Striving toward equity: A story of positioning and status

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Generosa Angela Leach

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ABSTRACT

This study explores how teachers construct equitable learning environments in primary school mathematics classrooms as a means of striving toward equity. The deep complexity of classrooms and numerous connected elements that influence students' access and opportunities for learning mathematics are highlighted. Under consideration are the different pathways teachers take as they develop and maintain responsive and adaptive approaches to position all students to learn mathematics in ways that meet the aims of equity in mathematics education.

A qualitative design research methodology was employed to explore the complexities and challenges of teacher learning and change within primary school classroom settings. The design approach supported the development of a model of professional learning and frameworks of teacher instructional actions to establish and maintain mathematics classrooms focused on equity. Data collection over the school year included study group meetings, participant observations, video-recorded observations, documents, and teacher and student recorded reflections and interviews. Retrospective data analyses drew the results together to be presented as cases of two teachers, their classrooms, and students.

The findings show that constructing equitable mathematics learning environments is a gradual and complex process. It involves teachers reconstructing their beliefs and enacting specific instructional actions to position all students to learn mathematics. Reconceptualising mathematics teaching and learning requires transforming the social and organisational structures within classrooms and disrupting assumptions of uniformity across all students from a strength-based approach. Of importance is how the findings highlight possible ways of meeting the needs of diverse, and often marginalised groups of students in New Zealand schools.

Significant implications based on these findings include how the aims of equity in mathematics education can extend beyond policy and into practice within primary school mathematics classrooms in the New Zealand context.

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CHAPTER ONE

INTRODUCTION TO THE STUDY

1.1 INTRODUCTION

Significant reform has been proposed for mathematics education in recent times. The goals of reform are founded on a belief that all students are capable of working mathematically and learning in mathematics classrooms. To meet these goals, educational systems must be structured so that students are provided equitable access and opportunities to learn worthwhile mathematics and come to view themselves as capable mathematicians. Enabling students to access mathematics requires the teacher to use instructional actions which position all students as capable sense-makers of mathematics. It is within this frame that this study, “Striving toward equity: A story of positioning and status” was conducted. The primary aim of this thesis is to examine and tell the story of the journey two teachers undertake as they work at constructing equitable learning environments in primary classroom settings which position all students to learn mathematics.

This chapter identifies the key aim of this study. In Section 1.2 the background context outlining international and national calls for reform and equity in mathematics education is presented. Section 1.3 provides the motivation for this research and highlights the urgent need for change in mathematics teaching and learning within the context of Aotearoa, New Zealand. Section 1.4 outlines the research aim. An overview of the thesis is provided in Section 1.5.

1.2 BACKGROUND CONTEXT OF THE STUDY

Attention to complex classroom structures and instructional practices that aim for equitable outcomes in mathematics education has been recognised increasingly as important (e.g., Esmonde, 2009b, 2009c; Martin, 2009; Nasir & Cobb, 2007; Stouraitis et al., 2017; Tate, 2013; Zavala, 2014). For over two decades, there have been heightened calls for reform in mathematics education both internationally and in New

Zealand. Reform-oriented approaches advocate a shift away from traditional teacher-centred mathematics classrooms emphasising procedure, speed, and accuracy, to learning environments promoting student engagement in mathematical reasoning (Hunter & Hunter 2018b; Ing et al., 2015; Yackel et al., 2007). When learning environments provide opportunities for all students to engage effectively in collective mathematical activity and discourse characterised by mathematical explanation, justification, and argumentation; they open potential for equity (Esmonde, 2009b; Hunter, 2013).

To meet the goals of equity, teachers must hold high expectations and beliefs that all students are capable of learning mathematics successfully. For some practitioners this means reconceptualising mathematics learning as developing from collective student engagement in mathematical activity and discourse, as opposed to the view that mathematical learning is acquiring a static body of knowledge from a mathematical authority, most often the teacher. To provide opportunities for all students to engage effectively in mathematical discourse teachers are required to establish and maintain effective classroom communication and participation norms (Cobb et al., 2011; Hunter, 2007b; Yackel & Cobb, 1996).

Historically, goals for educational systems such as schools are driven by policy. For example, international aims for reform are reflected in Australian educational policy (Australian Association of Mathematics Teachers, 1998; Australian Curriculum, Assessment and Reporting Authority (ACARA), 2021; Willis, 1991) which encourage teaching mathematics in ways that promote student discussion about mathematics within relevant contexts. In the U.S.A., the National Council of Teachers of Mathematics (2000) advocates for instructional approaches that stimulate students' fluency, creativity, resourcefulness, insightfulness and understanding, and classrooms that support conceptual learning of mathematics. In the United Kingdom, policy documents highlight the need for teachers to position students to reflect and collaborate on differences, difficulties, and successes in their mathematics understanding (Department for Education, 2013; Office for Standards in Education (OFSTED), 2003). Mathematics education in New Zealand is also driven by Ministerial policy. For example, the New Zealand Curriculum (Ministry of Education, 2007) calls for students to learn mathematics in ways that promote the development of creative, critical, and logical thinking, where mathematical problems can be solved in flexible ways. In New Zealand, the response to calls for reform resulted in a large-scale mathematics education professional learning initiative. In 2000, the Ministry of Education launched the Numeracy Development Project (NDP) aimed at supporting teachers across New Zealand to teach mathematics in ways that positioned all students to actively engage in collaborative sense-making and to explain and justify mathematics ideas.

However, despite the call for teachers to move from teacher directed instruction to collective sense-making practices, many researchers (e.g., Anthony & Hunter, 2017; Bell & Pape, 2012; Webel & Platt, 2015) show the difficulties in shifting pedagogical practices within mathematics classrooms. Fundamental changes have been difficult to enact in the pedagogical practices of individual teachers, and mathematics classrooms continue to be dominated by initiate-response-feedback interaction patterns emphasising procedures and accuracy. Hiebert, (2013) contends that supporting change in teaching practice is one of the most difficult and enduring problems in education. This challenge can be attributed to multiple factors including that teachers are unsure how to interpret research into practice; they lack accessible models of mathematical discourse and argumentation, key components of reform-oriented mathematics learning; or individual teacher beliefs and knowledge about teaching and learning mathematics and the goals of reform may not be aligned (e.g., Maass, 2011; Manouchehri, & Goodman, 2000; Warfield et al., 2005; Wilson & Goldenberg, 1998).

A further challenge facing teachers in New Zealand is the perpetuation of structural inequities for some students. Significant teacher action is required to alleviate these inequities and meet the goals of equity and reform in mathematics education. However, there is clear evidence that many teachers need support in enacting instructional approaches that best meet the needs of all students in New Zealand. Therefore, it is time to examine ways that can provide support for teachers as they navigate these challenges.

1.3 RATIONALE FOR THE STUDY

Despite the New Zealand Ministry of Education stating an intent to "shape an education system that delivers equitable and excellent outcomes" (Ministry of Education, 2018, p. 6) and calling for teaching to be "adaptive, evidence-based, equitable, and inclusive" (p. 8), there continues to be ongoing challenges within the New Zealand education system. Equity remains a persistent and serious issue, with insufficient progress toward equity of educational outcomes, particularly for Māori and Pāsifika students (Ministry of Education, 2018, p. 10). Recent international and national data (e.g., Educational Assessment Research Unity and New Zealand Council for Educational Research, 2018) report ongoing underachievement in mathematics for specific groups of students. Over the last decade, attempts have been made to prioritise and identify equitable ways to improve outcomes for Māori and Pāsifika students, for example, Ka Hikitea Māori Education Strategy (Ministry of Education, 2013a) and the Pāsifika Education Plan (Ministry of Education 2013b). However, equitable outcomes have not yet been achieved. Despite large-scale professional development projects, many New Zealand students, particularly those from marginalised backgrounds (for example, Māori, Pāsifika, and students from non-dominant cultural backgrounds) continue to face inequitable access, a lack of opportunities, and resulting poor achievement outcomes in mathematics learning.

Within the New Zealand education system, Pāsifika students have been identified as the most at-risk group with regards to academic achievement when compared to other New Zealanders (Sharma et al., 2011). In relation specifically to mathematics achievement outcomes, the recent report (2021) by the New Zealand Royal Society Te Apārangi Expert Advisory Panel on Mathematics and Statistics emphasised the over-representation of Pāsifika students among lower achievers as evidenced in both the Trends in Mathematics and Science Study (TIMSS) and the National Monitoring Study of Student Achievement (NMSSA) data (see Figure 1).

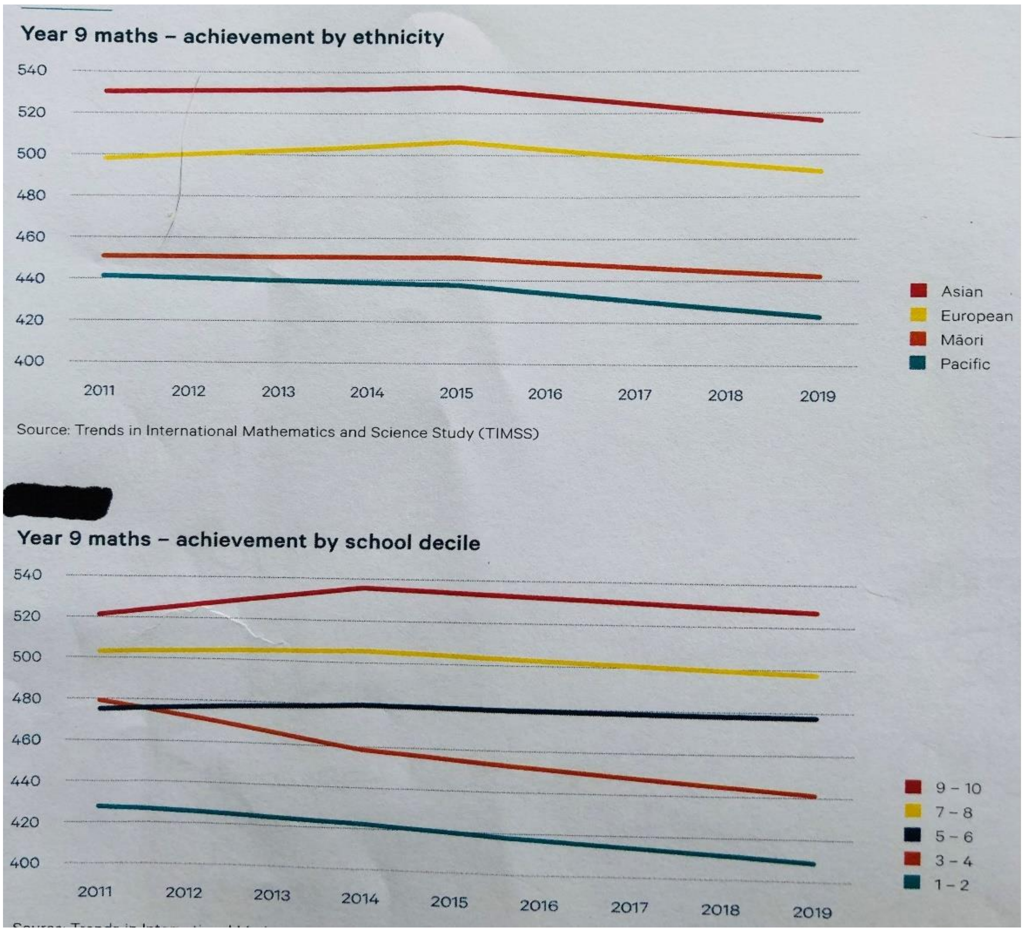

Figure 1: *New Zealand Mathematics Achievement Data by Ethnicity and by School Decile*

In the context of New Zealand, Pāsifika is a homogenous label for multi-ethnic groups with heritages deeply rooted in island nations in the South Pacific. This includes, Samoa, Tonga, Cook Islands, Niue, Tokelau, Tuvalu, Fiji, Kiribati, the Solomon Islands, and French Polynesia. Two-thirds of the total Pāsifika population of New Zealand live in the Auckland region (Statistics New Zealand, 2018). Data show that in comparison to European and Asian students, students of Pāsifika descent continue to achieve more poorly across most levels of the school system (Ministry of Education, 2019). More Pāsifika students leave school with lower levels of qualifications than non-Pāsifika students. More often the explanation offered to explain the different outcomes for the different ethnic groups in New Zealand has been to attribute the poor outcomes to the learners themselves or to their impoverished circumstances (Samu, 2015; Turner et al., 2015). However, recent research (e.g., Adiredja & Louie, 2020; Hunter & Hunter, 2018b; Rubel, 2017) identifies specific factors contributing to ongoing underachievement of non-dominant students in New Zealand; namely, deficit theorising and structural inequities within the education system.

Deficit theorising and low expectations for marginalised students within mathematics classrooms are evident when teachers state that specific groups of students are unable to do mathematics, as they bring no prior mathematics to the school classroom (Allen et al., 2008; Nieto, 2002; Siope, 2011; Rubel, 2017). Many Pāsifika students, shaped by their experiences in New Zealand classrooms, hold a deficit view of their own culture in relation to mathematics (Hunter & Hunter, 2018b). Other students indicate that to be successful in mathematics they believe they need to enter what has been described as 'white-space', the organisational space in schools representing the dominant middle class Pākeha or 'white' cultural group's ideologies (Barajas & Ronnkvist, 2007; Milne, 2013). Within these spaces, specific groups of students are required to negotiate the different expectations upheld for students who either reflect the dominant group and those who do not (Barajas & Ronnkvist, 2007).

Clearly, studies of instructional practices that support participation of traditionally marginalised students are needed if we are to develop equitable outcomes for all students. As highlighted by Bartell et al., (2017), this research needs to focus on explicit instructional practices in specific contexts, such as school classrooms. With a greater emphasis on reform approaches to mathematics teaching and learning, a major challenge facing teachers is that they themselves lack experiences with equity focused pedagogical approaches to mathematics teaching and learning (Yackel et al., 2007). Striving toward equity in mathematics education requires teachers to intentionally pursue instructional practices that will address inequitable access and allow all students to engage with complex mathematics (Cohen & Lotan, 1997; Dunleavy, 2015; Esmonde, 2009b, 2009c; Turner et al., 2015). Over time, how students are positioned to participate in collective mathematical activity affects not only what they learn, but how they come to view themselves as learners, and conversely, students' beliefs in their capabilities to learn mathematics, significantly affects how they interact and participate in school settings.

Recently, the New Zealand report completed by the Royal Society Te Apārangi (2021) stated that "all children in Aotearoa New Zealand, irrespective of their individual life circumstances, must be given opportunities to develop sufficient competence and confidence in pāngerau mathematics and tauanga statistics that enables them to lead rewarding and fulfilling lives." (2021, p. 6). To meet these goals, structural inequities in school systems need be addressed through re-inventing ways of teaching and learning which provide equitable access to mathematics learning for all students, particularly those who are often marginalised in New Zealand. Currently, there is limited research undertaken within New Zealand on how to position students equitably in primary school mathematics classrooms.

It is against this background and for these reasons that this study, "Striving toward equity: A story of positioning and status" was conducted.

1.4 RESEARCH AIM

This study aims to examine how teachers construct equitable learning environments in primary school mathematics classrooms. The focus of the investigation is on the different journeys teachers take as they (re)construct their beliefs and classroom practices to position all students to learn mathematics. Specifically, this study explores the teachers' pedagogical actions and how these actions position students to have equitable access and opportunities to engage in collective mathematical reasoning. This study also seeks to understand how teachers address status. A final focus of this research investigates how teachers delegate mathematical authority to all students which serves to position them equitably as legitimate knowers and doers of mathematics.

1.5 MOTIVATION FOR THIS STUDY

There were two motivations underpinning the rationale for this research. The first motivation concerned my experiences growing up in South Africa during the Apartheid Era[1]. The second related to my encounters as a mathematics learner. Each motivation is discussed next.

[1] The apartheid era in South African history refers to the time when the National Party led the country's white monority government, from 1948-1994. Apartheid is an Afrikaans language term that means "apartness" and was

I was born in South Africa and grew up there. My parents were not of South African Heritage. My father was an immigrant from Spain and my mother an immigrant from the United Kingdom. As a child of “other” parents and culture, I often did not feel a sense of belonging in many settings. However, my white skin meant that I was not marginalised in society and education as millions of others were. I grew up in a place where racism and marginalisation occurred at its extreme. Systematic structural and educational inequities were ways of being that we were conditioned to believe were important to uphold a privileged white/European way of life. Much of this conditioning occurred through an education system which habituated the citizens of South Africa to accept and uphold mandated racism and segregation at all levels of life. These life experiences have had a significant effect on my passion for equity, particularly equity in education.

The second motivation for this study involves my experiences as a learner in the bottom or lowest ability group in mathematics classrooms throughout my secondary schooling. Being in the bottom group I felt I was afforded fewer and less diverse ways to learn mathematics effectively, and that these were more procedurally based. Memorisation and recall of rote-learned procedures and formulas were called for, which I either comprehended or did not. I did not feel that any of my mathematics teachers held high expectations for the long-standing members of the lowest ability groups to succeed in mathematics. In class, I was a silent, passive receiver of knowledge from the only mathematical authority in the classroom-the teacher.

This research is driven by my experiences growing up in a society rife with marginalisation and inequities, and as an unsuccessful learner of mathematics. Through my encounters as a teacher and postgraduate student and researcher, I have realised that my lack of success as a learner of mathematics need not have occurred and that there are ways to counter some of the instructional approaches of teaching mathematics that result in inequitable outcomes for certain groups of students in Aotearoa, New Zealand.

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the term used for the National Party’s racial segregation policies which built upon the country’s history of racial segregation between the ruling white monority and the nonwhite majority.

1.6 OVERVIEW OF THE THESIS

This thesis is presented in seven chapters. This chapter has provided the research aim, background, and rationale for this study. Chapter two reviews relevant research literature. The use of a qualitative design approach and the methods used to collect and analyse the data are outlined in Chapter Three. In Chapters Four and Five, the findings of the teachers' participation in the professional learning are presented. Drawing on two cases, the outcomes are analysed to illustrate how the teachers (re)constructed their mathematics learning environments to reflect the aims of equity. Chapter Six presents the discussion of the findings in relation to relevant literature. Chapter Seven provides the conclusion of the thesis, and implications and recommendations for future research are presented.

CHAPTER TWO

LITERATURE REVIEW and THEORETICAL FRAMING

2.1 INTRODUCTION

The previous chapter introduced the background and rationale to the study. This chapter examines relevant literature linking mathematics teaching and learning to themes of equity in mathematics education. Throughout the current study, the researcher draws on Esmonde's (2009b) definition of equity as "a fair distribution of opportunities to learn for all students" (p. 1008). Cultural connectedness and culturally responsive pedagogical approaches are highlighted as important ways of ensuring all students are provided with access and opportunities for learning mathematics successfully. Critical elements of a sociocultural perspective are examined to situate mathematics teaching and learning as occurring within equitable and strength-based communication and participation patterns. Positioning theory and status generalisation theory are presented to form the theoretical framework for critically examining the complex and dynamic nature of social episodes in classroom systems.

Section 2.2 highlights the goals of equity in mathematics education and the importance of enacting these, particularly within the context of education in New Zealand. In Section 2.3 the sociocultural perspective is outlined. Section 2.4 explores the role of the teacher as mediator in mathematics education. Section 2.5 examines literature pertaining to communication and participation practices in mathematics classrooms. Section 2.6 focuses on grouping practices in mathematics education. In Section 2.7 the theoretical framework is discussed.

2.2 EQUITY IN MATHEMATICS EDUCATION

Equity in mathematics education is possible when educators believe that all students are capable of learning complex mathematics and provide access and opportunities for them to do so. However, a significant body of recent research (e.g., Civil et al., 2019; Ministry of Education, 2019; Stinson, 2013) has identified historical and ongoing structural inequities, both internationally and within New Zealand, whereby certain students, often those described as culturally or ethnically diverse are marginalised in mathematics education. While attention to complex practices reflecting the aims of equity has become a growing priority in mathematics education (e.g., Bartell, 2013; Esmonde, 2009b, Gutiérrez, 2002) many students are not achieving successful outcomes. Reasons for this disparity have been highlighted by several researchers (e.g., Bishop et al., 2003; Ladson-Billings, 1994; Webber, 2013) who have identified a connection between teacher expectations and student ethnicity, where lower teacher expectations were evident for non-dominant students (e.g., indigenous and minority groups) than for dominant (e.g., European) students.

Previous research (e.g., Bartell et al., 2017; Esmonde, 2000c; Gutiérrez, 2007) has outlined how equity in mathematics education can be achieved when schools and teachers utilise pedagogical approaches that emphasise access, opportunities, and successful outcomes for all students, regardless of ethnicity, race, gender, cultural background, or socioeconomic status. Of importance is that equitable pedagogical approaches require the enactment of specific instructional actions that reflect teacher beliefs and expectations that all students are capable of learning complex mathematics (Boaler & Staples, 2008; Langer-Osuna, 2018). These approaches call for classrooms to be reconceptualised as spaces where high expectations are established and all students are afforded opportunities to engage in extended mathematical discourse comprised of mathematical explanations, argumentation, and justification, and where the teacher consistently presses for deep mathematical reasoning (Greeno & Gresalfi, 2008; Hunter, 2007b).

Teachers and educators need to develop learning that builds on and reflects students' experiences, cultural backgrounds, and funds of knowledge (Gutiérrez, 2012; Hunter & Hunter, 2018a) and makes connection through language (Rubel, 2017; Turner et al., 2013). When teaching and learning are planned for in these ways, those students who are most often marginalised both internationally (e.g., African American, Latinx, people of colour) and within New Zealand (Māori and Pāsifika) are provided equitable opportunities for learning mathematics. These instructional approaches are deeply embedded in ideas around culturally responsive pedagogy.

2.2.1 Culturally responsive pedagogy

Culturally responsive approaches premise that when academic knowledge and skills are situated within the lived experiences and frames of reference of students, they are intrinsically motivated to learn. However, defining culturally responsive pedagogy is problematic as it is open to interpretation. Gay (2010) defines culturally responsive pedagogy as “using the cultural characteristics, experiences, and perspective of ethnically diverse students as conduits for teaching them more effectively” (p. 106). Berryman et al. (2018) highlighted how the term culturally responsive is defined, understood, and enacted very differently across New Zealand. While Berryman et al. agreed that culturally responsive pedagogy was grounded in cultural relationships and responses to the prior knowledge and experiences of the students, they sought to explore a deeper understanding of this pedagogical approach. Berryman et al. highlighted how often when discussion ensues around culturally responsive pedagogy within New Zealand, it is relationship based. These researchers cautioned that further thought must be given to the nature of these relationships to avoid tokenistic relationships that “continue to place certain students in partnership with the dominant culture whose power differentials are upheld as more important” (p. 9). Berryman et al. outlined how learning is not simply linked to the relationships between people but is deeply embedded in the types of relationships that exist between teachers, students, and their wider whanau (family). These researchers call for thinking about “cultural relationships for responsive pedagogy” to provide opportunities for students to develop strong and secure cultural identities. Of particular importance for Māori students is to be strong and secure in their identity as Māori. Therefore, success for Māori students, would mean they could “walk confidently and with mana (presence) in the two worlds of Aotearoa, New Zealand” (2018, p. 8). For these reasons, these researchers offered a “renaming of the culturally responsive space as cultural relationships for responsive pedagogy” (2018, p. 4). Similarly, other researchers in New Zealand (Hunter & Hunter, 2018a; Hunter et al., 2019) have advocated using pedagogical approaches which incorporate the core Pāsifika values to shape the norms of the classroom. These researchers have looked to expand the ideas of cultural responsiveness to include advocating for culturally sustaining practices which include being responsive to students' cultural ways of being but extending these concepts to also sustain students' values through pedagogy.

Developing deeper understanding of culturally responsive or culturally sustaining pedagogy within mathematics education requires exploring important ideas around cultural connectedness and mathematics teaching and learning.

2.2.2 Cultural connectedness and mathematics teaching and learning

Culture is a social structure that exists and is learned within local contexts where people interact with each other (Fecho & Clifton, 2017; Nasir & McKinney de Royston, 2013; Nieto, 2010). New Zealand is home to Māori, a heterogeneous group of people comprised of unique groups and cultural identities (Greaves et al., 2015). Another important group of peoples deeply connected to New Zealand are those from island nations in the South Pacific, for example, Samoa, Tonga, Cook Islands, Niue, Tokelau, Tuvalu, Fiji, Kiribati, the Solomon Islands, and French Polynesia (Coxon et al., 2002; Samu, 2015). Pāsifika is a homogenous label for peoples belonging to multi-ethnic groups with heritages deeply rooted in these island nations. Both Māori and Pāsifika peoples share many common values, including reciprocity, respect, service, inclusion, relationships, spirituality, leadership, love and belonging (Ministry for Pacific Peoples, 2018; Ministry of Education, 2013b). Above all, family, communitarianism, and collective responsibility are central to daily life for both Māori and Pāsifika peoples (Allen et al., 2008; Ministry for Pacific Peoples, 2018).

Collectivism and communitarianism in the classroom context refer to ways of working together, for example, engaging in group discussions, that show respect of others (Ministry for Pacific Peoples, 2018). Many researchers (e.g., Civil et al., 2019; Turner et al., 2013) have agreed that for this to occur, teacher actions should draw from and on the cultural values of their students, and in such ways, teaching could become culturally responsive or sustaining. Furthermore, a focus on enacting culturally responsive teaching aligning with students' core cultural values meets the goals of equity in mathematics education (Boaler, 2006, MacFarlane, 2004). When teachers create classroom environments where all students are expected to engage in collective mathematics activity, attention should be given to the beliefs and prior experiences diverse students bring to expected ways of participating and communicating (Gutiérrez, 2012; J. Hunter et al., 2020). Therefore, establishing effective and mutually respectful ways for student collaboration require careful consideration of what collectivism and communalism mean for these students outside of school.

A New Zealand study by Siope (2011) examined the level of cultural responsiveness Pāsifika students' (aged 14-18 years old) had experienced at school. Specifically, Siope highlighted the importance Pāsifika people placed on feeling a sense of belonging and purpose in all aspects of their daily lives, including at school. This researcher emphasised that students' learning was enhanced when they believed they belonged and what they were doing had purpose. These findings align with other research (e.g., Bishop & Glynn, 1999; Nuthall, 2007; Siope, 2011) that highlighted the responsibility of the teacher to create structures for inclusion and belonging. Furthermore, in their research, Hunter and Hunter (2018b) outlined how teachers and schools could support diverse students to learn mathematics in more equitable ways. Drawing from findings from a large professional development project involving thirty-two New Zealand primary schools (students aged 5-12 years old), these researchers illustrated how teachers could support a diverse range of students, including Māori and Pāsifika students, to learn mathematics by engaging in collective mathematical discourse (2018b). These researchers drew attention to how teachers modified their mathematics learning environments to respectfully align with the core cultural values of Māori and Pāsifika students, including reciprocity, respect, service, inclusion, family, relationships, collectivism, and belonging.

Enacting culturally responsive teaching requires willingness for some teachers to change their practice when they recognise that what they are doing is not working for all students. This call for change has been highlighted in Spiller's (2012) exploration of the complex nature of classroom interactions across a range of diverse cultures. Spiller's study involved classroom observations of a range of Pāsifika students aged 13 years old from a low socio-economic school in New Zealand, and interviews with their teachers. Many of the teachers espoused the importance of engaging students in learning by providing opportunities for groupwork and classroom discussions. However, despite knowing that these opportunities would encourage student engagement, these teachers did not adapt their practice to support this way of learning. A range of reasons was offered, and Spiller outlined how these stemmed from teachers' assumptions and beliefs about Pāsifika culture (2012). These included apportioning blame to important Pāsifika values. For example, some teachers asserted that poor behaviour or lack of engagement were caused by Pāsifika values themselves and that disengagement in class was linked to Pāsifika children not being involved in decision making at home. Other teachers linked lack of students trying to the Pāsifika value of humility, claiming students did not want to look smart in front of their peers, so were reticent in participating in classroom discussions. Spiller highlighted that the best ways of learning for Pāsifika people reflects best practice for all students. Her analyses of Pāsifika students' voices strongly emphasised that these students' academic outcomes were greater in learning environments where teachers responded in culturally authentic ways to how these students learn best; thus, providing what Spiller termed "dignity in their learning" (2012, p. 65).

Internationally, the importance of drawing on the core cultural values of a range of diverse students in teaching and learning has also been recognised. Civil and Hunter (2015) reported on research undertaken by Civil in the U.S.A. (Arizona) with students (aged 12 years old) of Hispanic heritage. Civil worked alongside the teachers to support student participation in mathematical discussions. To accomplish this, Civil purposefully drew on these students' core cultural values of "familia" (Family), "confianza" (mutual trust), and relationships. In doing so, she forged strong relationships with these students and their families in and outside of the school classroom. Developing these relationships provided the students with a sense of belonging and inclusion in classroom learning. Civil reported that after her intervention these students were able to engage in moments of "rich mathematical discussion and argumentation" (Civil & Hunter, 2015, p. 304). These types of learning environments where teachers enact culturally responsive and sustaining practices that strive to meet the goals of equity in mathematics education reflect a sociocultural perspective.

2.3 A SOCIOCULTURAL PERSPECTIVE

Sociocultural perspectives seek to examine the ways that activity in learning environments is socially and culturally organised, and how these social supports and cultural arrangements affect learning. Sociocultural approaches have progressively been used to understand learning and development by highlighting the influence of culture on human activity and thought and exploring the significance of local settings, and how individuals use artefacts, tools, and social interaction to learn (Bruner, 1996; Saxe, 1999).

Sociocultural theories of learning developed from the work of Lev Vygotsky, a Soviet psychologist of the early 1900's who emphasised the dynamic nature of learning. Key components of Vygotsky's work highlighted how students learn through interaction with others and their setting (Vygotsky, 1978). In addition, sociocultural theories identify culture as a system of meaning making that is continually being formed and re-formed in local settings, activity, or cultural practices across generations. Distinctly different from psychological perspectives seeking to understand human behaviour and cognition at an individual level, sociocultural frameworks examine human behaviour as cultural practices and activity (Bruner, 1986; Brown & Palinscar, 1989). This view of activity emphasises that how students participate in settings is influenced by how they view themselves as learners. Furthermore, how students construct these beliefs and participatory patterns is largely influenced by what opportunities are provided for them in settings such as schools (Hunter & Civil, 2021; Vygotsky, 1986).

Several studies (e.g., Boaler & Sengupta-Irving, 2016; Gresalfi et al., 2009) drawing on a sociocultural framing in mathematics education have highlighted how understanding is developed when students work collectively on mathematical activity and engage in mathematical discourse comprised of explanation, conjecture, justification, and generalisation to reason about important mathematical concepts. Within such a frame social interaction itself is viewed as a means of teaching and learning. A body of research (e.g., Lave & Wenger, 1991; Sfard & Kieran, 2001) emphasises mathematics learning as a cultural process that draws on cultural tools such as taken-as-accepted ways of participating and communication that are developed through social interaction in the setting**.** Furthermore, other studies (e.g., Cobb et al., 1991; Cobb et al., 2011; Yackel, 1995) have outlined how when students are supported to learn mathematics through collaborative sense-making, they can be supported to develop positive mathematical beliefs, dispositions, and intellectual autonomy. These kinds of mathematics learning environments are described as reform-oriented classrooms.

2.3.1 Reform-oriented classrooms

Reform mathematics learning environments reorganise structures to provide students opportunities to actively engage in learning through doing and talking mathematics (Alton-Lee et al., 2012; Askew, 2012). These elements contrast with traditional views of mathematics classrooms which emphasised predictable teacher directed instruction in an initiate-response-feedback interaction pattern (Bell & Pape, 2012; Mehan, 1979). The mathematics reform movement as outlined in international and national research (e.g., Australian Curriculum, Assessment and Reporting Authority (ACARA), 2021; Department for Education, 2013; RAND Mathematics Study Panel, 2003; Royal Society Te Apārangi, 2021) advocates for classrooms where teachers strive to provide opportunities for students to develop and extend their mathematical reasoning by engaging in mathematical discourse embodying mathematical explanation, argumentation, and justification.

Underpinning reform-oriented mathematics classrooms is the emphasis on student and teacher collaboration and communication as essential components of developing conceptual understanding (Hiebert et al., 2005; Hunter & Civil, 2021). Developing conceptual understanding requires students look for and understand mathematical relationships, reconstruct prior knowledge, and make connections to new ideas. Furthermore, when given the opportunity to reason about mathematical concepts in a supportive environment, students can create conjectures, reflect upon, and evaluate them, and try to convince others to accept these justifications. To illustrate these ideas, in the U.S.A., Mueller (2009) conducted a study involving twenty-four students of African American and Latinx backgrounds, exploring how mathematical reasoning develops. These students were grouped in mixed groups of four to collectively work on open-ended mathematics problems which could be solved in several different ways. They were invited to build and justify solutions collaboratively and then share their ideas with the larger group. What Mueller noticed over time was that at first, interaction revolved around students merely repeating what others had stated. However, through consistent teacher expectations for deep mathematical reasoning, the students began to listen carefully to each other and consider each other's ideas and eventually engaged in mathematical argumentation and justification.

There is growing evidence for the ways in which pedagogical approaches can be used to build on all students' capabilities to engage meaningfully in complex mathematics tasks. Within these asset-based approaches, teachers focus and draw on students' collective strengths when designing complex mathematics activities requiring students to engage in collaborative sense-making (Esmonde & Langer-Osuna, 2013; Hunter et al., 2018; Wood et al., 2019). When students are provided with opportunities to learn mathematics in collaborative ways, they are able to enhance their conceptual understanding of mathematics (Cobb et al., 2011; Ing et al., 2015). For students to learn mathematics in these ways, communication and participation patterns need to be specifically constructed that provide all students with opportunities to learn mathematics equitably and successfully.

The role of the teacher in establishing and maintaining effective mathematics learning environments, where all students have access and opportunity to learning complex mathematics is critical.

2.4 TEACHER ROLE AND BELIEFS

Teachers play an important role in mediating students' interaction and engagement in learning. Several studies (e.g., Goos, 2004; Yackel et al., 1991) have examined teacher actions which facilitated the restructuring of the mathematics classrooms to reflect reform-oriented learning spaces. These studies highlighted that in the first instance, teachers engaged in deliberate acts of teaching such as, modelling processes, structuring social interactions, and linking important mathematical concepts to mathematical language and symbols. Teachers within these classrooms held high expectations for students to reflect on their mathematical reasoning and pressed for student agency by requiring students to ask each other questions and explain and justify their own ideas. The emphasis was on the teacher providing multiple opportunities for students to develop their reasoning through collaboration with their peers.

However, a teacher's choice of pedagogical approach for teaching and learning mathematics in classrooms is often influenced by what they believe about the nature of mathematics itself and how it ought to be taught and learned (Rubie-Davies, 2016; Turner et al., 2015). Some teachers view mathematics as a static body of knowledge to be learned and focus teaching on transferring this body of knowledge to passive recipients (Horn, 2008). On the other hand, other teachers believe that mathematics teaching and learning should be structured around important and connected ideas that students actively engage with (Resnick, 1988; Richards, 1996).

There have been studies that explored whether teachers' beliefs and observed classroom practices were aligned. For example, in the U.S.A., Stipek and her colleagues conducted a study (2001) examining beliefs and practices related to mathematics instruction with 21 teachers from a range of different elementary schools. At the beginning of the study, the teachers completed an initial survey regarding their beliefs about mathematics and teaching. Classroom observations were then recorded capturing the teachers' instructional actions and practices. The findings highlighted that the teachers held a connected set of beliefs related to what they believed mathematics to be, and how it should be taught and learned. Furthermore, these sets of beliefs could be placed at opposite ends of a spectrum. For example, some teachers' beliefs centred around mathematics being a fixed body of knowledge and skills to be learned and speed and accuracy were important. These teachers believed that the teacher was the sole mathematical authority in class and should direct all instruction and all students could be motivated to participate in class when offered extrinsic rewards. In addition, these teachers also believed in an innate and finite mathematical ability. At the other end of the spectrum, other teachers believed in the dynamic nature of mathematics, that students should be supported to understand mathematical concepts, hold some agency over how they learned these concepts, and could be intrinsically motivated to learn mathematics if tasks were meaningful to them (2001, p. 222). Classroom observations identified that teachers' pedagogical approaches closely matched their beliefs. For example, those teachers who held traditional views about the nature of mathematics and mathematics teaching and learning utilised traditional teaching approaches where speed and accuracy were valued as opposed to learning and understanding and vice versa, when teachers held reform-oriented beliefs, their pedagogical approaches were aligned with these (p. 223.). In addition, more traditional beliefs in mathematics and teaching reflected less teacher self-confidence and enjoyment of mathematics, while conversely, more confidence and enjoyment were associated with teachers employing inquiry-oriented approaches. The implications for these findings highlighted how teachers needed support to implement reform-oriented goals of teaching and learning mathematics, particularly if these goals conflicted with their beliefs about mathematics. Furthermore, Stipek et al. concluded that when teachers were supported to critically reflect on their instructional actions when enacting professional learning oriented toward reform mathematics education, they could begin to view mathematics teaching and learning through a different lens and effective teacher change becomes possible.

The effect of teacher beliefs on mathematics teaching approaches highlights the importance of understanding how teachers construct these beliefs. Furthermore, to attain the goals of equity in mathematics education, comprehensive teacher professional learning is required to support teacher learning.

2.4.1 Teacher learning for change

Implementing teacher professional learning into practice presents several challenges. Firstly, there is often a lack of specific guidance on how to implement policy aims. A further challenge relates to how a teacher’s embedded beliefs about teaching and learning can inhibit pedagogical change. Earlier research studies (e.g., Cohen, 1990; Kosko et al., 2014) have outlined significant challenges affecting teacher change in mathematics teaching. While policy documents and reform-oriented goals both internationally and in New Zealand outlined the rationale and details for pedagogical change, specific guidance for implementing these goals in classroom settings was not provided. Internationally, Hiebert (2013) discussed a U.S.A. mathematics reform document that aimed to bridge the ideological divide by providing an outline of student learning goals. This document stated five connected ways for students to achieve mathematical proficiency, for example, through adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency. Although this statement gained widespread acceptance and endorsement across the U.S.A., these goals were stated without specific instructions on how they should be implemented. Consequently, each of the five connected strands was widely interpreted across the teaching sector. Similarly, in New Zealand, the Ministry of Education's (2018) statement of intent aimed to raise teaching quality and student outcomes by implementing practice-based improvements through more targeted and effective learning and development programmes. However, schools and teachers did not receive specific guidance on how to enact these aims.

A further challenge affecting teacher pedagogical change are the notions of professional and disciplinary obligations guiding teacher practice in schools. Through a sociocultural lens, teaching and learning are viewed as cultural activity. The complexity of the cultural nature of teaching and learning can therefore create tension for enacting teaching change. Webel and Platt (2015) conducted a study investigating why secondary mathematics teachers' expressed goals or beliefs did not match their classroom practice. Their findings highlighted the role disciplinary obligations relating to teaching mathematics played in this misalignment. Webel and Platt asserted that within the cultural activity of teaching and learning, there were several rules which governed actions and practices. These rules have been referred to in other literature (e.g., Herbst & Chazan, 2012; Herbst et al., 2011) as practical rationality. Within the lens of practical rationality, underpinning these rules are norms and obligations. Norms being the actions and practices so embedded in the culture of teaching that most teachers would not feel the need to justify them, while obligations are the aspects of teaching that teachers would refer to if required to justify their pedagogical approaches. Within educational settings, teachers potentially feel bound by professional obligations to teach in certain ways advocated by their school. Webel and Platt highlight how disciplinary obligations relating to how mathematics should be taught, challenge teachers' expressed goals. The teachers, in their study emphasised they felt compelled to teach in ways that met the disciplinary obligations of mathematics, and rather than creating opportunities for students to develop creative and flexible ways of reasoning with mathematics, they felt obliged to teach students to use procedural strategies and symbolic notation methods which reflected precision. In contrast, teachers who utilise pedagogical approaches that support students to engage dynamically in learning mathematics pay particular attention to structuring the learning environments around productive discourse patterns.

2.5 COMMUNICATION AND PARTICIPATION

Classrooms structured around mutually respectful and effective communication and participation practices strive to meet the goals of equity in mathematics education. Establishing and maintaining these kinds of patterns require careful consideration of several key factors. Firstly, teachers need to develop and embed positive social norms that support all students in enacting taken-as-accepted and mutually respectful ways of interacting with each other. Successfully enacting positive social norms provides the foundation for students to learn mathematics through collaborative mathematical discourse. Students can learn how to make sense of mathematics together when teachers attend to the complex and dynamic nature of discourse patterns that can emerge in classroom settings. Furthermore, when teachers establish sociomathematical norms that support students to know and use important mathematical practices, students are positioned to critique and reason with complex mathematical concepts.

2.5.1 Social norms

Social norms are the taken-as-accepted interaction patterns within local settings, such as listening to others, explaining, and asking questions. These norms shape teacher and student participation and communication in classroom settings and are an important tool for organising student engagement in collective activity (Gutiérrez, 2007). Several studies (e.g., Civil & Hunter, 2015; McClain & Cobb, 2001) demonstrate how the establishment of effective interaction patterns are possible when social norms focus on positive relationships, mutual trust, and respect between all participants.

When norms centred on these aspects, there were opportunities for all participants to develop constructive relationships while interacting with each other.

Other research studies (e.g., Ichinose & Martinez-Cruz, 2018; Jacobs & Empson, 2016) highlight that when social norms are used to support students to reason with each other's ideas, classrooms become centres of collective engagement and productive discourse. Productive mathematical discourse provides opportunities for students to collectively deepen their mathematical understanding and engage in learning complex mathematics.

2.5.2 Learning mathematics through collaborative discourse

Discourse communities are learning environments where students are supported to enhance their participation and communication patterns to reason and learn. Student participation and engagement in collective mathematical activity provide students affordances to share their ideas, consider differing ideas, and reason about a range of mathematical representations (Alton-Lee et al., 2012; Bell & Pape, 2012; Stein et al., 2008). A study by Sfard (2007) highlighted how student learning in relation to mathematical concepts was evident in change in their discourse. For example, if a student began engaging with a problem unsure about the definition of area or how area contrasted with perimeter, and by the end was able to describe and measure area as the number of square units in a figure, then learning could be said to have taken place. Sfard added that discourse need not only be viewed as verbal utterances, but included other forms of communicating ideas, such as drawings, non-verbal communication, and symbolic notation.

In discourse communities, it is also important to examine discourse patterns that the teacher uses within the class (e.g., Barnes, 1998; Mercer & Littleton, 2007). Wood (1998) highlighted a discourse pattern called focusing. Focusing discourse patterns emerged when the teacher drew other students' attention to a peer's intellectual contribution. Rather than validating the student's statement by repeating it to the class, the teacher used "questioning" to focus attention on reasoning with the idea. Consequently, all students were expected to engage in sense-making by actively engaging with mathematical contributions made by their peers. In contrast, funnelling (Bauersfeld, 1980) is a communication pattern of discourse characterised by the teacher attempting to guide a student step-by-step to the correct solution through specific questions. In this type of communication patten, it is the teacher rather than the student who engages in cognitive thought.

The type of classroom discourse used by students has an impact on the overall communication patterns. Thomas (1994) suggested that student talk could be categorised as either “non-task-related” or “task-related” talk. Task related talk was further classified as “socially-oriented” and/or “cognitively oriented”. Thomas noticed that while some students appeared to engage in discussions, their utterances, while task related, were not cognitively oriented. Rather, these students’ discourse patterns appeared to stem from compliance to meet the teacher’s expectations that they “talk to each other” but did not allow for deeper mathematical exploration or understanding. On the other hand, Thomas found that several other students’ statements dominated classroom discussions. These types of discourse patterns have been described by researchers as cumulative (uncritical repetitive responses) or disputational (argumentative, competitive) talk (Hunter & Civil, 2021; Mercer & Littleton, 2007; Mercer et al., 1999).

Facilitating students to engage in productive discourse requires specific teacher moves. This includes talk moves (e.g., Chapin & O’Connor, 2007; Chapin et al., 2003, 2013; O’Connor et al., 2015; O’Connor et al., 2017; O’Connor & Michaels, 1993, 1996), and teacher invitation moves (Franke et al., 2015). Chapin and O’Connor’s (2007) study demonstrated ways in which teachers equitably position students to engage in “academically productive forms of talk” (p. 113) which have a positive effect on student learning. The four-year study examined one low socio-economic urban area in the U.S.A. and involved 18 teachers and 400 marginalised students aged 9-13 years old. Over time, learning environments which centred on developing academically productive student discourse, such as explaining one’s mathematical reasoning were established and maintained. National normed testing highlighted improved outcomes in mathematics for these students. In addition, by consistently engaging in meaningful discussion, significant development was noted in the students’ language capabilities. Five specific discourse-based instructional tools called talk moves; revoice, repeat, reason with other's ideas, adding on, and wait time, were utilised by the participants to develop productive discourse. Chapin and O'Connor emphasised that classrooms reflecting productive discourse communities required high expectations for all students to participate in mutually respectful ways. Centring student attention on their individual and collective reasoning required the establishment of a secure nurturing learning environment where all students could engage in argumentation about each other's ideas (p. 125). When academically productive interaction and communication patterns were established, mathematics sense-making could flourish.

Developing productive discourse and engagement with mathematics also requires further teacher actions beyond initial moves. Franke and her colleagues (2015) gathered data over one school year in 12 classrooms with students aged 5-11 years old. They found that, the teachers employed both what they termed invitation moves, inviting students to reason with others and supporting moves, supporting students to engage in collective mathematical reasoning. Interestingly, invitation moves did not always provide enough support for students to develop rich, productive mathematical discourse. For example, at times, the invitation moves resulted in passive participation with students agreeing or disagreeing without cognitively engaging. Franke at al. proposed that in-the-moment instructional tools, such as probing, scaffolding, and positioning are required to support students to engage critically with mathematical sense-making.

By engaging in productive discourse students are positioned with opportunities to learn complex mathematics. When teachers hold and maintain high expectations for student engagement in productive discourse practices, over time, and with consistent support, students can fulfil these expectations. To achieve these goals, students need to be supported to learn how to work specifically on mathematics activity. To provide the necessary support, teachers establish sociomathematical norms which focus on connected ways that students can engage with mathematical concepts.

2.5.3 Sociomathematical norms

Reform-oriented mathematics classrooms require a shift away from the transmission of knowledge to a model of participating in the culture of using mathematics, or a culture of mathematising as a practice-this practice involves “learning when to do what and how to do it” (Bauersfeld, 1993, p. 4, as cited in Yackel & Cobb, 1996). Sociomathematical norms provide the support students need to be able to learn the practice of mathematising. Sociomathematical norms, which differ from social norms “in that they are specific to the mathematical aspects of students’ activity” (Yackel & Cobb, 1996, p. 458) form the foundations for how students can learn to make sense of complex mathematical ideas while working with their peers (Carpenter & Lehrer, 1999; McLean & Cobb, 2001; Voigt, 1996) can be established.

Sociomathematical norms are developed and maintained by specific teacher actions that focus student attention on the mathematics and negotiation of mathematical meaning as they interact in collaborative groups on mathematical activity (Yackel, 1995). Yackel et al.’s (1991) study involving 20 students aged 7 years old examined how classroom norms were developed as a means for teaching students how to work together on mathematics activity and the opportunities for learning that arose during student collaboration. These researchers specified that the norms for collaboration were mutually developed while students worked together to solve mathematics problems. The teacher closely monitored student interactions during small group-work and only intervened to “encourage cooperation and collaborative dialogue or to discuss the children’s solution attempts” (1991, p. 393). During the large group discussion, the teacher supported the students to explain their reasoning and develop mathematical explanations. Consequently, the students were expected to actively participate in collective sense-making and mathematical reasoning and sought solutions and mathematical understanding rather than expecting a “more knowledgeable” peer or teacher would explain the mathematics to them. In this way, notions of who held the mathematical authority shifted away from any one individual to the collective authority of the group to sense-make.

2.5.4 Mathematical practices

Mathematical practices are actions that successful mathematicians know and use to engage effectively in mathematical reasoning. These actions include making mathematical explanations, developing justifications through friendly argumentation, and generalising mathematical concepts (e.g., Ball, 2003; NGA, & CCSSO, 2010; NCTM, 2000). Mathematical practices are grounded within collective practices occurring in social and cultural activity systems and among multiple participants (e.g., Coombes & Sook Lee, 2017; Frost & Coombes, 2014; Moschkovich, 2013). Research studies (e.g., Foreman & Ansell, 2001; Moschkovich, 2002; Rogoff, 1995;) demonstrated that students learn these practices through appropriation by participating in a set of cultural practices through joint activity and by engaging in collaborative mathematical discourse practices. Teachers take a key role in facilitating students to engage in mathematical practices in the classroom (Ball et al., 2005; Boaler, 2002; Makar et al., 2015).

Making mathematical explanations is an important mathematical practice that supports conceptual understanding. Research studies (e.g., Forman et al., 1998; McClain & Cobb, 2001; Wood et al., 2006) highlight teacher actions such as revoicing that support students to develop explanations. However, Esmonde (2009a) emphasises the importance of teachers developing broad understanding of what making mathematical explanations involves. She advocates for understanding the value of students working together to co-construct explanations in order to develop equitable collective mathematical reasoning rather than expecting detailed explanations from individual students.

Collective reasoning about mathematics involves negotiating and renegotiating important mathematical ideas. Several researchers (e.g., Engle et al., 2008; Forman et al., 1998) have highlighted how students developed and used mathematical argumentation to reach consensus about important mathematical concepts. Wood (1999) emphasised that mathematical argumentation was a way for students to understand how mathematics is “a discipline that relies on reasoning for the validation of ideas” (p. 189). Mathematical argumentation develops when teachers hold students accountable for mathematical sense-making, including pressing students to justify their thinking and providing opportunities to agree or disagree. Within classrooms characterised by mutually respectful communication and participation patterns, students can be supported to develop friendly argumentation to prove important mathematical claims and concept. These kinds of interaction patterns are only made possible when the teachers give students autonomy to sense-make. When students reason collectively, they are positioned to rely on each other as sources of mathematical validation (Brown & Renshaw, 2000; Connor et. al., 2014; Turner et al., 2013). In such ways, mathematical arguments are co-constructed to provide proof through the negotiations within mathematical discourse (Mueller, 2009).

Explicit and clear teacher instruction is required to establish mathematical practices in the classroom. Selling's (2016) study examined two teachers' actions across three different mathematics classrooms with students aged 11 years to 15–16-year-olds to explore how teachers could specifically create affordances for developing mathematical practices. A key finding was that simply providing opportunities to learn mathematics practices is not sufficient to support all students. Instead, the teachers successfully used several kinds of reprising moves that served to make mathematical practices explicit in classroom discourse. This included naming the mathematical practice students were using in-the-moment and explaining the importance of using that particular practice. This supported students to view the practice as an important way of learning mathematics. This kind of support requires teachers to notice and explicitly call attention to the mathematical practice that students are engaging in, for example highlighting that mathematical argumentation requires justification. By positively evaluating the mathematical practice a student has used, students develop understanding of how using these practices support them in learning mathematics.

Within the New Zealand context, Hunter and colleagues (e.g., Hunter, 2007a, 2007b, Hunter & Hunter, 2018a, Hunter et al., 2018) have worked with teachers in high poverty areas to explore how Māori and Pāsifika students can access and learn mathematics equitably. Central to their work is the use of a Communication and Participation Framework (Hunter, 2007b); a tool specifically designed to scaffold teachers to engage students in mathematical practices within communities of mathematical inquiry. An important component of this tool is the way in which teachers can use it adaptively, flexibly, and in culturally responsive ways (Hunter & Hunter, 2018a) to empower students to develop or critique mathematics claims through mathematical explanation, justification, and argumentation.

To engage in mathematical practices, students use prior knowledge and build new mathematical understandings. By reasoning collectively through questioning each other, explaining their ideas, and justifying mathematical claims, all students can be arbitrators of mathematical sense-making and deepen their understanding of mathematics (Civil & Hunter, 2015; Selling, 2016). When students understand the importance of utilising such practices to develop their mathematical understanding, notions of any one person (teacher or students) holding authority for validating mathematical ideas can shift (Nathan & Knuth, 2003). Delegating mathematical authority as a means of striving for equity is well documented in research (e.g., Dunleavy, 2015; Esmonde, 2009b; Langer-Osuna, 2016).

Developing and maintaining effective and mutually respectful communication and participation patterns are important components of students learning mathematics in equitable ways. A fundamental dimension of this process involves the grouping approaches in mathematics classrooms.

2.6 GROUPING

Grouping approaches in mathematics classrooms affect not only what students learn, but also how they come to view themselves as learners and determines how they participate in educational activity and settings (Boaler, 2013, 2014; Nasir & Hand, 2006). While reform-oriented mathematics education strives for equity, ability grouping practices present a persistent challenge to these ideals when beliefs of individuals holding innate abilities to learn mathematics perpetuate.

2.6.1 Ability grouping

Grouping by ability is a common institutionalised practice in many mathematics classrooms both internationally and within New Zealand (Anthony & Hunter, 2017; Hallam, 2012). However, ability grouping in the mathematics classroom is a contested practice with some viewed this practice as a way of managing student diversity, while others caution that only gifted and talented students benefit from this practice (e.g., Kulik & Kulik, 1992). Furthermore, a growing body of research (e.g., Hartas, 2017; Hodgen et al., 2020) highlights that ability grouping neither supports all students nor raises achievement. Of significance and well documented in other research (e.g., Adiredja & Louie, 2020; Chestnut et al., 2018) was how grouping by ability implies a belief in the inherent ability of some individuals to learn mathematics better than others.

Zevenbergen (2003) highlighted that research on grouping often focuses on ideas around innate mathematics abilities or intelligence. She argued that those kinds of beliefs perpetuate deficit views around certain students' capabilities to learn mathematics successfully, which in turn limited their opportunities for learning. In addition, ideas of an innate ability to learn mathematics resonated with many peoples' own schooling experiences. While some people achieved academic success in other curriculum areas, achievement in school mathematics was not experienced, therefore, the idea of only certain students holding an innate ability to learn mathematics was established and embedded. When the idea of a fixed inherent ability to learn mathematics is upheld by teachers, they are less likely to believe that all students are capable of learning mathematics successfully.

The perpetuation of an innate ability to learn mathematics upholds the idea of mathematics as a gatekeeper. Several researchers (e.g., Crespo & Featherstone, 2012; Moses & Cobb, 2001; Zevenbergen, 2003) have identified that grouping practices were linked to preconceived assumptions about certain students' backgrounds, including gender, socioeconomic status, language, and cultural background. Furthermore, deeper analyses of ability grouping practices have highlighted societal categories whereby middle to upper class students were grouped in higher groups while certain students from lower socioeconomic backgrounds were often marginalised and relegated to the lower groups (Parsons & Hallam, 2014). These inequitable structures permit educational institutions to lay the blame for some students' academic failures elsewhere.

Ability grouping practices also replicate and perpetuate social and cultural disparities where cultural capital (Bourdieu & Passeron, 1973; Grenfell, 2012) affects access and opportunities to learn mathematics. This is significant as the kinds of learning environments that are established through ability grouping influence how students view the world around them, which Bourdieu (1977) termed a habitus. A habitus mirrors the lived realities of students, their distinct experiences, and their possible opportunities and outcomes (Harker, 1985; Maton, 2012). Those students whose habitus fits with the dominant practices of mathematics teaching and learning are more likely to have successful outcomes, those who habitus does not fit become systematically and institutionally marginalised. When grouping practices reflect societal marginalisation, mathematics classrooms become inequitably structured.

Contrasting with arguments that ability grouping is necessary to manage a wide range of students and allow them to learn relevant mathematical content that is academically within their reach, studies highlight detrimental impacts for students at both ends of achievement. For example, in the U.K., Marks (2012) explored grouping experiences of two primary students aged 10-11 years old with one in the high ability stream and one in the bottom ability stream. Both students experienced negative impacts from being streamed. The student in the high ability group developed anxiety and became fearful of publicly making errors in her mathematics class where a culture of speed and procedural accuracy was evident. Consequently, this student consciously chose not to engage in classroom discussion to avoid making errors and feeling humiliated in front of her peers. The student in the low ability stream was provided with concrete, kinesthetic approaches, unchallenging tasks, and a slow pace of work. Subsequently, this student was not provided with access or opportunities for learning complex mathematics and showed little progress.

Further research by Marks (2013) found that even in classrooms that did not use explicit grouping and labelling, both teachers and students had the idea of an inherent fixed ability to learn mathematics. Marks investigated the ways in which "ability" is conceptualised and how this label affected teaching and learning mathematics in schools by analysing data from two primary schools, she concluded that no matter how classroom groups were organised, group labels centred on beliefs that all students held innate or limited capacities to learn mathematics.

Within New Zealand, the implementation of a nation-wide professional development project called the Numeracy Professional Development Project (Ministry of Education, 2004a, 2004b) advocated for ability grouping based on normed testing (Golds, 2014; Tait-McCutcheon, 2014). Students were grouped in strategy stages and taught content limited to the expectations of the stage at which they were perceived to be working. This meant that some students were denied access to higher levels of learning. Despite the efforts of the Numeracy Development Project to provide accelerated achievement in mathematics, the project failed to provide equitable outcomes for all students, particularly Māori and Pāsifika students (Hunter & Hunter, 2018a; Young-Loveridge, 2010).

2.6.2 Alternative grouping approaches

Despite the strong focus on ability grouping in mathematics, we can look to a considerable body of research that highlighted alternative approaches to ability grouping (e.g., Esmonde & Langer-Osuna, 2013; J. Hunter et al., 2020). Boaler's (2006) study investigated outcomes for students from an ethnically diverse, low-socioeconomic school in the U.S.A. where teachers employed heterogenous grouping practices for mathematics teaching and learning. Findings from this longitudinal study emphasised the development of mathematics classrooms focused on high expectations, equity, mutual respect, and positive relationships. Students were scaffolded to work together in mixed ability groups by the assignment of specific roles, such as facilitator, team captain, recorder, or resource manager. Status issues were alleviated by the teachers publicly assigning competence to lower status students and holding high expectations for students to be responsible for their group's learning. A significant outcome of Boaler's work was the development of what she termed "relational equity" (2006, p. 45). She described relational equity as students interacting and participating in mutually respectful ways and where students learned to appreciate the intellectual contributions of their peers.

Other research has been conducted (e.g., Dixon et al., 2002) exploring whether teaching could ever be free from constructs of ability. Based on historic classroom grouping practices in the United Kingdom education system (mirrored in many other countries, including New Zealand), Dixon and her colleagues extensively examined whether different approaches to teaching could produce teaching unimpeded by forms of ability labelling. These researchers worked with nine teachers across primary and secondary school levels. These teachers had refrained from ability grouping and instead, developed alternative grouping approaches. Data collected over one year (e.g., classroom observations and teacher and student interviews) yielded several approaches markedly free from ability labels. One of these approaches emphasised the idea of "transformability" as a potential instructional approach that "consciously rejects ability labelling" (Dixon et al., 2002, p. 8). The notion of transformability signified every student had equal rights to learn, and enabled teachers to design teaching and learning centred on providing equitable learning opportunities for all students. Currently, in New Zealand, a professional learning project called Developing Mathematical Inquiry Communities is grounded in the idea of transformability. Within this project, culturally sustaining practices and mixed ability grouping are specific pedagogical approaches used to strive for equitable outcomes for all students, in particular Māori and Pāsifika students, who are most marginalised in New Zealand society (Hunter, 2013; Hunter & Anthony, 2011).

Effective communities thrive and succeed when diversity is celebrated, and individual strengths are acknowledged and drawn on. Building such communities involves a continuous and adaptive process where the generation and sharing of ideas are developed within a supportive environment. Askew (2012) called for learning environments that provided access and opportunities for a wide range of students to work together. To create these kinds of classrooms, teachers were required to believe in all students' capabilities to learn and understand mathematics and shift away from notions of an innate ability. Furthermore, Askew argued that when educators accepted the status quo for ability grouping practices, an implicit agreement with the perspectives that different children have different mathematical capabilities was implied and upheld. As such, Askew offered an alternative of creating mathematics-focused classroom where students were expected to explore the ideas and solution pathways possible in the mathematics. This kind of learning environment shifted away from teacher-or-student centered classrooms where emphasis on the smartness or correctness of one student’s solution were the focus. Askew suggested that within such learning environments both mathematics learning and classroom communities could be enhanced (2012). Aligning with Askew’s ideas, Sergiovanni (1996) described successful communities as places focusing on the collective good and connection through common goals and shared values. Within such communities, children could learn about how and why they might want to work with others.

However, despite a body of research advocating for a move away from ability grouping practices in mathematics classrooms these kinds of grouping practices remain. Highlighted in a body of research (e.g., Civil et al., 2019; Gutiérrez, 2012; Langer-Osuna, 2018) is the connection between ability grouping and inequity, including differences in how certain students responded to or ignored the ideas and contributions of their peers. In classroom settings, when some students’ ideas and utterances were deemed more important than others or were completely ignored, status hierarchies developed which have a detrimental effect on access and opportunities for learning mathematics.

The themes highlighted in the previous sections relate to ideas of equity in education. Exploration of topics related to equity in mathematics education require viewing social episodes with multiple participants in classroom settings through a particular lens. In the next section (see Section 2.7), the theoretical framework underpinning the current study is discussed.

2.7 THEORETICAL FRAMEWORK

Within the current study, positioning theory and status generalisation theory are drawn on to form a synthesised lens through which to observe and theorise ways to establish equity in mathematics learning environments. In the next section, an overview of positioning theory is provided with specific links to how this theory can be used to provide structure to understanding social episodes in mathematics classrooms.

2.7.1 Positioning theory

Positioning theory provides a structure for analysing the dynamics of social interactions (e.g., Bishop, 2012; Esmonde, 2009c; Langer-Osuna, 2016; Yamakawa et al., 2009) and has been defined as the "discursive process whereby selves are located in conversations as observably and subjectively coherent participants in jointly produced storylines" (Davies & Harré, 1990, p. 48). This framing of positioning accepts that individuals use storylines to understand intricate social events (van Langenhove & Harré, 1994). Positioning theory developed partly from emerging dissatisfaction with role theory, a theory popular in the 1950'and 1960's which centred on the belief of a fixed nature of the psychological foundations of social interactions (Davies & Harré, 1990; Harré & van Langenhove, 1991). Within role theory, the emphasis was placed on discerning the rules that direct interactions, as opposed to examining the changeable and dynamic nature of interactions. Consequently, positioning theory developed as an alternative lens for viewing social interactions.

In education, positioning occurs through communication and participation patterns that position participants in specific ways, including relations of power, competence, and moral standing (the right to explain or the duty to listen) (Cobb et al., 2009; Gresalfi et al., 2009; van Langenhove & Harré, 1994). Through the lens of positioning theory, the social aspect of interactions is emphasised and is understood as the ways students come to understand the local moral order, that is the accepted rights and duties of the setting. Within a setting, participants learn to interact in ways that either promote or hinder individual agency and opportunities for learning (Davies & Harré, 1990; Pinnow & Chval, 2015). Positioning theory also provides a means for making sense of individuals' assumptions in social episodes (Harré & Slocum, 2003; Harré & van Langenhove, 1999a, 1999b; Pinnow & Chval, 2015). For example, what people are taken to mean by what they say and do relies to some extent on what the other people involved in the social episode believe this person has the right to say and do. Within positioning theory, these rights are called positions.

2.7.1.2 Positions, storylines, and speech acts

Positions are defined as the rights, duties, and obligations an individual is believed to be entitled to have within a social episode. For example, children are not always afforded equal speaking rights as adults and are positioned to receive disciplinary cautions as opposed to being sources of these. In classrooms, teachers are positioned as having the right and duty to assign grades, while students are positioned as obligated to accept these gradings (Harré & Slocum, 2003). Positions are also relative to one another, and individuals are assigned positions as a social episode unfolds. For example, in the mathematics classroom, if one student is positioned as a mathematics expert, simultaneously, others can be positioned as less competent at mathematics. Moreover, the student positioned as the expert, is believed to have the right and duty to instruct others who, in turn are expected to listen. If one student positions themselves as mathematically smarter than their peers; they may communicate this position and perception of their peers' less competent positions by taking control of the mathematical activity and directing what the others should do (Langer-Osuna, 2011; Wood, 2013). Positioning is not always deliberate but can be unintentionally allocated, given the fluidity of social interactions. In any given interaction within a setting, individuals assign positions or can themselves be positioned by others as either competent or incompetent, dominant or submissive, decisive or passive (Harré & Slocum, 2003; Pinnow & Chval, 2015). Every position or positioning act evolves within a storyline.

Within positioning theory, storylines are the patterns of interactions that unfold in certain social arenas and are used to make sense of complex social events and possible related positions. Storylines are a means for individuals to interpret who people are and why they act in certain ways (Langer-Osuna, 2016; van Langenhove & Harré,1999; Wood, 2013). Furthermore, storylines are viewed as figured worlds in that they are socially constructed realities. A storyline frames the event and the context, in turn, also framing other participants and assigning positions for everyone (Davies & Harré, 1990). Within mathematics classrooms, for example, one student may have a storyline about school in which there are certain students who are mathematically smart and others who are not. Within this storyline are expected ways of behaving, responding, and being. For example, the student positioned as competent is expected to provide the correct answers and direct others (Langer-Osuna, 2011, 2016; Wood, 2013). These storylines are not usually directly stated and are instead implied and assumed through the activities or speech acts of the storyteller (an individual). For example, one student may not directly state that they think their peers are less competent, rather their actions in controlling and directing the mathematical activity, or tone of voice will convey this storyline. Simultaneously, these performed storylines offer a way for individuals to negotiate their positions. In other words, some students may not accept another's assumed position as being mathematically smarter than them and might refuse to follow their directions. In taking these actions they successfully dispute that student's positioning of themselves and them, and, in turn they reposition themselves and others (Cobb et al., 2009; Harré & Slocum, 2003; Langer-Osuna, 2011, 2016).

Examination of social episodes within a classroom setting provides a way to focus both on individuals and the relationship between learning and the individuals participating within the episode. Therefore learning, if it transpires, would be evident in the activities and interactions of students as they participate in the lesson (Sfard & Kieran, 2001, Sfard & Prusak, 2005; Wood, 2013). Storylines are a useful tool for examining students' interactions as they address the dynamic evolution of social interactions that can make prior or new stories available to participants, and where what has unfolded before can affect the future narratives of the participants (Pinnow & Chval, 2015; Wood, 2013). Analysing existing or evolving storylines requires deconstructing patterns of communication. Within positioning theory, patterns of communication are called speech acts.

Speech acts are the patterns of communication, which may or may not be verbal, but will include gestures, gazes, or physical movement that are familiar as socially important and are used by participants to contribute to or induce storylines-known or new (Davies & Harré, 1990; Pinnow & Chval, 2015). Speech acts are particularly useful when examining classroom interactions as they provide a means for understanding the semiotic resources used by students to communicate and interact with each other (Pinnow & Chval, 2015).

In mathematics classrooms positions, storylines, and speech acts are often mutually constructed around ideas of mathematical competence and often result in developing power struggles and issues of status as diverse positions of authority are assumed and assigned (Cobb et al., 2009; Gresalfi et al., 2009; Langer-Osuna, 2016). Positioning theory, therefore, provides a structure for analysing the positioning experiences of students who have been consistently marginalised in our current education system; specifically, Māori and Pāsifika students.

2.7.2 Status generalisation theory

Complementing the use of positioning theory to examine the classroom context is status generalisation theory. This theory proposes that status generalisations are made about characteristics related to an individual or group's intellectual ability, social advantage, and cultural preference (Featherstone et al., 2011; Shah & Crespo, 2018). Historically, status generalisation theory has been used in sociology research to examine issues of social inequities in a range of contexts (e.g., school playgrounds, jury rooms, and work groups). Within educational settings, this theory offers a way of understanding what attributes are valued in the classroom and how students generalise their own reactions and responses for other student's contributions (e.g., Cohen & Lotan, 1997; Dunleavy, 2015; Kalkhoff & Thye, 2006) and provides a critical lens for investigating patterns of inequitable participation and the generalised expectations of competence in collaborative learning settings (Cohen, 1997b; Cohen et al., 1999).

When teachers and students believe that only some individuals can learn mathematics well, status problems emerge. Status issues arise when generalisations made by peers relate to certain characteristics connected to perceived notions of intellectual ability, social advantage, and cultural preferences (see e.g., Boaler, 1997a; Crespo & Featherstone, 2012). Status issues are also caused by inequitable relationships of authority and power among peers (Cohen, 1997a, Langer-Osuna, 2016) These issues impact significantly on equitable learning opportunities for all students. Addressing these challenges requires understanding the dynamic nature of status, that is, that status fluctuates depending on who holds what power and when. These status hierarchies affect classroom participation as they render some students as small and incompetent in relation to their peers, and often disrupt student collaboration (Langer-Osuna, 2016; Shah & Crespo, 2018; Wood et al., 2019).

2.7.2.1 Status hierarchies

In education, narratives exist that some students have more ability than others, or that there are fast and slow students (Boaler, 2014; Boaler & Wiliam, 2001). These narratives create and perpetuate hierarchies of students' academic potential by inequitably allocating intellectual status between students, and hierarchies of social status (Dunleavy, 2015; Featherstone et al., 2011; Wood et al., 2019). Inequitable classroom interactions related to issues of control, competitiveness, and language proficiency are all ways that status hierarchies are perpetuated. These hierarchies disrupt classroom communication and participation patterns and influence who participates in learning and how they participate. Furthermore, it is often the students for whom it is of the utmost important to engage in productive discourse who are marginalised and excluded in mathematics classrooms (Rubel, 2017; Rubie-Davies, 2016).

Marginalisation occurs across many contexts, including race, gender, and language. Wortham's (2006) study highlighted how racial and gender narrative in a secondary school English classroom positioned students of colour as outcasts, which affected participation and communication. Similarly, Stinson (2013) highlighted how racial hierarchies of mathematical ability occurred in a classroom setting where students of colour were believed inherently inferior to their peers. In mathematics education, gender is also a status characteristic that is matched with perceived mathematical capability, and boys are often seen to be mathematically superior to girls. In many primary school classrooms when the teacher directs, instructs, or praises students, emphasis on gender occurs, for example, "good job, boys and girls" (Shah & Crespo, 2018). As such, these kinds of interactions repeatedly reinforce the idea of two types of students: boys or girls. This is problematic as it emphasises a false gender binary. This incomplete idea of gender becomes a status generalisation linked to who is and who is not capable of learning mathematics.

Marginalisation of students also occurs around language proficiency. Placing too much emphasis on vocabulary instead of mathematical reasoning and communication can affect the status of additional language learners. For example, should a teacher ask if everyone knows what 'shortest' means, through the lens of status generalisation, some students' mathematical competence and English proficiency may be called into question (Civil, 2011; Turner et al., 2013). Within the New Zealand education context, language-based equity issues are a restricting factor. Although many Pāsifika students start school fluent in their own language and bring a rich background of knowledge and experiences, within a short time of schooling, where the language of instruction is English, they join the high number of Pāsifika students who are systematically marginalised within the current education system (Anthony & Hunter, 2017). However, when teachers provide opportunities for students to shift between their first home language and the dominant language when working together, more equitable access to learning is possible (Gutstein et al., 2005; Anthony & Hunter, 2017; Turner et al., 2013).

Status generalisation theory provides a means to examine which students' intellectual contributions are valued higher than others when teachers regulate responses. For example, teachers' reactions to right and wrong responses can emphasise that learning mathematics is intrinsically about the quest for correct solutions. In mathematics classrooms with an emphasis on pursuing correct solutions as quickly as possible, a culture of competitiveness is established. Students in these classrooms also accrue status by being openly acknowledged for providing the correct answer (Featherstone et al., 2011; Shah & Crespo, 2018). In addition, students raising hands and being called on by the teacher also strengthens the narrative about mathematics learning being an individual pursuit. When status hierarchies are disrupted, all students are granted access and opportunities to learn and institutional marginalisation can be alleviated (e.g., O'Connor, 1998; Sfard & Kieran, 2001). One specific teacher action that mitigates status hierarchies is assigning competence.

2.7.2.2 Assigning competence

Assigning competence is a public action a teacher takes to deliberately highlight the intellectual contribution a student or group has made (Cohen, 1997b; Cohen & Lotan, 1995; Wood et al., 2019). A teacher is only able to assign competence when they notice and attend to specific elements of student thinking that enhance students' mathematical reasoning (Choy et al., 2017). This kind of teacher noticing has been highlighted in previous literature as productive noticing (Choy et al., 2017) and noticing for equity (van Es et al., 2017). When teachers notice students' ideas in these ways, they recognise students as sources of knowledge (Gonzalez & Vargas, 2020). Assigning competence supports all students to appreciate their own intellectual contributions and those of their peers. When low-status students are noticed and acknowledged as sources of valid mathematical knowledge, they can be positioned as capable knowers and doers of mathematics. Over time, how students are positioned to participate in collective activity affects not only what they learn, but how they come to view themselves as learners, and conversely, "how students view themselves as learners can greatly influence how they participate in educational activity and settings" (Nasir & Hand, 2006, p. 467). This is important, because, to ensure students feel confident engaging in mathematical sense-making, they must believe themselves capable of learning mathematics successfully (Dweck, 2008; Esmonde et al., 2009; Gresalfi & Cobb, 2006).

While assigning competence has the potential to position students as legitimate participants in learning mathematics, its use needs to be in response to genuine mathematical contributions (Cohen & Lotan, 1997; Cohen et al., 1999; Dunleavy, 2015). Assigning competence also provides students access to new mathematical ideas. This is an advantageous outcome, as when students are encouraged to grapple with new ideas, they are pressed intellectually to view an aspect of mathematics in a different way and connect ideas within and across mathematical concepts (Dunleavy, 2015; Wood et al., 2019). This process deepens and extends mathematical learning while equalising status in the classroom.

Status generalisation theory provides a powerful lens through which the connections between participation and status may be explained, and more specifically, how students' engagement in mathematics learning may be built in equitable ways.

2. 8 CHAPTER SUMMARY

This chapter has explored important themes of equity in mathematics education. Woven through the sections is the emphasis on all students having the opportunities and right to access complex mathematics. Both international and New Zealand literature have identified how certain students are marginalised or excluded from learning high-level mathematics and how this marginalisation reflects societal inequities. Despite considerable research over the last few decades that has outlined how specific pedagogical approaches and transformation of embedded beliefs about innate abilities of individuals to learn mathematics successfully, there are specific barriers to affecting change. While reform policy documents reflect the aims of equity in mathematics education, little guidance is provided for teachers on how to implement these goals. Throughout this field of literature, the complexities of classroom systems emerge identifying how mathematics teaching and learning involves multiple participants positioning themselves, each other, and mathematics in fluid and dynamic ways.

The following chapter describes the methodology of design research which is used in the current study. This was chosen as suitable for the theoretical framework and the overarching aim of developing a learning environment advocated by current theory as beneficial but not yet common practice (Design Based Research Collective (DBRC), 2003).

CHAPTER THREE

METHODOLOGY

3.1 INTRODUCTION

The previous chapter examined the international and national literature providing the theoretical framework through which the current study can be viewed. This chapter identifies and describes the methodology used in the study. A design-based research methodology was chosen as appropriate to the theoretical framework and the overall goal of developing equitable mathematics learning environments prompted by current theory as productive, but not yet common practice (Design Based Research Collective (DBRC), 2003).

In Section 3.2 the main research question and supporting questions are presented. The qualitative research paradigm underpinning the research is outlined in Section 3.3. In Section 3.4 an explanation of design-based research and justification for its use in the study are provided. The professional learning model is also given. Several supportive frameworks designed and used within the research are presented and discussed. Section 3.5 outlines and justifies the case study approach. Section 3.6 outlines the research setting and schedule, and the selection of the participants is justified. Ethical considerations are discussed in Section 3.7. In Section 3.8 the data collection and methods are outlined. Data analysis is discussed in Section 3.9. Section 3.10 outlines the data presentation. Validity and trustworthiness are discussed in Section 3.11.

3.2 RESEARCH QUESTIONS

This study addresses the key question:

How do teachers construct equitable learning environments in primary school mathematics classrooms?

To address this question, the study focuses on three smaller research questions:

- How do teachers (re) construct beliefs and classroom practices to position all students to learn mathematics?
- How do teachers address status?
- How do teachers delegate mathematical authority to all students?

These questions acknowledge the key role the teacher plays in enacting change in the classroom. Specifically, the study focuses on how, through collaborative and personal inquiry, teachers adopt and adapt key pedagogical strategies and classroom practices that support students to engage meaningfully with mathematics. While the focus of the study is on teacher actions, implicit in the research questions is the understanding that student engagement potentially changes when new practices and foci are introduced into the classroom.

3.3 THE QUALITATIVE RESEARCH PARADIGM

This study employs a qualitative interpretive research model and is grounded in a sociocultural perspective. This type of research is contextual, has a practical relevance, and is carried out within a naturalistic setting (Bogdan & Knopp Biklen, 2007; Cohen et al., 2018; Denzin & Lincoln, 2005). Consequently, it was suitable to employ a qualitative, interpretive approach to this study which was situated within different classrooms at one school.

Qualitative research focuses on gathering rich, descriptive data in natural settings, uses inductive thinking, and focuses on understanding and making meaning of the social occurrences in the setting (Bogdan & Knopp Biklen, 2007; Merriam, 1998, 2009; Yin, 2011). The aim is to capture the various facets within the setting and seek to understand how they work together to shape the whole (Merriam, 1998).

The role of the researcher, within qualitative, interpretive studies, is to be personally involved in designing, implementing, and interpreting the research. This complements design research where practitioners and researcher work together to introduce innovative practice and develop change in the context of practice (DBRC, 2003). In the current study, the researcher drew on design-based methodology to work alongside both teachers and their students to design, develop, implement, and evaluate an innovation focusing on equitable positioning of all students to engage in mathematical reasoning. Within this frame, close relationships exist between all participants (Cohen et al., 2018).

3.4 DESIGN-BASED RESEARCH

Design-based research (DBR) is being increasingly used in recent years in mathematics education to study innovations in teaching and learning in a wide range of naturalistic settings. Using design-based research involves linking theoretical research to practice. This method offers promise for creating innovative teaching and learning environments by designing and doing research about teaching and learning in classrooms with teachers. DBR may also add to existing theory through detailed description of “instructional interventions and their effects across multiple settings” (DBRC, 2003, p. 8).

Design-based research aimed at sustaining teachers’ learning involves repetitive cycles of design and research where speculations about the learning of the teachers and the means of supporting them are continually tested and revised during ongoing interactions. Participants in DBR investigations are regarded as co-participants in the design and at times the analysis (Anderson & Shattuck, 2012; Barab & Squire, 2004; Gorard et al., 2004). A key aim is for the teachers to participate in professional learning sessions with the goal of supporting their pedagogical approaches in the classroom (Kazemi & Franke, 2003; Zhao & Cobb, 2006). This entails sustaining learning across two different settings-the professional learning setting, and the classroom. For the current study to meet its aims, it was essential to explore teaching approaches which focused on positioning all students equitably for learning mathematics. To accomplish this, the design included teacher participants and the researcher undertaking activities in one setting-the professional learning sessions (study group), to support the overarching goal for teachers to restructure their activity in a different setting-the classroom. This included developing teacher understanding of pedagogical actions, classroom norms, and mathematical practices which support positioning all students with equitable access and opportunities to learn mathematics.

The strong relationship between theory and practice is emphasised in design-based research studies. Theory is used as the foundation for designing systems which utilise detailed planned teaching practices to enable learning (Anderson & Shattuck, 2012; Walker, 2006). Gravemeijer and Cobb (2006) outline the need for researchers to develop local instruction theories. This can be achieved by examining existing research literature and then speculating about the trajectory of the possible learning process and how this learning process can be supported in the classroom through the actions of the teacher.

There are three phases in design-based esearch: preparing for the innovation, experimenting to sustain learning, and conducting reflective analyses (Cobb et al., 2003; Walker, 2006). The preparation phase involves outlining a conjectured learning pathway that includes goals and the starting point for collaboration. In addition, conjectures about the ways in which teachers' learning in professional development might affect their classroom practices (or vice versa) are included. A central assumption is that teachers' practices in their classroom could and should function as a vital resource from which researchers can draw to inform professional development design (Zhao & Cobb, 2006). The second phase in DBR involves implementing and revising proposed conjectures about both the potential learning process of teachers and the means of supporting them. Participation and learning are analysed and used to evaluate the validity of the conjectures which form the local instruction theory and to revise specific aspects of the design. This process requires classroom observations and interviews with teachers for a shared insight of lessons and changes to be made (Gravemeijer & Cobb, 2006; Zhao & Cobb, 2006). In this phase, the assumptions held about the relationship of teachers' learning in the professional learning sessions and their practices in the classroom continue to shape the ongoing design conjectures. The final phase of the design experiment involves carrying out retrospective analysis. The goal of this phase is to develop practical knowledge about restructuring instructional practice (DBRC, 2003; Fishman et al., 2004; Zhao & Cobb, 2006).
Aligned with design-based research, in this project, the researcher assumed the role of participant observer (Cobb et al., 2003; Cohen et al., 2018). As such, the researcher took the lead role in all the professional learning session activities, providing the relevant research articles and facilitating critical reflection of classroom lessons. In the

classroom, the role of the researcher was mostly as a passive observer. Intervening in the lesson in any form was always at the request of the teacher. This dual role allowed the researcher to develop a comprehensive understanding of the events. Positive relationships and relational trust developed throughout the study as the researcher was positioned as both participant and observer. At the same time, the researcher sought to be objective in all aspects of the investigation. While there has been argument (e.g., Barab & Squire, 2004) that the close involvement of the researcher in all cycles of the design research can present challenges for the credibility and trustworthiness of findings, others (e.g., Anderson & Shattuck, 2012) highlight that many qualitative research methods utilise close researcher involvement and that researcher bias cannot be excluded. Furthermore, Anderson and Shattuck suggest that "researchers themselves (with their biases, insights, and deep understanding of the context) are the best research tool" (2012, p. 18).

3.4.1 Developing the professional learning model

The professional learning model in the current study drew on findings from the literature review chapters. Initially, the focus centred on exploring teacher perceptions of pedagogical actions supporting equity in mathematics education. The subsequent and ongoing re-design of the model drew on researcher observations and teacher reflections from the classrooms. For example, in one classroom, observations and teacher reflections identified a need for professional learning around mitigating status challenges. In response, a framework for raising status was designed to enable the teachers to explore different ways to resolve these issues. Further information was also drawn from the regular study group meetings, teacher discussions, and interviews. For example, in both the study group sessions and teacher discussions and interviews, the teachers requested support in facilitating collaborative and productive student discourse centred on mathematics. Subsequently, in the following study group session, detailed discussion and design centred on drawing on important cultural values familiar to the students as a means of supporting students to work respectfully and productively together. The focus for professional learning within the model can be seen in the framework as shown in Table 1.

Table 1: Framework of Teacher Professional Learning to Develop Equitable Learning Environments in Primary School Mathematics Classrooms

Retrospective analysis phase	Professional learning sequence	Outline of professional learning
Phase two	First session: Term 2, Week 2	Opportunity for teacher participant questions regarding the current study. Role of researcher further clarified-working alongside the teachers to implement the innovation and to reflect and refine the innovation (collaborative nature of the current study further emphasised). Concept maps completed. Instructional activities explored teachers' existing understanding and beliefs about positioning and status. For example, issues of status, assigning competence and ethic of care in mathematics classrooms. Research articles and extracts from articles shared with the teachers to extend their understanding of positioning, issues of status, and social constructs which support students to access mathematical opportunities to learn. Social Domain Framework drafted (see Table 2) The intended learning trajectory of the current study explored-explicit cycles of experimentation and reflected analyses planned. Reflective questions used to focus discussion on research articles. <u>Research articles</u> Planas, N. & Civil, M. (2010). Discourse processes in critical mathematics education. In H. Alro, O. Ravn, & P. Valero (eds.), *Critical mathematics education: Past, present and future.* (pp. 145-159). Sense Publishers. Reading focus: Constructing social structures that enable students to develop strategies that help maintain certain positions and reduce others. Bartell, T. G. (2013). Learning to teach mathematics for social justice: Negotiating social justice and mathematical goals. *Journal for Research in Mathematics Education, 44*(1), 129-163. Reading focus: To challenge and counteract societal stereotypes and inequities to which students and communities are subjected.
Phase three	Second session: Term 2, Week 6	Critical reflection on experimentation phase-all teachers and the researcher: Explicit links to social interaction episodes made, including reflective data from semi-structured

		interviews with teachers. Teacher reflections on recorded classroom observations. Reflection and re-design of Social Domain Framework. Design of Positioning Framework (see Table 3) to support in-the-moment noticing and responding for positioning students for engagement in discourse. Identifying episodes where teachers constructed social structures that supported students' access to mathematics learning. Strength-based grouping activity completed (see Chapter 4.5.2, Chapter 5.6.4, Appendix E). The learning trajectory (the next cycle of experimentation) collaboratively modified and re-designed in response to critical reflection in this meeting. The following research article was used to support teacher learning and design: <u>Research article</u> Franke, M. L., Turrou, A.C., Webb, N. M., Ing, M., Wong, J., Shin, N., & Fernandez, C. (2015). Student engagement with other's mathematical ideas: The role of teacher invitation and support moves. *The Elementary School Journal, 116*(1), 126-148. Reading focus: Teacher invitation moves to position students for learning.
Phase four	Third session: Term 3, Week 1	Critical reflection on experimentation phase-all teachers and the researcher: Supportive framework-Raising Status (see Table 4) collaboratively designed to address status issues that had arisen since the last professional learning session. Framework design based on supporting teachers to notice and identify instances of where competence could be assigned; where status could be changed; and where status explicitly effected opportunities to learn. The following research article was used to support teacher learning and design: <u>Research article</u> Featherstone, H., Crespo, S., Jilk, L. M., Oslund, J. A., Parks, A. N., & Wood, M. B. (2011). *Smarter together! Collaboration and equity in the elementary math classroom*. National Council of Teachers of Mathematics. Reading focus: P. 87-92. Assigning competence to support students' repositioning of one another.
Phase five	Fourth session: Term 3, Week 6	Critical reflection on experimentation phase-all teachers and the researcher:

		Mathematical practices Framework (see Table 5) designed. Resources drawn on for design: Alton-Lee, A., Hunter, R., Sinnema, C., & Pulegatoa-Diggins, C. (2012). *Best evidence synthesis exemplar 1: Developing communities of mathematical inquiry.* Ministry of Education. Wood, T., & McNeal, B. (2003). *Complexity in teaching and children's mathematical thinking*. Paper presented at the Proceedings of the 27th annual meeting of the International Group for the Psychology of Mathematics Education, Honolulu, Hawaii.
Phase six (Final phase)	Fifth session: Term 4, Week 3	Reflection on each phase of the research-all teachers and the researcher. Identifying what had worked and why. Completing final teacher questionnaires (see Appendix C).

The study group sessions formed part of the six retrospective analyses and re-design phases of the design research study. Several frameworks (see Section 3.5) were designed within these sessions which along with the study group sessions were tools which mediated the continued press toward the construction of equitable learning environments in primary school mathematics classrooms.

3.4.2. The Frameworks

Several frameworks influenced by the teachers' and researcher's critical reflection of classroom lessons were designed throughout the duration of the research. Between each study group session, the teachers' taught most of their mathematics lessons without the researcher's participation. Reflective discussion in the study group sessions identified the need for teacher guidance and support at times when the researcher was not in the field. Consequently, the frameworks were designed as supportive tools for instruction in the classroom. In each instance, research articles were used to support the design.

A Social Domain Framework was drafted early in the study to provide guidance for teachers in establishing effective social norms and prosocial interaction and communication patterns. The design was influenced and adapted from several research articles[2]. This framework was repeatedly modified before the draft (see Table 2) was finalised.

Table 2: The Social Domain Framework

Social Domain	Definition
Social Support	
Creating the space-developing the norms (with support from the Positioning Framework)	Teacher establishes high expectations for learning, pre-empts non-social behaviours, establishes positive social norms, creates a respectful, interactional space for students.
Inclusivity	Teacher creates expectations for and invites all students to participate in collective mathematics activity.
Attending to language and culture	Teacher refers to cultural or everyday situations, including embedding mathematics problems in culturally appropriate ways.
Engagement Support	
Verbal communication practices	Teacher utilises and sets communication practices that focus on active engagement of/with mathematics. These include inclusive language (we, us), mathematical language, extending and synthesising students discussions; sustained and engaged interactions; teacher noticing-and-responding to students.

[2] Alton-Lee, A., Hunter, R., Sinnema, C., & Pulegatoa-Diggins, C. (2012). *Best evidence synthesis exemplar 1: Developing communities of mathematical inquiry.* Ministry of Education.

Bartell, T. G., Wager, A., Edwards, A., Battey, D., Foote, M., & Spencer, J. (2017). Toward a framework for research linking equitable teaching with the standards for mathematical practice. *Journal for Research in Mathematics Education, 48*(1), 7-21.

Chapin, S. H., & O'Connor, C. (2007). Academically productive talk: Supporting students' learning in mathematics. In W. G. Martin & M. E. Strutchens (Eds.), *The learning of mathematics* (pp. 113-128). National Council of Teachers of Mathematics.

Franke, M. L., Turrou, A.C., Webb, N. M., Ing, M., Wong, J., Shin, N., & Fernandez, C. (2015). Student engagement with other's mathematical ideas: The role of teacher invitation and support moves. *The Elementary School Journal, 116*(1), 126-148.

Turner, E., Dominguez, H., Maldonado, L., & Empson, S. (2013). English learners' participation in mathematical discussion: Shifting positionings and dynamic identities. *Journal for Research in Mathematics Education, 44*(1), 199-234.

Non-verbal communication practices	Teacher fosters communication practices that may focus less on talk and more silent, yet intent participation-these include, body language, gestures, facial expressions, representation of mathematical reasoning, observation.
Talk moves	Teacher employs the use of talk moves: Repeating, revoicing, reasoning, adding on, and waiting.
Organisational Support	
Grouping	Teacher utilises a strength-based approach to grouping. All students are grouped in multiple-strengths-based groups.
Small-group interactions	All students collaborate with peers in making sense of mathematics through explaining, listening, and questioning (refined further in mathematical practices framework).

The Positioning Framework (see Table 3) designed in phase three of the research was adapted from and incorporated aspects from one research article[3]. This framework supported teachers to notice-and-respond in-the-moment to position and reposition students for engagement in mathematical discourse. Positioning actions include physical, intellectual, emotional positioning; status-related; speech-acts-verbal, non-verbal (gestures, gaze, facial expression) positioning acts. Furthermore, this framework supported the establishment of the social domain.

Table 3: Positioning Framework

Positioning action	Definition
Invitation for a student to share their thinking	Teacher notices a student's idea and invites that student to share their thinking.
Invitation for a group to share thinking	Teacher notices a group's idea and invites that group to share their thinking.
Invitation to consider an idea	• Teacher invites another student to consider a student's idea • Teacher invites another student to consider a group's idea • Teacher invites a group to consider a student's idea

[3] Franke, M. L., Turrou, A.C., Webb, N. M., Ing, M., Wong, J., Shin, N., & Fernandez, C. (2015). Student engagement with other's mathematical ideas: The role of teacher invitation and support moves. *The Elementary School Journal, 116*(1), 126-148.

	• Teacher invites other groups to consider a group's idea
Teacher statement	Teacher positions a student through an explicit statement. These include statements about: • A student's intellectual response to mathematics • the validity of individual students' ideas or strategies • the capability of a student to help another student or group
Student statement	Student positions another peer via a statement. These include statements about: • A peer positioning another student's idea as valid or invalid • A peer positioning another student as having mathematical foundation for actions
Self-positioning	A student makes a statement to position self. These include statements such as: • needing help • being sure or unsure • belief in own capability to learn/do mathematics • an evaluation of another's idea
Revoicing	Teacher or peer revoices an idea or statement and in doing so positions the idea or statement favourably/unfavourably.
Verbal emphases, inflection	Teacher or student uses verbal emphases to position others or self.
Non-verbal emphases	Teacher or student uses gestures, gazes, or physical movements to position others or self.

In phase four of the research, the Raising Status Framework (see Table 4) was designed in response to status challenges mitigating the establishment of respectful and productive communication and interaction patterns in the classroom. The specific purpose of this framework was to raise status by assigning competence in a range of different ways. Several research articles[4] supported the design of this framework.

Table 4: Raising-Status Framework

Domain	Definition
Acknowledging students' intellectual contributions	Teacher responds to an individual student's or a group's ideas and work. Teacher's response values a student's or groups' intellectual contribution.

[4] Bartell, T. G., Wager, A., Edwards, A., Battey, D., Foote, M., & Spencer, J. (2017). Toward a framework for research linking equitable teaching with the standards for mathematical practice. *Journal for Research in Mathematics Education, 48*(1), 7-21.

Featherstone, H., Crespo, S., Jilk, L. M., Oslund, J. A., Parks, A. N., & Wood, M. B. (2011). *Smarter together! Collaboration and equity in the elementary math classroom*. National Council of Teachers of Mathematics.

Acknowledging students as mathematical resources	Teacher purposefully designs tasks requiring the mathematical strengths of students who have low status with their peers.
Assigning competence to raise status	Teacher deliberately responds to low-status student's intellectual contributions with the intention of publicly raising that student's intellectual status in front of peers.
Assigning competence through questions	Teacher asks questions that provide opportunities for students to justify statements.
Assigning competence through perseverance	Teacher presses all students for understanding.
Assigning competence through accountability	Teacher holds all students accountable for how they work together; this includes for individual responsibility and group accountability.

In the fifth phase of the research, the Mathematical Practices Framework (see Table 5) was designed to support the teachers to develop students' understanding of the importance of, and how to use important mathematical practices to learn mathematics. This framework was adapted from and incorporated aspects from several sources[5].

Table 5: Mathematical Practices Framework

Mathematical Practices Framework	Definition What the students will be doing or saying
Make sense of problems and persevere in solving them	• Work collaboratively to understand what a problem is about (context) • Work collaboratively to understand what a problem is expecting them to find out • Work collaboratively to agree on the entry point for possible solution strategies • Work collaboratively to find possible solution strategies for solving the problem-this includes making conjectures,

[5] Bartell, T. G., Wager, A., Edwards, A., Battey, D., Foote, M., & Spencer, J. (2017). Toward a framework for research linking equitable teaching with the standards for mathematical practice. *Journal for Research in Mathematics Education, 48*(1), 7-21.

Hunter, R. (2007b). *Teachers developing communities of mathematical inquiry* [Unpublished doctoral dissertation]. Massey University.

NGA (National Governors Association Center for Best Practices) & CCSSO (Council of Chief State School Officers). (2010). *Common Core State Standards for Mathematics.* National Governors Association Center for Best Practices & Council of Chief State School Officers.

Wood, T., & McNeal, B. (2003). *Complexity in teaching and children's mathematical thinking*. Paper presented at the Proceedings of the 27th annual meeting of the International Group for the Psychology of Mathematics Education, Honolulu, Hawaii.

	developing hypotheses, or working backwards from a guess/hunch/possible solution • Work collaboratively trialling or checking different solution pathways • Monitor, evaluate, and change strategy if necessary • Continually ask themselves and each other, “Does this make sense?” • Explain clearly what they are doing or thinking
Co-construct mathematical explanations	• Communicate clearly and precisely • Co-construct mathematical explanations for group solution strategy • Use mathematical language to explain mathematical reasoning, ideas • Build on each-others’ ideas • Mathematical explanations may include using concrete referents such as objects, drawings, diagrams, actions
Check for precision or reasonableness	• Make sense of quantities and their relationships in problem situations • Consistently monitor reasonableness of solution pathways • Work backwards from a solution
Construct viable arguments and critique the reasoning of others	• Understand and use stated claims, definitions, and solution strategies in building arguments • Make conjectures and build a logical progression of statements to prove these • Can break conjectures, definitions, or solution strategies down into parts and generalise across a range of instances (contexts and numbers) • Justify their conclusions, communicate them to others, and respond constructively to others’ arguments • Can construct arguments using concrete referents such as objects, drawings, diagrams, and actions • Can listen to and compare the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments
Model with mathematics	• Draw on and use the mathematics they know to engage in collective mathematics activity • Can identify important patterns and relationships in contextualised problems and map these using a range of tools such as materials, pictures, diagrams, tables, graphs, formulas • Can analyse those relationships mathematically to draw conclusions • Consistently think about and reasons with the mathematics in the context of a situation and reflect on what make sense
Look for and make use of patterns and relationships	• Noticing patterns or relationships and drawing on these to seek possible solutions or to make further mathematical connections

Design-based research utilises techniques used in other research paradigms, for example, gathering thick descriptions, rigorous analysis, and interpretation of data. An iterative process of continuous cycles to allow the design to be adapted and modified as required addresses issues of validity. Triangulating multiple sources of data and the repetition of analyses addresses reliability of findings (DBRC, 2003). Maintaining objectivity is paramount, as design-based researchers often hold dual intellectual roles of advocate and critic; within design-based research this is viewed as a “necessary tension arising from the coupling of empirical research to design” (DBRC, 2003, p. 7). Maintainable innovation requires understanding how and why an innovation is effective within a setting over time and across settings. Therefore, utilising comparative case study method in which different teachers’ actions and instructional goals are examined is justified.

3.5 CASE STUDY

Qualitative case studies are frequently used in educational research (Merriam, 1998). This methodology emphasises collecting rich, detailed data through fieldwork in bounded settings to illustrate findings and support theoretical conclusions (Berg, 2009; Check & Schutt, 2012; Yin, 2011). The focus in case studies may be on a single case or involve multiple cases or sites (Lodico et al., 2010; Merriam, 1998). Specifically, this study utilised a comparative case study whereby multiple cases were examined to develop understanding of both common and unique aspects between each case (Berg, 2009; Lodico et al., 2010).

Case study aims to capture specific examples of real people in authentic settings through multiple sources of data collection (Bloor & Woods, 2006; Bogdan and Knopp Biklen, 2007; Johnson & Christensen, 2000). The use of a comparative case study was appropriate in this study, given it includes a detailed investigation within a bounded setting (two distinct classrooms) where the focus was on deepening and increasing the understanding of how teachers construct equitable mathematics learning environments. A detailed description of the rationale for selecting the two case study teachers is outlined in Section 3.6.

3.6 RESEARCH SETTING AND PARTICIPANTS

This section describes the research setting and the participants of this investigation. The setting selected for this study was one urban primary school in New Zealand. This school provides education for students from Year Zero to Year Six (aged 5-10 years old). Students at this school mainly come from low-to-middle socio-economic backgrounds. The school is culturally diverse and represents over 20 cultures (including-Māori, Pacific nations, Pākeha, African, Indian, Middle Eastern, Asian, Southeast Asian) with many students speaking English as an additional language. Sixty-seven percent of the students are of Pāsifika heritage and 13% are Māori. Classes are not streamed, and the students have the same teacher throughout the day.

Four teachers from the school were selected to participate in this study, this included one Year Four to Six (students aged 8-10 years old) teacher, one Year Four (students aged 8 years old) teacher, and two Year Five to Six teachers (students aged 9-10 years old). The Year Four to Six class was a Samoan bilingual class, where instruction was given dually in Samoan and English. Participants agreed to be part of the study after an outline of the intended study was presented to them in person and in writing. These teachers expressed personal interest in being part of an investigation aimed at implementing and sustaining equitable pedagogical actions in the classroom. They were also interested in participating in innovative professional learning and trialling theoretical ideas which position students as agentive problem-solvers and mathematically capable. Furthermore, each teacher expressed interest in the opportunity to critically reflect on their own pedagogical approaches, their students' experiences, and to modify these if required.

In Chapter Four and Five two teachers are featured as cases. These two teachers were selected due to the distinctly different pathways they took in adopting and adapting key pedagogical strategies and classroom practices in their everyday mathematics lessons. Whilst both teachers had engaged in similar activity during the professional learning sessions, one teacher's path followed the linear timeline of the study, while the other teacher encountered challenges which resulted in a more circuitous route. To preserve the anonymity of the two case teachers and their students, it is adequate to say that both teachers were regarded as experienced classroom teachers (each with more than six years of teaching experience). The students in both classrooms represented the rich diversity of the school. Meliame[6] (case reported in Chapter Four) taught 32 Year Five to Six students working at Level Three of the New Zealand Curriculum. Alisi[7] (case reported in Chapter Five) taught 24 Year Four students working at Level Two of the New Zealand Curriculum.

3.7 ETHICAL CONSIDERATIONS

Conducting social research necessitates careful consideration of ethics. In designing and undertaking this study, the researcher adhered to Massey University's Human Ethics Committee's *Code of Ethical Conduct for Research, Teaching, and Evaluation involving Human Participants* which provided the ethical guidance for educational researchers. These guidelines ensured that the tenets of informed consent, protection from harm, confidentiality, privacy, sensitivity, and honesty were always maintained.
The following sections describe how these tenets were considered for this investigation.

3.7.1 Informed consent

Informed consent is an essential element of ethical research and involves all participants fully understanding and agreeing to participating without any coercion or pressure prior to the commencement of the study (Berg, 2009). The current investigation was an overt study, and its purposes were made clear to all the participants (and the students' guardians) and informed and written consent was sought and obtained before the study was undertaken. An important ethical consideration involved all participants being aware of the collaborative nature of this study, the significant time commitment involved in participating in professional learning meetings, and reflective discussions and other activities all participants would engage in.

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[6] A pseudonym
[7] A pseudonym

Permission to undertake the research in their school was sought and granted by the school's Board of Trustees (see Appendix G). Prior to the commencement of this study, detailed discussions with each of the teacher participants included explicitly outlining the aims of the project, participation expectations, and opportunities for teachers to seek clarity on any aspect of the project. During these discussions, the researcher also highlighted several factors considered to possibly cause harm to the participants. These involved aspects such as discomfort or awkwardness from lessons being observed and recorded by the researcher, and from both individual and collective critical reflection and review of teacher actions during the retrospective analyses' phases of the study. Consent was regularly revisited and discussed throughout the study. The researcher also highlighted the benefits of participating in the research which provided consideration for both the risks and potential results.

After the initial discussion, information sheets about the research (see Appendix H) were presented to the teacher participants. The information sheets emphasised that participants held the ongoing right to request the recording devices be turned off, any footage be excluded or destroyed, or to withdraw themselves from participation at any point in the study. Written consent was obtained from the teachers once the information sheets were read and understood.

Informed consent was also sought from the student participants and their guardians. This was an important consideration as this provided the children (aged 8-10 years old) with some independence, control, and confidentiality. Discussions were held with the students in each of the teacher participants classrooms prior to the commencement of the study. During these discussions, information sheets were provided for each student which outlined the aim of the project. These aims were discussed, and opportunities were given for students to ask questions and seek clarity. Information sheets and consent forms (see Appendix I) for both the students and for their guardians were taken home. All the students in the two case study classrooms consented to participating in this research. All students who had provided written consent were asked for verbal consent prior to any observation, recordings, or interviews throughout the research duration. The students were also advised that they could refuse to participate or answer any questions at any part of the study. Verbal permission was consistently sought to use classroom artefacts. Students and guardians held the right to withdraw from participation in this investigation at any point.

3.7.2 Protection from harm

Potential harm to all participants was reduced by fully informing all participants in the study the intentions and expected duration of the study. The impact of the researcher's presence in each classroom on the daily routines of the school setting was discussed. Harm to the students was reduced by the research taking place in the students' classrooms during regular mathematics lessons. The researcher spent time in each classroom prior to the commencement of the study which provided opportunities for the students to become familiar with them, the nature of the researcher/teacher interaction, and the recording equipment. Potential harm to all participants was reduced by open and honest communication throughout the study. Researcher objectivity was a key factor as the study progressed. While relational trust was developed and sustained throughout the investigation, the researcher maintained an objective distance from the participants. The researcher allowed the teachers to set appropriate times for reflective discussions and the professional learning sessions. This action allowed the researcher an objective lens through which to view and analyse the data. Above all, respect for all participants as individuals and not research objects ensured that professional relationships were not compromised, and trust was sustained over the duration of the study.

3.7.3 Anonymity and confidentiality

In educational research, high importance is placed on ensuring participant anonymity and confidentiality (Berg, 2009; Bloor & Woods, 2006; Cohen et al, 2018). Anonymity amongst the school participants could not be guaranteed as the participants were known to each other. To ensure anonymity within the wider distribution of the project, pseudonyms were assigned to the teachers and students. Further anonymity has been provided with only pertinent information being given about the school, the case teachers, and their students. No data collected throughout the duration of the research was shared with others outside of the study in any form. This action supported maintaining the privacy and confidentiality of individuals. Further anonymity and confidentiality were sustained by the sole participation of the teachers and researcher in collective review of mathematics lessons, recorded footage, classroom artefacts, and group interviews.

3.8 DATA COLLECTION

This section outlines the data collection timeline and the data collection methods utilised in this study. Data were collected through video-and-audio recorded observations, discussions, individual and group interviews, written teacher and student questionnaires, fieldnotes, classroom and professional learning session artefacts.

The research schedule for the duration of the study from the initial observations to the point of withdrawal from the school is outlined in the Table below (see Table 6). The programme of observations and study group meetings are detailed. The timeline is structured into six distinct phases.

Table 6: Data Collection Timeline

2017 Term 1	
April	Initial meeting with school senior leadership team to discuss research project. Invitation extended to teachers to join the research project. Information sheets and consent forms given to Board of Trustees, teachers, students, and guardians (see Appendices G, H, I). All consent forms signed and obtained.
PHASE ONE	
Week 9	Initial classroom observations (Lesson one). Data collected via video and audio recordings; fieldnotes; classroom artefacts; audio recorded teacher/student conversations/semi-structured interviews. Purpose: Gaining insight into teacher participants initial classroom practice in mathematics teaching and learning environment. Becoming a familiar presence in the classroom. Initial teacher and student interviews completed (see Appendices A, B).
2017 Term 2	
Week 1	Initial classroom observations (Lesson two). Data collected via video and audio recordings; fieldnotes; classroom artefacts; audio recorded teacher/student conversations/semi-structured interviews.

	Purpose: Gaining insight into teacher participants initial classroom practice in mathematics teaching and learning environment. Becoming a familiar presence in the classroom. Initial teacher and student interviews completed (see Appendices A, B).
PHASE TWO	
Week 2	First professional learning (study group) session-whole day session. (see Table 1 for outline).
Week 4	Experimentation Phase: Data collection in classrooms (Lesson three). Data collected via observations, video and audio recordings; fieldnotes; classroom artefacts; audio recorded teacher/student conversations/semi-structured interviews.
Week 5	Experimentation Phase: Data collection in classrooms (Lesson four). Data collected via observations, video and audio recordings; fieldnotes; classroom artefacts; audio recorded teacher/student conversations/semi-structured interviews.
PHASE THREE	
Week 6	Retrospective analysis and re-design phase: Second professional learning (study group) session (See Table 1 for outline).
Week 8	Experimentation Phase: Data collection in classrooms (Lesson five). Data collected via observations, video and audio recordings; fieldnotes; classroom artefacts; audio recorded teacher/student conversations/semi-structured interviews.
2017 Term 3	
PHASE FOUR	
Week 1	Retrospective analysis and re-design phase: Third professional learning (study group) session (See Table 1 for outline).
Week 2	Experimentation Phase: Data collection in classrooms (Lesson six). Data collected via observations, video and audio recordings; fieldnotes; classroom artefacts; audio recorded teacher/student conversations/semi-structured interviews.
Week 4	Experimentation Phase: Data collection in classrooms (Lesson seven). Data collected via observations, video and audio recordings; fieldnotes; classroom artefacts; audio recorded teacher/student conversations/semi-structured interviews.
PHASE FIVE	
Week 6	Retrospective analysis and re-design phase: Fourth professional learning (study group) session (See Table 1 for outline).
Week 8	Experimentation Phase: Data collection in classrooms (Lesson eight). Data collected via observations, video and audio recordings; fieldnotes; classroom artefacts; audio recorded teacher/student conversations/semi-structured interviews.
2017 Term 4	
Week 1	Experimentation Phase: Data collection in classrooms (Lesson nine).

	Data collected via observations, video and audio recordings; fieldnotes; classroom artefacts; audio recorded teacher/student conversations/semi-structured interviews. Final student interviews-written questionnaire.
Week 2	Experimentation Phase: Data collection in classrooms (Lesson ten). Data collected via video and audio recorded lessons; fieldnotes; classroom artefacts; audio recorded teacher/student conversations/semi-structured interviews. Final student questionnaires (see Appendix D) completed.
PHASE SIX (FINAL PHASE)	
	Retrospective Analysis Phase: Final professional learning (study group) session. (see Table 1 for outline). Finalising all frameworks. Final teacher questionnaires (see Appendix C) completed.

Table 6 has highlighted the six phases of the research. Within each phase, the method of data collection was presented. Each phase outlined the professional learning (study group) sessions, including retrospective data analysis and re-design subsequent experimentation phases. The aim of the initial professional learning session was to prepare the teachers for the cyclical research design. As research design consists of iterative cycles of preparation, experimentation, and retrospective analyses, it was important to capture data as the teachers and the researcher co-constructed the conjectured learning trajectory and engaged in reflective analyses of classroom practice. Subsequent professional learning sessions cycled through retrospective analyses phases in which modifications to the learning trajectory were made. One set of data were collected during professional learning sessions (study group) with the teachers. Further data were collected during the participants' regular classroom mathematics lessons (implementation phase) as the students engaged in collective mathematical activity.

The research focused on the collective mathematical activity for several reasons. Firstly, it was anticipated that students would engage in discourse as they worked collectively in mathematical activity. Discourse affords opportunities for students to explain and justify solution strategies, pose questions, and articulate connections between mathematical ideas. Importantly, engaging in discourse provides opportunities for students to publicly take on agentive problem-solving roles and to participate in ways that can impact their mathematical understanding and, over time, their sense of themselves as knowers and doers of mathematics.

Additional data were drawn from the following sources: observational field-notes; video-and-audio-recordings of all professional learning (study group) sessions and classroom lessons; documents and artefacts; teacher and student questionnaires, teacher and student individual and group interviews. All video-and-audio recorded data were wholly transcribed.

3.8.1 Interviews

Gaining deeper understanding of alternative meaning and developing insight into multiple perspectives can be obtained through interviewing (Berg, 2009; Walford, 2009; Yin, 2011). Teacher and student interviews captured additional data about teacher and student beliefs about learning mathematics. Semi-structured individual and group interviews were conducted to develop the researcher's understanding of the perspectives of the teachers and the students regarding notions of positioning and status. Informal semi-structured interviews took place after each lesson observation during each school visit. In these interviews, the teachers and researcher reflected on and discussed the lesson, including the student participation and notions of status. These interviews were used to promote reflection on practice and to support the re-design phases of the study.

The teachers and students both completed two written questionnaires (see Appendices A, B, C, D). The purpose of the first teacher questionnaire (see Appendix A) was to explore beliefs and approaches to teaching and learning mathematics. This questionnaire was conducted at the start of the study and provided baseline data for each teacher's pedagogical approach and expectations for their students' learning of mathematics prior to the experimentation phase. The final written questionnaire (see Appendix C) was conducted in the final phase of the research design and aimed to capture the teachers' reflections on their participation in the study, including any pedagogical changes, student engagement, challenges, and possible future pedagogical foci. The initial student questionnaire (see Appendix B) took place at the start of the research and focused on capturing students' experiences, beliefs, and understanding about learning mathematics. The final student questionnaire (see Appendix D) was completed in phase five of the investigation and aimed to capture and articulate the students' experiences and perceived changes in instruction. These questionnaires formed part of the interview data and the document data (see Section 3.8.3).

In addition, individual and focus group interviews were conducted throughout the study. These interviews regularly followed each lesson and provided multiple opportunities for the researcher to consider the communication and interaction patterns within the lessons from the point of view of the participants. Open-ended questions were asked during these interviews, for example, "what was happening when…?" Or "What were you thinking? Why?". These questions served as prompts for critical reflection on teacher practice, student engagement, communication, and participation patterns within the lessons.

Recorded discussions captured during the professional learning sessions provided reflective data from the teachers on their participation in the investigation and any perceived changes in their pedagogical approaches or beliefs about mathematics teaching and learning.

3.8.2 Observations

Observations form an integral part of qualitative research in the field (Basil, 2011; Mertler, 2016; Yin, 2011). In the current study, semi structured classroom observations provided the researcher with flexibility to work alongside the teacher participants and attend to the dynamic nature of the classroom system. Systematically capturing the range of data in naturalistic settings was possible using recorded observations (Basil, 2011; Wang & Lien, 2013; Yin, 2011). Video-recording the lessons provided a way of capturing the participants' communication and participation patterns during social episodes, including conversations, discussions, scaffolding, explanations, modelling. Videorecording is advantageous in that this method captures verbal emphases, non-verbal emphases, physical gestures, and facial expressions. This method of observation provided another set of eyes within the research field, as the researcher was not able to observe every aspect within the field (Yin, 2011). Within naturalistic research settings involving one researcher, there are considerable challenges in capturing participant behaviour throughout (Powell et al., 2003; Wang & Lien, 2013). Using more than one camera provided the means to obtain a broad view of interactions. Specifically, during each lesson, one fixed camera was positioned to capture the whole class, while the researcher used a mobile camera to focus on specific groups at particular times. In addition, detailed field notes supplemented the video-record.

All study-group sessions, and interviews following each lesson were video-and-audio recorded. Records of group discussions during the professional learning sessions allowed for reflective and retrospective data analyses involving the researcher and teacher participants. Audio-recordings provided additional evidence to support video-captured data. Utilising both video-and-audio recorded data allowed the researcher to undertake detailed analyses of the broader and smaller visual and verbal social occurrences (Wang & Lien, 2013). Ongoing review of all recorded footage, and triangulation with detailed fieldnotes occurred throughout the study. This was an advantageous way to mitigate the limitations of reliability of recorded observations (Wang & Lien, 2013).

3.8.3 Documents

Within this research, collecting documents from the field strengthened the development of a rich account of events. In design research, field artefacts support tracking changes and provide sources of evidence about how learning was initiated and supported (Cohen et al., 2018; Yin 2009). In this study, teacher and student written questionnaires formed part of both the interview and document data. In addition, artefacts from the professional learning sessions included the teacher participants responses to activities, such as concept maps, annotated lesson plans, grouping records, and teacher reflections. Documents were also collected from the classrooms, and these included students' work samples, teachers' lesson plans, and photographs of student groups mathematical representations both on paper and on the board. Physical artefacts complemented the others forms of data collected throughout the study and strengthened the validity of the data analysis process.

3.9 DATA ANALYSIS

A significant amount of data was generated through capturing the teachers and students' dialogue and actions during the professional learning sessions, classroom lessons, and individual teacher and student interviews. All professional learning sessions, classroom lessons, and interviews were recorded. These recording were wholly transcribed. Completed transcriptions were uploaded through QSR International's NVivo 12 qualitative software programme (2018) and used to identify and analyse specific themes and codes within episodes of interest. The codes are discussed further in this section (see Section 3.10.2).

Data analysis occurred both in the field and retrospectively.

3.9.1 Data analysis in the field

Data analysis in the field took place in several different ways. Firstly, after each lesson, the teacher participants individually viewed the recorded footage of their mathematics lessons. This allowed each teacher to critically reflect on their instructional actions and students' responses without the influence of the researcher or other participants. These actions provided early analysis of captured data from the participants' perspectives. In each study group session, the teacher participants were invited to discuss these views collaboratively and with the researcher. The researcher also viewed the recorded study-group sessions after each meeting and wrote reflective notes. In addition, field notes were used to inform and test the speculations developed about teacher learning and to build further conjectures which informed the design of subsequent professional learning sessions. The collaborative discussions along with theoretical propositions (Yin, 2011) informed the design of each experimentation phase.

For the research design to be informed by evidence from the field, data collection and data analysis occurred concurrently. The reflections and field notes were also used to inform and shape ensuing frameworks (see Section 3.4.2). The focus of data analysis was on identifying episodes which highlighted pedagogical actions outlined in theory as supporting equitable positioning of all students for learning mathematics.

3.9.2 Retrospective data analysis

Retrospective data analysis began by returning to the research questions and identifying evidence addressing these questions. Tentative conclusions and hypotheses were drawn and then tested against other forms of evidence. Further examination took place by analysing the data captured in the professional learning sessions, including the participants' perspectives. Recorded footage was reviewed multiple times along with transcripts and field notes which either corroborated or contradicted earlier conjectures and codes, and other codes and conjectures were developed as needed.

Initial examination of the data yielded the following four codes (parent nodes): social structures, positioning, status, and delegating mathematical authority. Sub-nodes (child nodes) developed through repeated review of data. Analyses of classroom observations involved identifying social episodes, as defined by Forman and Ansell as being "units larger than individual's turns at talk" (2001, p. 138) that addressed the research questions. Coding of social episodes allowed the researcher to respond to the types of interactions in relation to these questions. Emphases were also coded and formed part of the child-nodes. Emphases included both verbal and non-verbal communication and physical gestures and facial expressions.

The first iteration of coding attended to the first parent code-social structures. Social episodes from classroom observations pertaining to this code, were analysed using the child-nodes highlighted in the following Table (see Table 7).

Table 7: Child Nodes for Social Structures

Setting emotional tone
Teacher expectations-utterances
Teacher scaffolding or modelling expectations
Attending to language & culture

Teacher communication through posture or gaze
Student communication through posture or gaze
Student collaboration-small group (2-4 students)
Student collaboration-bigger group (more than 4 students)
Talk moves

Initially social episodes were analysed and coded pertaining to teacher actions to construct the social environment and prosocial norms. Further review of the identified social episodes then attended to the students' actions and reactions.

The next iteration of coding focused on positioning as a parent code (see Table 8). Social episodes were identified that addressed the ways that the teacher positioned individual students or groups of students through verbal invitations. Within these episodes, further analyses of student positioning of self, and student positioning of peers took place.

Table 8: Child Nodes for Positioning

Teacher invitation for individual student to share idea
Teacher invitation for group to share idea
Invitation for students to consider a peer's idea
Student-self positioning through verbal emphases
Student-self positioning through non-verbal emphases

Status as a parent code (see Table 9) generated several child nodes focused on how the teachers framed students' capabilities to learn mathematics through utterances and/or non-verbal emphases. The child nodes further included assigning competence and acknowledgement of students' intellectual contributions. Teacher questions that focused on providing opportunities for students to make or justify statements were included.

Table 9: Child Nodes for Status

Framing student capability-utterances
Framing student capability-non-verbal emphases
Acknowledging students' intellectual contributions
Assigning competence
Teacher questions

Delegating mathematical authority was the final parent code. This code included nine child codes (see Table 10).

Table 10: Child Nodes for Delegating Mathematical Authority

Explanation
Questioning
Justification
Listening
Holding students accountable
Acknowledging students as mathematical resources
Pressing for understanding
Generalisations
Mathematical argumentation

Data analyses were triangulated through examination of recorded data, transcripts of recordings, transcripts of teacher and student interviews, classroom artefacts, and the observation and reflective field notes. The social episodes within the codes were comprehensively examined. The identified themes were used to develop an understanding of how teachers construct equitable learning environments in primary school classrooms.

3.10 DATA PRESENTATION

The findings from the data analysis were illustrated through two cases in subsequent chapters (see Chapter Four and Chapter Five) of this thesis. These cases focused on the teachers' engagement in professional learning, their mathematics teaching in their classrooms, and their students' participation in the lessons. The cases provided a detailed and thick, rich description of the distinct journeys the teachers took throughout the study (Bogdan & Knopp Biklen, 2007; Cohen et al., 2018; Willis, 2008). Teacher voice from professional learning sessions, classroom episodes, and interview data is used in the findings to develop an understanding of how teachers reconstructed their classroom practices to position all students equitable for learning. Vignettes and examples from the classroom are used to illustrate the actions of the students and the teachers. Findings are reported in several distinct themes beginning with an overview of classroom observations preceding the start of the professional development. The remaining sections report on the findings at the beginning, middle, and end of the investigation.

3.11 VALIDITY AND TRUSTWORTHINESS

The trustworthiness of the current research study can be validated through consideration of the reliability, credibility, transferability, and dependability of the qualitative research methods employed (Merriam, 2009; Mertler, 2016; Yin, 2011). Validity refers to the extent to which research findings may be generalisable to other situations and settings (Bogdan & Knopp Biklen, 2003; Merriam, 1998; Yin, 2009). Design research aims to "use the close study of learning as it unfolds within a naturalistic context that contains theoretically inspired innovations, usually that have passed through multiple interactions, to then develop new theories, artefacts, and practices that can be generalised to other schools and classrooms" (Barab, 2014, p. 151). In this manner, any innovative instruction developing from theory through the design research can be replicated in other contexts (Reimann, 2016), in this case to strive for equitable outcomes in mathematics teaching and learning. Within design-based research studies, the context provides the means through which conjectured outcomes can be achieved (Reimann, 2016). Utilising case study within design-based research further strengthens generalisability of findings, as generalising occurs by connecting the specific case to relevant theories (Reimann, 2016; Yin, 2009). Generalisation does not necessarily occur identically across different participants, instead the observation itself is considered more general. In other words, the hypothesised learning trajectories are viewed to be generalisable across different groups of participants (Gravemeijer & Cobb, 2006).

To increase construct validity, this research drew on multiple sources of evidence (see Section 3.8) which were explicitly linked to the research questions, thus forming the chain of evidence (Yin, 2009). The rigour of the data collection maintained in the current study has ensured the dependability of the findings and conclusions (Merriam, 1998). QSR International's NVivo 12 qualitative software programme (2018) was used to code the large amount of data which was accumulated through video-and-audio records, interviews, and field notes. Research validity was further enhanced through multiple sources of data collection and the triangulation of the data collected. Transcriptions of recorded footage, interviews, and other artefacts were collected, analysed, and compared repeatedly throughout the course of the study. Further validity was achieved by key informants, such as the teacher participants and the researcher's main supervisor reviewing the draft reports, this also supported removing elements of bias which can arise from utilising one research method (Cohen et al., 2018; Yin, 2009).

Qualitative studies aim to develop rich, deep understandings of a specific setting. Therefore, transferability is provided through the detailed descriptions of settings, participants, and themes (Mertler, 2016). Credibility of the research findings is further strengthened by the iterative cycle of design-based research methods. Consistent review and analyses of recorded observations, and ongoing group discussions throughout the investigation supported testing conjectures emerging from earlier phases of the study in later lessons.

3.12 CHAPTER SUMMARY

This chapter has outlined the methodology underpinning this research study. The main research questions and supporting questions have been presented. The research setting and the participants were introduced. Details of the professional learning model were identified and outlined. Several frameworks designed to support the teachers in the experimentation phases of the current study were presented. The data collection and analyses methods have been explained. Ethical considerations and the validity and reliability of the data have been discussed. In the next chapter (see Chapter Four) the first case is presented.

CHAPTER FOUR

THE CASE OF MELIAME

4.1 INTRODUCTION

The previous chapter described the methodology utilised in the current study. In this chapter and the next (see Chapter Four and Chapter Five), each case study is organised in distinct phases which relate to the phases of the design-based research cycle in which data collection occurred (see Table 6). The pedagogical actions of the two teachers (Meliame and Alisi), and the transformative changes which occurred in their learning environments are presented. This chapter reports on the first teacher, Meliame.

Section 4.2 describes Meliame's beliefs about doing and using mathematics and how these beliefs shaped her classroom learning context. Section 4.3 outlines Meliame's active participation in the professional learning sessions (study group sessions) and examines a concept map Meliame drew to illustrate her ideas of an effective mathematics learning environment. Section 4.4 presents the journey Meliame took in establishing and embedding specific social structures to position all students with equitable opportunities to learn mathematics. Section 4.5 describes the actions Meliame took to group her students as active participants in learning mathematics. In Section 4.6 Meliame's actions in providing her students with opportunities to use mathematical practices are presented.

The positioning of the teacher and students reported here is in the form of excerpts taken from mathematical lessons throughout the cycles of the design-based research in which data collection occurred. The excerpts include the talk, text, and actions of the teacher and students. A table is provided before each analysis and includes when in the current study the data were collected, the number of students, an introduction to, and explanation of the observed event. This is followed by an analysis that highlights the positioning of teachers and students, the evolving storylines and social acts, and a commentary that supports the analysis. The excerpts are coded according to the Table below (see Table 11):

Table 11: Excerpt Script Codes

Code	
Bold script	Teacher's name
Italicised script	Dialogue
(brackets)	Actions of participants

The next section describes Meliame's initial beliefs about mathematics and how these shaped her pedagogical practices and classroom environment.

4.2 TEACHER CASE STUDY: MELIAME

Meliame had taught for eight years. Currently, she taught 32 Year Five to Six students aged 9-10 years old. These students represented multiple ethnicities. They were described as achieving at a range of levels in mathematics-according to normed test results.

In a written questionnaire at the beginning of the study (Term 2, Week 1; see Appendix A), Meliame stated that she enjoyed teaching mathematics and felt confident in her ability to meet the needs of all her students. Meliame expressed that all her students were capable of learning mathematics. She also positioned herself as a learner of mathematics:

> *All students, everyone, me included can learn mathematics. We all may have a different starting point with different ketes[8] of knowledge, but we can all progress on our learning journey, teacher included*.

Although she positioned herself as a learner alongside her students in this statement, further in the same interview, Meliame positioned herself-the teacher-as the central authority in the mathematics classroom. She stated that the person who held the

[8] Kete is a Māori language word for traditional baskets made and used by New Zealand's Māori people. The term is used to describe knowledge a person holds.

mathematical power in the classroom was the teacher, as the teacher knew the mathematics and decided what would be learned.

Meliame also said that it was important for students to learn new mathematics by working with others and to be able to apply this new knowledge when working independently. Meliame believed that while collaboration was an important part of learning mathematics, ultimately, students also needed to be able to "do mathematics independently" to demonstrate their understanding.

While it appeared that Meliame held constructive beliefs about who could learn mathematics and how mathematics should be taught, these beliefs were not evident in her mathematics lessons. Initial classroom observations (lessons one and two) showed that during group mathematics problem-solving, teacher-student-teacher-student patterns of discourse (funnelling) occurred. Funneling or focusing (Wood, 1998) is a type of interaction pattern which directs the students towards one particular way of reasoning with the mathematics, usually, the one the teacher wants the students to use. Although Meliame had structured a learning environment in which the students were encouraged to talk in mathematics lessons, the patterns of discourse were teacher-directed. While she acknowledged the importance of students asking questions to support their learning, she tended to be the one asking most of the questions. Meliame maintained a central position in the classroom as the main source of mathematical authority and as a result, her classroom context was consistent with what Wood (2002) describes as a conventional culture. In addition, Meliame identified the teacher as having the highest status in the mathematics classroom. She identified those students with "higher mathematical ability" as having higher status than those requiring learning support from intervention learning programmes. Those students traditionally marginalised (see Chapter 2.2) in New Zealand classrooms occupied the group she described as requiring learning support from such programmes.

The next section begins to examine how Meliame's initial beliefs and practices about doing and using mathematics shaped her learning environment.

4.2.1 Initial classroom practice

Classroom observations (lessons one and two) prior to the first professional development session provided evidence that Meliame held strong beliefs about which of her students were capable of learning mathematics. Her mathematics lessons followed a regular structure requiring students to work in what Meliame termed mixed ability groups. She divided her students into two evenly mixed groups. She described the mixed groups as consisting of even numbers of high, middle, and low ability students. Each group of 16 students would work with the teacher on alternate days. The lesson would begin with a contextualised problem being read silently, followed by a short discussion facilitated by the teacher, checking for understanding of the problem. Students were then directed to work in smaller groups of four to solve the problem together. She ensured that in each group there was at least one student who could "do" the mathematics in the problems she designed. Then, if any of the other students were stuck, this student could show the others how to solve the mathematics problem (Meliame's grouping practices are discussed in more detail in Section 4.5). After 15 minutes, the small groups would reform as a bigger group. The teacher would then select one or two groups to share their solution strategy with the other students. This discussion would be facilitated by Meliame. The duration of each lesson was 50-60 minutes. The following Table (see Table 12) provides the outline of a typical mathematics lesson in Meliame's class.

Table 12: Outline of Meliame's Mathematics Lessons

Lesson-Phase	Duration	Purpose
Launch	5 minutes	Introducing the problematic task/contextualised problem to the group working with the teacher (half the class, mixed group).
Small-group work	15 minutes	Students work in groups of 4 to solve the problem.
Big-group discussion	20-30 minutes	The smaller groups come back together. Teacher selects and sequences which group shares back to the bigger group. Teacher facilitates mathematical discussion.
Connect	10 minutes	Teacher supports the students to make connections to important mathematics concepts and ideas.

The mathematics group not working with the teacher on that day would engage in purposeful mathematical activity[9], working independently and mostly silently in other parts of the classroom.

The next section outlines how Meliame began (re)structuring her learning environment for mathematics learning. Her participation in the study group sessions, and her deliberate and considered actions in positioning herself and her 32 students for learning are described and discussed.

4.3 TEACHER LEARNING

4.3.1 Meliame's concept map

From the onset, Meliame was an active member of the study group. She spontaneously and confidently shared her ideas. In the first study group meeting (Term 2, Week 2) she drew a concept map (see Figure 1) showing what she believed were important features of a classroom environment which supported mathematics learning. Meliame's concept map illustrated awareness of some aspects of a learning environment that supported all students to engage as active participants in learning mathematics. For example, she believed in developing social norms, such as students listening to each other and sharing ideas. By doing so, students would be supported to work collaboratively in respectful ways. Meliame also identified the importance of implementing and utilising sociomathematical norms: For example, making clear mathematical explanations, asking questions to support mathematical reasoning, developing mathematical justification and generalisation.

[9] These activities consisted of a variety of differentiated mathematics practice. For example, students would work independently on the same problem as the day before-the numbers used could be the same or have been changed to suit the level the teacher thought best suited the student. The students could work on online mathematics programmes, or practice worksheets.

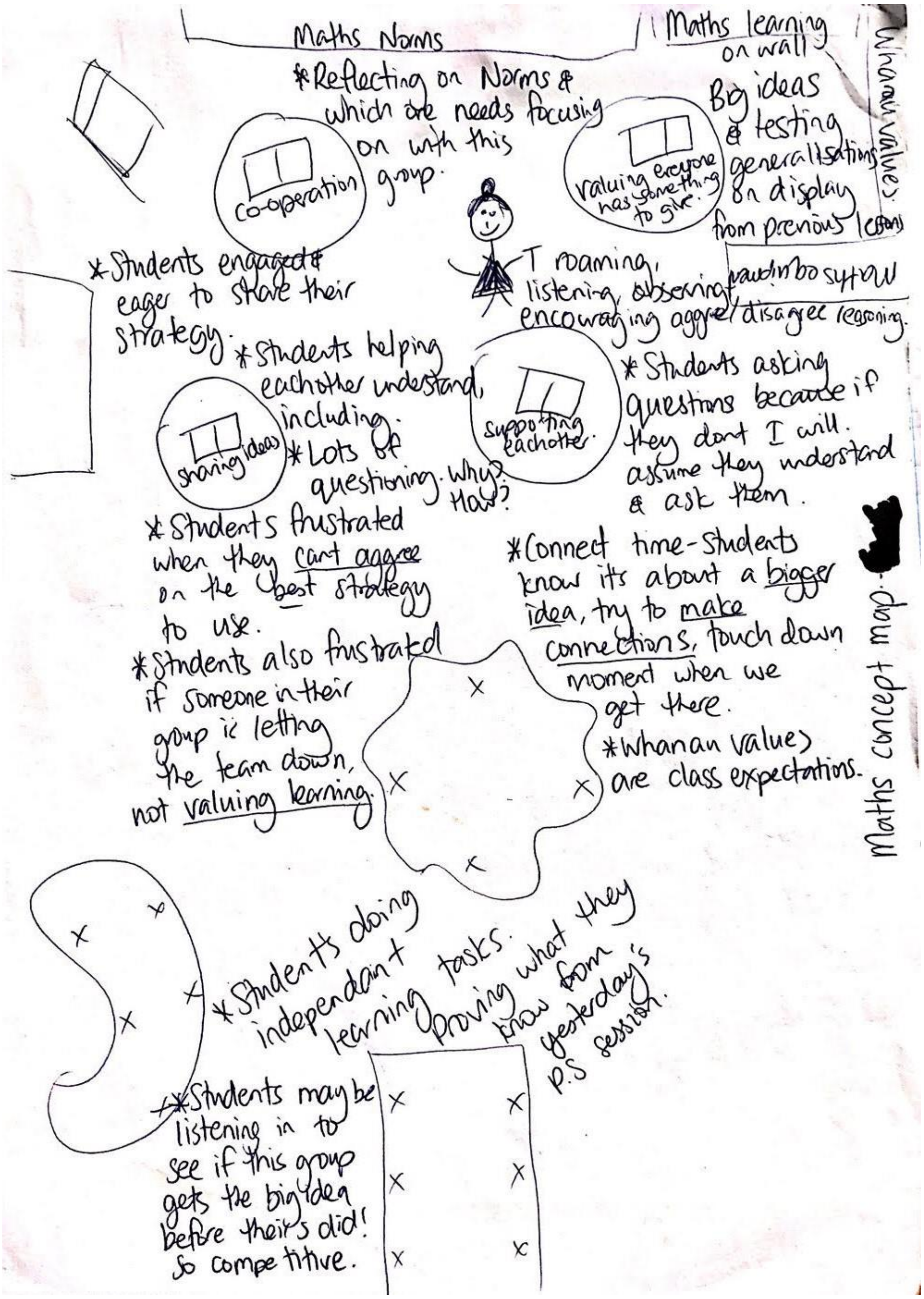


Figure 2: *Meliame's Concept Map*

Meliame reflected on her concept map and provided a written description to illustrate her thoughts and feelings about mathematics learning in her classroom:

> *I drew an action plan, and the messiness really reflects how messy I feel maths can be, and especially in my head, like I feel like all these things are just going off all the time, it's so messy, and I feel like it should be more structured, it should all look*

great, we should all make this light bulb moment at the end, and it just doesn't happen that way. I need to be more comfortable with letting that go, which is really hard.

Meliame believed that her mathematics lessons should be structured in a way that would result in all her students understanding the concepts she intended. However, she acknowledged that this was her construct, and that mathematics learning could be inherently messy-with students thinking and learning many different things; but that this messiness reflected student engagement and learning. She stated that ideally, it was acceptable for the teacher to relinquish control of each aspect of the lesson. Meliame acknowledged that when she provided opportunities for her students to work together, without her constantly directing what they should say or do, she noticed a higher level of student engagement. Meliame continued to reflect on student engagement in mathematics and stated the following:

When the students are engaged in their groups, there is a really high energy buzz, like we need to get there, we need to get there, we need to solve this problem, what's the best strategy?

Meliame had highlighted the importance her and her students placed on finding the answer to the problem in the most efficient way. While this suggests sound collaborative inquiry practices, potentially, the emphasis on finding a solution prevented some students from developing deep mathematical understanding and engagement in robust inquiry into the mathematics. While, at times, she noticed enthusiastic student collaboration, Meliame also identified students' frustration when agreement on one solution strategy was not reached. She added that her students placed importance on speed and accuracy, as well as competition, stating: "They're always listening to the other groups to see which group would get the answer first, be the first to finish."

Meliame considered her role during mathematics lesson and described it as follows:

The teacher is seen as just a roamer, I try not to get attached to any group, but I'm listening and observing, and encouraging agreeing or disagreeing, and the reasoning.

Meliame viewed her position as fluid, moving throughout the group monitoring the students' actions. Nevertheless, initial classroom observations revealed that she did in fact stop and work with most groups. Taking a critical stance, Meliame extended her reflection to her monitoring of students' reasoning. She acknowledged that her objective while students worked in groups was to support the students she identified as "low achieving". She would ask her "low-students" if they understood what the "high-students" were explaining. At times, she would ask the "low-students" to repeat what the "high-students" had explained and gauge their level of comprehension from their ability to repeat the explanation. She confirmed that she did not usually extend her mathematics lessons to justification and generalisation.

This section has provided insight into Meliame's initial ideas about teaching and learning mathematics, and her classroom practice at the start of the current study.

4.3.2 Analysis of teacher learning

Meliame had positioned herself as an active member of the study group. In the first study group meeting, her responses indicated an emerging exploration of elements of her teaching practice. While beginning to critically analyse her classroom practices, Meliame demonstrated some insight into deeper aspects of how students reasoned mathematically. Her concept map and commentary highlighted the emphasis she placed on students' actions. There was limited reference to what actions she could take to provide multiple opportunities for all students to develop deep conceptual understanding of important mathematics concepts. She acknowledged that changing her practice and learning environment would be an on-going process requiring continual self-reflection. Meliame's insights provided the foundations for the next phase of the design-cycle.

The next section presents data from the experimentation stages of the second phase of the research design cycle (see Table 1). In this phase Meliame began changing her pedagogical approach to teaching mathematics.

4.4 (RE)STRUCTURING THE LEARNING ENVIRONMENT

In this section, the ways in which Meliame initiated new instructional practices are examined. Meliame began establishing a learning environment that enabled students to feel a sense of belonging and one where they were supported to participate effectively in mathematics learning. The following section describes how Meliame set about (re)structuring her learning environment.

4.4.1 Establishing the classroom community

In classroom observations, Meliame began intentionally restructuring her mathematics classroom social environment. To begin setting the emotional tone for mathematics learning she facilitated a classroom discussion. This discussion focused on establishing expectations for participation in collective mathematics activity. The following excerpt illustrates an episode from a lesson (Lesson 3, Term 2, Week 4) in the first experimentation phase:

Lesson 3, Term 2, Week 4	
Participants	Dialogue & Actions
Meliame	*What are we supposed to do during problem solving mathematics?*
Malia	*Work out the problem and say how we did it*
Meliame	*Do you agree with what Malia is saying, everyone?* (All the students nod, some show a thumb up) Meliame continues: *Ok, tell us something more, Sue*
Sue	*Make sure everyone in your group understands* (Several students nod in agreement with Sue)
Meliame	*So, how do we make sure that everyone in your group understands?* (**Meliame** sweeps her hand across the group, not specifically at Sue)
T.J.	*By talking with the pen*
Meliame	*Tell us more about talking with the pen, T.J.*
T.J.	*Talking about the problem, explain…* (T.J. stops talking and looks around at his peers. Elisapeta picks up the discussion)

Elisapeta	*Learn new maths strategies*
Meliame	*Leilani, can you repeat what Elisapeta said, please?*
Leilani	*Learning new maths strategies, and like T.J. said, we talk about the problem, like, explain*
Meliame	*Ok, everyone, do you know what that means?* (Some students nod, others shake their heads. Meliame then directs the students to turn to the person next to them) *Talk to your buddy about what that means, what Leilani has just said*
Meliame	(Meliame waits in silence for 2 minutes while the students physically turn to a buddy and begin their discussion) (After 2 minutes, Meliame talks) *So, what did you and your buddy talk about, Simion?*
Simion	*We thought that it means, when you're in a group you listen to every other person's strategy, and you agree or disagree, and you have to use the maths norms*
Meliame	*Let's talk more about that. We've heard a lot of things now. Strategies and norms so let's talk about the maths norms, we'll come back to strategies, let's talk about the maths norms. What do you understand by that?* (Meliame indicates with a sweep of her arm that she wants all of the students to think about this, not just Simion and his buddy)

Meliame intentionally utilised inclusive speech patterns such as "tell us" and "we" signalling clearly to her students that she was part of the learning community and that all were expected to contribute to the discussion. The dialogue pattern shifted from what Mehan (1979) terms the initiate-respond-evaluate model of instruction typically seen in teacher-centred mathematics classrooms to increasingly fluid teacher-student-student-teacher patterns. By utilising the talk move of waiting, Meliame provided opportunities for students to think about what they had been asked to do, and then action those requests. She fostered a nurturing environment for students to begin risk taking by providing students with opportunities to think, talk to a peer, respond back to the whole group. Meliame supported her students to take risks by inviting them to publicly share their ideas. Furthermore, her non-verbal communication patterns-sweeping gestures and gazes when calling for participation in the co-construction of social norms, fostered risk taking. Notable was Meliame's positioning of the students as having the right (and duty) to tell *her* what the norms were, as opposed to traditional teacher-centred classroom where teachers are viewed as holding the authority (the right) to tell them. The discussion about norms for working together continued. The extract below, taken from the same lesson (Lesson 3, Term 2, Week 4) illustrates the students' understanding of what active listening involves and begins with one student stating that the norms for working together involved active listening:

Term 2, Week 4	
Participants	Dialogue & Actions
Heme	*The norms are actively listening*
Meliame	*Actively listening. What does that look like and what does it sound like, what does it mean?* (Two students raise their hands) Meliame notices and responds saying: *Let's have no hands, ok. Let's hear from some people we haven't heard from before. So, if we're talking about active listening, what does that look like? I'm going to give you time to think about that, and then you are going to talk to a buddy before we share back together* (Students sit silently for half a minute and then begin talking to others around them. Meliame continues to wait in silence for 2 minutes before speaking)
Meliame	*Sela, please tell us what you and Mele were talking about*
Sela	*Mele and I agreed it means not talking while someone's explaining*
Meliame	*Can we add more to that idea?* (As before, her gaze sweeps around the group on the mat and not at one specific student)
Simion	*Listening to the person that's showing you. Make eye contact.* (Most students nod in agreement and look at the teacher)

Meliame's words and actions are deliberate and purposeful, providing a scaffold for her students. By stating no hands up, she established an expectation for all to be prepared to share their ideas. In addition, she supported her students in preparing to share their ideas publicly in two ways. Firstly, by waiting in silence, she signalled the importance of using time to reflect, think, reason, and consider before sharing ideas. Secondly, Meliame created a secure emotional space by providing an opportunity for students to discuss their thoughts with a partner before sharing ideas in the big group discussion. When inviting students to share ideas publicly, both students were included in the invitation, for example: *tell us what you and your buddy were discussing.*

The students' responses in this episode highlighted a superficial understanding of what active listening involved. They had described active listening as sitting quietly and looking at the person talking, being respectful, and a good audience member. Meliame's ensuing actions supported her students further in establishing the role of actively listening as resource for reasoning with mathematics. She acknowledged that what had been discussed was important, but that they all needed to dig deeper into how actively listening could support learning. To embed this further, Meliame role-played *active listening*. The following extract illustrates what she did:

> *I'm going to tell you what goes on inside my head when I'm actively listening. In my head, I'm going, mmmm I'm hearing what the person's saying, trying to make sense of it. Some of it makes sense, yeah, most of it makes sense…or sometimes, I might, in my head be going, oh! I'm a bit stuck with that bit. So, it's more than just sitting there, you know you can see the person's mouth moving, and you can hear some words, and at the same time you're thinking about other things. It is actually you trying to make sense of the maths, like it's all about the mathematics and does this make sense in my brain? And if it doesn't make sense, what are things can I do so that it does make sense?*

Meliame modelled that active listening meant reasoning (thinking about) and grappling with important concepts. Directly modelling what active listening entailed provided her students the opportunity to make sense of this action. Meliame had highlighted that being an active participant did not always require talking. In addition, she identified that being confused was a natural state of learning and a precursor to acquiring new understanding. These were important elements for students to understand as Meliame proceeded to establish her mathematics learning environment.

This section has provided insight into the deliberate actions Meliame took in providing social support for her students. The next section provides deeper analyses of her actions.

4.4.2 Positioning to develop a community of mathematical inquiry

Meliame positioned herself as having the right and duty to establish the expectations for learning mathematics in her class. In Excerpt 4.4.1 Meliame supported the students in developing a shared understanding of the expectations for learning mathematics in class. By explicitly stating no hands, and using non-verbal gestures, such as sweeping her arm across the group, she positioned the students for participation. Notably, Meliame's consistent use of "us" and "we" firmly established her right to be part of the learning community. In addition, this speech act supported the construction of an inclusive learning environment where everyone, the teacher included, was positioned with the duty to participate in discussions. With this duty came the right to refuse to participate. Classroom observations highlighted that when students enacted this right, it was not for non-compliant reasons but a signal that more thinking time was needed before offering an idea. Students had a duty to respond to what Meliame was asking and the right to reflect on what was freely discussed in the group. These positions were readily accepted by the students and there were no examples of the students refusing the positioning or trying to position someone else to do the talking for them. As the students and teacher made sense of the words and actions a jointly constructed storyline emerged.

In this jointly constructed storyline, the classroom was a space where all members were expected to participate. Participation was not limited to speaking and included thinking and reasoning silently before sharing.

Meliame's teaching and positioning decisions with her students as she prepared them further to engage in collective mathematics activity are described in the following sections.

4.4.3 Individual responsibility and group accountability

As the first experimentation phase of the current study progressed, Meliame continued to develop communication practices that focused on active engagement of, and with mathematics. In Lesson 4 (Term 2, Week 5) before she commenced teaching, she stated that she had noticed that many students continued to defer to her as the authority in the mathematics classroom. Meliame stated her goal for this lesson (Lesson 4) was to focus on establishing the norms for group accountability and individual responsibility while the students worked in group mathematics activity. The episode highlighted below occurred just as the students were about to engage in the big group discussion section of the lesson. One group was preparing to explain their mathematical reasoning to the bigger group. Meliame seated herself within the semi-circle on the mat, alongside the other students. Just before the group began explaining, she reinforced the norms for this section of the lesson. She asked the students to consider their role during the big group sharing. The students explaining in front of the big group agreed that their role was to clearly explain their reasoning so that everyone listening understood. The students on the mat agreed that their role was to reason with the explanation of the mathematics and to ask questions, if needed, for clarity. The following excerpt illustrates how Meliame established these expectations:

Lesson 4, Term 2, Week 5	
Mathematics Task: Pene's dad worked at the markets sorting the fruit crates. One of the stall holders asked Pene's dad to let him know how many apples he had available. After sorting through the crate of fruit, Pene's dad realised that one third of the crate were apples and remainder of the crate was mandarins. How many apples and how many mandarins were in the crate of fruit?	
Participants	Dialogue and Actions
Lavinia	*Well, we knew we had to work out what two thirds of the crate of fruit were to figure out how many apples, so, first we drew three thirds and then, and each one third there were nine* (While Lavinia is explaining, Tupou is recording on the whiteboard, drawing a rectangle, and dividing it into three thirds. The other two group members, Leilani and Sela, are standing on the far side of the whiteboard)
Meliame	*Now ask us if there are any questions*
Lavinia	*Any questions?*
T.J.	*Where did you get the one from?*
Lavinia	*We did a whole and then we just put the apples in one and mandarins in the others* (Lavinia is looking at **Meliame** while answering the questions. Many of the students are looking at **Meliame. Meliame** purposefully says nothing and stares ahead. After a minute T.J. repeats his question, looking at **Meliame**, who keeps looking at the group in front)
T.J.	*Where did you get the one from?* (T.J. is looking at the teacher, as are the other students)
Meliame	*I am not the person explaining. T.J., you need to ask the group explaining* (T.J. looks at the group of four students at the front and asks his question again)
T.J.	*Where did you get the one from?*
Lavinia	*There is no one*

T.J.	*I'm confused* (All of the students are now looking at **Meliame**)
Meliame	*Why are you all looking at me? This is not my explanation. Ask the guys in front and ask them what they mean*
T.J.	*What do you mean there is no one?* (All of the students look at **Meliame. Meliame** looks at the group in the front and says nothing)
Lavinia	*I'm confused* (All students are looking at **Meliame** again. **Meliame** continues to wait in silence, looking at the group in front. After 30 seconds of silence, Lavinia begins speaking)
Lavinia	*T.J. I'm confused. I don't know what one you mean* (Lavinia looks at T.J., T.J. gets up and walks to the whiteboard and points to the one above the fraction symbol for one third-1/3. Lavinia looks at her other group members and Tupou begins talking)
Tupou	*Because there's only one piece and there is three of them* (Tupou points the rectangle divided into three parts she drew earlier. She points first at one part, then the next, then the next) (All of the students look at **Meliame. Meliame** says nothing but continues to look at the group at the front)
T.J.	*Oh.* (Pauses) *So, you mean that is the one, the one part of, one part of the whole third, oh, no, of the whole*
Leilani (from the group in the front explaining)	*Yes, so look, because two thirds is the mandarins, and one third is the apples, so look we drew the three baskets of fruit, but it is all the fruits together one whole crate that Pene's dad is looking at* (Leilani points to the rectangle and the three parts as she explains one part has the apples and the other two parts hold the mandarins. T.J. nods. Most of the students are looking at Leilani as she talks)
Carlos	*Oh, I get it now, they are showing us the crate, the fruit Pene's dad is looking at. But the crate is in three because thirds*

Meliame had positioned her students as having the right and duty to reason with the mathematics. The group explaining their strategy was positioned with the duty to clarify their mathematical thinking to the group on the mat. By insisting that the group explaining ask if there were any questions, Meliame had positioned the other students as having the right to ask questions if they needed to. Meliame's persistent use of non-verbal gestures (e.g., sitting silently; looking at the student talking) positioned all the students as being individually responsible and collectively accountable for making sense of the mathematical explanation by persisting with questioning. In addition, Meliame had shifted mathematical authority to all students by holding them individually responsible and accountable for explaining and making sense of the mathematics. The students had the right to refuse these positions, but none were observed to do so. Through these actions, Meliame provided the expectations for active engagement with mathematical reasoning. Through these positioning acts, a new storyline was possible, one where mathematics learning could take place in a positive emotional learning space.

Maintaining positions within this storyline allowed mathematical authority to be distributed through the group as students collectively sought clarity or made clear mathematical explanations. Meliame had also created a new storyline where the teacher was no longer the sole holder and keeper of mathematical knowledge. The students had been positioned and repositioned to view themselves as knowers and seekers of mathematics.

This section has focused on the explicit actions of the teacher in creating the supportive learning environment during the first experimentation phase of the current study (Term 2, Weeks 2-5). The ways in which Meliame established the norms for classroom participation were explored. The next section focuses on how Meliame changed her grouping practices to position all students for learning.

4.5 POSITIONING ALL STUDENTS FOR LEARNING

The data in the following sections were collected during the third and fourth phases of the current study. The ways in which learning environments are structured can provide distinctly different opportunities for how some students engage with learning and understanding mathematics. This section continues the investigation of Meliame's actions to (re) structure her classroom environment. Specifically, her utilisation of a strength-based approach for grouping students for mathematics learning is explored.

4.5.1 Grouping

At the start of the current study, Meliame's existing beliefs and practices in grouping her students were examined. The following Table (see Table 13) outlines Meliame's responses to the first written questionnaire (see Appendix A).

Table 13: Meliame's Responses to a Written Questionnaire

Question	Meliame's responses
How do you currently group your students for learning mathematics?	Completely mixed abilities. I look at their school testing results from the end of the previous school year and then split my class into two completely mixed groups of low, middle and top ability students-half in each group
What do you understand by the term *ability*?	What the students are capable of achieving in maths
How is ability measured?	By test results
Why do you group your students in this way?	So that there is always someone who knows how to do the maths and that person can help the others, especially the low ones, they can show them what to do

Meliame's initial practice of grouping her students in mathematics into groups of mixed ability reflected a school-wide practice of normed testing as the sole means of measuring students' capability for learning mathematics. In addition, her responses emphasised her belief that some students were innately better at learning mathematics than others.

Meliame's beliefs and practices about grouping her students for mathematics learning formed the basis of how she grouped her students for learning as the current study commenced. In Phase three of the current study, she began systematically changing the way she grouped her students, supported by a tool developed in the second study group meeting (Term 2, Week 6) of the design-cycle. The design and implementation of this tool is discussed in the next section.

4.5.2 Strength-based grouping tool

Although Meliame employed heterogenous grouping practices, notions of ability were consistently apparent during observed mathematics lessons. Her initial belief that some students were better than others at learning mathematics continually affected student participation and engagement during collective mathematics activity. The students she positioned as knowledgeable often dominated group discussions. Conversely, those who Meliame positioned as low ability tended to sit passively watching others complete the mathematics task. The actions taken to support teacher change in grouping for mathematics learning focused on taking a strength-based approach. During the second study group session (Term 2, Week 6), the teacher participants engaged in an activity designed to focus attention on the strengths individual students brought to learning (mathematics). Using a strength-based tool (see Appendix E) the teachers were asked to select four students to group together; and then identify the strengths each would bring to collective mathematics activity. During this activity Meliame said that she was struggling to complete this activity based on her prior views of certain students. She commented as follows:

> *I feel like I want to choose strategic, like not strategically, but I want to pick certain children because that group can problem solve as opposed to Leilani and them. I don't even know which group to use. What do we base this on? Some of my students, what are their strengths, I don't know.*

Meliame stated she found it difficult to view all her students from a strength-based perspective. Her beliefs about her students' perceived ability in mathematics were embedded and these presented conflict as Meliame completed the exercise. Realising further prompts were needed, the researcher suggested the following:

> *If you just think about the kids in your class. Look through your class list, look at the names and think which four If I put together could draw on each other as resources for working together to complete mathematics tasks?*

The prompt provided an opportunity for Meliame to review her grouping practice. The following Table (see Table 14) presents Meliame's grouping record:

Table 14: Meliame's Grouping Record

Name	Strengths	Other comments
John	Shares clearly, can explain what he is thinking	Patient with others and asks positive questions
Sepora	Good sharer	English is a second language. Doubts herself
Paulie	Art	Doesn't usually speak up or engage with others. Cannot read very well
Leilani	Can share others' thinking clearly	Avoids struggle. Does not see her own capability

Meliame had grouped four students for mathematics learning based on their individual strengths. Three of the students' strengths Meliame identified were important mathematical practices. For example, making clear (mathematical) explanations. Meliame had looked beyond traditional perspectives of what it means to be good at mathematics (e.g., accuracy and speed), instead, drawing on a range of mathematical practices good mathematicians know and use. One student's strength was drawn from another curriculum area. (Further discussion of Paulie will be presented in Section 4.5.3).

Meliame reflected that she had found identifying her students' strengths extremely difficult. She reflected that utilising a strength-based tool had compelled her to shift her views of students beyond their test results and/or reluctance to participate in mathematics lessons. She also identified how consistently perceived notions of ability influenced her grouping decisions and how embedded her beliefs and practices in grouping in such ways were. Meliame acknowledged the challenge in shifting her perspective and identified the grouping template as support. Her commitment to attend to grouping her students from a strength-based approach contributed to the development of a supportive and effective learning environment in class.

Meliame continued to employ strength-based grouping in class. The next section highlights the effects this approach had on providing opportunities for a marginalised student to learn mathematics.

4.5.3 Opportunities to learn: Extending strength-based grouping

The data presented in this section were collected in Term 3, Week 7, as the current study progressed into phase five. As Meliame extended her changed grouping practice, she not only paid attention to "what" the students would be learning, but also to "how" she could provide opportunities for all students to learn mathematics. In this section, her focus on supporting Paulie, a Tongan[10] boy who remained on the fringes of participation in class is presented. Meliame described Paulie as a struggling reader and a quiet, reluctant participant in collaborative mathematics activity. Prior to the mathematics lesson, Meliame considered who Paulie was as a person, what his interests and strengths were. Utilising the strength-based grouping tool, Meliame had positioned Paulie in a group with three other students (see Section 4.5.2). To extend

[10] The Kingdom of Tonga consists of more than 170 South Pacific Islands.

this grouping she drew on Paulie's identified strengths as the focus for a collective mathematics task. Meliame designed a Geometry task which required the students to identify and describe geometric shapes and angles present in a Picasso Artwork. She purposefully situated mathematical concepts within the context of Visual Arts, thus providing Paulie access to the mathematics through another area of learning. An immediate shift in Paulie's participation was evident. Meliame describes his engagement in mathematics learning in the following extract:

> *When a problem was written in the context of art, a huge shift was seen in Paulie. He was involving himself in group discussion, asking questions, taking the pen, and sharing his strategy. He also shared his group's ideas back with the class. He then also asked questions of other groups during the sharing process.*

Meliame had identified Paulie's strength in Art and utilised this to provide him access to reason with the mathematics in the task. In observation, Paulie's reaction to the task was notably different. Previously, Paulie had minimal interaction with other group members as they grappled with mathematical activity. Now he took a lead in the group's engagement with the mathematics activity. When the group explained their mathematical reasoning to the big group, he led the group explanation. For the first time also, Paulie asked other students questions when they shared their explanations. Meliame noted that this had never happened before.

Analysis of this episode is twofold: Firstly, Visual Art was the access to learning for Paulie. Secondly, his access, in turn, became multiple students' access to mathematics. In addition, the mathematics task did not involve calculation with numbers (a gatekeeper for some in mathematics learning). By utilising a strength-based approach, Meliame had provided access to mathematics learning not only for Paulie, but other students too. Meliame noted the effect her transformed grouping practices had on status in class:

> *Using the grouping template tool broke down all these hidden barriers that I wasn't aware are there that influence students' understanding of their mathematical capability.*

Meliame's realisation of the effect her grouping practices had on students' access to mathematical learning is significant. She noted that hidden bias had affected the way she grouped students for learning mathematics. She identified the planning framework as instrumental in supporting her transformation of grouping. Finally, Meliame acknowledged she was responsible for creating a multi-faceted learning environment that provided all her students access to learning mathematics.

4.5.4 Discussion: Positioning all students for learning

The episodes in the previous section described the actions Meliame took in positioning all her students to access mathematical reasoning. In addition to drawing on students' individual strengths as means to access the mathematics, opportunities arose for dealing with aspects of status. Paulie had been identified as someone who, previously had never actively engaged in mathematics lesson. Deliberately drawing on his strengths in spatial awareness, he was now given a gateway to conceptual mathematics learning. In addition, his status shifted from low to high as a direct result of Meliame's actions. This explicit action provided a previously marginalised student the means to develop belief in his capability to learn mathematics.

Positioning evolved from Meliame's deliberate structuring of strength-based groups. She positioned herself as having the duty to critically examine her knowledge of her students. In doing so she positioned them all as capable of learning mathematics; thus, resolving prior deficit views she held of her students. In turn, she positioned the students as having the right to be provided with opportunities to access the mathematical reasoning through multiple gateways. This right could only be upheld by the development of a new storyline.

The emerging storyline was explicitly constructed by Meliame deliberately positioning herself as having the ongoing duty to provide her students with access to the learning. In addition, her students participated willingly in this evolving storyline. Within this storyline, Meliame's students believed in their individual and collective capabilities as mathematicians. Subsequently, they were supported to be knowers and doers of mathematics. Maintaining this storyline remained contingent on it being jointly sustained. Meliame's verbal and non-verbal actions in mathematics lessons consistently supported students in maintaining these positions and sustaining this storyline.

This section highlighted Meliame's actions in positioning all students for learning through careful consideration to her grouping practices. As students were consistently positioned for learning and strength-based grouping practices were extended, Meliame began focusing on developing mathematical practices. Mathematical practices were an explicit focus of the fourth study group meeting. The actions Meliame took to embed these in her classroom practice are examined in the next section.

4.6 MATHEMATICAL PRACTICES

As Meliame's classroom practices changed, her students were now all expected to make clear mathematical explanations which were conceptually sound. Those listening were expected to consistently reason with others' explanations; to work out what was different or helpful. Sound conceptual explanations formed the basis for mathematical argumentation and the students in Meliame's class were encouraged to reason with mathematical explanations and build on from their own understanding. In doing so they were developing many ways to confirm their thinking. In classroom observations (Term 3, week 8), Meliame explicitly talked about 'friendly arguing about the maths, not arguing with the person' and how mathematicians use argumentation in this way to help them to reason, to understand the mathematics. Meliame identified that in order for her students to embed what was required to be successful in mathematics, they needed to know that there are many facets to being successful in mathematics and that using mathematical practices effectively was a way to be successful. To support further development of mathematical practices, a Mathematical Practices Framework (see Table 5) was developed in the fourth study group session (Term 3, Week 6).

4.6.1 Drawing on a supportive tool

This framework was designed to support the teachers in specific and deliberate ways to provide multiple opportunities for students to adopt a range of effective mathematical practices (see Section 2.5.4). The original framework incorporated aspects of Hunter's (2007b) *Communication and Participation Framework* and Wood and McNeal's (2003) *questions and prompts for defining classroom cultures*. As the current study developed, further investigation and broadening of what defined mathematical reasoning and how different mathematical practices support deep mathematics reasoning occurred. The original framework was continually refined during subsequent cycles of the current study. Ongoing discussions about appropriate and deliberate teacher actions to support the development of mathematical practices resulted in agreement by teacher and researcher participants that in order for students to learn and use a range of mathematical practices, these practices needed to be made explicit.

In the next section, the ways in which Meliame engaged her students in utilising several mathematical practices are presented. In addition, connections are made between the actions Meliame took in class to engage student participation in mathematical practices, and how student participatory engagement facilitated the development and use of mathematical practices.

4.6.2 Developing mathematical explanations and justification through friendly argumentation

In this section, Meliame's deliberate teacher actions in developing several mathematical practices are illustrated. She facilitated the development of clear mathematical explanations which, in turn, formed the basis for the development of justification through friendly argumentation. These actions are illustrated in the following extracts from classroom episodes in Term 3, Week 9.

In class, Meliame asks the students to think about which numbers come between 1 and 2.5. The students proceed to offer the next number. When they get to 1.9; one of the students claims that the next number is 1.10 (One-point-ten). Several students disagree with that claim and say the next number should be 2. Meliame pauses the students. The following extract illustrates her ensuring actions:

Term 3, Week 9
Emphasising the importance of engaging in mathematical argumentation and justification to support a mathematical claim

Participants	Dialogue & Actions
Meliame	*So, here we have reached a point where some of you are saying it is one thing and some of you are saying another thing. These are called* ***claims.*** *This is an opportunity to dig deep and do something good mathematicians do. They develop* ***mathematical arguments****-they friendly argue about the maths and use* ***justification*** *to prove their side of the argument or the claim they have made. We have to pull this apart, because if we do this while we are thinking about mathematics, we will be very good at mathematics* *Heme, you and Paulie and some others have made a claim that the number that comes after 1.9 is 1. 10. Pratesh, Tupou and some others think it is 2. You all need to think about it for a second. Is the next number 1.10 or 2-all of you have a think* (**Meliame** writes the claims on the board and then waits while the students take a moment to consider what she has asked and written. After a minute she continues) *Ok, you have all considered which claim you think is correct. Now think of how you would justify your reasoning. This means you have to say,* ***I think it is…because****….* *Take some time to think about it, because this is how mathematicians prove their ideas, their claims. After thinking in your head, turn and talk to the people around you about your thinking*

Meliame's first action was to "pause" the discussion when she noticed the students were unable to advance their claims. Her second action was to "explicitly identify and name" the mathematical actions of the students; namely making claims, mathematical argumentation, and justification. Meliame's final action was to "explain" that these actions are what mathematicians use and do, and these are the actions that make them successful at mathematics. By positioning the students from individual sense-making, she repositioned them into collective groups. These actions reinforced several important factors. Firstly, Meliame had high expectations that all students were capable of reasoning mathematically. Secondly, the students were positioned equitably, no individual student was singled out as having more mathematical authority or status than another. In addition, Meliame had specifically stated that the actions the students were taking in making claims were the very actions that successful mathematicians make when doing mathematics. Meliame had pressed further and added that in order to substantiate a claim, it needed to be justified mathematically. Furthermore, when Meliame revoiced the two groups' claims, she did not add her opinion, nor did her voice or facial expression reveal which claim she thought was correct. Additionally, she reinforced that there was no need to rush their thinking; thereby reinforcing successful mathematicians are thoughtful and deliberate in their mathematics reasoning.

Meliame continued to press the students towards developing mathematical justification, as illustrated in the excerpt (Term 3, Week 9) below:

Term 3, Week 9	
Mathematical justification as a means to prove the mathematical claim	
Participants	Dialogue & Actions
Meliame	*Simion, tell us what you and your buddies have been thinking*
Simion	*I think the next number after 1.9 is 1.10 because 10 always comes after 9*
Meliame	*Leilani, what do you think about the claim Simion has just made?*
Leilani	*I disagree. It can't be 10 because the number before isn't 9…it is 1* ***point*** *9* (Leilani stops. Several students nod their heads and several look confused. **Meliame** sits silently for a few seconds before speaking)
Meliame	*Who can add to the mathematical argument?* **(Meliame** waits in silence before continuing*) Pene, tell us what you think*
Pene	*We are not counting 1, 2, 3 like that. We are using decimals, so 1.1, 1.2 and that. So, then next number isn't 1.10, because we are looking for the numbers between 1 and 2* (Pene stops. Simion starts smiling)
Meliame	*Simion, what are you thinking now?*
Simion	*I get it now, I know why it can't be 1.10, it's because when we are in decimals after 9, we have to move to the next one, like it goes over to the next place*

Meliame had neither confirmed nor refuted the students' claims or justifications. Instead, she pressed the students to add to the mathematical argument. By deliberately positioning them to justify their thinking, the students had been provided with opportunities to extend their mathematical explanations. By positioning the students as responsible for proving their claims, individual responsibility, and group accountability for developing the mathematical argument had been reinforced.

Realising there was further opportunity to deepen students' mathematical reasoning in this moment, Meliame continues to press the students:

Term 3, Week 9	
Pressing for deeper reasoning	
Participant	Dialogue & Actions
Meliame	*There is more. Think about what the maths is that you are doing when you claim that the next number from 1.9 is 2. Work with the people around you.* (**Meliame** waits in silence. The students begin talking to others, some begin writing ideas down). After several minutes, **Meliame** asks two students to share their ideas
Meliame	*Tupou and Carlos, what were you thinking?*
Tupou	*We saw that when we wrote it down like on the line with all the spaces between the numbers it was 1.1, 1.2, 1.3 and so on, but when we got to 1.9 the next number was 2* (**Meliame** waits in silence for about 20 seconds before continuing)
Meliame	*Ok, now I am going to ask you all to think about what is the maths that makes the next number what it is? Ok, so how does 1.1 become 1.2 and so on? Turn to your buddy and discuss again* (**Meliame** waits in silence while the students work and talk together. All of the students are engaged in the task. For a few minutes, she observes each group closely, without intervening, before selecting a pair to share) *Sepora and Nikita, talk to us*
Sepora	*We saw that we are plussing the next number.*
Meliame	*You mean adding?*
Sepora	*Yes, we went 1.1 plus .1 made 1.2*

Simion	(**Meliame** records this on the white board as 1.1+.1=1.2) Then 1.2 plus .1 makes 1.3 (**Meliame** records as above and then directs the girls to ask the other students if there are any questions. When there are no questions, **Meliame** asks Simion to explain what the girls have done) *They are adding point 1 each time*

Meliame had pressed the students to think more about their mathematical claims and justifications. She had stated clearly that there was more mathematics to consider. Her next deliberate action was to invite the students to explain what they had considered. By repositioning the students to work in pairs or small groups, Meliame had provided access for all students to participate in developing mathematical reasoning. The students had been pressed to detail the mathematics action that provided the proof for the initial mathematics claim. The mathematical action of adding .1 was made explicit through Meliame's persistence. Pressing for justification supported the students to search for mathematical proof for the initial mathematics claim.

4.6.3 Discussion

Meliame's use of the process model for making mathematical practices clear provided support for classroom interaction. By deliberately pausing the classroom action, Meliame created a thinking space. By specifically naming the mathematics practice, she could focus on pressing the students further, Meliame positioned her students as having the responsibility to find the mathematical proof. By waiting in silence, Meliame positioned herself as having the right to expect her students to work collaboratively to investigate the mathematics further. Several potential storylines became possible through these positioning actions. One potential storyline was one where the mathematical authority had been delegated to all students. This storyline was enacted by Meliame positioning all students as capable of finding, representing, explaining, and justifying mathematical proof in a range of successful ways. A complementary storyline emerged where the mathematics itself became positioned at the centre of learning. By being provided many opportunities to reason creatively with how the mathematics worked, all of Meliame's students were granted access to "learning mathematics" successfully. The storyline that developed was one where students recognised that being "good" at mathematics involved much more than quick and accurate calculation, and where utilising each other as mathematical resources provided opportunity to inquire and reason deeper with mathematics.

4.7 CHAPTER SUMMARY

This chapter has presented the journey Meliame took in (re)structuring her classroom environment to consistently provide time and space to position all her students as capable of learning mathematics. The chapter began with a description of her initial classroom context and beliefs about which students could learn mathematics successfully. While Meliame espoused that collaboration and flexibility in finding solutions were important, observations of her classroom practice at the start of the current study highlighted the emphasis she and her students placed on "the correct answer" as being the most important characteristic of "understanding" the mathematics. Following her active participation in the study group sessions, Meliame began transforming her learning environment. She (re)defined her classroom parameters by utilising the social domain framework (see Table 2) to establish her learning community. Positioning all students to participate in a community of mathematical inquiry required intensive and deliberate (re) negotiating of teacher and student positions and existing storylines. Embedding and extending social and sociomathematical norms supported in students developing an understanding of the importance of taking individual responsibility and group accountability while learning mathematics. Meliame paid close attention to her grouping practices. Using a smart tool (see Appendix E), she transformed her prior views and approached grouping from a strength-base, identifying and acknowledging the strengths each of her students brought as resources to learning mathematics. In extending strength-based grouping practices, Meliame positioned her students for learning mathematics. Finally, she highlighted mathematical smartness as understanding and utilising key mathematical practices.

The next chapter presents the second case (Alisi) in the current study.

CHAPTER FIVE

THE CASE OF ALISI

5.1 INTRODUCTION

The previous chapter described the actions Meliame took in (re) structuring her learning environment for enhanced mathematics learning. This chapter documents the contrasting case of transformation in Alisi's classroom. Her journey was marked by two critical factors which significantly affected the students' participatory interactions: namely, status and conflicting core cultural values. Both issues affected students' participation and became barriers to mathematics learning. Many students found it difficult to walk in two worlds; namely, the embedded norms and values of home and the taken-for-granted norms and values of their school classroom. Each section in this chapter focuses on the considered actions of Alisi in dealing with these issues and how she developed a more equitable learning environment.

Section 5.2 describes Alisi's beliefs about doing and using mathematics and how these beliefs shaped her classroom learning context. Section 5.3 outlines Alisi's active participation in the professional learning sessions. Section 5.4 outlines the journey Alisi undertook to establish and embed the social structures. Section 5.5 describes the emergence of two barriers for learning, status issues and conflicting core values. Section 5.6 outlines how Alisi overcame these obstacles by creating affordances for learning mathematics. Section 5.7 provides a summary of the chapter.

As in the previous chapter (see Chapter 4), the positioning of the teacher and students reported here takes the form of excerpts from mathematical lessons throughout the cycles of the design-based research cycle in which data collection occurred.

The next section provides a background of the teacher and her pedagogical beliefs and practices prior to the current study commencing.

5.2 TEACHER CASE STUDY: ALISI

Alisi had taught for seven years. Currently, she taught 24 Year Four to Five students aged 8-9 years old. Alisi described her class as achieving at a range of curriculum levels in mathematics, according to normed test results.

In an interview at the beginning of the study (Term 2, Week 1; see Appendix A), Alisi stated that she believed that all students in her class were capable of learning mathematics, but that “they were just at varying degrees of where their mathematics learning and knowledge were.” Notable, was Alisi’s description of mathematics as “mathematics learning” and mathematics knowledge”. She confirmed that for students to grapple with mathematics, there was a certain body of mathematical knowledge they needed to know. For example: times tables, numbers bigger than 20, forwards and backwards counting of numbers up to 1, 000. Alisi added that she thought it was important that students know and use several social and sociomathematical norms (Hunter, 2010; McClain & Cobb, 2001; Yackel & Cobb, 1996) while learning mathematics. She listed these as: listening, sharing, explaining, justifying, questioning, making connections, and taking responsibility for their own learning. Moreover, Alisi stated it was important for her students to work collaboratively during mathematics lessons. She described collaboration as students talking to each other and recording the mathematics strategies used to solve the problem. Alisi said that by observing her students working collaboratively, she could assess which students were “able to do the mathematics”.

Alisi asserted that prior to the commencement of the current study, these mathematical norms had been established in her class. She described the enactment of these norms as students: actively listening, checking that everyone understands, using the talk moves, and giving everyone a chance to share their thinking. She added that mathematical argumentative language had been encouraged and modelled by her (but did not provide any examples of what this kind of discourse sounded like).

Initial classroom observations and discussions (Term 1, Week 9 & Term 2, Week 1) with Alisi revealed that these elements were neither embedded nor enacted in her classroom. Alisi acknowledged that unlike previous classes she had taught, she had struggled to implement and embed these norms with her current class. She described that she routinely chose the norm the students needed to work on, based on what had occurred in the previous mathematics lesson. She stated that while she chose the focus at the start of the lesson, she tended to forget to hold the students accountable for enacting the particular norm throughout the lesson, or to reflect on it at the end of the lesson.

When asked to describe who held status in her mathematics class and why, Alisi identified several students and herself as holding status. Her general description of the high-status students was that they were loud and confident. Initial classroom observations and student interview data confirmed Alisi's description (As issues of status form a significant part of Alisi's journey, I do not discuss these further here. Elaboration can be found in Section 5.5).

The next section begins by outlining Alisi's initial beliefs and practices about doing and using mathematics, and how these shaped her learning environment.

5.2.1 Initial classroom practice

Prior to the current study commencing, initial classroom observations (Term 1, Week 9 & Term 2, Week 1) highlighted that Alisi's mathematics lessons followed the same structure as Meliame's (see Chapter 4.2). Alisi described her grouping as "heterogenous". Her students were divided into two evenly mixed groups. She described the mixed groups as always "including one student who would know how to solve the mathematics problem". The duration of each lesson was approximately 50-60 minutes long.

The next section highlights how Alisi began transforming her mathematics learning environment. Her participation in the study group sessions, and her deliberate and considered actions in positioning herself and her students for learning are presented.

5.3 TEACHER LEARNING

5.3.1 Alisi's mathematics concept map

Alisi actively participated in all the study group sessions, sharing ideas freely and willingly. In the first study group meeting (Term 2, Week 2) she drew a concept map. Her concept map (see Figure 2) illustrated her beliefs about what she considered to be important parts of an effective mathematics learning environment.

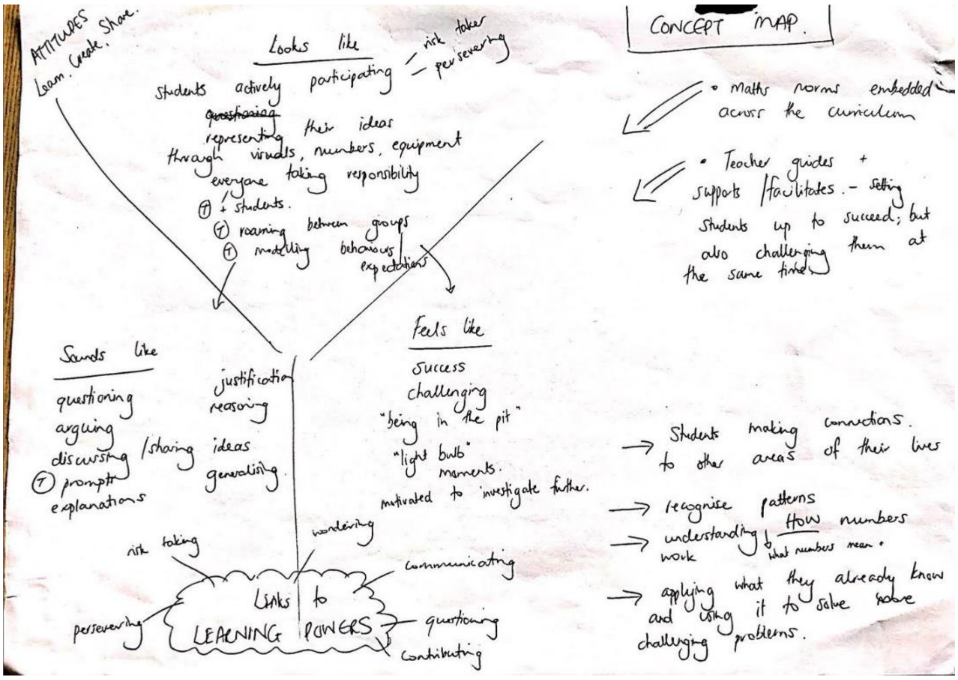


Figure 3: *Alisi's Concept Map*

Alisi identified several elements of what she considered were essential for an effective learning environment. Specifically, she differentiated between what she expected her students to be doing, and what she expected the teacher to be doing. These ideas are illustrated in the Table (see Table 15) below:

Table 15: Alisi's Hypothesised Mathematics Classroom

Looks Like	Sounds Like	Feels Like
Students: • Actively participating (risk taking, persevering)	Students: • Questioning • Arguing • Justifying • Reasoning	Teacher & Students: • Success • Challenging • 'Being in the pit' • 'Light bulb' moments

• Representing their ideas through drawing or with equipment	• Discussing/sharing ideas	• Motivated to investigate further
Teacher: • Roaming between groups • Modelling behaviours & expectations	Teacher: • Prompting • Explaining • Generalising	
Student Attitudes: LEARN-CREATE-SHARE		
Students: • Making connections to other areas of their lives • Recognising patterns • Understanding how numbers work, what numbers mean • Applying what they already know and using it to solve more challenging problems		
Links to Learning Powers • Risk taking wondering • Communicating • Persevering • Contributing • Questioning		
Mathematics norms need to be embedded across the curriculum while the teacher guides and supports/facilitates setting the students up for success; but also challenging them		

Alisi visualised a learning environment where students actively participated in collective mathematical activity by drawing on sociomathematical norms and mathematical practices, for example, mathematical representation and argument. By utilising these norms and practices, the students and teacher would both experience success in learning. Alisi conceptualised her role as monitoring the groups and engaging in deliberate acts of teaching, such as modelling expected behaviour and practices. Consequently, her students would be supported to reason successfully with mathematics by making connections and applying their understanding in flexible ways.

5.3.2 Analysis of teacher learning

Conceptually, Alisi had positioned all her students as capable risk-taking, resilient participants in learning mathematics. Reflexively, she had positioned herself as a facilitator of their success. A potential storyline was envisioned wherein students could draw on a wide range of mathematical skills, processes, and practices to reason as highly critical mathematicians.

In practice, (as seen in the classrooms observations in Term 1, Week 9 & Term 2, Week 1) Alisi positioned all her students as capable of learning mathematics, only if they possessed a set of fixed mathematics knowledge and enacted appropriate social and sociomathematical norms. Furthermore, her students were positioned as having the duty to enact the norms they claimed were essential. Reflexively, Alisi positioned herself as having the responsibility to model expected behaviours and expectations. Subsequently, the storyline that had developed was one where students failed to uphold their duties to enact what they claimed were important ways of participation, and Alisi struggled to secure her intended outcome.

Alisi acknowledged she required support in restructuring her learning environment. The next section outlines how she began establishing social structures to support mathematics learning in class, and the barriers that emerged.

5.4 RESTRUCTURING THE LEARNING ENVIRONMENT

In this section, Alisi's efforts to create an effective learning environment are presented. In the first study group meeting (Term, 2, Week 2) two frameworks were developed: The Social Domain Framework, and the Positioning Framework (see Table 2 and Table 3). Alisi drew on both frameworks as supportive tools.

5.4.1 Establishing the social domain

After the first study group meeting (Term 2, Week 2), the study progressed into the first experimentation phases (Term 2, Weeks 4, 5). During this time, Alisi used prompts from the Social Domain Framework as support in establishing effective participatory practices in class. She stated clear expectations for learning, including attending to the social norms. In observation (Term 2, Week 4), at the start of each mathematics lesson, Alisi stated the particular social or sociomathematical norm students were expected to focus on. She used specific "Talk Moves" (Chapin & O'Connor, 2007) such as repeating and revoicing to ensure her students knew what these expectations were.

Alisi also attended to inclusivity by issuing the instruction that all students were expected to participate in collective mathematics activity, working together in small-group interactions, and bigger group discussions. She also drew from the Positioning Framework as support for (re)structuring her learning environment. Alisi extended multiple invitations for all students to consider the thinking of others, or to share their own ideas while working on collective mathematical activity. By specifically drawing attention to the validity of individual students' ideas or statements, she expected her utterances to position her students as valid learners of mathematics. Instances of Alisi implementing these actions were evident from classroom observations (Term 2, Week 4 & 5) and reviewed recorded footage of these lesson. She had attempted to provide many opportunities for her students to develop effective participatory practices.

However, despite Alisi's efforts to create the learning environment she had conceptualised earlier in her concept map, by lesson five (Term 2, Week 7), Alisi acknowledged she was struggling to embed the participatory components required. Alisi critically reflected on her lack of success at establishing an effective learning environment in class, stating the following in the third study group meeting (Term 3, Week 1):

> *My class know the norms, they can rattle them off, but they don't know how to do them. Some of them are better than others, but walking the talk, they are not that good at it. Some of them just won't talk, and those that do just repeat whatever someone else has said without really understanding or thinking about it. I think they are just saying anything because they know I am expecting them to say something.*

Here, Alisi had identified that her students "knew" what they should do but were unsure "how" to enact what they knew. She had noticed that some students struggled to accept invitations to share their reasoning, while others, in accepting invitations to talk, merely repeated what others had said. Alisi posited that her students seemed to only talk because they knew she expected them to do so. She did not believe they were intrinsically motivated to engage in collective discussion.

5.4.2 Digging beneath the surface: Highlighting challenges to effective participation

Alisi’s reflection highlighted several important factors. Firstly, blame for the failure of establishing effective participatory norms had been apportioned to the students. Alisi had positioned her students as failing in their duties to enact what she had spent much time reinforcing each lesson. Superficially, the students fulfilled their duty to listen to the teacher and follow her prompts to repeat what others had said. However, without Alisi’s prompts, many students opted to remain passive onlookers. Attempts at establishing, extending, and owning the norms had stalled.

Additional clarification was required to facilitate the pathway forward for Alisi. In examination of further data, two critical factors emerged as impeding mathematics learning for Alisi’s students: namely, issues of status, and conflicting core values. Both factors are investigated in more detail in the following sections (see Section 5.5) of this chapter. They are presented in two different sections for clarity of presentation and analyses, and to demonstrate the significance of the scope and complexity of each issue. However, the emphasis is on these two factors that are inextricably linked as barriers to learning. In subsequent sections of this chapter (see Section 5.6), the resolutions for overcoming these obstacles and the actions Alisi took in providing affordances for her students in learning mathematics are presented.

5.5 BARRIERS FOR LEARNING MATHEMATICS

In this section, two significant obstacles preventing some of Alisi’s students from participating in learning mathematics are discussed. Firstly, issues of status became apparent from review of recorded mathematics lessons and how these issues manifested are described and analysed. Secondly, analysis of Alisi’s reflections on her attempts to restructure her classroom environment highlighted differences in core teacher and student values. For example, what Alisi considered were important ways to learn mathematics at times conflicted with some students’ core values and beliefs. A presentation is provided of how these issues emerged as barriers for learning for some of her students.

5.5.1 Status issues

Recorded footage from Alisi's observed lesson (Lesson five, Term 2, Week 7) were reviewed during the third group study session (Term 3, Week 1). Noted were many instances of several students dominating classroom discussions. Whenever Alisi extended an invitation to all students to participate in discussions about the mathematics, these students would talk loudly over others, offer opinions, and reject others' statements. The more these students dominated, the less the other students contributed. Alisi described these students as being loud and self-confident. She said they could either give the answer very fast or believed only their ideas were valid and so would not give other students opportunities to share during small group work or big group sharing. Alisi described these students as assigning themselves high status in class (not only mathematics lessons); as well as being given high status by the other students who lacked the confidence to challenge any of their statements, opinions, or ideas. Alisi defined status in the following way:

> *The mana[11] they have in their classroom. How they are perceived by other students, but also how they perceive themselves.*

Alisi had described status as authority, control and influence being afforded on one person by another, dependent on what the other felt or recognised about the other person, and of themselves. Accordingly, positions had been assigned reflexively by individuals and interactively by other participants. The high-status students had been positioned as having the rights to uphold and enact their authority over others in classroom discussions and the duty to maintain that authority by ignoring and discounting what others said. Conversely, the remaining students, were positioned as having low status. The low-status students had the right to defer authority to the high-status students and the duty to remain silent.

Examination of Alisi's students' initial interviews (Term 1, Week 9 & Term 2, Week 1) provided further evidence of how some students positioned themselves as mathematics learners, and how this positioning affected status. Alisi had identified four

[11] In Māori culture, mana is used to describe an individual's presence. Mana can mean prestige, authority, control, power, influence, status, spiritual power, charisma.

students as being assigned high-status by themselves and the other students in class. The following Table (see Table 16) illustrates their responses:

Table 16: Initial Interview Responses: High-Status Students

Participants: Four students: Two students identified as being from Syria. One student identified as being from India, and one student identified as being from Pakistan	
Question	Responses
Do you think you are good at maths? How do you know?	All responded that yes, they thought they were good at maths Reasons • I am confident • I know the answers first • I do lots of worksheets at home • I go to extra lessons for maths • I know my times tables • I am already good at maths • I do not need the teacher
Do you like working in a group in mathematics class? Provide reasons	• No. It is boring and there are other people • No. I get people who do not even know what they are doing • No. I am put with silly people, and I am the person who knows, and they don't listen • No. When we work like this, some people don't contribute to the work

These students all believed they were good at mathematics. Their reasons centred on the following themes: speed and accuracy of calculation; attending private mathematics tuition; and completing practice homework sheets. Apparent was their belief that support for learning mathematics came from places outside the classroom. None of these students responded positively to working in groups. Their reasons focused on their perceived abilities to learn mathematics and the inability of their peers to do so. Those they considered unable to learn mathematics were described as incompetent, silly, and unwilling to listen to their authority. None of the high-status students identified requiring assistance from peers. Additionally, one student had stated that even the teacher was not needed for support in learning mathematics. Reflexively, these students positioned themselves with high-status. The rights they accorded themselves were to continually demonstrate their authority by being the first to provide answers confidently, without support from others. Their duties included preserving their authority by safeguarding their high-status. They, in turn, positioned the other students as having low status. The low-status students, in this storyline, were assigned duties to listen and respond to the authority of the high-status students, without the rights to challenge. The storyline created was one that emphasised value in responding loudly and quickly to teacher questions (teacher-student-teacher communication patterns). Sustaining this storyline would uphold these positions.

In contrast, the following Table (see Table 17) provides insight into the ways many other students in Alisi's class saw themselves as mathematics learners.

Table 17: Initial Student Interview Responses: Low-Status Students

Participants: The remaining 20 students in Alisi's class. Of these, 15 identified as being Pāsifika	
Question	Responses
Do you think you are good at maths? How do you know?	A range of responses was received: • Yes. If I know the answer (x 5 students) • Yes. The teacher says well done • Sometimes. When someone helps me (x 2 students) • Sometimes. I am still learning my times tables • No. I do not listen • No. The teacher does not ask me (x 2 students) • No. I cannot say it fast (x 3 students)
Do you like working in a group in mathematics class? Reasons	A range of responses was received: • Yes. Someone can help me (x 5 students) • Yes. I can speak if I am with my friends • Yes. If I can be in a nice group • Yes. Other people, if they are nice, they can help me • Sometimes. It depends on the other people in the group • Sometimes. I like to help my friend if he is stuck, I can help him • Sometimes. We can talk to them if it is hard • No. I am shy in my group • No. I am too scared, the others are too quick (x 2 students) • No, I am nervous. I can't say it

The students responded in a range of ways to the first question. Those who believed they were good at mathematics attributed this to knowing the correct answer, or the teacher praising their effort. Others described being good at mathematics in instances of receiving help. Some students identified needing specific knowledge, such as times tables being necessary for achieving success in mathematics. The students who declared they were not good at mathematics stated that success was only possible if acceptable behaviour, speed, and accuracy of calculations were evident, or if the teacher invited them to share their ideas with the class. All reflexive positioning occurring within this group credited success in mathematics to external influences. The students positioned their teacher's actions as a determining factor in their perceptions of their capabilities to learn mathematics. If the teacher did not call on them, they perceived she did not consider them capable of learning mathematics successfully.

The students' responses to questions about working collaboratively highlighted how perceptions of others affected participation. Those who liked working in groups stated that collaboration was beneficial as assistance could be received or offered "if" the other group members were friends or "nice". Fear, shyness, and inadequacy in speed of calculation were stated as reasons for those not enjoying collaboration. Reflexive positioning relied on the interactive positioning of peers. Students who were positioned as "nice" had the duty to help those who needed assistance. Those who positioned themselves as needing help had the right to expect assistance from peers but would only fulfil this right if those offering help were perceived as "nice".

The storyline created by this group was one where the dominant students knew the mathematics and others did not. The low-status students conceded they needed support but were unwilling to ask for or receive help from the high-status students. The ongoing storyline formed a formidable impediment to learning.

5.5.2. Section summary

Distinct positioning between the two groups of students was evident. The high-status students emphasised belief in their individual capability to be successful at mathematics. Contrastingly, the low-status students attributed any success in learning mathematics to external support, such as the teacher, or correct answers. The high-status group made no reference to helping others or needing help to support them in learning mathematics. On the other hand, the low-status students were willing to offer or receive assistance in learning mathematics, but this was dependent on their perceptions of peers. The teacher's help was not required by the high-status group, and her inaction in drawing the low-status students into discussions confirmed this group's belief in their inability to learn mathematics successfully. Two complementary storylines developed which confirmed each groups' positioning and expectations. Some students believed they were duty bound to dominate mathematics learning time. Others withheld from effectively interacting with each other, or the mathematics. An inequitable setting had been established which created a barrier to learning mathematics.

Furthering the obstacles for learning mathematics in Alisi's class were concerns of conflicting core values. In the next section, the ways in which this impeded mathematics learning for some of Alisi's students are described and analysed.

5.5.3 Conflicting core values

A high proportion of Alisi's students identified as Pāsifika (62%). The remaining 38% students identified as Syrian, Pakistani, Fijian-Indian, Indian, Iraqi. Considering the core values of these students is important, for the following reasons: Firstly, actions or behaviours which are valued or widely accepted in one culture, may not be considered the same in another. Secondly, nearly two-thirds of Alisi's students belonged to what is considered a marginalised group in New Zealand (see Section 2.3 for further detail). In thinking about who her students were, Alisi considered what she was developing in class and whether her actions and expectations aligned with some of her students' core values and beliefs. Alisi reflected as follows:

> *I've been thinking, how do our mathematics norms fit in with cultural norms, of things like, in terms of within a cultural setting, my Pāsifika students are not supposed to talk out of turn unless they've been asked to speak. Then maybe they're not expected to share, so I just wonder how much kind of juxtaposition is going on in these kids' heads, of like, I'm supposed to do this here, but I'm not allowed to do this at home, and they want me to talk here, but at home I'm not allowed to share my opinions.*

In this excerpt, Alisi had identified a potential conflict of interest in what she was establishing, and how this was affecting some of her students. The setting she referred to in this instance was the students' mathematics learning environment at school. In this environment, from a Eurocentric point of view, taken-for-granted sociomathematical norms of students explaining their mathematics thinking, justifying their ideas, asking questions for clarity, and engaging in mathematical argumentation were expected and acceptable. Alisi now questioned whether these norms aligned with core cultural values and beliefs of all her students. She had specifically identified potential clashes for her Pāsifika students in enacting certain values: For example, from a school perspective (dominant cultural viewpoint) a core value such as respect would be enacted by students listening to the teacher, complying with social norms the teacher stated were acceptable, and enacting what the teacher stated was expected. In the current study, Alisi spent considerable time stating the norms and expectations for learning at the start of each lesson and expected the students to fulfill these expectations. However, she had not considered that for her Pāsifika students, being respectful was enacted by students not talking without being invited to; or looking directly at others, particularly, older people while they were talking; or questioning what others say; but rather, remaining respectful and humble by being silent. She acknowledged that her students were expected to enact certain values in specific ways at school but that these expectations might not be acceptable in their homes. Alisi specifically identified that she expected her students to talk in class and share their ideas freely, but for many of her students this would be considered unacceptable behaviour at home. She continued to reflect how differences in cultural values were a potential barrier to learning mathematics for some of the students in her class:

> *But, it's even that like the difference between say maths norms and cultural norms, but it's any kind of norms, it's like the difference between maths norms and behavioural norms; behaviour norms you don't challenge, like if you're put on timeout, you're not allowed to challenge that, and yet in maths we're expecting them to flip that switch and go, actually, I'm allowed to challenge now, and I'm allowed to question…and that whole assumption that what we've said is clear to them.*

Alisi had recognised the difficulty some students might have in reconciling different core cultural values. She had identified that socially, in class, it was not appropriate for students to challenge authority, yet she had expected them to understand that challenging mathematical ideas was acceptable. Furthermore, Alisi had identified that her expectations may not have been clear and concise. In conflict with some of her students' core cultural values were the social norms widely accepted as taken-for-granted in school. Essentially, Alisi had highlighted that she expected some of her students to flexibly travel between two-worlds, namely, home and school. Her critical reflection served as a significant catalyst for transforming her subsequent pedagogical actions.

In the preceding sections, two inextricably linked factors had been identified as significant obstacles for learning for the marginalised students in Alisi's class. Alisi had recognised and acknowledged how assumptions she had made about her students had impeded the establishment of the learning environment she had envisioned.

The next section presents the opportunities Alisi created which served to reposition her marginalised students to access mathematics learning.

5.6 OVERCOMING BARRIERS: CREATING AFFORDANCES FOR LEARNING

In this section, the deliberate and considered actions Alisi took in removing barriers for learning are presented. These actions included reconciling conflicting core values; extending prosocial norms; addressing status issues; and transforming grouping practices.

5.6.1 Reconciling conflicting core values

Following the third study group meeting (Term 3, Week 1) Alisi took immediate action to attend to the conflicting cultural core values that had emerged as an obstacle to learning. In class (Lesson six, Term 3, Week 2) she invited her students to participate in a discussion where she outlined the obstacles to learning they faced. The following extract highlights key elements from this discussion:

> *We need to work together to get these challenges we're seeing out of the way. We have agreed that our groupwork is not working well right now. Too many are just sitting waiting to repeat stuff other people have said. Some of you are talking, but it is not being done in a respectful way, and other people don't have confidence to share if they're put down when they speak. We've listed the things that are problems, lack of respect and not joining in, not participating. We have to fix this together, otherwise we are going to keep struggling to get to the maths. If we can't share our ideas respectfully, we are going to miss out on the learning we could be getting to.*

Alisi addressed the challenges impeding mathematics learning directly. Respect was identified as a key component of productive discourse. She stressed that without respectful collaboration, barriers to mathematics learning would endure. By utilising the collective term, "we", Alisi positioned herself as part of the group facing these problems. This deliberate positioning action accorded them all the duty to respect others' ideas, and the right to speak freely in discussions. A potential storyline was created wherein everyone engaged respectfully in learning mathematics and ideas were freely exchanged. The discussion continued and Alisi presented a possible solution, as highlighted below:

> *We have to find a way of working together successfully. I want you to think about how we could work together as a whānau[12].*

Alisi had again positioned them all as one group charged with finding new ways of collaborating. She identified the core Pāsifika value of family as a metaphor for working together. Continuing, they identified key elements of what working together as family might look and sound like. Alisi stressed they focus on prosocial elements only. Over several sessions, in class, her and the students persisted at brainstorming and examining values all agreed were mutually meaningful, relevant, and worth upholding. The following Table (see Table 18) outlines their ideas:

Table 18: Working Together as Whānau

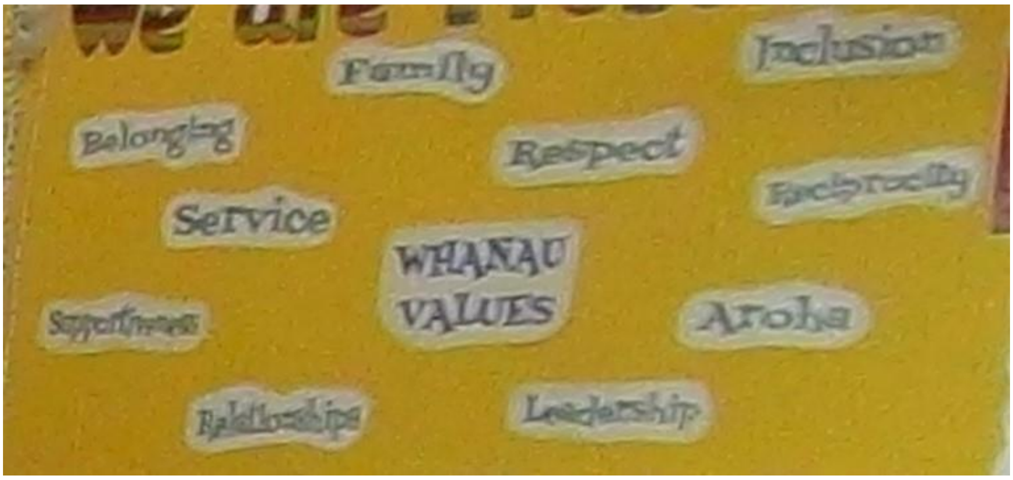
Working as whānau looks and sounds like this:
• Being respectful • Everyone belongs to the family

[12] Whānau is a Māori-language word for extended family.

- Being kind when you speak
- Being kind when you do something
- Listening to what other people say
- Thinking about what other people say
- Responding with respect when other people share their ideas
- Helping someone if they are confused or stuck
- Asking for help if you are confused or stuck
- Being patient
- Showing love
- Being humble

Using the metaphor of family had provided a mutually understood image for working together. By dissecting key components of family, several core Pāsifika values were identified that were relevant to all the students in class, regardless of cultural background. Namely, respect, being of service, family, reciprocity, aroha[13], belonging, inclusion, relationships, leadership. These values were ones all students could identify with. In class, a wall display (see *Figure* 3) was created which served to remind everyone what they had agreed was worth upholding.

Figure 4: Alisi's Classroom Wall Display of Whānau Values



Alisi had provided opportunities for her students to collaboratively develop mutually agreed ways of engaging in collective mathematics activity. Moving forward required these values to be enacted as taken-for-granted actions for accessing learning. Utilising these values as a foundation for transforming the participatory interactions in class, Alisi began extending the prosocial norms.

[13] Aroha is a Māori-language word meaning love.

5.6.2 Extending positive social norms

Over several weeks (Term 3, Weeks 2-4) in class, Alisi utilised these core values to transform her communication practices. Drawing on these and utilising prompts from the Social Domain Framework (see Table 2), she was able to (re) establish high expectations for learning. Alisi describes this process in the following extract from a semi-structured interview following an observed lesson (Term 3, Week 4):

> *I realised it was my job to make the norms explicit. I did this by focusing on positive behaviours and norms that students were exhibiting, e.g., listening, questioning, explaining etc. But I had to do this every day and all through the day.*

Alisi had identified the need for constant reinforcement of the expectations and behaviours for learning. Students' attention was consistently drawn to productive ways for engaging in learning and collaboration. Alisi stressed the need for all students to engage effectively with each other and the mathematics, regardless of culture, gender, language, or disability. Her reflection continues below:

> *To get them to all join in, it was my job to make sure they understood that the expectation was that everyone participates. Participation was expected constantly throughout the day and was reinforced through the regular use of group work. It became just expected, everyone participates, and they got that. But only after we broke everything down and started again.*

Alisi had recognised her pivotal role in establishing and extending effective participatory norms. She consistently positioned students to work together in groups to engage in mathematics activity. She regularly reinforced the expectation and focus on participation. Alisi was able to extend an environment where prosocial norms could be freely enacted, and students could engage in intellectual and stimulating productive discourse.

Alisi was developing effective communication practices that focused on active engagement with mathematics. Alisi now turned her attention to creating further affordances for learning by resolving the issues of status. The next section illustrates several examples of how Alisi raised the status of marginalised students by assigning competence. Also presented is how she used a tool designed to support teachers in assigning competence in class.

5.6.3 Raising status by assigning competence

A framework was developed in the third study group session (Term 3, Week 1) aimed at mitigating status challenges by assigning competence (see Table 4). In class, in the ensuing experimentation phase of the current study (Term 3, Weeks 2-4). Alisi systematically began using this tool to resolve status issues in mathematics lessons. She continued to use this tool as the current study progressed into phase five (Term 3, Week 6 to Term 4, Week 2). This section presents examples of this process.

The example below shows how Alisi raised a marginalised student's status by acknowledging his contribution. In Lesson eight (Term 3, Week 8), Alisi had selected one group of four students to explain their mathematical reasoning during the big group discussion. The students had worked in small groups on collective mathematics activity. The task involved working out all the possible scores that could be made if three darts were thrown at a dartboard with three rings. A hit on the outer ring scored 9 points, the middle ring scored 5 points, and the centre ring scored 2 points. Three of the four students confidently took turns explaining parts of the group's strategy in the big group discussion. One group member, Anjelo, stood to one side throughout. At first it appeared he was not contributing to the group's explanation. However, it soon became apparent he was actively engaged with the explanation. He listened intently to what his peers were explaining. Each time the explainer talked about the numbers or totals, Anjelo would whisper *"the score from throwing the dart, don't forget to say"* or *"you have to say the name of the boy throwing the dart, use the story".* Alisi was prompted (by the researcher) to call a pause as an opportunity to assign competence (by acknowledging the student's intellectual contributions) to a marginalised student had been presented. The following extract illustrates what happened next:

Lesson 8. Term 3, Week 8	
Raising status by acknowledging student contributions	
Participants	Dialogue & Actions
Alisi	*I want to pause the lesson here. I have noticed something really smart that has been happening while this group explains. Anjelo, I want everyone to know that I have noticed your contribution to this group's*

	explanation. Firstly, you have been doing all the things that really smart mathematicians do. You have been listening carefully to what your group is saying, and you have been listening very closely. I know this, because every time someone in your group talked about the numbers in the problem, you would remind them to talk about the story, the real problem you were working on together (**Alisi** paused and waited. Anjelo's facial expression was wide-eyed and surprised. The other students were watching Anjelo. **Alisi** continued speaking) *What Anjelo is showing us is that good mathematicians are always paying attention to what is being explained and trying to make sense of it. Anjelo has reminded us that if we want to make sense of this problem, we need to always be bringing the problem in, the story, not just the numbers. Well done, Anjelo, you are showing us how to be really good at mathematics* (Anjelo is smiling broadly. His group members are also smiling)

By pausing the action and making her observations public, Alisi had positioned Anjelo's general capabilities to do mathematics. In doing so, she had provided an opportunity for a marginalised student to be positioned as being capable of contributing legitimately to constructing a reasonable mathematical explanation. By continuing to do this regularly, students would be given opportunities to realise that learning mathematics comprised of more than calculating the numbers correctly.

In another lesson (Lesson nine, Term 4, Week 1), Alisi was afforded an opportunity to raise status by pressing the students to justify their thinking. In class, she wrote several mathematical statements on the board and asked the students to prove whether they were true or false. The following excerpt outlines a section from this episode:

Lesson 9. Term 4, Week 1	
Raising status via expectations for students to justify their reasoning	
Participants	Dialogue & Actions
Alisi	*How can we prove that 2 + 3 = 5? Because we all agreed it was true, but how do we prove it. Turn and talk to your partner and see what you come up with* (The students talk to each other and make notes. Each pair has a one piece of paper and one pen between

	them. **Alisi** observes the students. After a few minutes she calls on one pair to respond)
Farid	*I said its true because you just add them, so you know*
	(**Alisi** writes what they are saying on the board and revoices their statement)
Alisi	*Ok, so Farid said it is just true. Who else thought the same as these two?* (Many students raise their hands)
	Ok, so think a bit more because, remember, just because we say it is true, it doesn't really prove that it is true. Have another go and see if you can prove it is true
	(The students work together; **Alisi** observes what they are saying and recording and then calls on two students to share)
Alisi	*Fetu and Lotu, tell us how you can prove that 2 + 3 = 5*
Fetu	*We wrote under it, if 2 + 2 = 4, then 2 + 3 must be 5*
Lotu	*Yes, because you are adding 1 to the other 2 to make it 5*
Alisi	*Everyone, think about what Fetu and Lotu have shown us. What do you think?*
	(The students share back that they agree with what the boys have represented as proof)
Alisi	*These two boys, Fetu and Lotu have proved to us how they know that 2+3=5. They have used other bits of mathematics that they know to justify their thinking. Justifying thinking is a really important way of showing you are good at maths. If you can justify, you can prove your thinking. It means you really understand the maths. Great proving, both of you*

Alisi had taken the opportunity to assign competence to two marginalised students by asking them to justify their reasoning. Earlier in the episode she had recorded the statement that most of the students agreed was proof. Alisi had noticed that these two students had justified their thinking differently. By asking the group to think about how they could prove the statement, she positioned them as having legitimate contributions to make to the mathematical discussion. The students had been positioned with the duty to respond to Alisi's position, but also with the right to refuse to participate. The boys accepted their duty and refuted the right they had to not respond. Alisi was developing a storyline where all students were positioned as capable of effectively reasoning with mathematics, and as legitimate contributors to learning mathematics.

Continuing to utilise opportunities to mitigate status issues, Alisi continued to create affordances to do so. The following example (Lesson 10. Term 4, Week 2) presents an example of how competence has been assigned via accountability: In class, Alisi continued to develop communication practices that focused on active engagement of and with mathematics. While the students were working in small groups on collective mathematical activity, Alisi monitored their actions. She roamed from group to group noticing and responding to what she heard and saw. A status issue became apparent when Alisi noticed a group where one student was becoming frustrated in his multiple attempts to explain his mathematics reasoning to his peers. Alisi paused the lesson, recognising an opportunity to raise status by accountability. The except below captures the dialogue and actions that followed:

Lesson 10. Term 4, Week 2	
Raising status via accountability & framing capability	
Participants	Dialogue & Actions
Alisi	*Something we all need to work on is making sure that everyone understands what we are explaining. They want to hear what your idea is, what your strategy is. So, how do we do this?* (She sweeps her gaze across the group)
Ilisapesi	*Give everyone a chance to share the pen*
Koloa	*Yeah, so we are not hogging the book or the paper, we need to share*
Laveni	*Check that everyone understands*
	(The students stop talking and look at **Alisi**)
Alisi	*Who can explain more what that one is all about?* (**Alisi** sweeps her gaze across the group)
Eseta	*If someone is confused, you need to say it* (**Alisi** waits in silence)
Laveni	*You have to show it, so they get it*
Alisi	*Yes, so, sometimes when we are working in our small group and someone doesn't understand, and you know it and are checking that everyone understands. And they're sitting there, and you know they don't get it. Now what as a group can you do? Instead of saying, do you understand? Yes, or no? What can you do as a group to help that person understand?* (no one

	speaks for a while. The students look at **Alisi** and say nothing. **Alisi** then speaks further) *Turn and talk to the people around you about this* (After 2 minutes, **Alisi** asks a group to share their ideas)
Hiva	*Explain it in a different way*
Alisi	*Yes, explain it in a different way. And I have noticed that we aren't very good at that sometimes, sometimes we get into our little groups and say this is our way. But actually, there are lots of different ways and sometimes, people aren't always going to understand your way and you need to think of a different way to explain it, or a different way to show what you are thinking. It is your job to make sure someone understands what you have done.*
Alisi	*What about if you are the person confused? What do you do?*
Tomasi	*You ask them a question*
Koloa	*You ask the person telling, you ask them what they are talking about*
Alisi	*That's right. Because we only learn if we ask questions when we are confused and if we can explain so our group understands*

Alisi deliberately paused the action in class to attend to individual responsibility and group accountability. She had invited the students to participate in a discussion about what were vital actions to ensure everyone understood the mathematics. By stressing this problem belonged to the group, and using "we" throughout the discussion, Alisi moved towards resolving the disparity in status. Her positioning actions had assigned everyone, including herself as having the duty to explain mathematical reasoning clearly, and to modify explanations if needed. All were positioned as having the right to expect clear explanations, or to ask questions if confused. Her utterances as positioning acts constructed a new storyline where individual responsibility and group accountability were emphasised. In addition, the resulting storyline was also one where "all" students were recognised as capable of explaining clearly and asking questions if confused. In this storyline, if all students understood their responsibilities and capabilities, inequity could be mitigated.

At the same time as assigning competence in various ways to attend to status issues, Alisi transformed her grouping practices. During the second study group meeting (Term 2, Week 6) Alisi had participated in completing the strength-based grouping activity (see Section 4.5.2). The next section focuses on the data collected during the experimentation phases following this session (Term 2, Weeks 7 to Term 3, Weeks 2-8). The data presented focus on those regarding Alisi's transformed grouping practice and the impact this had on the teaching and learning of mathematics in her class.

5.6.4 Raising status through strength-based grouping

Following the second study group meeting (Term 2, Week 6), Alisi implemented changes in her grouping practices and continued to employ these changes in class. Note must be made that grouping was dynamic. This meant flexibility-changes to grouping could be made in-the-moment or for each lesson. The episodes highlighted in this section illustrate the impact strength-based grouping had in Alisi's class. Specifically, the role a strength-based approach played in raising status for a group of four marginalised students is illustrated.

In class, (Lesson 8, Term 3, Week 8) Alisi listed the strengths of the four students and other characteristics considered important. The Table below (see Table 19) shows Alisi's completed grouping:

Table 19: Alisi's Grouping Record

Student	Strengths	Other
Langi	Good listener, friendly, and tries to work with her group	Very low-status, high learning needs-short memory, learned behaviour to talk very slowly, or off topic so others give up listening or taking the pressure off what she is being asked
Marama	Works well with others, listens, good at explaining others' strategies	Prefers to draw/use materials to prove she is right
Tomasi	Looks for connections to previous big ideas, always trying to make sense of	Learning to check for reasonableness of different strategies

	mathematics. Strong number knowledge, works well with anyone, great patience, and asks questions of anyone	
Fetu	Strong reader and can use that to understand and explain the problem. Beginning to share back with group	Low self-esteem and confidence in mathematics. Will not take risks and attempt to answer the problem using his own strategy. Low number knowledge

Alisi's rationale for grouping these four students together for this particular lesson (Lesson 8) is captured in the following statement:

> *I wanted to try a mix of students who were strong in different areas, but also had some other issues affecting their participation in group activity. So, I thought about who would actually get on with each other, work together, no matter what their strengths were, so the social side of things.*

Alisi's strength-based approach to grouping included considering the social dynamics of the students. During the lesson (Lesson 8) Alisi expressed surprise at the success of this group's interactions. Her observation is noted as follows:

> *I did not ever expect to see such a change so quickly, just because of how I grouped them. Langi has never worked this well in a group before. I can't believe what I see happening.*

The effect of the new grouping strategy exceeded Alisi's expectations. Specifically, Alisi seemed most surprised at one particular student's transformative participation. To investigate further, the researcher conducted an semi-structured-group interview with these students immediately following the mathematics lesson (Lesson 8). The interview excerpt follows below:

Semi-structured group interview (Term 3, Week 8; following Lesson 8) Langi, Marama, Tomasi, Fetu	
Participants	Dialogue & Actions
Researcher	*How did you feel working in this group today, just have a bit of time to think?*

Langi	Excited
Researcher	*How come?*
Langi	*Because it was like, it felt like I was in groups I hadn't been in before*
Tomasi	*I was excited and happy, because two of us have been in the same group before, but not with her* *(Pointing to Langi) Yeah, and when she came it got easier*
Marama	*We're good at working together because we always share our strategy*
Fetu	*I felt really good working in a team. When Langi came in, she made it kinda logical and these ideas popping up, it was really good as we were working as a team*

In this episode, the students had identified the strengths Langi brought to the group. Her strengths as a good listener and as someone who tries working with a group emerged as resources for developing the mathematical understanding of the group. The other group members specifically identified Langi's actions in supporting them to make sense of the mathematics, and in turn, explain their mathematical thinking to others in the class. At no point did issues of status arise for either Langi or the other students; yet Alisi had described Langi as having very low-status and high learning needs (which she had). It could be argued that her short memory emerged as a strength within the dynamics of this group, as her ongoing questions for clarity propelled the group members to dig deeper into their mathematical thinking to refine and justify their mathematical explanations.

The interview continued. The group not only identified Langi's strengths, but the strengths of all members of the group as outlined below:

Continuation of semi-structured group interview (Term 3, Week 8; following Lesson 8) Langi, Marama, Tomasi, Fetu	
Participants	Dialogue
Researcher	*Do you think you are good at maths, Marama?* (Marama drops her gaze to the floor and shakes her head)
Marama	*No* (Langi begins giggling and Fetu and Tomasi both smile broadly)

Researcher	*Why are you laughing, Langi?*
Langi	*Because Marama IS good at maths, she comes up with ideas, she is like the expert*
Fetu	*Yeah, and like, she can explain, she's not shy and keeps it in*
Tomasi	*Yeah, she explains clearly*
Researcher	*Can you understand when she explains?* (Tomasi, Fetu, and Langi all nod their heads. Marama is smiling broadly and looking at them)
Tomasi	*When Marama joins in, she always wants to help, she likes to help with things that make it logical, and she makes stuff easy, and she is really good at mathematics*
Langi	*She helps us break stuff down. If it was something hard, she'd be like "put it in a different way"*
Researcher	*Let us talk about Tomasi. Why is he good at maths?*
Marama	*When he is explaining he goes back to the problem and double checks it to make sure we're getting it. He's always in a clear voice and he would be like, oh like let's go back*
Fetu	*Yea, he would say, guys let's go back and make sure we are correct, or is it really that?*
Researcher	*What about Fetu?*
Tomasi	*He's good at maths, because whenever we don't have it, he's like oh, I have an idea for that.* *When the paper is just like clean and white, he will just pop up with a unique plan for us to get there, and he will just write it down.*
Marama	*He explains it to us, he's good at explaining*

Notable was the effect that Alisi's positioning of her students had. Her strength-based grouping of these four students, had positioned them with the means to recognise and acknowledge strengths others brought to learning. Furthermore, these students utilised these strengths as resources for learning. The students' enactment of what it meant to be successful at mathematics finally aligned with what Alisi had initially conceptualised (see Section 5.3.1). The students had identified that being able to explain the mathematics clearly was an important component of learning mathematics successfully. In addition to explaining clearly, was the belief that it was important to be able to explain your thinking in a range of different ways if others didn't understand.

Furthermore, the students highlighted that thinking logically and sequentially supported them in reasoning with the mathematics. The students added that rechecking, finding proof and justifying were important ways of helping each other understand the mathematics. Here, too were references to the importance students placed on the core values the class had identified earlier. Being of service by helping a family member served to position the student to enact important sociomathematical norms.

As Alisi began extending her grouping approach, she highlighted the thought and effort that she applied when grouping her students from a strength-base. The following extract (data captured from her final teacher interview; Term 4, Week 3) illustrates Alisi's critical reflection on the changes she noticed in what she thought of her students' capabilities to learn mathematics:

> *To begin with, it was never random, but it was more ability based, I think. Who are my higher ability kids, can I put them with my medium and low? What behaviour issues do they have-there wasn't the same thought process behind what strengths they bring. The thought process that I put behind grouping now is more purposeful. And their status, I didn't really think of the kid's status as such. Now I feel I can group anyone, but they're more able to be grouped with different people with different status.*

Alisi had identified that a key facet to changing her grouping strategy was a shift in her thinking and prior beliefs about some of her students. Her grouping selection shifted away from ideas around traditional mathematical strengths to social skills. Alisi continued further to describe what she had noticed about her students' statuses:

> *The kids look at themselves differently. Their attitudes have shifted, and I think socially we've seen a huge improvement in those kids*.

Alisi had identified considerable shifts in her students' statuses overall. Not only was she viewing her students in new ways, but the students themselves had noticed a change. Alisi was pressed to consider how these changes had transpired. She highlighted the effort she had put into consistently facilitating direct discussions about status and the actions she and the students took to transform them.

> *Real unpacking of status, what that is, and their perception of status. We talked about how you feel, what you think about your own status, and then what you think your peers think your status is. Then they had to go around and tell each other what their strengths were, that really opened their eyes to actually, I am a valued member of this group, I do bring other strengths that I didn't even know I had, but people see*

that in me. So, it really united them a bit more. In hearing feedback from their peers, is so much more valued than just the teacher saying well done, you did good.

Alisi had identified that as part of the process of establishing new ways of doing things in class, she needed to discuss these directly with her students. In this process, it became apparent that some students held deficit view of themselves and that, for some, individual perception was low. By providing opportunities for students to freely acknowledge each other's strengths, individual students were afforded the means to reflexively (re) position their beliefs about their capabilities. Through this process, many students had been interactively (re) positioned with higher status. In this transformed positioning, students now had the responsibility to view each other from a strength-base. Students had the right to reject others' view of them, but none did. A new storyline had been created which centred on acknowledging others from a strength base. Within this storyline, students had the right to enact those strengths and expect their peers to do likewise. Verbal and non-verbal acts consistently demonstrated enhanced self-awareness. For example, students smiled broadly upon hearing what others perceived their strengths to be.

Not only had Alisi revised her perceptions of her students, but the students themselves had been given the chance to do so. By providing this opportunity, the students were able to begin viewing each other as legitimate and valued resources in learning mathematics.

5.7 CHAPTER SUMMARY

The focus of the work Alisi undertook in the current study was to develop an equitable learning environment which provided multiple opportunities for all students to access mathematical reasoning. Although Alisi participated in the same way as Meliame, her journey took a far more circuitous route. One of the first challenges to arise was dysfunctional group dynamics. Despite norms being agreed upon initially, Alisi struggled to see these enacted successfully. Marginalised students found it difficult to engage in collective mathematics activity as dominant personalities sabotaged collaboration. Two significant barriers to learning mathematics were identified, namely, conflicting core values and issues of status. The actions taken in deconstructing these obstacles were deliberate and systematic. The metaphor of family was established as a firm foundation for students to build effective participatory practices. Issues of status were directly and explicitly examined and resolved. Assigning competence utilising a supporting framework and approaching student grouping from a strength base were effective tools in developing the equitable learning environment Alisi had initially conceptualised.

This chapter presented the Case of Alisi and the journey she took in providing multiple opportunities for all students to actively engage in mathematical reasoning. The next chapter presents the discussion with relevant links to literature.

CHAPTER SIX

DISCUSSION

6.1 INTRODUCTION

This chapter reflects on important aspects of the findings in relation to the primary objective of the study and discusses these with reference to the literature. The key aim of this study was to explore how teachers construct equitable learning environments in primary school mathematics classrooms. The three research questions that guided this study are:

1. How do teachers (re)construct beliefs and classroom practices to position all students to learn mathematics?
2. How do teachers address status?
3. How do teachers delegate mathematical authority to all students?

The data collected during the current study are integrated and discussed with a specific focus on the themes that emerged. These themes are discussed alongside relevant literature. Section 6.2. discusses the findings in relation to how teachers can construct equitable learning environments in primary school mathematics classrooms. This includes specifically examining the (re)construction of teacher beliefs and classroom practices that provided all students with opportunities to learn mathematics. In addition, the challenges faced in enacting teaching change and the actions taken to support teacher change are explored. The final part of this section discusses a strength-based grouping approach. Section 6.3 addresses how students were (re)positioned for learning mathematics when issues of status emerged as a barrier to learning. The importance of drawing on the students' core cultural values and beliefs is also discussed. Section 6.4 explores the process of delegating mathematical authority to students. This includes a focus on developing and embedding important social and sociomathematical norms; assigning competence; developing individual student responsibility and group accountability; and developing mathematical practices. Finally, Section 6.5 presents a summary of this chapter.

The two teachers in the current study took different journeys to enact change in their practice. One teacher's pathway to (re)structure her classroom environment followed a linear trajectory, aligning with the timeline of the current study. She willingly participated in all aspects of the current study. Importantly, her commitment to critically reflect on aspects of her practice, with support from research articles and in-class-support from the researcher, served as conduits into each phase of the current study. Furthermore, by utilising each framework as guidance for (re)structuring her classroom participatory norms, the teacher provided all students with equitable opportunities to learn mathematics. As she enacted change in her teaching practice, her students willingly accepted new positions and collectively created new storylines reflecting reform-oriented mathematics classrooms.

The other teacher's journey took a different path. Whilst she too willingly participated in all aspects of the current study, significant obstacles to enacting change emerged. Status issues amongst her students challenged the (re)construction of a learning environment where all students had equitable opportunities to learn mathematics. To overcome these challenges, this teacher critically reflected on her practice in class and in the study group meetings. In class, she used the frameworks as supporting tools for in-the-moment noticing and responding to students' engagement and participation. Through a more circuitous route, she enacted several constructive elements that effectively addressed the barriers to equity and served to (re)position the students for learning.

The pedagogical actions both teachers took highlighted several common themes that are discussed in this chapter. In the next section (Section 6.2), the first theme from the current study is presented. This theme addresses the question of how teachers (re)construct beliefs and classroom practices as a means of positioning all students for mathematics learning.

6.2 (RE)CONSTRUCTING CLASSROOM PRACTICES AND BELIEFS

In the first phase of the current study, data were collected to ascertain initial teacher beliefs and classroom practice. The design of the subsequent phases focused on the teachers' critical reflection of their beliefs about teaching and learning mathematics and how these beliefs were enacted in class. Analyses of the teachers' reflections highlighted a belief in a sociocultural approach to teaching and learning mathematics. Deeper analyses through classroom observations further revealed that many of the teachers' beliefs were espoused and not enacted. This section discusses these findings and hypothesises possible reasons for why constructive teacher beliefs were not enacted in practice (see Sections 6.2.1 and 6.2.2). Section 6.2.3 presents the discussion of how the teachers were supported to enact change. The final part of the section (see Section 6.2.4) discusses the development and use of a strength-based approach to grouping that supported (re)positioning all students equitably for learning mathematics.

6.2.1 A sociocultural approach: espoused and enacted teacher beliefs

Initially, in interviews and discussions, both teachers in the current study identified several common beliefs about teaching and learning. One of these beliefs was that collective reasoning (student groupwork) was an important component of learning mathematics. This belief aligns with a sociocultural frame where emphasis is placed on teaching and learning (mathematics) through social interaction (Hunter, 2007b; Sfard & Kieran, 2001). Another belief expressed by the teachers was that all students should be encouraged to ask questions and explain their mathematical thinking. These ideas parallel other research (e.g., Bell & Pape, 2012; Boaler, 2006) advocating for students to engage in mathematical discourse to develop and make connections with different mathematical ideas. While both teachers expressed constructive beliefs about teaching and learning mathematics, initial classroom observations highlighted misalignments with these beliefs and enacted teacher practice.

In class, although students were grouped to learn mathematics collaboratively and encouraged to engage in discussions, discourse patterns followed more traditional forms characterised by teacher-student-teacher patterns of communication. Within sociocultural learning environments, teachers typically refrain from adopting these forms of communication and instead, strive to position students to engage in rich mathematical discourse and collective sense-making practices (Franke et al., 2015; Hunter, 2007a). In both participants' classrooms, while students were encouraged to take part in discussions, none were observed to critically reason with each other or the mathematics; rather, the discourse was dominated by teacher-voice.

In addition, both teachers maintained mathematical authority during mathematics teaching and learning; or delegated authority to those students who they considered "knew the answers and could "do" the mathematics". Like findings in other research (e.g., Hunter & Anthony, 2011), in the current study the positioning of the teacher and certain students as unchallenged mathematical authorities meant that what mathematical know-how got shared and by whom was largely predetermined. Conversely, other students were positioned as passive recipients of mathematics which parallels what other researchers have found (e.g., Goos, 2004; Hunter, 2010).

Initially, both teachers stated that all students were capable of learning mathematics successfully. These beliefs positioned them as holding high expectations and wanting to provide equitable learning experiences for all students. Educators who strive toward equity intentionally pursue pedagogical practices that support students to view themselves and others as capable of mathematical reasoning (Adiredja & Louie, 2020; Esmonde, 2009b). The question to be asked here is what had prevented these teachers from enacting their constructive beliefs about the teaching and learning of mathematics?

In the next section (Section 6.2.2) two possible reasons are hypothesised as to why these teachers' initial classroom practice was misaligned with their expressed beliefs about important components of effective mathematics teaching and learning.

6.2.2 Challenges to enacting teacher change

In this section, two possible reasons for the mismatch between teacher beliefs and classroom practice are discussed. Firstly, the challenges teacher face in implementing policy-into-practice and secondly, how the inherent beliefs teachers hold about the nature of mathematics itself affect their pedagogical approaches.

As highlighted by previous research (e.g., Hiebert, 2013; Webel & Platt, 2015) affecting change in teachers' approaches to teaching mathematics can be difficult. While policy documents (e.g., ACARA, 2021; Royal Society Te Apārangi, 2021) clearly outline intent for best practice, many of these documents do not specify how these changes can be implemented. So, while policy aims for equitable and excellent outcomes for students, support for teachers in enacting these aims is required. In the current study, the two teachers specifically identified their lack of knowledge and experience in how to construct mathematics learning environments reflecting the aim of the New Zealand Ministry of Education (Ministry of Education, 2018). In striving to support the teachers in specific ways to achieve the policy goals of equity and excellence, the findings of the current research add new knowledge about how to provide possible pathways from policy-to-practice.

In addition to challenges in implementing policy-to-practice, the teachers also voiced beliefs in the disciplinary nature of mathematics teaching and learning. For example, while the teachers espoused the importance of students working together to reason about mathematics, they believed that students needed to individually know the answer and strategy, so that one person in each group knew what to do and could then show the others, rather than believing that with genuine collaboration, the students could develop a solution to an unknown problem. The teachers emphasised that if the students "did not know", then the teacher was obliged to tell them. These ideas about disciplinary obligations hindered the teachers from enacting their constructive beliefs of mathematics teaching and learning as a cultural activity (as was evidenced from initial classroom observations). These findings parallel Webel and Platt's (2015) investigation into why teachers' expressed goals or beliefs did not match with their classroom practice. These researchers concluded that teacher obligations often extended beyond individual beliefs and were grounded in perceptions of how mathematics should be taught. Similarly, in the current study, while the teachers had expressed a desire for more student agency; fulfilling their need to ensure the students "knew what to do and how to do it" resulted in teacher-directed mathematics instruction.

The findings in the current research highlight that enacting policy intentions cannot be achieved solely by providing teachers with documentation. Rather, by purposefully guiding teachers to understand the cultural nature of teaching and learning mathematics as intended by reform policy documents, teachers can be supported to enact change. In the next section (see Section 6.2.3) the ways in which the teachers were supported to enact changes in their practice are discussed.

6.2.3 Supporting and enacting teacher change

As shown in this thesis, creating an equitable learning environment for mathematics required firstly, a focus on (re)constructing the social environment. This process involved attending to three interrelated components. Namely, social support (social norms); engagement support (communication practices); and organisational support (grouping for learning). Providing specific scaffolding for the teachers in how to construct these components was important. Initial support comprised of reviewing research documents together in the study group meeting. Over the course of this investigation, several articles (see Table 1) were used in these meetings to introduce the teachers to relevant research literature that focused on how students can be positioned to learn mathematics effectively. Professional learning for teachers that centred on evidence-based research parallels actions evidenced in other studies (e.g., Hunter, 2007b; Stein et al., 2008). The current study builds and extends on these studies by the careful design and use of several frameworks aimed at connecting theory-to-practice (see Tables 2, 3, 4, 5). Essentially, while the research articles provided the evidence-based theory for best-practice in teaching and learning mathematics, the frameworks provided specific structure for the teachers in how to enact these concepts and ideas.

In class, the teachers used the frameworks to implement specific pedagogical actions. The teachers' instructional decisions about "what" to engage in and "how" to do so were continuously modified in response to their students' reactions. This kind of responsive teaching builds on what has been documented previously in research (e.g., Alton-Lee et al., 2012; Hunter & Hunter, 2018a; Jacobs & Empson, 2016) by adding to the field of responsive and adaptive teaching expertise from a New Zealand perspective. In addition, each framework was uniquely designed to fulfil the specific needs of the teachers as they arose while implementing theory-into-practice. Providing detailed structures to support enacting policy-to-practice offered the means for the teachers to begin (re)constructing their classroom practices and (re)positioning their students for learning mathematics.

As shown, the teachers' actions to position students for learning were deliberate, considered, and consistent. Emphasis was placed on supporting students to (re)consider what learning mathematics involved. High expectation was placed on all students learning mathematics by participating in mathematical discussions (collective mathematical reasoning). To position all students to engage in collective mathematical discussions, the teachers modified their discourse patterns, shifting away from the initiate-respond-evaluate model of teaching-centred mathematics classroom towards a discourse-centred approach. The potential to enhance students' mathematical learning through a discourse centred approach is firmly documented in research (e.g., Chapin et al., 2013; Franke et al., 2015; Stein et al., 2008). The current study expands on the body of previous research by providing a multi-level analysis of teacher and student actions to construct such mathematics learning environments.

One particular action utilised by the teachers in the current study, was the use of talk moves such as reasoning, repeating, revoicing, adding-on, and waiting to establish the foundations for discourse-centred teaching and learning. As has been well established in other research (e.g., Chapin et al., 2003; Chapin & O'Connor, 2007; Michaels et al., 2008; O'Connor & Michaels, 1993) the deliberate use of talk moves supported teachers in developing effective communication patterns promoting mathematically productive discourse. The findings of the current research specifically highlighted how wait-time, in particular, served to create an intellectual thinking space for all students. Providing all students time to think individually followed by collective reasoning embedded and maintained important communication practices that centred on engagement with mathematics.

By (re)positioning the students in the current study to communicate their mathematical reasoning in different ways, a new storyline evolved in which mathematics teaching and learning were reconceptualised. In this new storyline, the teacher and students communicated their ideas and understanding about mathematics freely. In sharing ideas, students were positioned to reflect on and reason about their own and others' mathematical ideas. These acts of considering one's own and others' ideas provided affordances for students to make connections or to develop new ideas This new storyline reflected and aligned with the ideals of the mathematics reform movement as outlined in international and national research (e.g., ACARA, 2021; RAND Mathematics Study Panel, 2003; Royal Society Te Apārangi, 2021).

In the current study, maintaining this storyline relied on it being jointly sustained. To jointly sustain this new storyline, organisational support was required. In the next section the ways in which organisational support was developed are discussed.

6.2.4 Grouping for equity

Organisational support in the current study involved creating learning structures that provided opportunities for all students to learn mathematics. This required considered attention to classroom grouping practices. Initially, the teachers grouped their students into what they termed "mixed ability" groups. Teachers utilised school-wide test scores to group students into groups where at least one student "knew and could do the mathematics" along with other students "who were not achieving at mathematics". These grouping practices align with well documented international and New Zealand research (e.g., Anthony & Hunter, 2017; Dixon et al., 2002; Marks, 2012) that highlight ability grouping in mathematics classrooms as a common practice.

The ability grouping practice of both teachers in the current study emphasised their beliefs that some students were better than others at learning mathematics. The belief in an inherent ability of some individuals to learn mathematics better than others parallels what many other researchers have documented (e.g., Adiredja & Louie, 2020; Chestnut et al., 2018). Although the teachers in the current study grouped their students into two heterogeneous group, these groups were founded on levels of perceived ability. These findings align with Marks (2013) conclusions that even in classrooms not subjected to explicit grouping and labelling, the idea of an inherent ability to learn mathematics was a fixed mindset of teachers and students alike. As the current study progressed, what became apparent was that the term "ability" served to perpetuate the teachers' beliefs of a fixed capacity of individuals to learn mathematics.

The findings of the current research suggested that the teachers' grouping practices were linked to their own educational experiences. Both teachers had participated in the final stages of the New Zealand Numeracy Development Professional Development project which advocated for ability grouping based on normed testing (Golds, 2014; Tait-McCutcheon, 2014). Later, the teachers participated in the Developing Mathematical Inquiry Communities Professional Development project which promoted mixed ability grouping (Hunter et al., 2018b). Whether their grouping practices were founded on the ability grouping practices of the Numeracy Development Project, or on the mixed ability grouping of the Developing Mathematical Inquiry Communities Project; the teachers' grouping practices continued to reflect embedded beliefs in notions of inherent ability. It is evident that supporting teachers to alter their beliefs in ability grouping for mathematics requires a purposefully designed alternative approach to grouping that rejected ability labelling.

Reflecting the overarching aim of the current study, all mathematics teaching and learning in this investigation was founded on a strength-based approach. This included grouping practices. The current research expands on previously documented research on grouping practices in mathematics classrooms through the specific design and use of a smart tool, namely a strength-based grouping template (see Appendix E). This smart tool was fundamental in assisting the teachers to group all students from a strength-base. It also supported the teachers to dispel their beliefs in inherent or fixed ability for learning mathematics, and they were able to view their students' capabilities from a growth mindset. The strength-based grouping approach implemented in the current study builds on and significantly extends earlier research (e.g., Askew, 2012; Dweck, 2008; Anthony & Hunter, 2017) that shows that when teachers are supported to develop a growth mindset, they hold high expectations for all students to learn mathematics successfully. Consequently, in the current research, the notion of a fixed or innate ability to learn mathematics transformed to one where the teachers viewed all students as capable of mathematical reasoning. With the increased emphasis on creating equitable learning environments that provided and extended learning opportunities for all students, teachers shifted to grouping practices that were flexible, dynamic, and explicitly planned. Further teacher support for grouping and planning was provided through the subsequent design and use of a mathematics lesson planning template (see Appendix F). This framework provided structure for explicit mathematics lesson planning that focused on consideration of important mathematical ideas and grouping students from a strength-base to access learning about these concepts.

Section 6.2 has discussed a range of answers to the question of how teachers (re)construct beliefs and classroom practices to position all students to learn mathematics. Illustrations are provided to show how the current research builds on and extends previous research in new ways. In the next section (see Section 6.3) the ways that the teachers in the current study addressed status are discussed.

6.3 ADDRESSING STATUS AND (RE)POSITIONING ALL STUDENTS FOR LEARNING MATHEMATICS

In this study, while the teachers worked to establish high expectations for collaborative sense-making, barriers to learning emerged in one class. In this class, while the teacher's instructional actions focused on positioning all students for collective reasoning in mathematics, some students resisted efforts to accept and enact these positions. These outcomes align with findings from Franke et al. (2015) who proposed that merely inviting students to work effectively together did not necessarily mean that they would accept the invitation to do so. In the current research, to find ways of addressing this dilemma, the question was asked: what factors had inhibited the construction and enactment of effective participatory norms? Two interrelated challenges were identified, namely, status issues and conflicting core values.

In the next section (see Section 6.3.1) the issues of status and how these were resolved are discussed.

6.3.1 Status

It was evident in this study that some students' interaction and participation practices became challenging and served to prevent all students from accessing mathematics learning. Analyses of classroom episodes highlighted that student participation in classroom discourse was either passive or domineering. In analysing the passive students' engagement in group discussions, Thomas' (1994) classification of the content of talk was useful to build on. While some students appeared to engage in discussions, their utterances, while task related, were not cognitively oriented. Rather, these students' discourse appeared to stem from compliance to meet the teacher's expectations that they talk to each other but did not allow for deeper mathematical exploration or understanding. On the other hand, several other students' statements dominated classroom discussions. These findings also align with Barnes' (1998) investigation of power relations among students. Similar to her findings, the ways in which the dominant students in the current study authorised the flow of the classroom discourse affected the teacher's attempts to construct an equitable learning environment. In the ensuing power struggle, some students assumed authority and were ceded authority by their peers. As a result, none of the students were afforded opportunities to engage in meaningful mathematical reasoning as all interaction and participation patterns of communication were conflicted. Central to the arising conflict were beliefs about who was considered "able" to learn mathematics successfully. Where the current study expands both on Thomas (1994) and Barnes (1998) research is that issues of status were identified as a cause of conflict among the students.

It was evident in analyses of the disruptive students' interaction patterns and beliefs that these stemmed from generalisations made by peers that related to certain characteristics connected to perceived notions of intellectual ability, social advantage, and cultural preferences. These findings reflect those of other research (e.g., Cohen & Lotan, 1995; Dunleavy, 2015; Featherstone et al., 2015; Langer-Osuna, 2016) that examined status challenges in mathematics classrooms. Inequitable relationships of authority and power among the students in the current study developed as status issues remained unresolved. These issues impacted significantly on the teacher's efforts in creating an equitable mathematics learning environment for all students. Addressing these issues required understanding the dynamic nature of status, that is, realising that status fluctuates depending on who holds what power and when. Status hierarchies affected classroom participation as they rendered some students as incompetent in relation to their peers and often disrupted student collaboration. Essentially, participation in mathematics learning depended on who was viewed as holding the “maths smarts” at a particular moment in time. Furthermore, certain students (those dominating the discussions) were consistently afforded higher status than their peers while other students were accorded lower status and were often ignored. These findings align with research (e.g., Crespo & Featherstone, 2012; Shah & Crespo. 2018; Wood et al., 2019) that documented that higher status students were seen as smarter and participated in classrooms discussions more often, while those accorded lower status were often marginalised. The current study builds on previous research by not only identifying effective ways to resolve status issues, but also searching for and adding new knowledge related to their causes within a unique New Zealand perspective. This will be further elaborated on in the following sections.

Importantly, the status issues exacerbating one teacher’s attempts to construct effective participation and interaction norms required deep exploration. Deeper exploration involves seeking possible reasons for the origin of these issues. In the next section (see Section 6.3.2) one of the possible reasons for the development and perpetuation of status issues, namely, the role that conflicting core cultural values and beliefs played is examined and discussed.

6.3.2 Culture matters

Many New Zealand classrooms are comprised of diverse student groups (for example Māori, Pāsifika, and other groups). In the current study, investigation into why status hierarchies had developed highlighted that the students and teacher were enacting differing core cultural values and beliefs. As previously documented in research (e.g., Hunter & Anthony, 2011; Hunter, 2013) establishing equitable learning environments required careful consideration of the core cultural values that all students (and the teachers) brought to the classroom community. Therefore, in this work, to strive for equity, expectations for and models of communication and participation were explicitly attended to in response to specific ways of knowing and being of the students from a cultural perspective. Furthermore, the findings highlighted the importance of attending to culture given that many of the students were of Pāsifika backgrounds. As illustrated by Hunter and Anthony (2011), positive learnng outcomes for Pāsifka students were possible when attention was given to understanding and drawing on the cultural background of all students in mathematics classrooms.

The students in this study reflected the diverse makeup of the school community with over sixty different ethnicities represented. In one class over 68% of students identified as being of Pāsifika heritage. While Pāsifika students form a diverse heterogenous group, there is a set of cultural similarities within Pāsifika values which include "respect, reciprocity, communalism, and collective responsibility" (Anae et al., 2001, p. 14). By recognising and acknowledging the importance of core cultural values the basis was formed for enacting culturally responsive pedagogical practices. Underpinning this foundation was the understanding that some facets of reform mathematics teaching and learning, such as students engaging in collaborative reasoning, mathematical argumentation and justification did not align with core cultural values of many of these students. For example, for some students, the core value of respect was shown by sitting quietly and listening to an acknowledged authority speak. For these students, being expected to engage in mathematical discussion or argumentation was viewed as being impolite.

In response to connecting with who her students were, the teacher in the current study modified her instructional approach and drew on important Pāsifika values while (re) constructing classroom practices. These values included reciprocity, respect, service, inclusion, family, relationships, collectivism, and belonging. Taking these actions builds on research in New Zealand classrooms (e.g., Hunter & Hunter, 2018b) that focused specifically on supporting schools and teachers to construct learning environments where a diverse range of students, including Māori and Pāsifika students, could learn mathematics utilising a variety of mathematical practices and engage in rich mathematical discourse.

In the next section (see Section 6.3.3) the process of drawing on the students' core cultural values and beliefs to support the construction of an equitable mathematics learning environment is discussed.

6.3.3. Drawing on important core cultural values and beliefs

To (re)construct the learning environment, the teacher purposefully acknowledged and emphasised the important cultural value of whānau (family), to develop mutually understood ways of togetherness. Deliberate action was taken to develop a sense of belonging among the students, as previous research (e.g., Bishop & Glynn, 1999; Nuthall, 2007; Siope, 2011) had highlighted the importance Pāsifika people place on feeling a sense of belonging and purpose in all aspects of their daily lives, including at school. The current study expands on previous research by identifying the specific pedagogical actions taken to achieve this sense of family and togetherness. One of these actions included explicitly utilising inclusive speech patterns such as: “tell us”; “we are all working together”; “can we add more to that idea?”; in doing so, the students were positioned into purposeful and connected ways of working together. The students were further positioned for mathematics learning when the teacher drew on the Pāsifika value of collectivism. Collectivism and communalism in the classroom context refer to ways of working together, for example, by engaging in group discussions, that show respect of others (Ministry for Pacific Peoples, 2018). The teacher drew specific attention to expected norms for working together and modelled what these should look and sound like. Analyses of classroom episodes highlighted how she directly addressed impediments to learning (such as disrespectful ways of communicating or passiveness) and carefully scaffolded all students to actively engage and participate collectively in mathematics activity. By specifically addressing these factors, all students were supported to shift from participation motivated by compliance or dominance to participation motivated by a set of commonly upheld values and beliefs. These outcomes support what Hunter and Hunter (2018a) described as connecting core values and beliefs in culturally responsive ways, which in turn, strengthened positive social skills and supported students in developing constructive and improved relationships, not only with mathematics, but with each other.

The increased focus on emphasising and drawing on the cultural capital of the Pāsifika students to frame expected behaviours for everyone (including the teacher) built on the goals of equity in mathematics teaching and learning highlighted by other researchers both internationally and in New Zealand (e.g., Boaler, 2006, Gutiérrez, 2012; J. Hunter et al., 2020). The current study expands further on the aims of equity by identifying how teachers can be supported to critically reflect on their assumptions and beliefs about certain students. By responding and adapting to students in culturally authentic ways, the teacher navigated the complex nature of classroom interactions across a range of diverse cultures; and in these ways, marginalised students were provided with what Spiller (2012, p. 65) termed "dignity in their learning".

This section has discussed important ways that status issues were alleviated for these culturally diverse students. Consequently, this discussion has provided answers to the second research question concerning how teachers address status. In the next section (see Section 6.4) the third research question, how do teachers delegate mathematical authority to all students, is attended to.

6.4 DELEGATING MATHEMATICAL AUTHORITY

Delegating mathematical authority to students as a means of striving for equity is well documented in research (e.g., Dunleavy, 2015; Esmonde, 2009b; Langer-Osuna, 2016). When teachers purposefully position and empower students to engage in mathematical reasoning, mathematical authority can be delegated. In the current study, the teachers followed a specifically designed process to guide and support the students to learn and use these practices through engagement in collective mathematical reasoning. These actions expand on other research (e.g., Hunter & Civil, 2021; Moschkovich, 2013; Selling, 2016) that documented students learning to know and do mathematics through practices occurring in social and cultural activity systems (classrooms) and among multiple participants (collectively).

In the current study, creating effective social and cultural activity systems that positioned all students as capable sense-makers of mathematics was initiated through the establishment of sociomathematical norms. In the next section (see Section 6.4.1) the significant role the development of sociomathematical norms played in supporting students to learn important mathematical practices is discussed.

6.4.1 Developing and embedding sociomathematical norms

As previously documented in research literature, (e.g., McClain & Cobb, 2001; Yackel et al., 1991; Yackel, 1995) developing effective sociomathematical norms required, in the first instance, the presence and enactment of effective social norms. Constructive and positive behavioural and participatory norms formed the basis for establishing effective interactions. Once these norms had been established, the teachers extended them to act specifically on mathematics, in this way they became sociomathematical norms.

The ways in which the teachers established social and sociomathematical norms builds on previous research in this field (e.g., Cobb, 1995; Yackel & Cobb,1996) that focused on how classroom norms were developed as a means of developing sociomathematical norms. In the current study, the teachers established, extended, and maintained effective social norms by positioning the students to work together in mutually respectful ways to solve mathematics problems. While the students worked together, the teacher monitored their actions by observing student interactions and only intervening if necessary. The focus of any intervention was to support the development of mathematical discourse and enhance mathematical understanding. Developing effective mathematical discourse involved students discussing their mathematical reasoning. These discussions centred on mathematical activity, providing pathways for students to engage in clarifying, explaining, disputing, and justifying their thinking about the mathematics. Learning to reason proactively with mathematics provided students the means to enact sociomathematical norms. During group discussions, the teacher would not offer value statements, but rather encouraged the students to think and reflect critically on the reasoning being offered. The teachers also deliberately used inclusive statements such as “tell us” and “we” and with these actions, the teachers (re) positioned themselves as part of the learning collective. Students’ attention would deliberately and steadily be drawn to highlight expected ways of behaving (social norms) and expected ways of acting on mathematics (sociomathematical norms). In this way, important ways of working together to think about mathematics were consistently negotiated and renegotiated.

As mathematical authority was delegated to all students through the process of engagement in collaborative and productive mathematical discourse, such engagement provided all students with opportunities to shape their own understanding. Delegation of mathematical authority became embedded when all students who actively engaged in collective sense-making and mathematical reasoning did not expect to have a “more knowledgeable” peer or “the teacher” explain the mathematics to them. Rather, they sought meaning for themselves by viewing each other as important resources in learning mathematics.

Maintaining these new positions required the teachers to draw attention purposefully and consistently to students’ intellectual strengths and contributions throughout each mathematics lesson. Drawing attention to individual student’s intellectual contributions was achieved through the specific instructional action of assigning competence

In the next section, the ways in which the teachers in the current study assigned competence as a means of distributing mathematical authority are discussed.

6.4.2 Delegating authority through assigning competence

In this study, assigning competence was a public action that deliberately identified and acknowledged a specific intellectual contribution a student made to the group activity. One example from the findings (see Section 5.6.3) highlighted how the teacher assigned competence by drawing all the other students’ attention to the specific actions of one low-status student. The teacher assigned competence authentically in response to a student’s insistence that the context of the mathematical activity formed part of the mathematical explanation being offered in a large-group discussion. In this way, the student was positioned as being a legitimate contributor to the co-construction of the mathematical explanation. The teacher specifically identified the actions of the student that made him a legitimate contributor. She emphasised that good mathematicians always pay attention to and try to make sense of what is being discussed. The teacher then explained why those actions were important as part of learning and understanding mathematics. In this way, assigning competence was a means to embed and extend mathematical authority, which in turn, served to support the construction of an equitable learning environment in the mathematics classroom.

By noticing and responding to students' intellectual contributions, however small; and publicly acknowledging and validating them as worthy mathematical contributions, students were positioned as capable knowers and doers of mathematics. As highlighted in previous research (e.g., Choy et al., 2017; Gonzalez & Vargas, 2020) how teachers notice and respond to students' ideas are powerful ways of acknowledging students as sources of knowledge and enhancing students' reasoning. While the teacher had initially assigned competence as a specific pedagogical strategy to raise the status of some students, this action supported all students to appreciate their own and others' intellectual contributions as valid. By deliberately assigning competence to some students, the teacher provided affordances for all students to build confidence and self-belief in their own capabilities to reason mathematically. The outcome of these actions highlighted that over time, how students were positioned to participate in collective activity affected not only what they learned, but how they came to view themselves as learners. The findings in this research expand on previous studies (e.g., Choy et al., 2017; Nasir & Hand, 2006; van Es et al., 2017) by identifying and illustrating specific ways that teachers can notice and respond to students' thinking that enhances mathematical reasoning and supports the development of positive student dispositions in learning mathematics. The more positive mathematical disposition a student holds, the more likely they are to participate more effectively in learning mathematics. For example, assigning competence was a powerful tool for focusing students on their peers' intellectual strengths and providing all students access to new mathematical ideas. When students were encouraged to acknowledge a new idea and grapple with it, they were pressed intellectually to view an aspect of mathematics in a different way and connect ideas within and across mathematical concepts.

The current study specifically addresses structural inequities in school systems. Within the New Zealand context, assigning competence can form part of re-inventing ways of teaching and learning which fit better with those students who are often marginalised in New Zealand schools. When students are positioned as capable knowers and doers of mathematics, opportunities to deepen and extend their mathematical understanding is enhanced.

In addition to assigning competence as a means of delegating mathematical authority, evident in the data is the way the teachers emphasised the need for individual responsibility and group accountability for mathematics reasoning. These actions provided the means to delegate mathematical authority amongst all participants. In the next section (see Section 6.4.3) the ways in which mathematical authority was distributed through the development of individual student responsibility and group accountability are discussed.

6.4.3 Delegating authority through individual responsibility and group accountability

Initially observational data revealed how the students repeatedly deferred to the teacher as the sole mathematical authority. For instance, rather than engage in collaborative activity or discourse with each other, the students repeatedly looked to the teacher to validate their utterances or actions. These reactions hindered the development of effective collaborative engagement in mathematics activity. The findings highlighted that the students had not yet grasped the value in collective exploration of mathematical concepts. In other words, the students had not recognised each other as valuable and legitimate resources in mathematical exploration and reasoning. In response, the teachers took considered pedagogical action to support students to understand the importance and process of working effectively with each other. One specific instructional action involved supporting students to acknowledge and enact their individual responsibility and group accountability when working with others.

In this research (see Section 4.4.4) the process used to develop individual responsibility and group accountability was presented. This process involved the teacher verbally stating the goal for the lesson, and then repeatedly reinforcing the goal by holding herself and all the students accountable for achieving the intended outcome. Accountability was established and developed by the teacher making statements before the group explanation was shared, outlining expectations for the group explaining their strategy to consider what their responsibilities and accountabilities were while doing this. During the group explanation the teacher maintained the expectation of accountability by repeatedly calling on the students to interact with each other whenever they attempted deferring to her to validate actions or statements. Previous research (e.g., Staples, 2008) focused on how verbal feedback was used to develop group accountability. The current study expands on this research. as the instructional actions highlighted included the teacher not only relying on verbal instructions to maintain expectations, but also using non-verbal gestures. For example, the teacher remained silent and still, using body language and gestures to direct students to seek acknowledgement, confirmation, or refutation from one another. By persistently positioning and re-positioning students for accountability in this way, the teacher succeeded in shifting mathematical authority to all members of the group discussion. In this way, the teacher was no longer viewed as the sole holder and keeper of mathematics knowledge, rather, the students were supported to view themselves and each other as knowers and doers of mathematics. Another important element of the process actioned in the current study included the teacher deliberately pausing the classroom activity to attend to repositioning acts. The students were consistently invited to share and discuss what they believed were important ways of interacting and collectively reasoning. The teacher emphasised that the responsibility for understanding mathematics rested both with each student individually, and the group collectively. Constant use of “we” and “us” also (re) positioned the teacher as part of the collective and thus not the sole mathematical authority. By regularly reinforcing expected student behaviours and actions, these actions became embedded and taken-as-shared appropriate ways of learning mathematics.

In the current study, effective and constructive participatory norms were enacted, and the teachers continued to press the students to reason mathematically. To support students to deepen their mathematical understanding the teachers established and developed the use of important mathematical practices. By drawing on these mathematical practices, mathematical authority was delegated to all students.

In the next section (see Section 6.4.4) the development of important mathematical practices that resulted in the delegation of mathematical authority is discussed.

6.4.4 Delegating authority by developing mathematical practices

In this section, the key actions taken by the teachers to develop mathematical practices and how these actions served to support delegating mathematical authority are discussed. The current research draws on definitions of mathematical practices as documented in previous research (e.g., Ball, 2003; NCTM, 2000; Selling, 2016) and includes actions such as making mathematical explanations, developing justifications through friendly argumentation, and generalising mathematical concepts. The current study draws further on research (e.g., Esmonde, 2009a; Moschkovich, 2018) that highlighted how students learned these practices by participating in collaborative mathematical discourse. In addition, the current research acknowledges previously documented evidence (e.g., Boaler & Sengupta-Irving, 2016; Chapin et al., 2013; Makar et al., 2015) of the key role teachers played in establishing mathematical discourse practices and how students were supported to learn about and use mathematical practices in these.

In this study, the teacher actions included (re)positioning students to consider the many ways of being successful at learning mathematics. These actions reflected a sociocultural perspective where mathematical ideas could be negotiated and renegotiated. The process for establishing and developing mathematical practices in the current study expands on other studies (e.g., Ball et al., 2005; Boaler, 2002; Selling, 2016) that highlighted the need for deliberate and clear instruction in how to learn and use these practices. Specifically, an explicit framework (see Table 5) is presented for teachers to use to establish, develop, and maintain mathematical practices in-class. This framework alleviates some of the challenges highlighted in previous research (e.g., Selling, 2016) that highlighted that providing opportunities for students to learn mathematical practices should be planned for, but that planning may not support all students to learn them; and secondly, that overtly teaching mathematical practices could be challenging if instructions become rigid. In the current study, a three-step-process was used by the teachers to position all students to draw on and use important mathematical practices. The first step focused on the teacher noticing in-the-moment use of mathematical practices; and then, secondly, explicitly drawing attention to the practice; and thirdly, justifying why knowing and using the practice could improve mathematical understanding. Monitoring student discourse and actions as they worked collectively on mathematical activity provided opportunities for teacher noticing-and-responding to how students engaged in collective mathematical activity. The second step provided the opportunity for the classroom dialogue and action to pause. Pausing provided an intellectual space for implementing and extending important ways for learning mathematic. Additionally, pausing signalled a time to focus on something important. In the intellectual space created, the mathematical practice could be clearly identified and named. The final action of this process consisted of the teacher identifying, explaining, and justifying why and how using the practice would enhance and deepen mathematical understanding.

Delegating mathematical authority to all students was possible through teachers positioning all students to learn and enact mathematical practices. Specifically, the students learned to co-construct mathematical explanations. The process of teaching the students to co-construct mathematical explanations builds on Esmonde's (2009a) study that emphasised the importance of teachers broadening students' understanding of what it means to make mathematical explanations. In the current study, all students were supported to co-construct explanations, as opposed to composing individual explanations. Co-constructing explanations provided the means for all students to participate equitably, as no one person was accountable for providing the complete mathematical explanations, rather, the students were effectively positioned to engage in collective reasoning. The teacher provided students with specific in-the-moment verbal and non-verbal prompts and feedback. For example, the student explaining their thinking would be paused in-the-moment (by the teacher) to ask the other students if they had any questions so far. Initially the students looked at the teacher for validation (delegated authority to the teacher), and the teacher would reposition them to ask the group explaining for clarification and not her. The teacher deliberately used statements such as: "Why are you all looking at me? This is not my explanation. Ask the guys in front and ask them what they mean"; "I am not the person explaining, you need to ask the group who is explaining" to delegate authority to the person explaining and not her. In addition to verbal prompts, the teacher used non-verbal prompts such as waiting in silence or looking ahead at the group explaining. These actions served to establish clear processes and expectations for the development of co-constructed student explanations. In turn, by being supported to co-construct clear mathematical explanations, all students were provided the means to develop mathematical justification through friendly argumentation (important mathematical practices and a means to further delegate mathematical authority).

The findings show evidence of an expansion of other researchers (e.g., Civil & Hunter, 2015; Connor et al., 2014) work that shows how collaborative mathematical argumentation is facilitated by student engagement in collective mathematical activity. The current study also builds on research (e.g., Hunter, 2007a; Yackel & Cobb, 1996) that illustrates the ways students develop and use mathematical argumentation to reach consensus about mathematical concepts. In this study, the teacher positioned and repositioned students to reason collectively about the validity of each other's claims. These positioning acts were facilitated by several key teacher invitation moves.

The key invitation moves used by the teacher extend the invitation moves described by Franke et al. (2015). Specifically, the teacher facilitated a group discussion where students were positioned to justify (argue) their mathematical claims regarding which numbers came between 1.9 and 2.5. The teacher's first action was to initiate an invitation for one student to share his group's mathematical thinking. She then extended an invitation for other students to consider the group's explanation (claim). The other students accepted this invitation and stated their disagreement with the original claim. These outcomes provide further evidence of how beneficial the in-the-moment teacher actions were in supporting students to engage in learning mathematics together. These results highlighted that while students engaged in collective mathematical reasoning, teachers could make in-the-moment decisions about actions could enhance mathematical understanding.

By making in-the-moment decisions about how students could deepen their mathematical reasoning, the teachers were delegating authority to the students. Analyses of classroom episodes highlighted specific actions teacher took to delegate authority to students. For example, by sitting in silence and waiting (non-verbal and non-evaluative action) the teacher remained neutral to the argument. This deliberate action served to position the students as having legitimate authority (permission) to make sense of the mathematical reasoning collectively. Students were further positioned when the teacher extended invitations for others to add to developing the mathematical argument (co-constructing). Invitation moves were extended without evaluating statements or gestures. The findings illustrated the importance of the teacher remaining neutral and extending invitations for further reasoning to delegate mathematical authority. By providing an unbiased intellectual space, the teacher positioned the students as genuinely able to justify their reasoning while simultaneously expressing the expectation that all students should contribute mathematical ideas. The teacher had essentially become what Staples (2008) termed a “gate-opener”.

In this section discussion has related to how mathematical authority was delegated through the establishment and implementation of important mathematical practices such as developing claims through mathematical explanation, justification, and argumentation. The teacher actions supported shifting notions of any one person (teacher or student) holding authority for validating mathematical ideas. By engaging in mathematical practices, students were provided with opportunities to use prior knowledge and build new mathematical understandings. By purposefully inviting students to reason collectively, to ask questions, to explain their ideas, and to justify mathematical claims, all students were positioned as arbitrators of mathematical sense-making. These actions expand on other research evidence (e.g., Hunter & Hunter, 2018b; Selling, 2016; Turner et al., 2013) that advocated that when students are supported to understand the importance of explaining and justifying, their mathematical reasoning strengthens.

In the current study, mathematical practices were explicitly taught in a responsive way. This meant that the students were supported to learn and use mathematical practices organically, in-the-moment as they participated in mathematical discourse practices.

6.5 CHAPTER SUMMARY

This chapter discussed the specific and deliberate pedagogical actions the two teachers took to position students as capable sense-makers and doers of mathematics. To support these changes, the teachers spent time examining research-based evidence on how to develop effective elements of effective mathematics teaching and learning. The teachers were further supported by several frameworks to connect theory to practice.

Connecting theory to practice involved constructing foundations for equitable participation in mathematics learning. Students were supported to work together to reason in mathematics activity. Grouping for mathematics teaching and learning was approached from a strength-base. Barriers for learning were addressed by attending to issues of status and drawing on the core-cultural beliefs and values of students often marginalised in mathematics classrooms. Resolving status issues (re)positioned all students to access mathematics in equitable ways.

Delegating mathematical authority to the students was achieved by purposeful and specific teacher actions. These actions included establishing and developing classroom norms for working together in respectful ways; establishing and developing sociomathematical norms; assigning competence; using non-evaluative and inclusive language and gestures; extending invitations to students to explain their ideas; holding students accountable for justifying their reasoning to one another; and developing important mathematical practices as a means of delegating mathematical authority in the classroom. Utilising these actions positioned students in authentic engagement in collective mathematics activity and provided useful processes for constructing equitable learning environments.

By developing specific classroom structures the teachers positioned students as capable thinkers and doers of mathematics. These structures provided the means for mathematical authority to be delegated to the students. In other words, they were positioned as capable of engaging in collective mathematical reasoning and understanding. A learning environment was structured in which students were at ease to engage in collective mathematics activity. Through consistent attention to status issues and by assigning competence, the many capabilities of all students to learn and understand mathematics could be made visible to peers.

Conclusions and implications for how teachers construct equitable learning environments in primary school mathematics classrooms and potential areas of further research are presented in the next chapter.

CHAPTER SEVEN

CONCLUSION

7.1 INTRODUCTION

The aim of this thesis was to examine how teachers can construct equitable learning environments in primary school mathematics classrooms. Several important themes were woven through the literature review related to the ways equitable learning environments can be constructed in primary school mathematics classrooms. These were: (1) the goals and instructional approach of the reform mathematics movement; (2) important elements of the learning environment that support student engagement in mathematical reasoning; and (3) how teachers can be supported to develop and enact pedagogical practices that fulfil the aims of equity in mathematics education.

This study used a qualitative approach to provide data to explore the complexities and challenges of teacher change and enactment of changes within classrooms. Design-based research was selected to develop and extend understanding of how innovative learning environments such as those focused on striving for equity can be established and maintained. This research methodology provided the means to structure, support, and examine the instructional actions teachers took to construct equitable mathematics learning environments. A key factor of the design research was the role that several frameworks (see Tables 2, 3, 4, 5) played in supporting the teachers to establish, develop, and maintain essential classroom practices that positioned all students with equitable opportunities to access mathematics teaching and learning.

The findings presented in this thesis involve two contrasting cases specifying teacher engagement in professional development, the subsequent changes in their classroom practice, and shifts in students' participation in mathematical activity. In both cases, the (re)construction of teachers' beliefs and classroom practices as a means of striving for equity was a gradual process. This process involved transforming pedagogical actions; the social and organisational structures within classrooms, including grouping practices; attending to status challenges; and the delegation of mathematical authority to all students. Above all, the process emphasised positioning students for learning mathematics from a strength-base. The two cases highlighted the differences in how respective teachers engaged in the professional learning and subsequently enacted changes in their classrooms.

This study focused on one key question: How do teachers construct equitable learning environments in primary school mathematics classrooms? To address the key question, there were three smaller research questions:

- How do teachers (re)construct beliefs and classroom practices to position all students to learn mathematics?
- How do teachers address status?
- How do teachers delegate mathematical authority to all students?

Section 7.2 commences by summarising the pathway the two teachers in this research took in (re)constructing their beliefs and classroom practices to position all students to learn mathematics. This is followed by an examination of how status was addressed. Concluding this section is the summary of the pedagogical actions and classroom and mathematical practices which supported the teachers in delegating mathematical authority to all students.

Section 7.3 identifies the limitations of this study. In Section 7.4 the contributions this research makes to the research field are presented. Section 7.5 identifies the implications of the study and suggestions for further research. Section 7.6 provides a final conclusion to this research.

7.2 CONSTRUCTING EQUITABLE LEARNING ENVIRONMENTS IN PRIMARY SCHOOL MATHEMATICS CLASSROOMS

This research detailed the pathways two teachers took in constructing equitable learning environments in their mathematics classrooms. Within this study, both teachers participated in the same professional learning activities and utilised the same resources. However, the findings presented in Chapter Four and Five, show that each teacher travelled a distinctive path as they endeavoured to (re)construct their beliefs about the nature of mathematics and how it should be taught, and align these authentically with their classroom practices. To begin with, both teachers espoused similar beliefs about teaching and learning mathematics, however, initial classroom observations illustrated that these beliefs and practices were not enacted. At the end of the research study, both teachers had enacted important pedagogical strategies that resulted in the (re)construction of equitable mathematics learning environments.

7.2.1 (Re)constructing teacher beliefs and classroom practices to position all students to learn mathematics

Initially, both the teachers identified constructive beliefs about teaching and learning mathematics that aligned with the goals of reform mathematics. The teachers highlighted the importance of students working together on mathematics activity and engaging in mathematical discourse practices as effective means of learning mathematics. However, these teachers also stated that they were unsure how to structure their learning environments to position all students to be able to engage and interact effectively to allow such engagement. This finding highlighted the need for teachers to be supported to implement policy-into-practice. The deliberate choice of the research methodology selected for this study provided potential for supporting teachers to enact recommendations from policy and research into classroom practice.

Utilising design-based research methodology provided consistent cycles of design (for the innovation), implementation (of the innovation), and refinement (of the innovation) throughout the study. Design and refinement phases occurred during study group sessions in which both teachers keenly participated. Following each design phase, the teachers implemented or enacted innovated pedagogical actions aimed at meeting the goals of equity in mathematics education. Implementation phases included the researcher working alongside the teachers in class as participant-observer. Mathematics lessons during these phases were dynamic. This meant that both the researcher and the teacher might notice-in-the-moment an aspect of teacher or students' interaction or participation requiring attention. The teacher and the researcher would work together to respond-and-adapt-in-the-moment. In lessons where the researcher was not present, teachers would use specifically designed tools (see Tables 2, 3, 4, 5) to support ongoing responsive and adaptive teaching. The outcome was that the implementation cycles involved consistent in-the-moment innovation, reflection, and consideration of potential pathways to constructing equitable learning environments in primary school mathematics classrooms. The cycles of design and reflection phases involved critical teacher reflection (both individual and collective) on aspects of long-held beliefs and classroom practice. Consequently, critical consideration resulted in collective innovation for subsequent phases of the investigation.

The importance of establishing a positive and constructive learning environment was agreed upon from the start of the study and required attending to several essential elements that encompass the complex nature of school classrooms. Focus centred on establishing mutually respectful interaction and participation patterns. Attention was given to critically analysing and reflecting on common organisational practices of mathematics classrooms. Specifically addressing ability grouping practices played a significant part in shifting teacher and student beliefs away from an innate ability to learn mathematics. The impediment of utilising any form of ability-labelling (e.g., mixed ability, ability) when grouping students for mathematics learning was identified. This impediment was overcome by the design and use of planning for learning from an asset-base. As the teachers purposefully and carefully reconstructed their mathematics learning environments from a strength-base, they reported noticeable changes in their embedded beliefs and views of which students were capable of learning mathematics successfully. As a result, all students were provided with more equitable opportunities to access and engage in learning mathematics.

Attending to the development of the social and organisational aspects of the classroom environment was shown to be instrumental in positioning all students with more equitable opportunities to learn mathematics. However, transforming embedded beliefs and instructional practices are not without complexity or challenge, but when teachers purposefully strive to position students for equity, possible ways of resolving challenges become possible.

The following section summarises how status issues emerged in one teacher's class and how these were addressed.

7.2.2 Addressing status

While one teacher progressively established and developed her learning environment, the other teacher encountered a complex challenge which hindered progress. This teacher reported that despite her endeavours to construct productive and respectful communication and participation patterns, many students resisted these attempts. Critical reflection and micro-analyses of classroom episodes highlighted that failure to recognise and understand the cultural backgrounds of all classroom participants had resulted in the formation of status hierarchies. Investigation into possible causes for the status issues highlighted a lack of understanding the importance of authentically attending to culture given that many of the students in the class were of Pāsifika backgrounds. As illustrated by Hunter and Anthony (2011), positive learning outcomes for Pāsifka students are possible when attention is given to understanding and drawing on the cultural background of all students in mathematics classrooms. Cultivating relationships that considered and strengthened all students' cultural identities required time and attention. Subsequently, the research focused on alleviating status issues in two unique ways. Firstly, important Pāsifika values were identified and discussed in a study group session. Later, in class, time and attention were provided for whole class discussions about these values and their importance. All students participated willingly and openly shared about how these values were enacted at home and the difficulties they experienced in reconciling their home-experiences with school. The teacher and the students began working together to identify and draw on meaningful cultural values such as belonging, family, and collectivism that would support respectful communication and interaction patterns.

Further support focusing on mitigating status hierarchies was provided. A tool (see Table 4, Raising Status Framework) was specifically designed to support raising status through assigning competence to low-status or marginalised students. Using this tool provided explicit support for the teachers to acknowledge students' intellectual contributions and then assign competence to these. In these ways previously marginalised or low-status students were publicly positioned as legitimate mathematical resources for their peers and overtime were able to reconsider their beliefs about their own capabilities to learn mathematics successfully. In addition, purposeful attention to noticing the mathematical strengths and capabilities of marginalised students demonstrated the respect the teacher held for her students' intellectual capabalities, and positioned all students as capable learners of mathematics.

Alleviating notions of innate abilities to learn mathematics for both the students and the teachers was an important outcome of this research and mitigating status issues opened the way for constructing a learning environment that centred on equity.

The summary provided in the next section addresses the third smaller question of this research and sums up how the teachers delegated mathematical authority to all students.

7.2.3 Delegating mathematical authority to all students

Delegating mathematical authority is recognised as a significant pedagogical practice that provides equitable learning opportunities for all students. In this research, four specific pedagogical actions formed the process taken by the teachers to enact this practice. The first action was to establish, develop, and maintain important sociomathematical norms. The gateway to establishing sociomathematical norms was possible through the development of positive, prosocial norms. When the students understood and enacted mutually respectful ways of working together, the focus could focus on developing productive discourse that centred on mathematical activity. Positioning all students to engage in mathematically productive discussions involved illustrating the importance of clarifying, explaining, and justifying their mathematical reasoning. This included reasoning with others' mathematical thinking and ideas also. A significant part of this process involved the teachers' use of waiting in silence and non-verbal gestures to persistently position and reposition students to reason collectively. Reasoning collectively was an important goal of this process, as when the students recognised and understood their roles as valid contributors to mathematical thinking, mathematical authority was delegated. Consequently, the teacher and certain students were no longer viewed as more knowledgeable or capable than others.

Another important way that mathematical authority was delegated was through the continued teacher action of assigning competence. By consistently and publicly identifying and acknowledging individual students' intellectual contributions, the teachers provided access and opportunities for all students to collectively construct and deepen mathematical understanding. To support the students in co-constructing mathematical reasoning and to delegate mathematical authority further, the teachers drew on additional pedagogical actions. These actions included consistently using verbal and non-verbal prompts, body language, and gestures that reinforced important ways for students to take ownership for collective reasoning. Taking ownership for collective reasoning became possible when students held themselves and their group members responsible and accountable for drawing on a range of ways to seek clarity of mathematical ideas. Teacher actions were persistent and stressed the importance of recognising peers as valid sources of mathematical support.

Mathematical authority was further delegated to all students through the implementation of several important mathematical practices, including co-constructing mathematical explanations and developing justification through mathematical argumentation. Developing these mathematical practices in-the-moment took a specific form. For example, when the teacher noticed a student or students beginning to use a particular mathematical practice (e.g., explaining or justifying) she would immediately pause the student action or dialogue and then explicitly name and identify the mathematical practice being used; the final step of the process involved the teacher explaining how and why successful mathematicians know and use these practices to enhance and deepen their understanding of mathematics. This process was consistent and persistent. Both verbal and non-verbal affirmations and validations were given to students to emphasise the importance of co-constructing meaning that built on individual and group understanding. By accepting authentic validation of their intellectual contributions, mathematical authority was able to be delegated to all students. Delegating mathematical authority provided a means for teachers to achieve the goals of equity.

This section has provided a summary of this research. The length of time it took to implement these pedagogical actions is important to stress. However, noticeably, and significantly, the positive student outcomes highlighted the benefits of spending this time (re)constructing the mathematics learning environment. Alleviating inequity in mathematics education requires ongoing and persistent teacher action to provide access, affordances, and positive outcomes for all students.

7.3 LIMITATIONS OF THE STUDY

While adding new knowledge to the field at a range of levels, any research has its limitations. The outcomes of this study centred on empirical analyses of a small sample of teachers and students, in one school, in one urban area of a city. The nature of the size of this sample inherently limits the extent of generalisability of the findings. Nevertheless, the detailed illustrations of the teachers' pedagogical actions and the specific nature of the various supporting framework provide others the means to try a comparable investigation.

Schools and classrooms are naturally complex settings; therefore, interpreting the outcomes of this investigation provide a developing insight into the pedagogical actions teachers can draw on to construct equitable learning environments. Although triangulation of data occurred, there is always the potential for bias in the research outcomes. The documented findings reflect one researcher's interpretation of data collected from audio-and-video recordings of study group sessions and classroom observations, interviews, teacher reflections, written questionnaires, and field notes. The possibility exists that other interpretations may be made from the same data; although the data collection is strengthened using design-based research methodology where analyses and discussion of classroom episodes and teacher views occurred collectively, and subsequently informed each phase of the research cycle.

Attention was given to alleviating the effect of the research process on the day-to-day running of the school and classrooms. The impact of both the presence of the researcher in the setting, and the use of recording equipment were considered. The researcher prioritised establishing a collaborative relationship with the teachers. Several meetings with the teachers took place before the commencement of this study which established mutual and relational trust. In addition, the researcher's role within this investigation as participant-observer (discussed in Chapter Three) provided further opportunity to develop and maintain constructive relationships with the teachers and the students. Prior to the start of this study, the researcher spent time in each class to familiarising the students and teachers with video-capture for the upcoming research investigation.

The actions and reactions of the students to the pedagogical changes the teachers enacted in class were analysed and discussed and formed a significant part of each phase of the design-based research cycle. However, the research presented in this study does not include exploration of individual student's views or perceptions. The parents', caregivers' and other community members' viewpoints were also not studied. Further limitations occurred through the nature of the length of time spent in the school. Extending the study to the following school year may have provided opportunity to examine the teacher actions with a different cohort of students and community members over a longer timeframe. However, extending the duration of this research was not possible due to the teacher participants leaving the school. It is accepted that the process of enacting pedagogical change is on-going process involving commitment and time.

7.4 CONTRIBUTION TO THE RESEARCH FIELD

The uniqueness of this research is the cultural frame. Drawing on cultural framing has highlighted specific ways to alleviate structural inequities experienced by certain groups of students who are most at risk of underachieving in mathematics. Furthermore, the emphasis on cultural framing directly addresses the gap in previous work related to status and positioning. Emphasised in this study is how deliberate and purposeful positioning of students for mathematics learning within a cultural frame was shown to significantly support all students to communicate and interact with each other in respectful and accepted ways, and at the same time, mitigate status issues. Specifically, the teachers were supported to acknowledge and recognise the cultural backgrounds and values of their Pāsifika students and situate mathematics teaching and learning within important Pāsifika values of belonging, family, and collectivism. These findings have relevance not only within Aotearoa, New Zealand, but internationally. Many students in mathematics classrooms around the world encounter mathematics teaching from a cultural capital. That is, mathematics teaching and learning are presented from the perspective of the dominant culture in a country. For many groups of students who come from culturally diverse backgrounds, mathematics presented from this perspective is a gatekeeper. This research has explicitly illustrated how teachers can open the gate to mathematics learning by authentically drawing on their students' cultural values and situating teaching and learning within a meaningful cultural frame.

This research offers educators a clear and effective model of professional learning focused on equity in mathematics education. Within this study, clear implications are provided for thinking about ways in which teachers can construct equitable learning environments in primary school mathematics classrooms. With the increasing focus being placed on the importance of practice-based forms of professional learning for educators, this study adds to the research field by illustrating specific ways for teachers to develop adaptive expertise. A clear contribution to research in the field of professional learning are the four frameworks (see Tables 2, 3, 4, 5), the Multiple-Strengths Grouping Tool (see Appendix E), and the Mathematics Lesson Planning Template (see Appendix F), and how they have been integrated and used. Each framework and tool clearly identify and outline specific teacher actions underpinning the construction of each component of an equitable learning environment. These components include developing the social domain of the classroom through, for example, social, engagement, and organisational support. These elements from the Social Domain Framework (see Table 2) provide a process for developing asset-based classroom practices that provide opportunities for student engagement in mathematical activity. The Positioning Framework (see Table 3) offers specific structure for productive teacher noticing in mathematics learning: This includes support for how teachers can notice-respond-adapt in-the-moment to position and reposition students for equitable engagement in collective mathematical activity and mathematical discourse. The Raising Status Framework (see Table 4) provides an explicit process for raising the status of low status or marginalised students in mathematics learning and focuses on recognising and acknowledging all students as valid knowers and doers of mathematics. In addition, this framework supports the development of students' perseverance and accountability, both important aspects of student agency. The Mathematical Practices Framework (see Table 5) frames teacher actions for developing mathematical practices which support mathematical reasoning. The Multiple-Strengths Grouping Tool (see Appendix E) offers an alternative approach to grouping for mathematics learning with the focus centred on students' strengths. An asset-focused lens positions all students as capable of learning mathematics successfully and alleviates the deficit views held by institutions and educators for certain groups of students (additional detail on the significance of this contribution is discussed further in this section). The final tool offered is the Mathematics Lesson Planning template (see Appendix F) which provides teachers with specific support for planning for teaching, learning, and grouping in mathematics classrooms. Previous research has predominantly focused on either one or two of these elements. These frameworks and tools expand and extend work to explicitly support teachers to notice strengths of diverse groups in multi-facetted ways by providing teachers with clear structure and specific support to integrate essential interconnected facets of mathematics education and classroom practices.

This study is unique within Aotearoa, New Zealand with its focus on strength-based grouping practices as it clearly rejects ability grouping, a key element in the enduring inequity in mathematics achievement in this country. Currently, in New Zealand, in response to the ability grouping practices widely accepted and implemented in mathematics classrooms nationwide, the Ministry of Education is calling for mixed-ability grouping in mathematics teaching (see Meyer & Slater-Brown, 2022). This research substantially extends this call and contributes to the research field by offering specific teacher guidance for grouping students for mathematics learning devoid of any form of "ability" label. Previous research (Bishop & Kalogeropoulos, 2015) has shown that labelling students perpetuates stereotyping which significantly impacts student disposition and learning. By offering an alternate approach to grouping that is strength-based, this study provides an important addition to the research literature both internationally and within Aotearoa, New Zealand. Specific guidance on how teachers can group students for mathematics learning by focusing on individual student's strengths is provided. The focus on individual student's strengths is significant, as the actions of identifying and acknowledging individual student's strengths positions them as valuable resources and legitimate contributors to learning mathematics. When students are positioned in these ways, their status is raised.

Raising marginalised or low-status students' status is an essential component of equity in education. The asset-based grouping approach developed in this research directly mitigates deficit teacher and institutional theorising of certain groups of students, their families, and their circumstances, where often the blame for failure to achieve is attributed.

Finally, these contributions to the research community provide significant insight into how a strength-based approach to constructing and organising equitable learning environments can address structural inequities in schools.

7.5 IMPLICATIONS AND FURTHER RESEARCH

This study took place in one urban primary school with students from low-to-middle socio-economic backgrounds whose cultural backgrounds reflected the diverse population of the city. The teachers who took part in this research were experienced teachers who were genuinely interested in striving for equity, were willing to critically reflect on their long-held beliefs and classroom practices and were willing to change these if necessary. Further research which expanded this investigation beyond these contexts would be beneficial. Specifically, it would be useful to extend understanding to how teachers with different levels of experience, beliefs, and attitudes, in different types of schools in varying locations could be supported to construct equitable learning environments.

Highlighted in this research were the distinct pathways each teacher took in navigating their professional learning. Examining the teachers' perceptions and learning as participants in professional development would be useful. In addition, closer study of elements of a professional learning cycle which provide opportunities for development or are catalysts for change would be beneficial. For example, examining way of working alongside teachers to enact pedagogical change in classrooms and the impact of utilising research-based tools. This study focused mainly on the teachers' pedagogical transformations over three-school terms. A longitudinal study of teachers' mathematics content knowledge learning alongside their pedagogical content knowledge development is required and would be significant.

Additionally, while data were collected on the students' practices and perspectives, these were not the emphasis of this study. Therefore, further examination centred on a longitudinal micro-analysis of the students' beliefs, perspectives, and responses within changing classroom environments such as evidenced in this research would benefit the field.

This research offers options for alternate considerations for teaching and learning mathematics in New Zealand primary school classrooms. Evidence within this investigation highlights the significant role of the classroom teacher in constructing equitable student access to learning mathematics. Initially, it was apparent that the teachers in this study espoused views reflecting the aims of the reform mathematics movement, however, classroom practice highlighted a deep-rooted traditional view of teaching and learning mathematics. This research provides explicit support for possible ways of navigating the challenges of developing equitable mathematics learning environments. It also explores the deliberate and specific pedagogical actions required by teachers to affect change to how students view themselves as doers and knowers of mathematics.

7.6 CONCLUDING THOUGHTS

The purpose of this research was to investigate and explore how teachers can construct equitable learning environments in primary school mathematics classrooms. This study highlights the deep complexity of classrooms and how many connected factors influence students' participation, interaction, and collaboration. The significant role the teacher plays in aligning these factors has been established and emphasised is the need for explicit and sustained teacher professional learning to negotiate and reconcile these interrelated elements. This calls for a responsive and adaptive perspective of teaching. Such a perspective is beneficial for developing and maintaining a pedagogical approach positioning students equitably for learning mathematics. Working within a responsive and adaptive pedagogical approach provides teachers with a full complement of mutually supportive actions, tools, and resources that are instrumental in bringing mathematics learning to all students.

Of importance is how the findings of this research provide possible ways of meeting the needs of diverse, and often marginalised groups of students In New Zealand schools. Traditionally, mathematics teaching assumes a level of uniformity across all students. Meeting the aims of equity in mathematics teaching and learning means disrupting these kinds of assumptions and paying close attention to who our students are when they enter our mathematics classrooms. As a result, not only will students learn mathematics effectively, but their cultural identity can remain strong and secure.

This design-based research investigation yielded rich data concerning teacher learning and change in practice. Evidence illustrated the varying ways that teachers can navigate new learning despite complex challenges. Moreover, this study supports and explicitly extends the current body of literature that seeks to understand the complexities of striving for equity in mathematics education and illustrates how the aims of equity in mathematics education can be extended beyond policy and into practice in practical ways within primary school mathematics classrooms in the New Zealand context. It is hoped that the findings of this research provide a productive model for researchers and designers of professional development to use to support teachers in constructing mathematics learning environments that strive for equity.

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APPENDICES

APPENDIX A: INITIAL TEACHER PARTICIPANT QUESTIONNAIRE

Name:

Year Level:

1. Which students in your class do you believe can learn mathematics? Reasons?
2. How do you group your students for learning mathematics?
3. Describe what mathematics problem-solving looks like in your classroom: • Who asks the questions? • Who answers the questions? • Who explains? • Who justifies the reasoning? • Who makes the connections explicit?
4. What do you understand by the term 'mathematical authority'? Who holds the mathematical authority in your class?
5. What social structures or norms for mathematics learning do you create in your mathematics class?
6. Is it important for you that your students participate in mathematical discussion during collective mathematical activity? Why/why not?
7. What do you believe is important for your students to learn as they participate collective mathematical problem-solving?
8. What do you understand by the term status?
9. In your mathematics class, who has status and why?
10. What do you understand by the term *mathematical practices*? List as many of these practices as you can.

APPENDIX B: FINAL TEACHER PARTICIPANT QUESTIONNAIRE

Changes in Pedagogy

Overall, how has your participation in this study impacted on the way you teach mathematics in your class? Please provide examples where possible.

In what ways do you now think differently about how students learn mathematics in your class?

What aspect of what you have learned about has had the most impact on how you now think about teaching mathematics?

What has supported you in making changes in your pedagogy?

Were there any parts of this study which were not particularly helpful?

Social Structures/Classroom Norms

Using the Social Domain framework, discuss how you developed your class collaborative problem solving norms?

How was this different to how you set the norms in previous teaching experience?

Frameworks

Over the year we have developed and used four frameworks. Using these attached frameworks as references, discuss how each has supported you in enhancing student learning in mathematics problem-solving in your class in term of the following:

- Positioning students as competent mathematicians
- Assigning competence (developing issues of status)
- Mathematical practices (refer to points on framework)

How has using these frameworks supported you in developing your mathematical pedagogical practice/how have these helped your teaching practice?

What was the most helpful part of each framework?

What would you change in each framework/or what was least helpful?

Grouping

How do you group your students for participation in collaborative problem solving?

How is this different to how you grouped children before participating in this study?

What has supported you in making changes?

Big Mathematics Ideas and Planning

Describe how you plan for math problem solving in your class.

How is this different to how you planned before participating in this study?

What has supported you in making changes?

Any other comments/reflections

APPENDIX C: INITIAL STUDENT PARTICIPANT QUESTIONNAIRE

Name:

Class:

1. Do you believe you can learn mathematics? Why/why not?
2. Do you think you are good at mathematics? Why/why not?
3. Can you describe how you know if someone is good at mathematics?
4. Where is your family from? 5. How does it feel to be ________________ in your mathematics class?
6. How are you grouped for learning mathematics in your class? 7. Do you like being grouped like this? Why/why not?
8. Describe what mathematics problem-solving looks like in your class: • Who asks the questions? • Who answers the questions? • Who explains the maths? • Who justifies the reasoning? • Who makes the connections clear?
9. Who holds the mathematical authority in your class (who do you think is in control of the mathematics in your class?) Why do you think that? *Because…*
10. Do you believe it is important for you to join in when your group is solving a mathematics problem? Why/why not?
11. How do you join in when solving a problem in groups? • What do you say? • What do you do?
12. What do you believe is important for you to learn when you work in your group to solve mathematics problems?
13. What do you understand by the term *status*?
14. In your mathematics class, who has status and why?

APPENDIX D: FINAL STUDENT PARTICIPANT QUESTIONNAIRE

Name:

Class:

6. Do you believe you can learn mathematics? Why/why not?
7. Do you think you are good at mathematics? Why/why not?
8. Tell me about the things that help you work well with your group when you are solving problems together.
9. Do you think the other people in your group think you are good at maths problem solving? Why/why not?
10. How are you grouped for learning mathematics in your class? 11. Do you like being grouped like this? Why/why not?
12. Describe what mathematics problem-solving looks like in your class:
13. Who do you think is in control of the mathematics in your class? Why do you think that? *Because…*
14. Do you believe it is important for you to join in when your group is solving a mathematics problem? Why/why not?
15. How do you join in when solving a problem in groups? • What do you say? • What do you do
16. What do you believe is important for you to learn when you work in your group to solve math problems?
17. In your mathematics class, who are the people you listen to the most and why?

APPENDIX E: MULTIPLE-STRENGTHS GROUPING TOOL

USE THE TEMPLATE BELOW

- **Step One:**
  - List all student names

- **Step Two:**
  - List each student's strengths beside their name-focus solely on strengths (these may be social, academic, other-e.g., resilience, reading skills, spatial skills, mathematical modelling, questioning etc)

- **Step Three:**
  - Complete the other comments column: The skills or needs listed here are necessary to support you in choosing which students to group together

- **Step Four:**
  - Select students for each group
  - Aim to complement and combine strengths for optimal engagement and learning of mathematics

MATHEMATICS GROUPING TEMPLATE-THINK STRENGTHS ONLY

All students	**Strengths**	**Other comments (i.e., status, language). These skills or needs will support you in grouping students together to complement strengths and needs**
e.g., Anna	Loves reading, can summarise ideas well	Holds low status in mathematics class, self-belief in capability to learn mathematics can be low. Speaks Samoan at home
Establish Groups		
Group 1		
Group 2		
Group 3		
Group 4		

APPENDIX F: MATHEMATICS LESSON PLANNING TEMPLATE

<table>
<tr><td colspan="6">Problem</td></tr>
<tr><td colspan="6">Why I chose this problem:</td></tr>
<tr><td colspan="6">Big mathematical idea:</td></tr>
<tr><td colspan="2">Anticipated strategies:
How might my students solve this problem?</td><td colspan="2">Who solved it this way?</td><td colspan="2">Which group should share today?</td></tr>
<tr><td colspan="3">Other strategies that emerged during the lesson</td><td colspan="3">Who solved it this way?</td></tr>
<tr><td colspan="6">What are the social goals for this lesson?</td></tr>
<tr><td colspan="6">What will I be looking to notice and respond to during this lesson?</td></tr>
<tr><td colspan="6" align="center">Strength-based student grouping for this lesson (groups of 4 or pairs)</td></tr>
<tr><td colspan="2">Group One:
Student name</td><td colspan="2">Student strengths</td><td colspan="2">Other comments-e.g., status, language</td></tr>
<tr><td colspan="2">Group One:
Student name</td><td colspan="2">Student strengths</td><td colspan="2">Other comments-e.g., status, language</td></tr>
</table>

APPENDIX G: BOARD OF TRUSTEES INFORMATION SHEET AND CONSENT FORM

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Students' positioning and status in primary school mathematics classroom

INFORMATION SHEET

Dear Board of Trustees

My name is Generosa Leach, and I am a PhD student at Massey University. My PhD study is titled *Students' positioning and status in primary school mathematics classrooms* and will be conducted within the context of primary school mathematics classrooms being places where diverse students can successfully learn mathematics. The main purpose of this study is to explore how students can be positioned as competent in primary school mathematics classrooms. The focus of the exploration will be on how teachers can construct equitable social structures to support students to learn. The study will also specifically monitor how student interactions mediate positioning of peers in mathematical activities. A final focus will be on examining notions of status change over time and how these changes affect opportunities to learn.

The Y4-6 teachers have agreed to participate in this study as the mathematics teachers of the students involved in this project. The duration of this study will be over four school terms. During this project, three-four mathematics lessons per term, during the normal classroom schedule, will be observed and filmed by me. The teacher, the students and their parents/caregivers will be given full information and consent will be requested. Specifically, permission to allow the students to be filmed and to participate in individual interviews will be sought from both the parents of the students and the students within each class. Interviews involving the teachers and their students will take place immediately following the observed mathematics lesson. The time involved for the teacher and students for each interview will be no more than 20 minutes. The interviews with the teacher and students will be audio recorded. Work samples from each lesson may also be collected and photocopied.

The time involved in the complete study for the teacher will be no more than 32 hours over a period of four school terms. There is no expectation that the usual classroom programme will be disrupted in any way.

All project data collected during individual interviews and filming will be stored in a secure location, with no public access and used only for this research and any publication arising from this research. After completion of five years, all data pertaining to this study will be destroyed in a secure manner. All efforts will be taken to maximize confidentiality and anonymity for participants. Names of all participants and the school will not be used once information has been gathered and only pseudonyms and non-identifying information will be used in reporting.

Please note that the Board of Trustees is under no obligation to accept this invitation. If you decide to participate you have the right to:

- Decline to answer any particular question;
- Withdraw from the study at any time;
- Ask any questions about the study at any time during participation;
- Provide any information on the understanding that your name will not be used unless you give permission to the researcher
- Be given access to a summary of the project findings when it is concluded.

If you have any further questions about this project, you are welcome to discuss them with me personally:

Generosa Leach. Phone: 021 709621. Email: G.Leach@massey.ac.nz

Or contact my supervisors at Massey University

- Associate Professor Roberta Hunter (09) 414 0800 ext. 43530. Email. R.Hunter@massey.ac.nz Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

- Dr Jodie Hunter (09) 4140800 ext.43518. Email. J.Hunter1@massey.ac.nz
  Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

- Professor Glenda Anthony (06) 3505701 ext.84406. Email. G.J.Anthony@massey.ac.nz
  Institute of Education, Massey University Manawatu (Turitea), Palmerston North 4474

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named above are responsible for the ethical conduct of this research.

If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher(s), please contact Dr Brian Finch, Director, Research Ethics, telephone (06) 356 9099, email humanethics@massey.ac.nz .

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Students' positioning and status in primary school mathematics classrooms

CONSENT FORM: BOARD OF TRUSTEES

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

We have read the Information Sheet and have had details of the study explained. Our questions have been answered to our satisfaction, and we understand that we may ask further questions at any time.

We agree to ____ participating in this study under the conditions set out in the Information Sheet.

Signature: ____ **Date:** ___________

Full Name – printed ____

APPENDIX H: TEACHER INFORMATION SHEET AND CONSENT FORM

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Students' positioning and status in primary school mathematics classrooms

INFORMATION SHEET

Dear

My name is Generosa Leach, and I am a PhD student at Massey University. My PhD study is titled *Students' positioning and status in primary school mathematics classrooms* and will be conducted within the context of primary school mathematics classrooms being places where diverse students can successfully learn mathematics. The main purpose of this study is to explore how students can be positioned as competent in primary school mathematics classrooms. The focus of the exploration will be on how teachers can construct equitable social structures to support students to learn. The study will also specifically monitor how student interactions mediate positioning of peers in mathematical activities. A final focus will be on examining notions of status change over time and how these changes affect opportunities to learn.

I am formally inviting you to be a part of this research as I examine the ways in which students can be positioned as competent learners of mathematics. Your role in this project will be as the mathematics teacher of the student participants.

Permission to participate in the study will be sought from both the parents/caregivers of the students in your class and the students themselves. The students and their parents/caregivers will be given full information and consent will be requested in due course. Consent will be twofold: one for permission to be filmed during mathematics lessons, and consent to be interviewed by me if randomly selected.

I will interview you and the students. The time involved for individual interviews will be no more than 20 minutes. Group interviews with all the teachers involved in this study may be conducted. These interviews will be 40-60 minutes and will take place as part of a study group session. The interviews for each student will also be no more than 20 minutes. The interviews with you and students will be audio recorded.

The duration of this project will be over four school terms. During this project, three-four mathematics lessons will be filmed each term. You and the randomly selected students will be interviewed following these lessons. Work samples from each lesson may also be collected and photocopied. The interviews and observations will take place in the classroom and be part of your normal mathematics programme.

The time involved in the complete study for you will be no more than 32 hours over the period of four school terms.

All project data collected during individual interviews and filming will be stored in a secure location, with no public access and used only for this research and any publication arising from this research. After completion of five years, all data pertaining to this study will be destroyed in a secure manner. All efforts will be taken to maximize confidentiality and anonymity for participants. Names of all participants and the school will not be used once information has been gathered and only pseudonyms and non-identifying information will be used in reporting.

Please note that you are under no obligation to accept this invitation. If you decide to participate you have the right to:

- Decline to answer any particular question;
- Withdraw from the study at any time;
- Ask any questions about the study at any time during participation;
- Provide any information on the understanding that your name will not be used unless you give permission to the researcher
- To ask for the audio or video recorder to be turned off at any time during the interviews and any comments you have made be deleted;
- Be given access to a summary of the project findings when it is concluded.

If you have any further questions about this project, you are welcome to discuss them with me personally:

Generosa Leach. Phone: 021 709621. Email: G.Leach@massey.ac.nz

Or contact my supervisors at Massey University

- Associate Professor Roberta Hunter (09) 414 0800 ext. 43530. Email. R.Hunter@massey.ac.nz Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

- Dr Jodie Hunter (09) 4140800 ext.43518. Email. J.Hunter1@massey.ac.nz Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

- Professor Glenda Anthony (06) 3505701 ext.84406. Email. G.J.Anthony@massey.ac.nz Institute of Education, Massey University Manawatu (Turitea), Palmerston North 4474

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named above are responsible for the ethical conduct of this research.

If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher(s), please contact Dr Brian Finch, Director, Research Ethics, telephone (06) 356 9099, email humanethics@massey.ac.nz .

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Students' positioning and status in primary school mathematics classroom

CONSENT FORM: TEACHER PARTICIPANT

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to the interview being sound recorded.

I agree/do not agree to the interview being image recorded.

I agree to participate in this study under the conditions set out in the Information Sheet.

Signature: .... **Date:** ....

Full Name - printed ....

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Students' positioning and status in primary school mathematics classroom

FOCUS GROUP PARTICIPANT CONSENT FORM

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I understand that I have an obligation to respect the privacy of the other members of the group by not disclosing any personal information that they share during our discussion.

I understand that all information I give will be kept confidential to the extent permitted by law, and the names of all people in the study will be kept confidential by the researcher.

Note: There are limits on confidentiality as there are no formal sanctions on other group participants from disclosing your involvement, identity or what you say to others in the focus group. There are risks in taking part in focus group research and taking part assumes that you are willing to assume those risks.

I agree to participate in the focus group under the conditions set out in the Information Sheet.

Signature: .... **Date:** ....

Full Name - printed ....

APPENDIX I: STUDENT AND CAREGIVER INFORMATION SHEET AND CONSENT FORM

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Students' positioning and status in primary school mathematics classrooms

INFORMATION SHEET

Dear Students and Parents/Caregivers,

My name is Generosa Leach, and I am a PhD student at Massey University. My PhD thesis title is *Students' positioning and status in primary school mathematics classrooms*. The main purpose of my study is to explore how students are positioned as mathematically competent in primary school classrooms and the effect this has on notions of status. The focus of the exploration will be on how teachers can construct equitable social structures to support students to learn mathematics.

I would like to invite you with your parent's permission to be involved in this study. Your classroom teacher has also agreed to participate in this study. Mrs Beth Noakes, the school principal has also given her approval for me to invite you to participate, and for me to do this research in your classroom.

I will be observing you participating in three-four mathematics lessons in your classroom each term. Your classroom teacher will be teaching you at this time and these lessons will be part of your normal mathematics programme, whether you agree to be in the study or not. These lessons will also be filmed, and you may at any time ask that the camera be turned off and any comments you have made deleted. With your permission I might sometimes collect copies of written work or charts you make to support your mathematical thinking. You have the right to refuse to allow the copies to be taken.

If you agree to be involved, I may interview you about what you know about being a mathematician. If you are selected to be interviewed, the interviews will take place immediately after your mathematics lesson. The interviews will take about 20 minutes each. The interview will be audio-recorded, and you may ask that the recorder be turned off and that any comments you have made be deleted if you change your mind or are not happy about what you said.

Taking part in this research will not in any way affect your learning, but rather may help you clarify what you know about being a mathematician.

All the information gathered will be stored in a secure location and used only for this research. After completion of the research the information will be destroyed. All efforts will be taken to maximize your confidentiality and anonymity which means that your name will not be used in this study and only non-identifying information will be used in reporting.

I ask that you discuss all the information in this letter fully with your parents before you give your consent to participate.

Please note that you have the following rights:

- To say that you do not want to participate in the study
- To withdraw from the study at any time
- To ask for the audio or video recorder to be turned off at any time during the lessons or interviews and any comments you have made be deleted
- To refuse to allow copies of your written work to be taken
- To ask questions about the study at any time
- To participate knowing that you will not be identified at any time
- To be given a summary of what is found at the end of the study

If you have any further questions about this project, you are welcome to discuss them with me personally:

Generosa Leach. Phone: 021 709621. Email: G.Leach@massey.ac.nz

Or contact my supervisors at Massey University

- Associate Professor Roberta Hunter (09) 414 0800 ext. 43530. Email. R.Hunter@massey.ac.nz Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

- Dr Jodie Hunter (09) 4140800 ext.43518. Email. J.Hunter1@massey.ac.nz Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

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If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher(s), please contact Dr Brian Finch, Director, Research Ethics, telephone (06) 356 9099, email humanethics@massey.ac.nz

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Students' positioning and status in primary school mathematics classrooms

CONSENT FORM: STUDENT PARTICIPANTS

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to being sound recorded.

I agree/do not agree to being image recorded.

I agree to participate in this study under the conditions set out in the Information Sheet.

Child's Signature: .... **Date:** ....

Full Name - printed ....

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Students' positioning and status in primary school mathematics classrooms

CONSENT FORM: PARENTS/CAREGIVERS OF STUDENT PARTICIPANTS

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to ______________________________ being sound recorded.

I agree/do not agree to ______________________________ being image recorded.

I agree to ______________________________ participating in this study under the conditions set out in the Information Sheet.

Parent's Signature:	....	**Date:**	....
Full Name - printed	....		