# Icy tornadoes in the quantum world - Josephson junctions of Bose-Einstein condensates

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# Abstract

This thesis presents a theoretical study of ultra-cold gases in, or close to a Bose-Einstein condensed phase. A system of two coupled Bose-Einstein condensates shows strong resemblance to Josephson junctions, consisting of two superconductors linked via a thin insulating barrier. In these systems quantised ring currents across the barrier (Josephson vortices) have been detected. We concentrate on similar macroscopic quantum states in our systems of linear and annular coupled Bose-Einstein condensates, and test them towards their potential for showing macroscopic quantum tunnelling.

For the linear system we present a very detailed stability analysis of the stationary solutions, the vortex and the soliton, using Bogoliubov-de Gennes theory. An analytic approximation of the unstable mode is provided. We show that the transition between vortices rotating in opposite directions is possible and propose an effective potential separating these two states. For the annular system of two coupled ring shaped condensates of different radii, we review the parameter regimes for finding vortices in the ground state. We show that pinning of vortices via a repulsive external potential is possible and suggest further steps towards the detection of macroscopic quantum tunnelling.

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# Contents

# List of Figures

# List of Tables