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Icy tornadoes in the quantum world - Josephson junctions of Bose-Einstein condensates

A thesis presented in partial fulfilment of the requirements
for the degree of

Master of Science
in
Mathematical Physics

at

Centre for Theoretical Chemistry and Physics,
New Zealand Institute for Advanced Study,
Massey University, Albany,
New Zealand.

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Auckland,
June, 7, 2012

Abstract

This thesis presents a theoretical study of ultra-cold gases in, or close to a Bose-Einstein condensed phase. A system of two coupled Bose-Einstein condensates shows strong resemblance to Josephson junctions, consisting of two superconductors linked via a thin insulating barrier. In these systems quantised ring currents across the barrier (Josephson vortices) have been detected. We concentrate on similar macroscopic quantum states in our systems of linear and annular coupled Bose-Einstein condensates, and test them towards their potential for showing macroscopic quantum tunnelling.

For the linear system we present a very detailed stability analysis of the stationary solutions, the vortex and the soliton, using Bogoliubov-de Gennes theory. An analytic approximation of the unstable mode is provided. We show that the transition between vortices rotating in opposite directions is possible and propose an effective potential separating these two states. For the annular system of two coupled ring shaped condensates of different radii, we review the parameter regimes for finding vortices in the ground state. We show that pinning of vortices via a repulsive external potential is possible and suggest further steps towards the detection of macroscopic quantum tunnelling.

Acknowledgements

I would like to thank my supervisor Joachim Brand for offering me the possibility to work on an interesting topic in his group, for all the fruitful discussions and for proofreading my drafts.

Special thanks goes to Oleksandr Fialko whose passion for physics was truly inspiring and motivating. Thanks for the countless number of discussions and the thoughts you put into my work.

I am thankful for the financial support received from the Marsden fund which made a full time study possible. It also financed the participation at the VSSUP 2011 in Melbourne and the Dodd-Walls symposium 2011 in Dunedin.

Last but not least I would like to thank all my colleagues for their warm welcome to the group and the positive and friendly working environment. You turned my stay in New Zealand into one of the best times of my life.

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