning tools, valued as a means to examine and analyse reasoning, rather than cause loss of self esteem. Moana's approach to errors as a way to widen discussion and questioning is similar to the approach used by teachers in

Kazemi and Stipek's (2001) and White's (2003) studies. The previous focus of the classroom on provision of answers to predetermined solutions had established a learning culture in which knowing mathematics was "associated with certainty: knowing it, with being able to get the right answer, quickly" (Lampert, 1990b). Moana wanted to address this pattern and so she emphasised a need for the students to stop, think, and reconsider the reasoning as she explained: *I was reinforcing rethinking because calling out has been the prevalent means but they need to learn to hold back and do some thinking first.* Moana's actions scaffolded the students to use a more 'mindful' approach to listening and using specific questions to understand the mathematical reasoning. As Lampert illustrated "teaching is not only about teaching what is conventionally called *content*. It is also teaching students what a lesson is and how to participate in it" (p. 34).

7.3.6   SUMMARY OF THE FIRST PHASE OF THE STUDY

In this section I have outlined the many hurdles Moana encountered as she laid the foundations of the interaction norms of an inquiry community. Moana had appeared to support the use of the communication and participation framework to plan out shifts in the classroom community's interaction patterns but the novice status of both her and her students in the inquiry environment meant that initially they lacked knowledge and experience of the many roles they were required to take in this culture. Many researchers (e.g., Ball & Lampert, 1999; Franke & Kazemi, 2001; Huferd-Ackles et al., 2004; Mercer, 2000; Rittenhouse, 1998; Sherin, 2002b; Wells, 1999) report similar challenges when teachers shift their classroom culture from a conventional to an inquiry classroom. In an interview at the end of the research Moana described what this first phase was like for her. She explained the effect of the shifts in classroom communication and participation patterns: *I was slowly coming awake...in those initial stages thinking and I suppose when you are a teacher and you do professional development and you think I will try this... then I started seeing...and I was more impressed not by their maths but by their talking, how they were talking. Then I didn't feel so harsh on myself for focusing on the participation norms because they were actually talking. So I just started to settle down to the maths.* The reconstruction of the communication and participation framework into smaller more incremental steps supported a gradual change in the classroom interaction patterns. This in

turn appeared to provide Moana with confidence to continue making shifts in the learning culture.

The students now actively engaged in construction and examination of conceptual explanations—an important shift because according to Cobb and his colleagues (1997) being able to provide explanatory reasoning is an important precursor for supporting development of explanatory justification and argumentation. Moreover, the close attention Moana had placed on active listening, questioning and rethinking shifted the students towards sense-making within zones of proximal development (Forman & McPhail, 1993). They had begun to view their reasoning from the perspective of others which potentially provided an important foundation for future mutual engagement (Wenger, 1998) in collaborative activity. The classroom culture Moana had constituted had shifted toward what Wood and McNeal (2003) define as a strategy reporting discussion context.

7.4 FURTHER DEVELOPING THE COMMUNICATION AND PARTICIPATION PATTERNS OF A COMMUNITY OF MATHEMATICAL INQUIRY

Moana's pedagogical actions in the first phase had focused on establishing the socio-cultural norms which supported the students to provide mathematical explanations. Our joint discussion of the students' current communication and participation patterns prompted us to establish the next steps on a trajectory towards developing reasoned collective discourse. Moana considered that the students were ready to learn how to engage in mathematical inquiry and argumentation to justify their reasoning. However, she voiced her key concern that the students retain their growing mathematical confidence explaining that she thought that: *this was risky stuff moving them, upping the ante; I'll be hanging out there as much as they will be hanging out there but they are ready*. Together, we carefully analysed the communication and participation framework and constructed small specific steps (see Appendix K) to support the shift towards student use of mathematical inquiry and argumentation.

7.4.1 COLLECTIVELY CONSTRUCTING AND MAKING MATHEMATICAL EXPLANATIONS

In the first instance Moana focused on further developing how the students worked collaboratively. She restructured the daily lesson format so that she was no longer leading mathematical activity from a central position. Now in each lesson, after an initial short discussion the students worked in small heterogeneous problem solving groups and then returned to the larger group situation to conclude with a sharing session.

Moana's previous focus had been to develop individual student capacity to actively listen, explain their reasoning and make sense of the reasoning of others. Now Moana wanted to press group behaviour towards increased collaborative interaction. She stated that she wanted the students to discuss, negotiate, and construct a collective solution strategy. In shaping their interactions Moana emphasised their personal responsibility to engage and understand the reasoning used by other members. She established a pattern where the students began their small group activity with a mathematical problem which they were directed to read and think about individually, then discuss, interpret and together negotiate a solution strategy. Moana gave each group one sheet of paper and pen to use. She moved from group to group listening to their talk and only intervened to ensure all individuals contributed and could explain the developing reasoning. She explicitly established with them that they could only bring questions or problems to her if the whole group agreed that they required assistance. When a group member requested help she discussed with the group their prior actions, drawing from them ways they might solve their problems together. She explained that she wanted to assist the students to recognise and use their collective strengths to engage more autonomously in mathematical activity.

Although Moana required that the students develop a joint explanation for the larger sharing session she encouraged them to explore and discuss a range of ideas, then select the one they agreed they could all understand and explain. She guided their negotiation and selection of a shared strategy solution through establishing a set of ground rules (see Appendix J) for talk loosely based on those developed by Mercer (2000) but pertinent to this group of students. When she observed that they had developed a shared explanation she

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asked them to predict which sections their audience might find difficult, discuss and explore questions they might be asked, and rehearse ways they might respond. She said that she wanted to maintain their confidence but also to increase their shared understanding and prepare them to respond appropriately to questions. The following vignette illustrates how Moana facilitated discussion which supported the students to question and probe an explanation for sense-making but at the same time ensured that ownership of the reasoning remained with the explainer.

Scaffolding exploratory talk to explore an explanation

Moana joins a small group and is a listening participant. Anaru is making an explanation for the problem[4].

Anaru	I drew six cakes and my way was halving it.
Moana	Can you show us?
Anaru	[draws four squares and divides each one in half] And then I halved it.
Moana	Has anybody got any questions they want to ask Anaru?
Wiremu	Why did you cut them in halves?
Anaru	Oh because they said that there were four people and she was to half it so they have equal amounts.
Donald	What did you do next?
Anaru	Each person took three parts and there was one whole cake left and I halved it and then I halved another half and they took one each. That's four pieces they took.

Aporo watches closely and moves his hands as he counts the pieces.

Aporo	How could she [referring to Mrs. Dotty] half it again?
Donald	If it was all gone?
Anaru	Yeah. There was one whole cake left and then I halved it into fourths.

The listening students count the halves and then look at Anaru questioning.

Moana	Do you all agree? You have listened to her but remember you have to agree.
Donald	[points at the last two cakes] Can you tell us what you said in that bit?
Anaru	I said that I halved it and each person took three.
Rona	I know. So you are saying they got a whole and a half. There are two cakes left that are halves so they got a whole cake and a half.
Moana	Do you agree Anaru? Is that what you mean?

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[4] Mrs Dotty had just baked 6 chocolate cakes and suddenly she heard a knock on the door. Guess what? 4 friends had come to visit in their flash car! They were really hungry so she cut up the cakes and they shared them between them. She didn't eat any because she had already had lunch. If they each ate the same amount how much did each person eat? What fraction of the six cakes did each person eat?

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Anaru	No because I just found out that there was halves of a cake. I said in quarters but I meant in halves.
Hone	So if she cut all six cakes into halves and gave them three halves each that means one and a half or three halves each eh?
(Term 3 Week 2)	

Facilitating the talk, increased student autonomy, gaining consensus through exploratory talk

The pedagogical actions Moana took to establish collaborative group skills are similar to those used by Lampert (2002). The direct attention Moana gave to the group processes, as she stepped in and out of a range of roles, meant that her students learnt that both social and academic outcomes result from group work.

Evident in the data is a shift in positioning of all the members of the classroom. Moana analysing her actions in a video excerpt of a lesson observation recorded: *more korero (discussion) less teacher talk and I am really moving into a facilitating role.* Her more facilitative approach could be attributed to her direct attention to group processes. Like other researchers (e.g., Rojas-Drummond & Mercer, 2003; Rojas-Drummond & Zapata, 2004; Wells, 1999) who explicitly scaffolded a talk-format with diverse students, this resulted in a progressive shift toward increased exploratory talk.

The changes evident in Moana's classroom resonate with those Hufferd-Ackles and her colleagues (2004) maintain are important in the growth of "a math-talk learning community" (p. 87). These researchers described the developmental growth of a classroom community in four dimensions. These included questioning; explaining mathematical thinking; the source of mathematical ideas; and responsibility for learning. As Moana's students gained greater agency, increasingly they initiated questioning. Similarly, their mathematical thinking became an important source for mathematical discussion. Moana noted the confidence the students showed in explaining, elaborating on, and defending their reasoning when she recorded after a lesson: *Anaru shows confidence and she has a definite sense of trust about her way.*

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The classroom observations reveal that explanations had become mathematical arguments as a direct result of Moana's expectation—the negotiation of a collective view. Through the close examination of mathematical explanations the criteria for what the students considered acceptable as mathematical explanations was established. Not only were explanations required to be experientially real and relevant but the listeners also expected elaboration or an alternative explanation.

7.4.2 PROVIDING A SAFE RISK-TAKING ENVIRONMENT TO SUPPORT INTELLECTUAL GROWTH

Taking intellectual space to rethink reasoning had become an established practice in the first research phase and when Moana and I discussed the most recent observational data we noted that student talk had increased significantly. But Moana was concerned that the students: *haven't fully got to the aspect of valuing what the other person is saying because they are so busy gushing. They are so excited about making sense themselves. But that will happen, well it is happening I guess because when you go back and look at the videos of them in the beginning...now actually you see them really on track.* Together, we planned adjustments to the ground rules for talk discussed in the previous section and the pathway Moana had planned using the communication and participation framework. We wanted to extend how the students used the concept of 'rethinking' to provide them with cognitive space—time to shape responses to conjectures, respond to questions, or examine the reasoning of others. Moana introduced this shift by asking the students in their sharing sessions to use 'rethink time' to develop more questions to gain clarity. This action reinforced that they needed to be constantly thinking through every mathematical action and questioning until they had complete understanding. She emphasised that 'rethink time' provided a way to work from confusion to understanding. For example, as she watched students struggling to develop an explanation she said to the group *I can see you are confused. Me too, that's all right we can take some time...rethink about it. It's good to take some risks with our thinking sometimes.* In this statement and others like it, Moana modeled that she too was a learner and at times needed to persist with questioning her own thinking to comprehend explanations.

Moana's validation of the acceptability of confusion as important to mathematical sense-making caused a further shift in the ways in which erroneous thinking was considered in the community. Erroneous reasoning emerged most often when the students were using the more specialised discourse of mathematical language or when they over-generalised number properties. Moana drew on these as valuable learning tools, using them to explore alternative ideas, support rethinking and reformulation of conjectures, or to refine the use of mathematical language. At the same time, she remained respectful of student reasoning. For example, in the following vignette Moana facilitates discussion, providing opportunities for reconsideration and clarification of reasoning but she maintains the self-esteem of the explainer and ensures that the ownership of the reasoning remains with the explainer.

Rethinking and reconsidering to clarify reasoning

During sharing Moana records 3 x 4 and 4 x 3. Donald provides another explanation for the problem[5] using counters and an array.

Donald	Three times three equals six and another group of three groups of three equals six and then you just put them together and they equal twelve.
Moana	Are there any questions? Do you agree with that? I will just write it down [records 3 x 3 and 3 x 3].
Aporo	Oh he has done too much, like too much threes.
Donald	Oh true I forgot to say different. I said it wrong.
Moana	That's okay. So can somebody explain? I like the way you are thinking about it.
Jim	Three groups of four.
Moana	[points at the first recording of 3 x 4 and 4 x 3] We are just looking at this and trying to make some sense out of it and comparing it with this one here.[points at the recording of 3 x 3 and 3 x 3]. There is a disagreement happening. So we need to look at these and we need to sort it out. What I need you to do is look at these again and think carefully about what they look like. Donald can you please put three groups of three and three groups of three…show us what three groups of three and three groups of three look like please [points to the pattern]. Here we've got three groups of three and another three groups of three.
Donald	Oh because at first I said three plus but I accidentally said times. I got mixed up.
Moana	That's fine because now you are making it clear. So you have got three plus three and three plus three.
Donald	Yes I got mixed up first.

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[5] Mrs Dotty baked some muffins. She only has a very small oven. So she fits them in the oven by putting them in four rows with three muffins in each row. How many muffins has she baked?

Moana	[records 3 x 3 = 9 then 3 + 3 + 3 = 9] So you have mixed the times with the plus?
(Term 3 Week 2)	

Moana continued to emphasise need for communal support, placing responsibility back with a group to clarify reasoning including when they used erroneous reasoning. When she observed that a student had difficulties, she would require that another group member step in and support them. She always affirmed their self esteem and risk taking. This was illustrated when she told the students: *Caliph has taken a wonderful risk. She is out there and she wants some help.*

Through our discussions Moana was aware that often students reveal their misconceptions through extended mathematical dialogue. She introduced specifically designed problems that required the students to devise ways to explain and clarify mathematical situations for a character in them who was confused or had erroneous reasoning. The problems (see Appendix I) provided the students with public opportunity to examine and explore possible partial understandings the students themselves held. The students had the opportunity to work from a point of confusion or an erroneous position through to sense-making as valid mathematical problem solving activity.

Respectful interaction, extending independent student engagement with mathematical reasoning, partial understandings and interthinking in shared communicative space

Weingrad (1998) maintains that intellectual risk-taking requires an environment which is respectful of student reasoning. The carefully crafted interaction patterns Moana was building are those which many researchers (e.g., Boaler, 2006a, 2006b; Martin et al., 2005; Pourdavood et al., 2005) identify as important for diverse students if they are to develop intellectual autonomy. Moreover, Moana was enacting specific expectations which were designed to press the students toward independent engagement with mathematical ideas. Goos (2004) illustrated how a teacher in her study enacted similar expectations related to "sense-making, ownership, self-monitoring and justification" (p. 283) which made possible a later press toward extending more autonomous engagement with mathematical ideas.

The way in which erroneous thinking became considered as a valued teaching and learning tool reveals the significant shift which had occurred in the culture of the classroom community. Moana modeled norms which reinforced that rethinking and persevering to work through confusion was a sound mathematical learning practice (Boaler, 2006a). No longer were errors the cause of negative situations and loss of self-esteem. Instead, they had become catalysts for further problem solving or "springboards for inquiry" (Borasi, 1994, p. 169).

Evident in the data is how the different partial understandings held by participants in the dialogue contributed to the continual reconstitution of a shared zone of proximal development. These results are similar to those of other researchers when students are specifically scaffolded to engage with the reasoning of others (e.g., Brown, 2005; Brown & Renshaw, 2004; Goos et al., 1999; Mercer, 2000). In Moana's classroom the variable contributions pulled the participants into interthinking within a shared communicative space. Moana noted the shifts she had made as a participant and facilitator as she viewed a video excerpt and recorded: *I was involved in clarification and whole class joint shared understanding, revoicing and talking about multiplication. I always thought maths was so straight forward but now it's changing rapidly in my head.* She had noted her own shift towards considering mathematics as an ever-expanding body of knowledge, rather than a limited set of mathematical facts and procedures (Stigler & Hiebert, 1999).

7.4.3   POSITIONING STUDENTS TO PARTICIPATE IN THE CLASSROOM COMMUNITY

Moana continued to monitor closely how different students in the classroom participated in interactions. She viewed and discussed video excerpts of the Maori and Pasifika girls' emerging communication and participation patterns. She closely monitored how less confident or less able students managed within the heterogeneous grouping. Moana specifically focused on strategies to build these students' mathematical confidence. She actively positioned specific students. For example, as she listened to a group discussion she

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overheard a quiet comment by a less able student and responded by directing the other students to listen to him. She then said: *good thinking Tama, something to get you all going.* At another time she listened to the exchange of ideas then commented loudly: *wow Teremoana see how you have made them think when you said that? Now they are using your thinking.* She regularly halted explanations as they were being shared to draw attention to how a low achieving or unconfident students' reasoning had contributed. The following vignettes illustrate how Moana listened carefully as small groups interacted and then when required stepped in and positioned specific students to participate and contribute their reasoning.

Positioning the students to access and own the mathematics talk

Moana observes Anaru nodding her head in agreement so she questions Anaru directly:

Moana   You are saying yeah…what are you saying yeah to?
Beau   She's just been going…making up suggestions…dumb ones…she's just…

Moana without responding to Beau repositions Anaru as a valued group member:

Moana   Well it's not dumb ones. I have been listening and she's making you think because she is using the problem and making sense of it. She knows that you have to listen to each group member, listen to their thinking and make sense of what they think [turns and asks Anaru] So, what was your way? Can you explain it please?

(Term 3 Week 8)

A group is discussing partitioning a line segment into fractional pieces. Moana quietly questions a passive on-looker to ensure that he is accessing the reasoning.

Moana   Do you understand it Haitokena? Come on people talk about it.
Beau   So that one in the middle is half.
Caliph   [points at mid-point] Yeah you can just like draw a line through there.

Moana positions Haitokena so he can sense-make and then provide an explanation.

Moana   But we [turns to include Haitokena] are asking what are each of these bits? What is the fraction of those bits? Then Haitokena will be able to explain them to you.

(Term 3 Week 8)

Changing student status, shifting beliefs, changing roles

Through direct pedagogical actions Moana repositioned the low achievers, the passive and shy students, and Maori and Pasifika girls from their previous social and academic status. She assigned competence (Boaler, 2006b) to them through explicitly drawing other students' attention to the intellectual value of their reasoning. Moana recognised how her own actions and expectations for different students had shifted when she commented to me informally: *I have realised I did have low expectations because why would I be so excited when they say something that is relevant to what is happening. I thought I had high expectations for my class; well I had high expectations about certain people...so at first when someone would say something and I thought did you just say that? Now I am hearing what they all say and its relative and it's linking in...so when I hear them talk out loud I help them to be heard.*

Moana regularly reviewed previous video records and discussed the shifts she observed in the students' communication and participation patterns. She voiced specific concern about the tensions and contradictions the shifts in interaction patterns had caused for the Maori and Pasifika girls. She included the changes in her expectations of them, and their own expectations they now held, of their role in the classroom mathematical community. She voiced her observations of how they had changed their view of what 'doing' mathematics meant: *when you go back and look at the videos of them working in the groups at the beginning they were just pretending to be working in those groups. They were just parroting the words...but now you see them really on track. But then as they got hooked in I can see the changes, it's the girls who are really changing like Anaru. I have seen Anaru's behaviour...well it is...it is coming to an uneasy uncomfortable place. She is taking herself seriously now in maths. She's a Pacific Island girl and she has come to a crossroad and she is making a choice. (Term 3 Week 8)*

Within the sociocultural approach of this study, to understand Moana's perspective requires explicating the relationships between her actions and the "cultural, institutional and historical situations in which this action occurs" (Wertsch et al., 1995, p. 11). To explain the many internal tensions individuals in a community contend with Forman and Ansell

(2001) use the notion of "multiple voices" (p. 115). These multiple voices match the many communities individuals belong to, but also incorporate a historical dimension in which "memories of the past and anticipation of the future affect life in the present" (p. 118). Moana, as the old-timer (Lave & Wenger, 1991) was required to induct the students into the mathematical community but in doing this a historical voice included her own memories of her learning experiences in mathematics as a Maori girl. Her expectations for the possible future in mathematics of the Maori and Pasifika girls coloured her present interactions. Forman and Ansell drawing on social cultural theories explain this "discursive mechanism by which the past and future are drawn into the present as prolepsis" (p. 118); a form of anticipation of the existence of something before it actually does or happens. Moana's statements and interactions with the Maori and Pasifika girls can be understood as part product of her own experiences and also her future expectations for them.

7.4.4 PROVIDING EXPLANATORY JUSTIFICATION FOR MATHEMATICAL REASONING

Through the first half of the year the participation structures Moana made available to the students emphasised a need for collective agreement in the construction of explanatory reasoning. Through discussion with me Moana realised that when the focus of classrooms is toward teaching students to work together and develop collective consensus there is potential that they interpret this as always needing to be in agreement (Mercer, 2000). Providing justification or convincing others was a key feature of a part of the communication and participation framework Moana aimed to enact. As an important incremental step, Moana established that her students needed to learn 'polite' ways to disagree and challenge. She placed an immediate focus on requiring that listening students voiced agreement or disagreement with conjectures. She regularly halted explanations and positioned students to take a stance. For example, she told them: *at some point you are going to have an opinion about it. You are going to agree with it or you are going to disagree.* But she ensured that they knew that they needed valid reason to support their stance directing the students to: *think about what they are saying. Make sense of it. If you don't agree say so but say why. If there is anything you don't agree with, or you would like them to explain further, or you would like to question, say so. But don't forget that you*

have to have reasons. Remember it is up to you to understand. The emphasis she placed on their need to justify the stance they took reinforced their responsibility to actively listen and sense-make and provided a platform to shift the discourse from questioning and examining explanations toward questioning for justification.

In the study group Moana had examined the questions and prompts for justification described by Wood and McNeal (2003) and viewed examples in video records from her colleagues' classrooms. As she analysed video excerpts of her classroom observations she stated: *a real need to move, shift from the surface questions or practising how to...to take a good look up close and personal, shift thinking to challenging, justifying, validating, creating other possibilities.* To do this, before mathematical activity began she verbally emphasised the need for questions and challenge: *I want you people asking questions...throughout ask questions. Why did you come to that decision? Or why did you use those numbers? Or can you show me why you did that? Or if you say that, can you prove that that really works? Or can you convince me that this one works the best?* Through our study group discussions Moana knew that explanatory justification often required more than one form of explanation and so she prompted the students to consider development of multiple ways to validate their reasoning: *sometimes...remember yesterday we had like three strategies, three different strategies, all the same. You all came out with the same solutions but you did the three different ways and sometimes you need that to convince somebody.*

Moana used problems (see Appendix H) purposely developed in the study group which had as key component the requirement that the students closely examine their collective explanations and construct multiple ways to validate their reasoning and convince others. Similarly, she required that all group members be able to explain and justify the collective strategy and provide support if their explainer had difficulties responding to questions or challenge. She also strengthened how they responded to argumentation through requiring the small groups to examine and explore questions they could be asked when explaining to the large group: *not only are you asked to justify and explain but you are also thinking about what possible questions you might be asked and how you are going to go about*

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answering those questions. As illustrated in the following vignette Moana actively participated in group discussions facilitating how the students could respond to challenge and as a result the students examined, extended and validated their reasoning autonomously.

Facilitating justification to validate reasoning

Moana has listened as Mikaere explained and drew, and segmented a set of lines into halves; eighths and then 13 pieces, and then recorded 13/13 for the problem[6].

Moana	Can you all understand that? Now what you people need to do...other people are going to ask you questions about why you split these sections into these fractions. What are you going to say? So what questions do you think other people are going to ask you?
Beau	They will ask why...
Wiremu	How did you come up with that?
Moana	Okay. So they will come up with...how did you come up with that idea? Why did you use those fractions? You need to clarify exactly what you were thinking at the time. Think of different ways to answer.
Hone	What fractions do you have?
Wiremu	Why did you put thirteen thirteenths?
Mikaere	Because it's thirteen lines.
Wiremu	No. You mean thirteen bits. But why is it thirteen bits? How do we say?

[6] This little guy from outer space   is in your classroom. He is listening to Annie, Wade, Ruby and Justin arguing about sharing a big bar of chocolate.

Annie says that you can only share the bar of chocolate by dividing it into halves or quarters

Wade says he knows one more way of sharing the bar of chocolate

Ruby and Justin say that they knows lots of ways of sharing the bar of chocolate and they can find a pattern as well

The little guy from out of space is really interested in what they say so they start to explain all the different ways to him.

What do you think they say? In your group work out a clear explanation that you think they gave.

He needs lots of convincing so how many different ways can you use to prove what you think they were thinking. Make sure that you use fractions as one way to show him because he likes using numbers. Can you find some patterns to explain?

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Faa	Because it was a whole chocolate bar. If it was halves you have two bits. But you cut his one whole bar into thirteen pieces and you still have the one bar but thirteen thirteenths now, just littler bits than the halves.
Hone	But they might ask if they equal the whole.
Mikaere	They all equal one whole chocolate bar just the half bits are bigger but still one whole and we can show as many bits but still the same. But there is another pattern. The bigger the number goes the smaller the bit goes.
Wiremu	Cut it into halves that's the biggest.
Hone	But if the top and bottom are the same then you just have one whole, doesn't matter what they are if they are the same.
(Term 3 Week 8)	

Scaffolding inquiry and justification, student voice

Reviewing the video records of lesson observations provided insight into how Moana carefully scaffolded a shift in the classroom discourse from the need for explanatory reasoning to explanatory justification. The pedagogical strategies she used are reported as important by other researchers (e.g., Kazemi & Stipek, 2001; Lampert et al., 1996; Lampert, 2001; Nathan & Knuth, 2003). The use of problematic mathematical activity coupled with Moana's structuring of the discourse and requirement that the students take a stance positioned them to develop logical ways to refute or support reasoning. Ball (1991, 1993) described a similar outcome in her classroom—increased engagement in discipline specific inquiry and debate was matched with growth in student agency. The use of inquiry and debate supported development of joint zones of proximal development. The similar levels of reasoning but different pieces of understandings held by the individuals gave them experience in accommodating a range of perspectives and experiencing transactive dialogue (Azmitia & Crowley, 2001).

Consistently over the duration of the research Moana voiced conflicts related to the pedagogical role and actions she needed to take. When considering introducing a press for justification she noted: *There have been significant changes for me. I started off thinking should I write those questions up so they could just pick and just answer them. I am glad that I didn't do that because it would have replaced their voice. It would have just replaced their voice with a card that they read off and that would have been something else because they do have their own language and they have to learn that it is okay in certain places and*

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not in others. I deduced from her comment that she had developed her own sense of trust in her actions in constructing an inquiry community. Moreover, she recognised that her students were constructing a mathematical discourse, as another speech genre or social language, along side their individual voice (Bakhtin, 1994). Her actions made possible opportunities for them to appropriate and explore, extend, expand and transform the language of inquiry into their own words and thoughts. Other researchers have indicated the importance of these actions for diverse learners (e.g., Gee, 1999; Gee & Clinton, 2000; Moschkovich, 2002b; Wertsch, 1991).

7.4.5 EXPLORING RELATIONSHIPS AND PATTERN SEEKING

Through the specifically designed problems and the search for multiple forms of justification the students began to tentatively discuss numerical patterns they observed. Moana had participated in discussions in the study group of generalisations students may use but she had given them little attention at the initial phases of the study. Now, the need to provide multiple levels of explanatory justification led to increased student recognition and voicing of numerical patterns. Moana began to use these with the students, often as position statements used to explore and extend numerical connections and patterns. In the vignette Moana extends discussion of a student-voiced observation to press the students to explore and examine patterns they observed in fractional numbers.

Shifting reasoning from justifying to pattern-seeking and exploring

The students in groups have discussed and explored the statement Aporo made "the bigger the denominator the smaller the bit". Moana began the large group discussion by positioning Aporo and his group to validate their conjecture.	
Aporo	[uses two segmented lines with their fraction equivalents recorded as symbols] Because that number is big [13/13] and this number is little [5/5] and you can tell the pieces because the five ones are bigger and the thirteen ones are smaller.
Moana	Questions? What have you got there on your…what is your fraction there Wiremu?
Wiremu/Pita	Four out of four, four quarters.
Moana	Okay. In comparison to five out of five, five fifths which bit is bigger? But what happens if you have thirteen thirteenths? Is what Aporo said true? Pita you just said 'different' why? Why? Why do you think it was different?

Rona	I know why [points at segmented lines and notation for 5/5 and 4/4]. That one [4/4] is a bit smaller that that one [5/5].
Moana	So what are you saying Rona?
Rona	Like that's…just a little number [4/4] and this is a kind of a little one [5/5]. But that one's got like the bigger piece [4/4] and this one has got the little pieces [5/5].
Aporo	Because you have got to cut this into five pieces and you have to cut that into four pieces so smaller number, bigger piece.
Moana	So if I had a chocolate bar and I said to you that you can have five fifths or thirteen thirteenths or twenty twentieths…
Rona	That will be the same.
Beau	They are all the same, because it's just smaller but still one whole piece.
Moana	So how does that work? Can anyone see the pattern? Is there any rule we can use?
Beau	Yeah. If the top and bottom are the same then you just have one whole doesn't matter what they are if they are the same.
(Term 3 Week 8)	

Pattern seeking and exploration, a collective zpd, importance of validating conjectures

Evident in the classroom observational data is how a shift from explaining to justifying increased the community listener-ship. Moana acknowledged the value of student contribution to progress collective reasoning noting after the lesson: *this session was interesting because it put me in the position of a sponge board or sponge board/spring board. I am revoicing, not in a negative or condescending way, what they are making sense of. I am not taking their words and changing them, but adding to a shared understanding for them and me. It helps me view things from their perspective.* Lerman (2001) describes how increased participation in discourse and reasoning practices pulls all participants into a shared zpd. This was evident in this classroom.

Moana's press on the students to validate conjectures scaffolded potential development of more generalised reasoning. Blanton and Kaput (2003, 2005) and Carpenter and his colleagues (2004b) note the importance of teachers using student validation of conjectures as a tool to mediate generalised reasoning.

7.4.6   USING MATHEMATICAL LANGUAGE

Although the students had developed an increased repertoire of questions to inquire and challenge they often still used short utterances and informal colloquial language to explain or respond to questions. We agreed that they needed richer ways to share their reasoning but Moana voiced concern that a push toward more extended responses might cause loss of confidence and withdrawal from participating. A research article[7] mediated Moana's next shifts as she used the teacher's actions in the article to map out her next steps. She increased her explicit models of mathematical talk, actively participating and describing the mathematical actions using informal, then formal descriptions. She listened carefully to their explanations and then revoiced and extended what they said using multiple layers of meaning. For example, after a student described cutting a chocolate bar into: *six bits* Moana responded with: *yes six bits, sixths and six of them, six equivalent pieces all the same size, six sixths of the one whole chocolate.* Then she asked: *is there anyone else who can model another equivalent fraction? Good Rona for taking a risk like this. Just go ahead and construct another fraction which is the same, equivalent.* Gradually Moana's phrasing of questions and responses were appropriated. The students used Moana's models, often rephrasing and using terms and concepts she had previously introduced.

Increasing and extending fluency in mathematical discourse

The discourse and communication patterns had been appropriate for their former situated classroom context (Gee & Clinton, 2000; Moschkovich, 2003; Nasir et al., 2006). Moana modeled her actions on Khisty and Chval's (2002) description of teachers who inducted students into more fluent forms of mathematical talk through use of specific models of rich multiple layers of mathematical words and statements. Moana's actions were designed intentionally to shift students from using colloquial talk which had been accepted previously when the answer was the focus but which now limited how they explained and justified their reasoning. Meaney and Irwin (2003) describe how informal and imprecise

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[7] Khisty, L. L., & Chval, K. B. (2002). Pedagogic discourse and equity in mathematics: When teachers' talk matters. *Mathematics Education Research Journal, 14*(3), 154-168.

use of a mathematical discourse eventually restricts students' mathematical reasoning. Similarly, Latu (2005) emphasises the importance of extending student understanding of mathematical concepts beyond the exact context in which they are learnt. Moana recognised student growth of communicative competence (McCrone, 2005) when she informally commented: *when we come to maths it's like these little antenna go up. They go right we are in maths what's the language and they start to think about the language, the strategies, like they always talk about what strategies are you using or why. Like when I was watching them working in a group the other day and one of them said 'just prove that then' and it was said so naturally, just part of what they say all the time.*

7.4.7   SUMMARY OF THE SECOND PHASE OF THE STUDY

Threaded through this research study is the influence of Moana's own past experiences and beliefs she had constructed about doing and learning mathematics. Her observations of the positive outcomes which emerged for her students were key factors which convinced her to continue the shift in communication and participation patterns towards inquiry and argumentation. My collaborative support, study group activity, the communication and participation framework, research articles and video observations supported each tentative step. These occurred in conjunction with Moana's careful analysis of their effect on the students' self-esteem and the growing confidence in doing and using mathematics. Influencing how Moana scaffolded interactions were the presence of multiple voices, past and future, but also a present 'new voice' Moana had constructed as she guided development of the sociocultural norms of an inquiry community.

Moana's many pedagogical actions to scaffold student engagement in more proficient mathematical practices parallel those described by other researchers who studied teachers working with diverse learners (e.g., Boaler, 2006b; Khisty & Chval, 2002; White, 2003). Through these carefully considered actions Moana laid the foundations for a community of inquiry. Increasingly, the students participated in exploratory talk (Mercer, 2000) as they explained and justified their reasoning in classroom interaction patterns which had begun to resemble more closely what Wood and McNeal (2003) defined as an inquiry culture. The

students had increased agency within a more balanced intellectual partnership (Amit & Fried, 2005).

7.5 TAKING OWNERSHIP OF MATHEMATICAL PRACTICES IN A COMMUNITY OF MATHEMATICAL INQUIRY

The gradual development of a community of mathematical inquiry had been a long, steady, change-process. At this point Moana noted her facilitative role and that the students: *expect everybody to make sense of what they are saying…they ask lots more questions all the time of each other, they just expect that they have to justify what they are thinking and that they can use words and other ideas to back up what they are saying.* Participation in communication of mathematical reasoning had become an integral part of what it meant to 'know and do' mathematics in the classroom community. Moana outlined to me that she considered being able to participate in mathematical discourse a fundamental right of her students. She metaphorically linked the way in which Maori are privileged when they know how and when to speak on a Marae to her facilitative role in mathematics lessons: *they all have the right to have that privilege. I am making sure that these children all know that they have got the right to talk and be heard in maths.* Moana wanted to sustain the press on mathematical inquiry and argumentation but also ensure ownership of it was vested in the contributions and reasoning of all community members.

7.5.1 USING MODELS OF CULTURAL CONTEXTS TO SCAFFOLD STUDENT ENGAGEMENT IN INQUIRY AND ARGUMENTATION

Whilst maintaining an expectation that individual students engage actively in inquiry and argumentation Moana directed attention to their responsibility to each other. She re-visited notions of the whanau (family) and emphasised the strengths inherent in being a member. She made direct links to the students' family context, for example emphasising that family members take different roles to make a Cook Island haircutting ceremony successful. When she observed groups working together she drew attention to their collaborative actions, paralleling these with the actions of a Kapa Haka group (Maori cultural group) who induct, support and challenge members until they achieve similar levels of expertise. She drew attention to similarities in the role of the tuakana (elder brother or sister or cousin in a

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whanau), linking these to specific individuals she saw taking leadership and actively supporting group members to challenge thinking or to promote development of collective reasoning. This was illustrated when Moana told the class: *There's really interesting korero (talk) going on. I really spent most of my time with this group because they were having problems and arguments and Wiremu was really good...you were really good in that position Wiremu, you were helping your group and you weren't giving out the answers and that's really good but you were pushing them to think. Yes you had everyone talking about and discussing how they were going to sort out the ideas. You were challenging and other people were following your lead so the arguing was really kapai (good).*

The students appropriated Moana's view of collective responsibility. They drew attention to their rights or responsibilities and readily stated their lack of understanding or need for support. They expected other group members to actively listen and engage in the reasoning and when explaining reasoning they would stop, wait, and then ask for questions or challenge. If other group members appeared inattentive they challenged their behaviour as illustrated when a student saw another scribble on the recording sheet, she said: *don't, man, you listen and ask or you aren't even learning, man.* On another occasion when a student responded to a conjecture with a disputational comment another student responded with: *don't dis her, man, when she is taking a risk.* Their recognition of their collective responsibility extended to their provision of explanatory justification in the larger sharing group. This is illustrated in the following vignette when a collective explanation is provided by group members stepping in and out to support each other. They then provide justification for their selection of the strategy solution explaining that it was the one they all understood and could explain.

Collective provision of explanatory justification
Anaru explains a solution strategy for 36 + 47 as her group clusters around tracking closely and quietly discussing her actions. She draws a numberline, records 47, draws a line from the 47 to 57, records + 10 above it, draws a line from 57 to 77, records +20. Hone   What are you doing? Anaru   Oh. I am adding...I am adding up my numbers. Wiremu   How is yours different from Mahine's?

Anaru	Different numbers.
Beau	What different numbers?
Anaru	Like the 20 and the 10.
Wiremu	Why have you only got one ten?
Anaru	[points at the 20] There are two tens.
Moana	[steps in to clarify further] What did you do with the first ten?
Anaru	I added…oh… I added it.
Hemi	[a group member extends the explanation] She added it to the 57. Plus another 20 were the two tens. That gave her 77.

Anaru records +3, 80 on the line, completes the solution recording +3 and 83 as group members Jim and Hone verbalise her actions.

Jim	Eighty…
Hone	Then you have got three so eighty three.
Donald	Why did you want to do it that way?
Anaru	Because it was the easiest way. Yes for me and for Alicia to understand.
Jim	[points at a previous explanation] Because it was a bit of a shorter way from there too.

(Term 4 Week 3)

Inquiry culture, ethnic socialisation, the role of groups

From the on-going interview data and reflective notes Moana made, I interpreted that she had constructed a clearer view of inquiry classrooms and what the learning in them looked like. Observational data reveals that she confidently drew on her situated knowledge of Te reo and Tikanga Maori and used her understanding of the "ethnic socialisation" (MacFarlane, 2004, p. 30) of her students. Her knowledge of their background supported their understanding of the changes she enacted in the interaction patterns. Within the sociocultural perspective of this study, groups were used as instructional agents. Moana guided the group members' appropriation and use of mathematical understandings of those more knowledgeable to advance collective understanding and access to the mathematical discourse. Within the Maori and Pacifica dimension Moana drew on, she positioned students to consider the more knowledgeable students not as individuals but rather as a knowledge component of a whanau (a collective). In turn, the students responded with an increased sense of interdependence as whanau members.

7.5.2 FURTHER DEVELOPING STUDENT AGENCY OF THE MATHEMATICAL DISCOURSE

Moana observed the students' growth in collective responsibility but she noted that in her presence the students behaved less autonomously and still looked to her to lead. Whilst she had addressed the former pattern of interjecting by requiring that students put their hands up Moana recognised that the students needed to contribute more to the management of the flow of discussion. After our analysis and discussion of a video record of a classroom observation she stated: *opportunities here exist for children to ask and dispute and so I need to let the children guide their own questioning and discussion more.* Moana introduced the use of koosh balls[8] to scaffold student management of the inquiry and debate. These were placed in the middle of the discussion circle; the students picked them up indicating a question or challenge. The following vignette illustrates how the koosh ball mediated the discourse, slowing discussion down and making questioners more accountable. Although picking up the ball indicated a question or challenge, the self esteem of the explainer was protected and they were provided with cognitive space to take time to think, and then respond.

Using the koosh ball as a tool to facilitate explanatory justification

Hone explains as he notates a collective addition strategy for 236, 219, 221, and 214:	
Hone	We added 236 and 214 which equals 450.
Aroha	[picks up a koosh ball then points at 219 and 221] Why didn't you add those two?
Hone	Because it tidies up these numbers. Because there is a six and four which equals ten.
Aroha	But why didn't you use the other ones?
Moana	[steps in to press for further clarification and justification] But why didn't you just add the 236 and 219?
Hone	Because then it will be harder…because it won't…like not equal…it won't like be tidied up.
(Term 4 Week 3)	

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[8] A soft ball that did not roll but which fitted into the students' hand

Intellectual partnerships, student agency and autonomy, teacher facilitator

The students were provided with a predictable structure to engage in inquiry and argumentation and the introduction of the koosh ball increased their agency to question and inquire. An intellectual partnership had been established and Moana noted the facilitative role she now held as she analysed a video record: *I am more guiding and refining the explanatory process…or making the understanding available to everyone. I am identifying children who still have misconceptions, or those who are not fully convinced, or those who know but want to know why.* Martin et al. (2005) illustrated that both the culture and structure of classroom communities positively influences student beliefs and attitudes toward mathematics and provides diverse "students with opportunities to gain deeper mathematical dispositions" (p. 19).

Discussion driven by student talk and contribution positioned Moana as much a participant in the discussion as she was the facilitator of it. Moana was drawn into what Lerman (2001) describes as a "symbolic space" (p. 103). Within this zpd, she too was required to engage with and work to understand the perspectives of all other participants. She acknowledged her role when she recorded the following reflection after a lesson: *Making sense of the process as much as they are. Sometimes my questions are to support me making sense as much as they are to show the kids what sorts of questions they need to be asking to make sense.* In further discussion Moana explained that she needed to question to sense-make and then she used the students' ideas to progress collective understandings. As Martin and colleagues (2005) describe "the centrality of students' voices provides teachers with opportunities to develop and modify their instruction" (p. 19).

Interaction patterns had shifted and the predominant form of talk was exploratory (Mercer, 2000). Mathematical discussions were extended as the students negotiated and renegotiated their reasoning to develop a shared perspective founded in the variable contributions of different members. At times, their partial understandings caused dissension, doubt and confusion but they now persistently explored, explained, and argued until all members understood and were convinced. These findings are similar to those reported by a number of researchers (e.g., Azmitia & Crowley, 2001; Brown & Renshaw, 2004; Goos et al.,

1999; Goos, 2004) when students of equal levels of understanding work collaboratively in zones of proximal development.

7.5.3   PROBLEM SOLVING AND A SHIFT TOWARDS GENERALISING

Moana introduced the use of a series of problems specifically constructed to support the students making connections from day to day and across problem situations. Previously Moana had carefully controlled how the students worked with problems but our discussion of video records of classroom observations prompted her to reconsider; now she gave the problems directly to the groups and asked them to conjecture solution strategies and then use their 'rethink time' to pattern-seek. To support them, Moana modeled questions they could use to compare, evaluate, and make connections between the conjectures. She told the students: *in your group there are going to be different strategies and different ways and you are going to pool all your combined collective knowledge to solve these problems. You need to be sharing the knowledge. You need to be asking questions. Questions like why did you use that number, why did you do that, what strategy are you using, is it more efficient than that one, which is the most efficient way, is my way more efficient than yours, why is it, why isn't it, is that the easiest way to understand it?*

Moana's requirement that the students examine and compare solution strategies provided them with opportunities to construct mathematical relationships—relationships which extended mathematical reasoning beyond the context of the immediate problem. A problem solution became applicable across problems as Moana worked with the students to identify and connect patterns and regularities. They were pressed to connect with their prior understandings and use this to construct new ways of reasoning. For example, after an explanation and before the next one Moana told the students: *what is happening here is that you are comparing the two strategies in your head. Okay? Then you might be able to try those strategies out. Or add them to the knowledge that you already have, the maths knowledge that you already have. So that's why this is really important, because you are really learning from each other...you just add to your existing base of knowledge. Think about those strategies. Think about your strategy and how you would apply them...and which one you liked...and which one you thought worked best...which one you understand.*

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Not only were students pressed to make connections to their own understanding, Moana also required them to analyse, compare, and justify differences in the efficiency or sophistication of solution strategies. Through the shift Moana enacted, the students were positioned to analyse and compare solution strategies and justify their stance. The vignette illustrates how Moana's press led to pattern seeking and provision of explanations which included generalised number properties.

Validating mathematical reasoning autonomously

Aroha explains a solution strategy for adding of 43, 23, 13, 3. She records 43, 23, 13, 3 and then 3 x 4 = 12

Aroha	I am adding forty three, twenty three, thirteen and three, so three times four equals twelve.
Kea	Why are you trying to do that with those numbers? Where did you get the four?
Aroha	[points at the 3 digit on the four numbers] These threes, the four threes.
Donald	[another member of the group] All she is doing is like making it shorter by like doing four times three.
Hone	[the third group member] Because there are only the tens left.
Donald	Three times four equals twelve and she got that off all the threes like the forty three, twenty three, thirteen and three. So she is just like adding the threes all up and that equals twelve.

After an alternative explanation is provided Moana positions the students to compare strategy solutions.

Moana	Did they use the same strategy as Anaru?
Faalinga	I don't think so.
Moana	You don't think so? So they haven't used the same strategy? [Moana observes that Danny nodded his head when she asked if they had used the same strategy so she turns to him] Argue with them Danny.
Danny	They have used the same strategy because they have both added the ten and the twenty and the forty and they have both added the threes together.
Moana	Well done you guys. They all added the threes together. You prove that you did the same thing Anaru. You show what you did with the threes.
Anaru	[draws an array of four sets of three] Three plus three plus three plus three. [points at each set of three] Or four times three or three times four [points across the top of the array and down the side of the array].

(Term 4 Week 4)

Rethink time as an example space, interthinking

The classroom community was now one in which the students had learnt to suspend judgment in order to examine and explore a range of possible solution strategy steps in what Watson and Mason (2005) term 'example' spaces. The problem contexts and the careful attention paid to identifying relationships, regularities, and patterns, beyond the immediate surface features of problem situations were tools which mediated a shift towards generalising. Moreover, as they analysed and discussed the conjectures their use of exploratory talk (Mercer, 2000) was evident in the words they used (because, why, so, but and forms of either agreement or disagreement). Mercer outlines the importance of these words if students are interthinking within zones of proximal development.

7.5.4 JUSTIFYING EXPLANATORY REASONING THROUGH INSCRIPTIONS

Moana used the communication and participation framework as a reflective tool to analyse student engagement in proficient mathematical practices. In our discussion she outlined how the communication and participation framework and the questions and prompts framework developed in the study group: *helps if I am going to plan a focus. I say this is the mathematical practice I am going to focus on. So what do I need to do to help the children engage better in it? What prompts do I need to use and what prompts could I help them use because I look on myself as a total facilitator, facilitating them engaging in mathematical practices.* She then described the limited attention she attributed previously to student notation and how she planned an explicit focus on them in this phase.

Moana established a requirement that the students illustrate their reasoning using a range of symbolic schemes, including informal notational schemes, diagrams and pictures. She modeled the use of different notational schemes, notating as students explained, or asking students to specifically justify their reasoning using their inscriptions. As a result, the following vignette illustrates how the symbolic schemes became sense-making tools which mediated students' analysis, justification and validation of their reasoning.

Notating to validate mathematical reasoning and clarifying terms

Donald is explaining a solution strategy for 63 – 26 using an empty number-line

Donald	First it was 26 so I took away 20 and that equals 43.
Mahine	Hang on. Can you hear what you are saying? 26 take away 20?
Donald	63. I mean. 63 take away 20 equals 43. Take away 3. That equals 40?
Caliph	You tidied that up then?
Mahine	[points at the beginning of the numberline] Why did you start there? It's take away?
Donald	It doesn't matter if I add up or subtract but if it's easier for you to see I will take away, so take away twenty, take away three.
Mahine	But why three now?
Donald	Because it is easier and it takes you to the nearest ten. Do you understand that way so far? Okay then I take the last three so 37. Got it? If you don't well just ask.
Mahine	So where did the…oh yes I see the other three was from the six.
Donald	Because I was subtracting twenty six so first it was the twenty then the three…

Caliph/Mahine [in unison add] And then the other three.

(Term 4 Week 3)

Inscriptions as public reasoning tools, informal notation schemes

The explicit focus placed on inscriptions pressed the students to examine and analyse whether their informal notation scheme represented their reasoning. During this dialogue explanatory steps were enlarged or modified and other notation forms devised in preparation for provision of explanatory justification as the inscriptions became valued as public reasoning tools. Sawyer (2006) illustrated that different representational forms students use are important in that they make mathematical reasoning public and accessible to an audience. As Cobb (2002) has described previously, in Moana's classroom the obligation to make available multiple explanations of reasoning influenced how students learnt and used representations. Clarification of explanations and the need to further justify were powerful reasons to re-present and recast notation schemes. In addition, many of the invented notation schemes closely approximated more standard procedures while all the time retaining a conceptual sense.

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7.5.5 SUMMARY OF THE THIRD PHASE OF THE STUDY

The enacted participation and communication patterns had established an inquiry community which supported student engagement in a range of mathematical practices. Learning and using mathematical practices developed steadily as the students gradually gained access to productive mathematical discourse. Inquiry and the need for multiple forms of justification led to increased focus on the need for engagement in generalising and notating reasoning.

All the roles in the classroom had shifted significantly. Moana outlined the view she held of her role toward the end of the research explaining: *I had that big debate; are you a teacher, are you a facilitator, but now I see that as long as they are learning I don't care what I am.* My interpretation was that she had confidence in the intellectual community as a site which provided the students with many opportunities for participation in rich learning situations. She had constructed a new set of beliefs about doing and learning mathematics which valued mathematical communication and mutual engagement in collective reasoning. Moana had drawn on her own knowledge of the ethnic socialisation of her students to accommodate the social and cultural practices the students were accustomed to in their homes and communities, and those being constituted in the classroom context.

7.6 SUMMARY

This chapter has mapped out the gradual, often circuitous and challenging journey Moana and her students made as the foundations of a community of mathematical inquiry were constructed. How the use of the communication and participation framework, the framework of questions and prompts, research articles, video records, study group activities and on-going discussion with me mediated a shift in beliefs and transformation of the sociocultural and mathematical norms of the classroom are illustrated.

The initial classroom culture was described as a conventional one. Shifting the interaction patterns towards a strategy reporting one (Wood & McNeal. 2003) caused many conflicts in Moana's and her students' beliefs about doing and using mathematics. A new set of roles

and scripts were needed for all participants. The communication and participation framework was reconstructed as small, incremental steps and used to provide foundations for a collaborative community. Constructing the norms of a strategy reporting classroom community took a full three months as the students learnt to actively engage in explaining, questioning and mathematically sense-making. Student ability to make conceptual explanations provided an important foundation from which other mathematical practices were learnt and used more proficiently. Questioning and challenge were important factors in deepening conceptual understanding and extending the collective reasoned discourse.

Moana used specific pedagogical practices to induct the Maori and Pasifika students and girls to actively participate in collective reasoning. In turn, these students' positive response to changes in the classroom context and mathematical activity prompted shifts in Moana's beliefs and supported her continued enactment of changes in the interaction patterns. Evidence was provided of the multiple voices (Forman & Ansell, 2001) which operated in the classrooms and which shaped the carefully measured pedagogical actions Moana took to press student engagement in inquiry and argumentation. At the conclusion of the study there had been significant changes in student autonomy and agency and this held potential for continued growth in the efficient use of interrelated mathematical practices. Moana had established her own voice and in doing so had constructed an intellectual community which placed value on the centrality of student voice.

The following chapter draws together this chapter and Chapter Six. The different pathways the two teachers took on their journeys to develop mathematical inquiry communities are discussed and the similarities and differences in the pedagogical practices they used are elaborated on. The contributions this research has made to the research field and the limitations and implications of this study are examined. A concluding section confirms that communities of mathematical inquiry can be constructed and these provide opportunities for students to participate in rich collective reasoning practices.

CHAPTER EIGHT
CONCLUSIONS AND IMPLICATIONS

As students engage in classroom practices, and in mathematical practices, they develop knowledge and they develop a relationship with that knowledge. Their mathematical identity includes the knowledge they possess as well as the ways in which students hold the knowledge, the ways in which they use the knowledge and the accompanying mathematical beliefs and work practices that interact with their knowing. (Boaler, 2003b, p. 16-17)

8.1 INTRODUCTION

The intention of this thesis was to examine how teachers construct communities of mathematical inquiry in which the participants collectively engage in the use of rich mathematical practices. The literature review examined the differential outcomes which result from the different patterns of participation and communication in mathematics classrooms. An important thread maintained in the review of the literature was the significance of constructing intellectual learning communities in which students learn to participate in, and use, reasoned mathematical actions and discourse.

The complexities and challenges teachers encounter in developing collective inquiry and argumentation within the classroom context was considered. As a result, the use of design research to simultaneously support and examine the pedagogical actions teachers take to engage students in proficient mathematical practices in communities of inquiry was proposed. A key element of the design research was a communication and participation framework which was used by the teachers to map out possible pathways they could take to construct the interaction patterns of inquiry communities.

Detailed descriptions were presented of the pedagogical actions two case study teachers took to constitute the sociocultural and mathematical norms which supported student engagement in reasoned communal mathematical inquiry and argumentation. Whilst the transformation of communication and participation patterns was a gradual process, the

impact on student engagement in mathematical practices was significant. Significant changes were revealed in both classrooms as the teachers enacted progressive shifts in the sociocultural and mathematical norms which validated collective inquiry and argumentation as learning tools. Higher levels of student involvement in the mathematical dialogue resulted in increased intellectual agency and higher intellectual levels of verbalised reasoning.

This research focused on one key question:

> *How do teachers develop a community of mathematical inquiry that supports student use of effective mathematical practices?*

Section 8.2 summarises the different pathways the two teachers took to construct mathematical inquiry communities. Key features of the constitution of the sociocultural and mathematical norms and how these resulted in the emergence and evolution of mathematical practices are explained. Section 8.3 describes the pedagogical practices the teachers used to support their students' participation in collective reasoned communication within inquiry communities. Section 8.4 examines and describes the critical features of the tools which mediated the teacher development of inquiry communities and its discourse.

Section 8.5 presents the contributions this research has made to the research field. In Section 8.6 and 8.7 the limitations and implications of this study are examined alongside suggestions for further research. Section 8.8 provides a final conclusion to this research.

8.2 THE PATHWAYS TO DEVELOPING COMMUNITIES OF MATHEMATICAL INQUIRY

This research documented the journey two teachers took as they developed their classroom participation and communication patterns as part of their goal to establish student engagement in effective mathematical practices in classroom inquiry communities. Within the research study both teachers participated in the same core professional development activities and used similar tools to mediate change but their development of mathematical

learning communities took different pathways. These may be attributed to the differing intermediary goal points on their trajectories and the "dis-coordination and resulting conflicts" (Brown & Cole, 2002, p. 230) the teachers experienced.

The way in which each teacher considered the discipline of mathematics was of central importance to how they instituted and maintained changes in their pedagogical practices and their students' learning practices. At the start of the research Ava readily adopted a view of mathematics as a discipline of "humanistic enquiry, rather than of certainty and objective truth" (Goos et al., 2004, p. 112). Her classroom culture reflected small shifts towards inquiry. This provided her with a foundation to further develop and refine a learning context founded in mathematical inquiry and argumentation. Moana, in contrast, viewed mathematics as a discipline of absolute certainty rather than humanistic inquiry. The conventional learning climate she had constructed supported her use of prescriptive teaching practices, her central position of authority, and a predominant use of univocal discourse. As a result, changing the classroom context and learning a new set of roles and responsibilities for a mathematical inquiry community presented Moana and her students with many contradictions and conflicts.

It was evident that the role the teachers adopted in the classroom communities shaped their response to initiating changes in the communication and participation patterns. At the beginning of the study Ava readily repositioned herself as a participant in the discourse. She modeled the process of inquiry, sense-making and self-monitoring, and emphasised the community's shared authorship of the mathematical reasoning. Moana's pathway to establishing a learning partnership with her students required significant reshaping of expectations and obligations in the mathematics community and took a lengthy period of time to realise. To achieve the shift Moana drew on her social and cultural understandings of a whanau (family grouping). She emphasised the students' responsibility for their own mathematical learning but also the need for collaborative support and responsibility for the learning of others as well as respectful listening and assertive communication. The teachers' observations that changes in the learning culture resulted in positive learning

outcomes for students, were powerful tools which reshaped their subsequent understandings of teaching and learning mathematics.

The constitution of a safe supportive learning environment—one which promoted intellectual risk-taking—was a critical component in the formation of the inquiry communities. Both teachers used a range of strategies to attend to their students' affective needs, including direct discussion of the need for collegiality and inclusion, risk-taking, and the repositioning of themselves and their students as risk-takers. The existing classroom culture within Ava's classroom supported her reconstitution of the interactional norms through direct discussion of her expectations and the students' obligations. Moana engaged in a lengthier process, initially needing to address some of her students' attitudes and behaviour during mathematical activity. She did this by closely engineering learning partnerships, specifically placing the Maori, Pasifika and female students in supportive pairs. When using larger groups she monitored the interactions and engendered learning competence through specific positioning of the shy, low achieving students, and the girls. She used her knowledge of the students' ethnic socialisation to provide models of the sociocultural norms she wanted enacted.

It was evident in both classrooms that an emphasis on independence, not dependence, was a key feature in the students' growth in mathematical agency.

8.2.1 SCAFFOLDING STUDENT COMMUNICATION AND PARTICIPATION IN MATHEMATICAL PRACTICES

Within the classrooms, communal construction and examination of mathematical explanations were an important precursor for supporting the development of explanatory justification and generalisation. The level of student engagement in productive discourse was the key factor which shifted the focus of mathematical reasoning past mathematical explanations to student communication and participation in many interrelated mathematical practices.

Initially, the most common form of talk used in both classrooms were those which Mercer (2000) terms cumulative or disputational. To change these unproductive forms of talk Ava immediately scaffolded collaborative small group construction and examination of mathematical explanations. In a larger group setting, she facilitated questioning and clarification of conceptual explanations, providing 'think-time' so that explanations and errors became reflective tools. Moana took a more gradual route with her initial focus concerned with addressing students' negative interjections and persistent attention to answers rather than sense-making. Her use of ground rules for talk established clear boundaries. Through small, incremental steps she gradually established active listening, questioning, explaining and rethinking mathematical reasoning. The deliberate focus both teachers placed on student analysis of their reasoning and the reasoning of others provided the foundations for developing the discourse of inquiry and argumentation.

It was evident in this study that enacting the norms which supported inquiry and argumentation caused all participants on-going conflicts and contradictions. The teachers acknowledged their own novice status in a mathematics environment which used inquiry and argumentation. They also expressed concern at what they perceived to be a lack of fit between the cultural and social norms of their students and the requirement that they engage in the mathematical discourse of inquiry and argumentation. The literature (e.g., Andriessen, 2006; Mercer, 2000; Wells, 1999) recognises that many students hold contrary views on argumentation and the teachers expressed similar views. They also recognised that mathematical inquiry and argumentation as a specific speech genre was not currently part of their students' repertoire of cultural practices and therefore developing it required careful attention. Of significance in achieving this was the explicit attention they gave to discussing and exploring with their students' their attitudes towards mathematical argumentation.

Almost immediately, using the communication and participation framework to map out a pathway, Ava scaffolded the foundations for what Wood and McNeal (20030 term an inquiry or argument culture. She scaffolded the use of questions and prompts and asked that the students construct multiple explanations, examine these closely, and rehearse possible

responses to questions or challenge. She pressed her students—revoicing and positioning them to take a stance to agree or disagree—but required that they mathematically justify their position. She provided models of mathematicians' use of 'maths arguing' as a tool to progress collective reasoning. These actions shaped the form and content of the discourse the students used to justify and validate their reasoning and provided them with a predictable framework for inquiry and argumentation.

In contrast, Moana maintained a lengthy focus on developing mathematical explanations and in her classroom the interaction patterns steadily shifted from a conventional classroom culture to what Wood and McNeal (2003) term a strategy reporting classroom. When Moana observed evidence of student ability to construct, explain, clarify and elaborate on their individual and collective mathematical reasoning she increased her expectation that they take a stance, explain and justify agreement or disagreement. Drawing on her observations of their growth in mathematical confidence and increased use of productive mathematical discourse, she further scaffolded the use of inquiry, all the time considering the students' home and school social and cultural contexts. The focus of questions shifted from questioning for additional information or clarification, to questioning for justification. This gradual shift took more than half the school year before the classroom culture could be described as an inquiry or argument context. However, in the final research phase it was evident that both Ava and Moana and their students readily accepted the need for extended mathematical discourse and exploratory talk. The students used mathematical inquiry and argumentation, during dialogic discourse, to examine and explore the perspectives of others' and ultimately achieve consensus.

In this study, clear evidence is provided of the difficulties both teachers had attending to and developing generalised reasoning in their classrooms. It seemed that neither had previously abstracted the fundamental numerical patterns and structures of numbers or operational rules, nor had they considered exploring these with their students. In the first phase of the study the teachers often did not appear to 'hear' student voiced intuitive generalisations. However, an increase in inquiry and argumentation supported them attending to, and building on, the students' observations of patterns and relationships. The

earlier use of inquiry and argumentation in Ava's classroom explains the earlier shift to the examination and use of generalised reasoning and why it was only in the latter stages of the research that this occurred in Moana's classroom. Although the increase in shared classroom talk prompted the teachers to afford explicit attention to developing generalised reasoning in the classroom communities other tools were also of importance. These included student provision of multiple ways to justify and validate reasoning, the use of numerical patterns to validate reasoning, the use of a specific set of questions and prompts for generalisations, position statements, the use of open-ended problems and a requirement that the students analyse and compare solution strategies for efficiency and sophistication.

Importantly, the students' increased participation in mathematical reasoning at higher intellectual levels was the prompt which caused the teachers to continue to press for inquiry and argumentation. In turn, their increased expectations provided the students with a platform to develop explanatory justification, generalised reasoning, the construction of a range of inscriptions to validate the reasoning, and a more defined use of mathematical language. Clearly apparent in this study were the differential outcomes which emerged as the frequency and complexity in the questioning and challenge used by the teachers and students increased. Higher levels of complexity in articulated reasoning were achieved earlier in Ava's classroom but both classroom communities at the conclusion of the research readily used interaction patterns most often premised in inquiry and argument and the use of exploratory talk.

 This section has described the pedagogical practices the case study teachers took to develop productive mathematical discourse which supported the development of mathematical explanations, explanatory justification, and generalised reasoning. However, it would be reasonable to suggest that the more difficult task which confronted the teachers was knowing which actions not to take—that is reducing cognitive challenge when introducing problems or doing the mathematical reasoning and talking for the students. Allowing the students' time to wrestle with confusion and erroneous thinking in what appeared messy and inefficient ways before attaining sense-making was challenging. Their complex role required 'on the spot' decision-making of pedagogical practices which best

facilitated cognitive and social opportunities in which the students come to know and use mathematical practices—practices which focus on not only "the learning of mathematics, but the *doing* of mathematics—the actions in which users of mathematics (as learners and problem solvers) engage" (Boaler, 2003b, p. 16).

8.3   SUPPORTING STUDENTS TO BECOME MEMBERS OF COMMUNITIES MATHEMATICAL INQUIRY

It was evident in this study that constituting mathematical inquiry communities conflicted with how the students had previously viewed mathematics—beliefs formed through their previous experiences in more prescriptive learning environments. The two case study teachers used a range of pedagogical roles and practices to shift the sociocultural and mathematical norms so that their students came to know and do mathematics as a process of inquiry and argumentation. The extended time and the pedagogical roles the teachers assumed to enact sociocultural norms which supported shared constitution of mathematical norms were important. Of significance were the changes in their pedagogical roles from a predominant use of univocal discourse to one in which they facilitated dialogic discourse. This supported a shift in the students' role as new identities were created. As the teachers adopted a range of varying roles their students gradually assumed mathematical agency within classroom communities premised on intellectual partnerships.

Important actions the teachers took to develop their students' mathematical agency included the expectation for the students to take ownership for communal responsibility for sense-making during mathematical activity. Activities, for example included the expectation of a group recording of the reasoning, and a flexible approach to pairing and grouping with consideration for the social and cultural context of the students. The Māori and Pasifika dimensions of the students were drawn on to establish key aspects of the whanau (family and collective) concept. These included assertive communication, the value of diversity and multiple perspectives, valuing effort over ability, assigning competence to individuals, and positioning the more knowledgeable as valued knowledge sources within the collective.

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The explicit framing of classroom interaction patterns so that all participants engage in the reasoning activity supported student 'interthinking' within shared communicative space. Scaffolding to consider mathematical reasoning within multiple perspectives occurred through the guidance offered by the teachers. This included requiring the groups to consider questions they might be asked, or sections of their explanations others might find difficult; and developing multiple ways to explain, elaborate, justify, and validate mathematical reasoning. Through the need to engage with group members' different perspectives, variable contributions, and partial understandings, multiple zones of proximal development evolved. In turn, in order to negotiate shared perspectives the students encountered mathematical situations which required transactive exploration and speculation— mathematical activity which often closely approximated those used by competent users of mathematics.

The teachers used many tools to mediate student communication and participation in mathematical activity. Of particular consequence was the provision of a predictable framework for strategy reporting, inquiry and argument. The teachers directly modeled ways to explain and justify mathematical reasoning. They ensured a gradual shift in the use of specific questions and prompts for the different mathematical practices so that increased levels of intellectual reasoning resulted. Consistent teacher revoicing, reshaping, and extending student use of informal terms and concepts while maintaining focus on the mathematical content, provided the students with access to a mathematical register and to knowledge of how to participate in mathematical discourse. The teachers ensured that the students developed understanding of inscriptions as sense-making tools to explain, elaborate, and validate mathematical arguments through direct modeling. Other tools included the problematic and open-ended tasks used during mathematical activity. A fundamental feature in the design of the problems focused on key mathematical content while at the same time emphasising enactment of specific participation and communication goals. Tasks were frequently supported with direct guidance to the students on how they were to engage in and communicate their mathematical reasoning.

Evident in this research is the significance of sustained time which supported student learning of key reasoning practices. Provision of 'time and space' was apparent in mathematics lessons in which examination of mathematical reasoning extended across both a lesson to lessons and a problem to problems. Partial understandings and misconceptions common to the student group were identified and extensively explored and as a result errors became valued tools used within the community to grapple with complex ideas and develop deeper conceptual understandings. Opportunities for sustained examination supported analysis and exploration of commonalities and differences and identification of sophistication or proficiency in solution strategies. 'Extended time and space' was also a tool the teachers used to mediate student appropriation of the communication and participation patterns of inquiry. They explicitly facilitated 'time and space' to support the students to explore, experiment with, examine, extend and innovate on models of the questions and prompts. Both teachers readily accepted and provided opportunities for the students to copy, practise, try out, and experiment with ways to use discipline specific dialogue. Similarly, they ensured space for the students to use, extend, and innovate on the inscriptions.

8.4 SUPPORTING TEACHERS TO CONSTRUCT COMMUNITIES OF MATHEMATICAL INQUIRY

In the teachers' construction of communities of mathematical inquiry there was no single unique factor which supported them although some influences proved particularly effective. The communication and participation framework (see 5.4.1) was a significant tool which threaded through every stage of the project. Initially, it was used as a tool to map out shared possible pathways the teachers could use to transform the communication and participation patterns in their classrooms. As the project progressed it served as a reflective tool, used by the teachers to analyse the development of inquiry communities and plan their next focus. It was a flexible scaffold which over the duration of the study supported backward and forward movement, the renegotiation of contexts and the development of different or more detailed pathways for individual teachers.

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The framework of questions and prompts for mathematical practices (see 5.4.1) was an additional tool which augmented the communication and participation framework. The framework focused the teachers' attention on the need to consider how the questions and prompts they and their students used, shaped mathematical activity and dialogue. It served as a more explicit reflective tool to help their analysis of classroom interactions and plan their next focus. It shaped their viewing of classroom interactions on video records and provided the teachers with important evidence that higher levels of complexity in questions and challenge were matched with higher levels of articulated mathematical reasoning.

Realisation of the positive student mathematical learning outcomes was a powerful factor in the shift of teachers' pedagogical practices. The video classroom records and transcripts provided clear evidence of how shifts in classroom interaction patterns advanced the students' learning and social goals. In the study group context maintaining a focus on identifying and analysing the communication and participation patterns, and questions and prompts, assisted identification of how these actions supported student engagement in mathematical practices, the emerging use of different mathematical practices, and the actions the teacher took to enhance their student's engagement in more proficient mathematical practices.

As illustrated in the previous section, of prime importance in the construction of classroom communities of mathematical inquiry were the opportunities the teachers gave the students to learn how to participate in collective inquiry and argumentation. Similarly for the teachers, participating in dialogic inquiry was a key factor in the transformation of their beliefs and attitudes towards knowing and doing mathematics within communities of mathematical inquiry. The teachers engaged in dialogic inquiry on many levels in a range of contexts, including formal staff meetings, study group meetings, and less formal discussion with their colleagues and me. Research articles, a DVD, and videos which illustrated aspects of inquiry classrooms and their sociocultural and mathematical norms were powerful tools. These provided opportunities for the teachers to reflect on the habitual patterns of interaction they used in their mathematics lessons. They suggested possibilities for change and formed the basis for objective discussions.

It was evident in this study that for the teachers the study group was an important learning site. Within this context the teachers developed deeper collegial relationships as they co-constructed through lengthy conversations the reshaping of their classroom mathematical communities. Although the primary focus of this group was the negotiation and renegotiation of the communication and participation patterns towards inquiry, the study group was responsive to other needs. These included extensive discussions of the tensions and contradictions the teachers encountered as they scaffolded inquiry and argumentation in their classrooms and exploration of solutions to these. The teachers used the study group context to review and reflect on their own history as learners and users of mathematics and reconcile this with the changes they were implementing. Another activity which evolved as an on-going need was the collaborative construction and exploration of mathematical problems. These problems gave the teachers' opportunities to examine and explore the small pieces of mathematical knowledge, used in mathematical activity, in detail. Examination of the problems also supported the teachers to anticipate the erroneous thinking which might emerge, or the possible strategy solutions, as well as the patterns and relationships inherent in the problems. The communally constructed problems were valued tools, used in the classrooms to advance both the mathematical content agenda and the social agenda.

In mediating change, 'time and space' was as important for the teachers as it was for their students. Time supported each teacher to develop their own pathway and approach in changing the interaction patterns in their classrooms. Space provided them with opportunities to draw on their own situated knowledge, reflect on it, and engage in dialogic inquiry about it with their peers and the researcher. Through taking 'time and space' they were able to construct their own perspective of what inquiry communities and the mathematical discourse and mathematical activities in them might look like. Moreover, it provided the room for exploration, while sustaining teacher agency.

8.5   LIMITATIONS

While the research contributes new knowledge to the discipline at a variety of levels, any research has its limitations. The results of this research are based on empirical analysis of a small sample of teachers and students, in one school, in one urban area of a city. Given the small sample the generalisability of the findings for teachers in the context of different classroom settings in New Zealand may be limited. However, the explicit outline of the participation and communication framework, the framework of questions and prompts for mathematical practices, and the clear descriptions of the teachers' pedagogical practices, allows others to trial a similar study.

Because of the complex nature of schools and classroom practices, interpretation of the results in this study can only provide an emerging understanding of the pedagogical practices teachers use to enact the communication and participation patterns of inquiry communities. Although triangulation methods were used, consideration needs to be given to the possibility of bias in the results of this research. The presented findings are based on one researcher's interpretation of data from audio records and notes from study group meetings, classroom video records, interviews, teacher reflections and field notes. Other interpretations are possible, although the interpretations are strengthened by the use of a wide range of data sources, the use of a grounded approach in the search for confirming and disconfirming evidence, and the prolonged engagement with the teachers as co-researchers.

Consideration needs to be given to the impact of the research process on the school and classrooms. Included in the impact is the level of intrusion the presence of a researcher/ university lecturer caused in the school and classrooms. Thought also needs to be given to the intrusive effect the video recording had on the classroom context and the behaviour of all participants. This issue—the way in which the presence of another person and video recording in classrooms influences change—was discussed earlier in Chapter Five and the steps to minimise the disturbance explained. These included the collaborative relationship the researcher established with the teachers, the role of the researcher as participant

observer, and the discussion and practice runs which familiarised the students to the use of video capture before the start of the study.

Although there was evidence of student resistance and conflict as the interaction patterns shifted, this was not a focus of the study. Nor did the research design encompass exploration or examination of individual student's mathematical learning, or include exploration of the individual student's views and attitudes. The parents' and other community members' views were also not examined. These are key factors which are regularly considered in conjunction with community of learners approaches. Furthermore, despite the extensive time in the school the research did not collect data related to continued development of mathematical communities of inquiry, or the use and refinement of mathematical practices within the community. It is acknowledged that the processes involved in teachers constituting communities of mathematical inquiry are on-going.

8.6   IMPLICATIONS AND FURTHER RESEARCH

Possibilities for different ways of thinking about teaching and learning mathematics in New Zealand primary classrooms are suggested by this study. Evidence within this study suggests that it is the classroom teacher who makes possible the development of the mathematical discourse of inquiry in learning communities. This study appears to be the first of its kind in New Zealand in that it simultaneously supports teachers to constitute intellectual learning communities in mathematics classrooms and at the same time explores the pedagogical practices teachers use to develop them. It is important to extend this base of knowledge beyond the urban, low decile, primary school in this study. Understanding needs to be extended to how mathematical inquiry communities can be developed in other age and year levels, types of schools, deciles, locations, and with different ethnic groupings.

It was evident in this study that the teachers held mathematical content and pedagogical knowledge grounded in more traditional forms of mathematics teaching and learning. Initially, they lacked experience in, and understanding of the pedagogical knowledge required to enact and maintain the shared discourse of mathematical inquiry and

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argumentation, communal learning partnerships, small group interaction patterns, the development of exploratory talk, and the sociocultural and mathematical norms of inquiry communities. Additional research needs to incorporate understandings developed in this study of the pedagogical practices teachers use to constitute the mathematical discourse of mathematical inquiry communities. In particular, it also needs to extend the examination of the ways teachers directly scaffold exploratory talk. The design component and the communication and participation framework which mapped out shifts in the interaction patterns in this study need to be further trialled with different teachers in different contexts.

Although teacher attitudes and beliefs were not intended as a focus of this study, their effect on the constitution of the mathematical discourse of inquiry and argumentation became a significant factor. Further research is needed in New Zealand to explore how the past experiences of teachers in learning and using mathematics influences their current pedagogical practices. In particular, exploration is required of the specific attitudes and beliefs teachers hold towards the value and use of mathematical inquiry and argumentation. Specific interventions like those used in this study need further trialling to analyse how, when, and which factors are significant in changing attitudes.

Within this study the different pathways the teachers took to construct inquiry communities resulted in part from their situated knowledge of the ethnic socialisation of their students. Further research is required which draws on Maori and Pasifika dimensions to explore optimal means to scaffold Maori and Pasifika students' participation in inquiry and argumentation.

The professional development of the teachers was not a focus of this study, but the study group activities in which the teachers participated had an important influence on the outcome of the research. Important factors included the year-long sustained focus, access to relevant research material, regular objective collegial discussions of video records, evaluation of the interaction patterns, analysis of the questions and prompts on the video records, the examination of mathematical content and the experience of being in a community of learners. Similarly, through directing attention on the examination and

analysis of critical incidents of students engaging in and using mathematical practices on the video records the teachers developed deeper understanding of the nature of mathematical practices. This finding suggests implications for the ways in which the professional development of teachers might be enhanced.

Within this study the findings provide clear evidence of the positive outcomes for student reasoning when students engage in interrelated social practices. However, it was evident in this research that initially the teachers consistently focused on teaching mathematical content knowledge (or specific solution strategies) and they demonstrated little awareness of how mathematical practices could support the learning. It was only when direct attention was focused on scaffolding the emergence and use of mathematical practices that the teachers began to understand and explore what they were and recognise their value. The communication and participation framework and the development of the framework of questions and prompts were significant in progressing the teachers' understanding of mathematical practices as individual and interrelated social practices. Both tools need to be further trialled with a range of different teachers, schools, ethnic groups and across different age groups to grow the knowledge base of how teachers can be supported to develop richer understandings of how to construct and maintain communities of mathematical inquiry.

8.7   CONCLUDING WORDS

The intention of this research was to explore and examine how teachers developed their mathematical classrooms to embrace inquiry communities in which the students come to know and use mathematical practices within reasoned communal dialogue. Mathematical practices in this study were shown to be collective and interrelated social practices which emerged and evolved within the classroom mathematical discourse. The study revealed findings that add to the research knowledge of the relationship between patterns of participation and communication in mathematics classroom and differential outcomes for student engagement in mathematical practices.

The rich data generated in this collaborative design study provided evidence that communities of mathematical inquiry can be constructed despite complex and difficult challenges. In particular, the research reported in this study resonates with and extends the current body of literature that seeks to understand the many factors involved in the construction of such classroom communities. It documents in detail a range of pedagogical practices which support the constitution of mathematical inquiry communities, the resultant changes and effects of shifts in communication and participation patterns on the participants' roles and attitudes towards doing and knowing mathematics, and the mathematical practices which emerge and are used. It has extended understanding of these practices as initiation into social practices and meaning, within a New Zealand context and in primary (elementary) school classrooms.

Of particular importance for New Zealand teachers is that the findings of this research provide possible models for ways teachers can draw on and use their Maori and Pasifika students' ethnic socialisation to constitute mathematical inquiry communities which align with the indigenous patterns of learning of these students. It is in that sense that the findings might act as a springboard for significant educational change.

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APPENDICES

APPENDIX A: TEACHER INFORMATION SHEET AND CONSENT FORM

My name is Bobbie Hunter and I am currently doing Doctoral research focusing on understanding mathematical practices and how those practices are developed by teachers and learned by students. I am writing to invite you to participate in collaborative research during Terms 2, 3 and 4 of this year. At the end of last year (2003) the management staff of Tumeke School expressed an interest to take part in the *Numeracy Practices and Change* Project funding by the Ministry of Education Teaching and Learning Research Initiative. The research on *Mathematical Practices* which I hope to undertake in your school is part of this larger project which aims to investigate equitable effects of numeracy and factors associated with sustained reform numeracy practices.

Mathematical practices refers to those activities and modes of thinking that successful mathematical learners and users actually do. To investigate the development and enhancement of effective mathematical practices this study will involve a group of teachers from your school in a professional development programme directly linked to your own classroom.

All Year 4-8 teachers at the school who participated in the New Zealand Numeracy Project will be invited to participate in the study. The professional development programme will involve you as co-researchers trialling activities, and evaluating your teaching in relation to mathematical practices. A particular focus will be on the communicative and interactive nature of the learning environment. To facilitate this inquiry you will be involved in reflective practices such as classroom observations (sometimes with audio/video records) and journal writing. Permission with regard to audio/video recording will be sought from both parents and children in your class. The focus of the study is on the teaching strategies, and thus it will be possible to organize the recording devices to avoid those students who do not consent to participate.

The project is comprised of four phases:
1. Term 1: Professional development will focus on the New Zealand Numeracy Framework and Teaching Model.
2. Term 2: Over a five week period teachers in each of the senior syndicates will work within a collaborative partnership to develop and trial strategies related to student mathematical practices.
3. Term 3: Building on the work in Term 2, a second Numeracy unit will be trialed focusing on key numeracy ideas and communication patterns. I will case study 3 teachers (self-identified) as they use an explanatory framework to examine their own and each others mathematical teaching practices related to mathematical practices. Video, audio and journal material will be used in reflective observations.
4. Term 4: The teachers and researcher as a group will refine the explanatory framework and evaluate its usefulness as a professional development tool for teachers. In addition, we will reflect and report on the process of teacher change and changes in student mathematical practices and learning.

The time involved in the professional development meetings for you will be no more than five school days out of classrooms. The proposed professional development is in accord with the school strategic plan for numeracy focus. Your teacher release time will be fully funded by the *Teaching and Learning Research Initiative Project: Numeracy Practices and Change*.

All project data will be stored in a secure location, with no public access and used only for this research and any publications arising from this research. After completion of five years, all data pertaining to this study will be destroyed in a secure manner. All efforts will be taken to maximize confidentiality and anonymity for participants. The school name and names of all participants will be assigned pseudonyms to maintain their anonymity. Near the end of the study a summary will be presented to you to verify accuracy, and following any necessary adjustments, a final summary will be provided to the school and teachers involved.

Please note you have the following rights in response to my request for you to participate in this study.
- decline to participate;
- decline to answer any particular question;
- withdraw from the study at any point;
- ask any questions about the study at any time during participation;
- provide information on the understanding that your name will not be used unless you give permission to the researcher;
- be given access to a summary of the project findings when it is concluded.

If you have further questions about this project you are welcome to discuss them with me personally:
Bobbie Hunter: Massey University (Albany), Department of Technology, Science and Mathematics Education. Phone: (09) 4140800 Extension 9873. Email. R.Hunter@massey.ac.nz; or contact either of my supervisors (co-directors of the *Numeracy Practices and Change* Project) at Massey University (Palmerston North), College of Education

Associate Professor Glenda Anthony: Department of Technology, Science and Mathematics Education. Phone: (06) 350 5799 Extension 8600. Email. G.J.Anthony@massey.ac.nz

Dr. Margaret Walshaw: College of Education. Department of Technology, Science and Mathematics Education. Phone: (06) 350 5799 Extension 8782. Email. M.A.Walshaw@massey.ac.nz
This project has been reviewed and approved by the Massey University Human Ethics Committee, ALB Protocol NO/NO *(insert protocol number)*. If you have any concerns about the conduct of this research, please contact Associate Professor Kerry P Chamberlain, Chair, Massey University Campus Human Ethics Committee: Albany, telephone 09 443 9700 x9078, email K.Chamberlain@massey.ac.nz.

CONSENT FORM: TEACHER PARTICIPANTS

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree to participate in this study under the conditions set out in the Information Sheet.

I understand that professional development will involve discussion of my own and other teacher's classroom practice and I agree to keep descriptions of specific classroom episodes confidential.

Signature: .... **Date:** ....

Full Name - printed ....

APPENDIX B: STUDENT INFORMATION SHEET AND CONSENT FORM

My name is Bobbie Hunter and I am currently doing Doctoral research focusing on understanding how mathematical practices are developed by your teacher and learned by you. Your teacher is one of several teachers in the school taking part in this research study during Terms 2 and 3 of this year.

As part of the research we will need to make some classroom observations and therefore I am writing to ask your permission for you to be audio or video recorded as part of your teacher's record of their practice. The focus of the recordings will be on your teachers' teaching strategies and so at no time will you be focused on or audio or video recorded for any length of time. The recordings would be of usual mathematics lessons and so you would not need to do anything special for the cameras or tape-recorder. In addition, your teacher may want to take a copy of some of your written work to help with their recordings.

All data recordings will be stored in a secure location, with no public access and used only for this research. In order to maintain anonymity the school name and name of all participants will be assigned pseudonyms in any publications arising from this research. At the end of the year, a summary of the study will be provided to the school and made available for you to read.

Please note you have the following rights in response to my request for you to participate in this study.
- decline to participate;
- decline to answer any particular question;
- withdraw from the study at any point;
- ask any questions about the study at any time during participation;
- provide information on the understanding that your name will not be used unless you give permission to the researcher;
- be given access to a summary of the project findings when it is concluded;
- have the right to ask for the audio/video tape to be turned off at any time during the observations;
- have the right to not allow copies of your written work to be taken.

If you have further questions about this project you are welcome to discuss them with me personally:
Bobbie Hunter: Massey University. Albany. College of Education. Department of Technology, Science and Mathematics Education. Phone: (09) 4140800 Extension 9873. Email. R.Hunter@massey.ac.nz

Or contact either of my supervisors:
- Associate Professor Glenda Anthony: Massey University. Palmerston North. College of Education. Department of Technology, Science and Mathematics Education. Phone: (06) 350 5799 Extension 8600. Email. G.J.Anthony@massey.ac.nz

- Dr. Margaret Walshaw: Massey University. Palmerston North. Department of Technology, Science and Mathematics Education. Phone: (06) 350 5799 Extension 8782. Email. M.A.Walshaw@massey.ac.nz

This project has been reviewed and approved by the Massey University Human Ethics Committee, ALB Protocol NO/NO *(insert protocol number)*. If you have any concerns about the conduct of this research, please contact Associate Professor Kerry P Chamberlain, Chair, Massey University Campus Human Ethics Committee: Albany, telephone 09 443 9700 x9078, email K.Chamberlain@massey.ac.nz.

CONSENT FORM: STUDENT PARTICIPANTS

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me.

My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to be audio taped during mathematical lessons.

I agree/do not agree to be video taped during mathematical lessons.

I agree to copies of my written work being collected.

I agree to participate in this study under the conditions set out in the Information Sheet.

Signature: .... **Date:** ....

Full Name - printed ....

APPENDIX C: BOARD OF TRUSTEES INFORMATION SHEET AND CONSENT FORM

To the Chairperson
Board of Trustees
Tumeke School

Dear Sir/Madam

My name is Bobbie Hunter and I am currently doing Doctoral research focusing on understanding mathematical practices and how those practices are developed by teachers and learned by students. I am writing to request permission to undertake collaborative research with teachers in your school during Terms 2, 3 and 4 of this year. At the end of last year (2003) the management staff of Tumeke School expressed an interest to take part in the *Numeracy Practices and Change* Project funding by the Ministry of Education Teaching and Learning Research Initiative. The research on *Mathematical Practices* which I hope to undertake in your school is part of this larger project which aims to investigate equitable effects of numeracy and factors associated with sustained reform numeracy practices.

Mathematical practices refers to those activities and modes of thinking that successful mathematical learners and users actually do. To investigate the development and enhancement of effective mathematical practices this study will involve a group of teachers in a professional development programme directly linked to the teachers' own classrooms.

All Year 4-8 teachers at the school who participated in the New Zealand Numeracy Project will be invited to participate in the study. The professional development programme will involve teachers as co-researchers trialling activities, and evaluating their teaching in relation to mathematical practices. A particular focus will be on the communicative and interactive nature of the learning environment. To facilitate this inquiry teachers will reflect on teaching practices through observations (sometimes with audio/video records) and journal writing. Permission with regard to audio/video recording will be sought from both parents and children in each teacher's class. The focus of the study is on teaching strategies, and thus it will be possible to organise the recording devices to avoid those students who do not consent to participate.

The project is comprised of four phases:
5. Term 1: Professional development will focus on the New Zealand Numeracy Framework and Teaching Model.
6. Term 2: Over a five week period teachers in each of the senior syndicates will work within a collaborative partnership to develop and trial strategies related to student mathematical practices.
7. Term 3: Building on the work in Term 2, a second Numeracy unit will be trialed focusing on key numeracy ideas and communication patterns. I will case study 3 teachers (self-identified) as they use an explanatory framework to examine their

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own and each others mathematical teaching practices related to mathematical practices. Video, audio and journal material will be used in reflective observations.

8. Term 4: The teachers and researcher as a group will refine the explanatory framework and evaluate its usefulness as a professional development tool for teachers. In addition, we will reflect and report on the process of teacher change and changes in student mathematical practices and learning.

The time involved in the professional development meetings for teacher participants will be no more than five school days out of classrooms. The proposed professional development is in accord with the school strategic plan for numeracy focus. Teacher release time will be fully funded by the *Teaching and Learning Research Initiative Project: Numeracy Practices and Change.*

All project data will be stored in a secure location, with no public access and used only for this research and any publications arising from this research. After completion of five years, all data pertaining to this study will be destroyed in a secure manner. All efforts will be taken to maximize confidentiality and anonymity for participants. The school name and names of all participants will be assigned pseudonyms to maintain their anonymity. Near the end of the study a summary will be presented to the teachers to verify accuracy, and following any necessary adjustments, a final summary will be provided to the school and teachers involved.

Please note you have the following rights in response to my request for your school to participate in this study.
- decline to participate;
- withdraw from the study at any point;
- ask any questions about the study at any time during participation;
- provide information on the understanding that the participants' names will not be used unless you give permission to the researcher;
- be given access to a summary of the project findings when it is concluded.

If you have further questions about this project you are welcome to discuss them with me personally:

Bobbie Hunter: Massey University (Albany), Department of Technology, Science and Mathematics Education. Phone: (09) 4140800 Extension 9873. Email. R.Hunter@massey.ac.nz; or contact either of my supervisors (co-directors of the *Numeracy Practices and Change* Project) at Massey University (Palmerston North)

- Associate Professor Glenda Anthony: Department of Technology, Science and Mathematics Education. Phone: (06) 350 5799 Extension 8600. Email. G.J.Anthony@massey.ac.nz

- Dr. Margaret Walshaw: Department of Technology, Science and Mathematics Education. Phone: (06) 350 5799 Extension 8782. Email. M.A.Walshaw@massey.ac.nz

This project has been reviewed and approved by the Massey University Human Ethics Committee, ALB Protocol NO/NO *(insert protocol number)*. If you have any concerns about the conduct of this research, please contact Associate Professor Kerry P Chamberlain, Chair, Massey University Campus Human Ethics Committee: Albany, telephone 09 443 9700 x9078, email K.Chamberlain@massey.ac.nz.

CONSENT FORM: BOARD OF TRUSTEES

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree to participate in this study under the conditions set out in the Information Sheet.

Signature: _____ **Date:** _____

Full Name - printed _____

APPENDIX D: INFORMATION SHEET FOR PARENTS AND CAREGIVERS

My name is Bobbie Hunter and I am currently doing Doctoral research focusing on understanding mathematical practices and how those practices are developed by teachers and learned by students. The management staff of Tumeke School has agreed to take part in the *Numeracy Practices and Change* Project funding by the Ministry of Education Teaching and Learning Research Initiative. The research on *Mathematical Practices,* which is taking part in your school, is part of this larger project.

To investigate the development and enhancement of effective mathematical practices this study will involve your child's teacher working with a group of teachers at Don Buck Primary in a professional development programme directly linked to their own classrooms. The focus of the study is on the teaching strategies that support classroom communication and mathematical practices—those activities and modes of thinking that successful mathematical learners and users actually do.

I am writing to formally request your permission for your child to be audio or video recorded as part of their teacher's record of practice. Your child's involvement will be no more than that which occurs in normal daily mathematics lessons. The video or audio recording would occur during Terms 2 and 3 during mathematic teaching involving numeracy work only. Sometimes a whole maths lesson will be recorded, but more usually the teacher would only want to record a part of the lesson. In all, the teacher would make a maximum of 8 recordings (it may be necessary to have a few trials recordings so that students get used to the video equipment). In addition, the teachers may want to take copies of student's written work to assist their recordings. The teacher would review the recordings as part of the professional development programme, and sometimes parts of the recording would be shared with the other teachers working in the study as they collectively develop and critique their teaching strategies.

All data recordings will be stored in a secure location, with no public access and used only for this research. In order to maintain anonymity the school name and names of all participants will be assigned pseudonyms in any publications arising from this research. At the end of the year, a summary of the study will be provided to the school and made available for you to read.

Please note you have the following rights in response to my request for your child to participate in this study:
- decline your child's participation;
- withdraw your child from the study at any point;
- you may ask any questions about the study at any time during your child's participation;
- your child provides information on the understanding that your child's name will not be used unless you give permission to the researcher;
- be given access to a summary of the project findings when it is concluded;
- decline your child being video recorded;
- decline your child being audio recorded;
- decline to allow copies of your child's written material to be taken.

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If you have further questions about this project you are welcome to discuss them with me personally:

Bobbie Hunter: Massey University (Albany), Department of Technology, Science and Mathematics Education. Phone: (09) 4140800 Extension 9873. Email. R.Hunter@massey.ac.nz; or contact either of my supervisors (co-directors of the *Numeracy Practices and Change* Project) at Massey University (Palmerston North), College of Education:

- Associate Professor Glenda Anthony: Department of Technology, Science and Mathematics Education. Phone: (06) 350 5799 Extension 8600. Email. G.J.Anthony@massey.ac.nz

- Dr. Margaret Walshaw: Department of Technology, Science and Mathematics Education. Phone: (06) 350 5799 Extension 8782. Email. M.A.Walshaw@massey.ac.nz

This project has been reviewed and approved by the Massey University Human Ethics Committee, ALB Protocol NO/NO *(insert protocol number)*. If you have any concerns about the conduct of this research, please contact Associate Professor Kerry P Chamberlain, Chair, Massey University Campus Human Ethics Committee: Albany, telephone 09 443 9700 x9078, email K.Chamberlain@massey.ac.nz.

CONSENT FORM: PARENTS OF STUDENT PARTICIPANTS
THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to my child being audio taped during mathematical lessons.

I agree/do not agree to my child being video taped during mathematical lessons.

I agree/do not agree to copies of my child's written material being collected.

I agree to my child participating in this study under the conditions set out in the Information Sheet.

Signature: .... **Date:** ....

Full Name - printed ....

APPENDIX E: THE FRAMEWORK OF QUESTIONS AND PROMPTS

	Teacher questions and prompts	Student questions and prompts
Make clear conceptual explanations	I'm confused? Would you tell us again what you thought? ●kay we have got some confused faces, any questions about what they did? I don't understand that bit can you explain it to us? Explain what you did? What did you do there…in that bit of your explanation? Does this make sense? Are there bits that don't make sense? You listeners need to ask questions about those bits so that they can clarify them for you. How did you decide this? Whose thinking did you use to build on? Now that you have heard their explanation can you explain their strategy? Can you show us what you mean by…?	I'm confused can you explain that bit. Can you use a different way to explain that? I don't get that, what do you mean by…? Can I just jump in here and ask… When we went…what happened? What do you mean by…? Why did you say…? I don't get it…could you draw a picture of what you are thinking?
Making explanatory justification	But how do you know it works? Show us… Convince us… Can you persuade them that perhaps what they have come up with might not be the best solution? Can you convince them that they might need to rethink their solution? Why would that tell you to…? Can we explore that further because I don't think we are all convinced yet. Can you justify why you did that? Why does that work like that? So what happens if you go like that? Do you agree? Do you disagree? Remember that you can agree or disagree but you must be able to explain mathematically why. Is that similar to what you were thinking? How?	Why did you… So what happens if…? Are you sure it's…? Can you prove that? You need to show that every bit works. But how do you know it works like that… What about if you say…does that still work? Can you convince us that… Can you draw a picture of that to prove what you are saying is true? What about if we…can that work? So if we….

|		
Looking for patterns and connections and making generalisations	Can you see any patterns? Can you make connections between…? That is a really interesting observation. . I wonder if we can find out how these patterns are related? Remember how… Think about what we were doing the other day. Why? Why would you do that? What is happening? What are the patterns you can see? Anybody do it a different way? Is there a different way you can do it? How are you going to find a quicker way? Can you generalise that? Can you link all the ideas you found in some overall way? Does it always work? Does it work for all of them? Can you explain the difference in…? Is it always true? Why does this happen? Can you give us another example of what you are explaining? Can you see how these solution strategies are the same? Can you see how these strategy solutions are different? Can you explain mathematically how they are the same…or different?	Can you/we check this another way? So why is it…? Is that the quickest way? What is a quicker way? But how are you going to…? But if you… Because if you… Is there a different way we can do that? How is this the same or different to what we did before? In what ways is this different from our last solution strategy? Let's work out if we can show how it always works. If you say that the… What happens if…? Can we do this another way? Does that work? What else could we do which uses the same way?
Validating reasoning	Do you want 'think time' in order to revise your thinking? Does this make sense? Why, why not? Does anyone want to comment on that? Questions for them, but remember you are not telling them the answer? Remember you need to agree or disagree but use because or if in your statement How are you making sure you can all understand and convince use of…	I don't agree because… I agree because… But if… What about if… If you say that the… Which bits do you think we might have questions about? How can we answer the questions?

APPENDIX F: SAMPLE TRANSCRIPT WITH TEACHER ANNOTATIONS

| Transcript	Teacher annotation
Moana: Okay we have all thought about it and discussed it? Can you tell me please K what you people have discussed in your group? Can you show me what two groups of four look like? The child laid out OOOO OO *Moana: Can someone ask K a question please...that's what K says two groups of four look like? Yes J.* *J: Would you like to rethink that?* *Moana: That was good J that you suggested that. You did not say you are wrong... but if you don't agree with what she is showing you need to ask a question which starts with why...* *S: Why did you do two lots of two and four?*	**Participation norms** Should have taken the opportunity to gauge consensus *probing encouraging questions* questions are not addressing why and how Trying unsuccessfully to shape questions...rethink what...why? None of this is easy to get these children to use. I am sure we all look lost together **Planning focus how, why, when** **But also move them from adding and skip counting** Actually covering lots of focuses maybe too many introducing of **language** or **too much** as well as understanding in context Disagreement, argument etc comparing
A later point in the lesson *Moana: Did you see how he solved that? He said something?* *R: Oh I know* *T P: He said three plus three equals six plus another three equals nine or three times three equals nine* *W: And it wont change* *J: What did you mean by that?* *T: Mean by what?* *J: It will always be that and it won't change?* *Moana: Can you explain that?*	**Focus on making explanations** Focusing closely on what is being explained and looking at the shift from materials I could see T P listening closely so rather than me re-saying it was good to get a girl speaking up. I am watching how much talk I do so good to see who can add explanations Trying to explore what he meant. I hoped he was making a generalisation but actually I am not sure.

BUILDING FENCES

The zookeeper is building a fence from half round posts,
for the zebras.
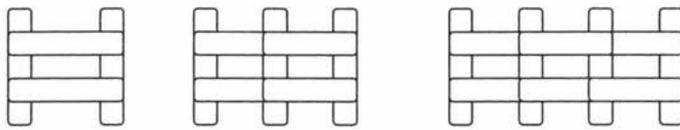
One section   Two sections   Three sections
takes four   takes seven   takes ten
posts.   posts.   posts.
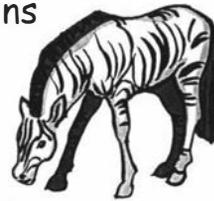
Using ice block sticks continue this pattern.
Complete the table below.

Sections	1	2	3	4	5	6	7	8
Posts	4	7	10					

Write the pattern you see as ordered pairs e.g. (1,4), (2,7)...

Can you find the rule?

Use your rule to extend your table to find out how many fence
posts would be needed for more sections of fence.
For example:
How many posts would be needed to build: 10 sections of fence?
15 sections? 20 sections? 32 sections? 1141 sections?

APPENDIX H: PROBLEM EXAMPLES DEVELOPED IN THE STUDY GROUP WHICH REQUIRED MULTIPLE WAYS TO VALIDATE REASONING

Mrs Dotty has made three cakes which look like this. She decided to cut them up. First she cut them into halves. Then she cut them into quarters. Then she cut them into eighths.

The Dotty children come home from school where they all have been learning about fractions and so Mrs Dotty decides to question them about the cakes she cut up.

Barbie Dotty says that if she cut the 3 cakes into halves she must have had 6 pieces or six halves or $^6/_2$
Maureen Dotty says that if she cut the 3 cakes into quarters she must have had 12 pieces or twelve quarters or $^{12}/_4$
Gaylene Dotty says that if she cut the 3 cakes into eighths she must have had 24 pieces or twenty four eighths or $^{24}/_8$

Baby Dotty who is just learning about fractions gets very confused about how they could have the same number and size cakes and yet get all the different fractional pieces. She doesn't know if her sisters are right. **Are all three of her sisters correct? How could you convince Baby Dotty?** *Can you work out different ways to convince her?*

Gollum says 'precious' 40 times every hour. How many times does he say 'precious' every day? Froddo thought that it was 1060. Sam thought it was 860. Who was right? Convince us about who is right using more than three different solution strategies.

Listen carefully to each member of your group and then together explore different ways to convince yourself and the larger group. Be ready to explain the tricky bits you think other people might find difficult in your explanation of this solution strategy.

Select one of your strategies which you think will work with any numbers and explore whether it does.

APPENDIX I: PROBLEM EXAMPLES DEVELOPED IN THE STUDY GROUP WHICH SUPPORTED EXPLORATION OF PARTIAL UNDERSTANDINGS

Peter and Jack had a disagreement. Peter said that 5/8 of a jelly snake was bigger than 3/4 of a jelly snake because the numbers are bigger.

Jack said that it was the other way around, that 3/4 of a jelly snake was bigger than 5/8 of a jelly snake because you are talking about fractions of one jelly snake.

Who is right? When your group have decided who is correct and why you need to work out lots of different ways to explain your answer. Remember you have to convince either Peter or Jack…and they both take a lot of convincing! Use pictures as well as numbers in your explanation.

A little guy from outer space is in your classroom and he is listening to Annie, Wade, Ruby and Justin arguing about sharing a big bar of chocolate.

Annie says that you can only share the bar of chocolate by dividing it into halves or quarters.
Wade says he knows one more way of sharing the bar of chocolate
Ruby and Justin say that they knows lots of ways of sharing the bar of chocolate and they can find a pattern as well

The little guy from out of space is really interested in what they say so they start to explain all the different ways to him.

What do you think they say? In your group work out a clear explanation that you think each person gave and then work out who is correct.
The little guy needs lots of convincing so how many different ways can you use to prove what your group thinks is correct. Make sure that you use fractions as one way to show him because he likes using numbers.

Can you find and explain any patterns you find in your explanation?

APPENDIX J: MOANA'S CHART FOR THE GROUND RULES FOR TALK

How do we korero in our classroom?

We make sure that we discuss things together as a whanau. We listen carefully and actively to each other. That means:

- We ask everyone to take a turn at explaining their thinking first.
- We think about what other questions we need to ask to understand what they are explaining.
- We ask questions 'politely' as someone is explaining their thinking; we do not wait until they have completed their explanation.
- We ask for reasons why. We use 'what' and 'why' questions.
- We make sure that we are prepared to change our minds.
- We think carefully about what they have explained before we speak or question.
- We work as a whanau to reach agreement. We respect other people's ideas. We don't just use our own.
- We make sure that everyone is asked and supported in the group to talk.
- We all take responsibility for the explanation.
- We expect challenges and enjoy explaining mathematically why we might agree or disagree.
- We think about all the different ways before a decision is made about the group's strategy solution. We make sure that as we 'maths argue' we use "I think …because…but why…or we use "If you say that then…"

APPENDIX K: EXAMPLES OF EXPANSIONS OF SECTIONS OF THE COMMUNICATION AND PARTICIPATION FRAMEWORK

Making conceptual explanations — Use problem context to make explanation experientially real.	Think of a strategy solution and then explain it to the group. Listen carefully and make sense of each explanation step by step. Make a step by step explanation together. Make sure that everyone understands. Keep checking that they do. Take turns explaining the solution strategy using a representation. Use equipment, the story in the problem, a drawing or diagram or/and numbers to provide another way or backing for the explanation.	Keep asking questions until every section of the explanation is understood. Be ready to state a lack of understanding and ask for the explanation to be explained in another way. Ask questions (what did you…) of sections of the explanation. Discuss the explanation and explore the bits which are more difficult to understand. Discuss the questions the listeners might ask about the explanation
Making explanatory justifications — Indicate agreement or disagreement with an explanation and have a mathematical reason for the stance.	Listen to each person in your group and state agreement with their explanation OR state disagreement with their explanation. Practise talking about the bits you agree with and be ready to say why. Ask questions of each other about why you agree or disagree with the explanation. Pick one section of an explanation and provide a mathematical reason for agreeing with it. Discuss the explanation or a section of the explanation and talk about the bits that the listeners might not agree with and why.	Provide a mathematical reason for disagreeing with the explanation or a section of the explanation. Think about using material or drawing pictures about the bit of the explanation that there have been a lot of questions about in the group. Ask questions of each other (why did you…how can you say…) Question until you understand and are convinced. Explain and using different ways to explain until you are ALL convinced.

APPENDIX L: A SECTION OF THE TABLE OF DATA OF THE ACTIVITY SETTING IN THE CLASSROOM

The teacher asks for collective responsibility to the maths community	OB:	8	9	10	11
Asks that all group members understand, share, record solution strategies	1		2 7 9 11		1,10
Asks for multiple solution strategies	1b		2 9	10	1,10
Asks other small group members to be able to continue an explanation	1c				
Asks for collective consensus reached on selected strategy solution	2a		2 9	10	1
Asks that all small group members understand each other's strategy	2b	4	9	10	3,11,13
Asks that each member of the small group describes a strategy	2c	4			11
Asks for small group collective development of strategy solution	3		9		18
Asks for turn taking in explaining strategies in small groups	3b				3,4,11
Asks for collective analysis to predict questions related to solution strategy	4		4, 9, 18		3,5
Asks group members to interject/support each other in explanation	5b	34			5
Asks students to analyse and explain own/or group erroneous thinking	34			20,21	
Asks for collective support when student states lack of understanding	7	2, 27			
Uses model of how small group reworked their thinking	7b		8,11		
Emphasises the value of working together	7c				
Examines with students how groups worked collaboratively	7d				
Uses body language to determine understanding of mathematical thinking	52	33			
Uses think time/wait time to support questioning/challenging	51	4	4		5
Provides wait time for other children to ask questions	54	7, 21		16	
Positions self as member of mathematics community with similar status	39				
Attributes mathematical thinking/explanation/generalisation to student	55b	18, 21			