# An Empirical Analysis of the Effects of the September 11 Terrorist Attacks upon the US Options Market

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# CHAPTER 1

# INTRODUCTION

On September 11, 2001 there were a series of terrorist attacks carried out on the World Trade Centre towers in New York and on the Pentagon in Washington by an extremist Muslim terrorist group, Al Quada. These resulted directly in the towers collapsing, serious damage to the Pentagon and the loss of thousands of lives.

These tragic attacks however, also sent shock waves throughout the world's economies causing far-reaching economic implications. The financial markets reacted to these attacks negatively with billions of dollars in wealth being eroded as share markets around the world fell as a result of these attacks. Volatility also increased as market participants developed pessimistic outlooks towards the future of world economies.

## *Aim and Scope of Research*

The September 11 terrorist attacks were unprecedented in history and never before had the world's financial markets reacted in such a dramatic fashion to external non-economic events. The aim of this study is to investigate the effects this dramatic event had upon the United States option market, more specifically options on the S&P 100 index and the S&P 500 index which are traded on the Chicago Board Options Exchange (CBOE).

Research is conducted into the affects that the September 11 attacks had upon the volume of the options traded and the patterns surrounding trading volume. Trading patterns surrounding market participants preferences towards put or call contracts will also be analysed in pre and post September 11 attack periods.

Volatility may be measured in two ways using historical volatility and implied volatility. Historical volatility uses past data to calculate volatility over a designated period and can be used in forecasting volatility. However, it only takes into account what has already happened not what is anticipated to happen. Implied volatility however, is what the market believes volatility will be in the future using the current information available to formulate this opinion. This opinion of what future volatility will be, is subject to change as new information becomes available to market participants. There has been previous research into discovering how the markets react to certain types of new information and how this affects implied volatility.

Previous research has looked at how the market reacts in formulating its implied volatility to certain types of information and whether the current economic conditions at the time that the new information becomes available, has an affect on the market's reaction. This study intends to investigate the effect these terrorist attacks had upon the level volatility of financial options.

There has also been much research conducted in the area of the efficiency of using implied volatility as an ex-ante forecast for future realised volatility. The relationship between implied volatility as a forecast of future realised volatility will also be investigated. Additionally whether the September 11 attacks altered the ability of implied volatility to forecast future realised volatility will be examined.

The CBOE's implied volatility indexes, VXO and VIX, will also be investigated to see which index is the more robust ex-ante forecast of future realised volatility. Finally the robustness of the results that the implied volatility indexes provide post the September 11 attacks is investigated.

## *Chicago Board Options Exchange*[1]

The option analysed in this study are options on the S&P 100 index (OEX) and options on the S&P 500 index (SPX), both are traded on the CBOE. The CBOE was formed by the Chicago Board of Trade in 1973. Prior to its formation, options were traded in an unregulated and informal basis, the formation of the CBOE provided then a formal and regulated market in which options could be traded. The time since the CBOE's formation has allowed it to become the second largest securities exchange in the United States and the largest options exchange in the world. The most popular options contracts traded on the CBOE are the OEX and SPX, which provide the data, used in this research.

Options on the S&P 100 index, OEX were first traded on March 11 1983 becoming the first cash settled securities product traded. OEX is an american style option, which provides market participants methods to reduce risk, increase leverage or to cheaply replicate the S&P 100 portfolio.

Options on the S&P 500 index, SPX were introduced two years after their OEX counterparts. SPX also is an american style option. SPX allows market participants to replicate the S&P 500 index and synthetically adjust their portfolios in relation to the S&P 500 index.

In 1993, the CBOE produced an innovative implied volatility measure, VIX. VIX soon became a benchmark for measuring stock market volatility with it soon being quoted in many reputable financial news sources. VIX was created to gauge the market expectations of near term future volatility of stock market indices as implied by OEX option prices. It soon picked up the nickname of the "investor fear gauge." VIX was based upon the volatility of an at-the-money option with a constant 22 trading days, or 30 calendar days until expiration

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[1] Information about the CBOE was sourced from http://www.cboe.com/AboutCBOE/History.asp

On September 21, 2003, a new methodology was formed and quoted by the CBOE for VIX. The various changes to the methodology now used in the calculation of the implied volatility index were to reflect the latest changes and developments in the index options market and in academia.

The S&P 500 index now underlies the most active stock index derivatives surpassing the S&P 100 as the most active index. It is also the United States domestic index that is tracked by volatility and variance swaps. The new VIX was also developed in response to demand for derivative products based upon volatility of which, VIX will provide the underlying index measure. The old methodology for calculating VIX is still quoted but has now assumed the ticker symbol of VXO.

## Contribution of Research

Currently there exists no literature on the effects that the September 11 attacks had on the option market behaviour. To fill this gap, this research provides an analysis and insight into the effects that the September 11 attacks had upon the volume of options traded. Also provided is an insight into the reactions of traders to the September 11 attacks and its economic consequences as illustrated through changes in the volume of options traded by market participants.

This research also extends analysis conducted into using implied volatility as an ex-ante forecast for future realised volatility. The latest research advances in econometric methodology for evaluating the effectiveness of implied volatility as an ex-ante forecaster for future realised volatility is used. This is done through using an implied volatility index as the measure for implied volatility opposed to using implied volatility of a single option, as done in previous research. Also contributed by this research is the effects that the September 11 attacks had upon implied volatility index measures and how these effects affected the robustness of using implied volatility indexes as an ex-ante forecast for future realised volatility.

Due to the newness of the latest CBOE implied volatility index, VIX there currently exists no literature making a comparison between the two methodologies for formulating implied volatility indexes. Therefore, this research contributes a comparison between the two indexes.

# CHAPTER 2

# LITERATURE REVIEW

There is little research that has been performed on the effects that actual events have on implied volatility and trading volume of options. However, there have been numerous studies on the effects that certain types of announcements have on the implied volatilities and trading volume of options.

## *Option trading volume reaction to events*

Kim and Verrecchia (1991) develop a model that is based on the intuition that traders trade on the preannouncement information before the public announcement is made. Once the announcement has been made, the traders re-evaluate their position and beliefs that they made before the announcement. They then trade on any new information or interpretation of this information after the announcement. It is therefore, the unexpected component of the news that causes traders to trade after the announcement. They also state that surprises in information announcements will also cause an increase in volume with a positive surprise resulting in an increase in price and a negative surprise resulting in a decrease in price.

Nofsinger and Prucyk (2002) examine the impact that 21 different types of macroeconomic announcements had on the options market with regard to volume. They find that negative news announcements resulted in quicker response in trading volume than did positive announcements.

In addition, Nofsinger and Prucyk (2002) stated the level of trading volume after an announcement depended upon the surprise factor and whether it was good or bad news. A high surprise factor in bad news announcement increased volume the greatest. Low surprise bad news and high surprise god news had the same effect increasing effect upon volume whilst a high surprise good news announcement actually decreased trading volume of options.

There is also literature documenting the relationship between volatility and volume. Sarwar (2003) report finding that historical option volume has a significant predictive power in with respect to implied volatiles in the currency options market Easley et al (1998) also report finding that option volume contains a significant amount of information option with regards to the future movements of the equity markets.

## Option implied volatility reaction to events

Ederington and Lee (1996) state that while traders acquire information prior to the announcement and form opinions based on this information there still exists some uncertainty as to whether or not traders have formed the correct opinions based on this pre-release information. Judgement on these pre-formed opinions is reserved until after the announcement has taken place. This results in increased uncertainty leading up to the announcement that is reflected in the options' implied volatility. They use their model to demonstrate that implied volatility should be higher before an announcement and decrease after the announcement when the uncertainty had been reduced.

Veronesi (1999) develops a rational expectations equilibrium model to illustrate that the state of the economy affects the reaction of the market to the type of news announcement being made. When a positive announcement is made during an economic contraction, it is found the market will overreact to this positive announcement. Conversely, the market will overreact to negative news in an economic expansion. They attribute this result to the market questioning whether the current economic climate is changing when the announcement is made. This questioning of whether the current economic climate is changing results in an increase in volatility. The extent which volatility increases depends on the magnitude of the uncertainty that surrounds the announcement, with the greater the uncertainty the greater the increase in volatility.

Barberis, Shleifer and Vishny (1998) develop a behavioural model of market's reaction to news announcements, which assumes that traders suffer from the cognitive error of representativeness bias. That is, the market believes that recent news announcements are an indication of what can be expected in the future. For example, a negative surprise news announcement will be followed by more negative surprise news announcements. This scenario creates uncertainty regarding the future. This in turn, creates additional volatility and results in investors taking positions in the market that reflect this pessimism.

In Nofsinger and Prucyk's (2002) examination of the impact that 21 different types of macroeconomic announcements has on implied volatility, they find negative announcements caused an initial increase in volatility, which revert back to normal throughout the day. Positive announcements however were followed by a persistent period of low volatility. There was also evidence pointing to increased levels of volatility being associated with increased volume levels.

## *Implied volatility as a forecast for future realised volatility*

In the Black-Scholes (1973) model for pricing options all the variables are observable except for volatility, which must be forecast. Two general methods are used to calculate volatility forecasts. The first approach is to calculate an historical realised volatility from past price data. The second is to calculate an implied volatility using current option prices to solve the Black-Scholes option-pricing model for volatility.

There is much conjecture as to whether the computed implied volatility is in fact a quality forecast of future realised volatility. Early studies suggest that this was the case. However, research undertaken by Day and Lewis (1992) and Lamoureux and Lastrapes (1993) provides evidence to the contrary. They conclude that implied volatility is inefficient and biased. Canina and Figlewski (1993) provide the strongest evidence against the robustness and efficiency of using implied volatility as a reliable forecast for future realized volatility. They state that there is no correlation between implied volatility and future realised volatility. They suggest that implied volatility had very little correlation with future realized volatility and that future volatility contains the information that implied volatility provides.

However, Christensen and Prabhala (1998) refute these findings stating that the results obtained by Day and Lewis (1992), Lamoureux and Lastrapes (1993) and Canina and Figlewski (1993) are marginalised by a combination of methodology and the data sets used. They carry on to illustrate that implied volatility outperforms historical volatility as a forecast for future realised volatility and that implied volatility incorporates all the information content from historical volatility because options markets are efficient. The implication that implied volatility is an efficient volatility forecast is a widely held notion. (e.g., Day and Lewis, 1988: Harvey and Whaley, 1992).

Fleming et al (1995) explores the use of the original Chicago Board of Options Exchange's (CBOE) market implied volatility index (VXO) as a forecast for future realised volatility. The idea of formulating a market volatility index originated from Gastineau (1973) after the introduction of exchange traded options. His idea was based upon the averaging of implied volatilities of at-the-money options of 14 stocks to form an index. Gastineau methodology was modified by Cox and Rubenstein (1985) who introduced the use of multiple call options on each stock and then weighting the calculated implied volatilities to form an option that was at-the-money with a constant length to expiration. Whaley (1993) then developed this methodology further to develop an index that was used as the original market implied volatility index as used by the CBOE. VXO essentially creates an at-the-money option with 22 trading days to expiration.

VXO differs from Cox and Rubenstein's index in that its uses options based upon a market index, S&P 100 and uses both calls and puts to form its multiple quote used in the calculation of implied volatility. The idea behind using a market index as the underlying asset for the options, as opposed to individual stocks, is that market participants are concerned mainly with systematic risk opposed to non-systematic risk. Firms risk still contains non-systematic risk but this is diversified away in an index. Therefore, using an index as a risk measure proxy, only systematic risk is measured (Fleming et al 1995).

The results obtained by Fleming et al (1995) indicate that VXO has a strong relationship with future realised volatility. They use this evidence to conclude that VXO is useful in the forecasting of future realised volatility and is a superior method than a first-order autoregressive volatility model. They attribute the reason for this being VXO has market expectations imbedded into it. It must however be noted that they used an overlapping sample data set which, Christensen and Prabhala (1998) have acknowledge that this methodology will have marginalised results due to high autocorrelation associated with this methodology.

# CHAPTER 3

# DATA AND METHODOLOGY

## *Data*

Data on volume trading of OEX options is collected from SIRCA from their Reuters database. The time period covered originates from 1st June 2000 through to 7th November 2002.

Also sourced from SIRCA's Reuters database are the inputs required for the calculation of the VXO index. All closing bid ask quotes for OEX options, the closing S&P 100 index level and the United States T-Bill rates that are used as a proxy for the risk free rate. VXO estimates that fall outside the 1st June 2000 trough to 7th November time period was accessed from the CBOE website to form a data set from 1 January 1998 through to 17 October 2003. VIX data was accessed in its entirety from the CBOE's web site for the period from 1 January 1998 through to 17 October 2003.

Days upon which exchanges are closed, no values are assigned to them. Interpolation techniques are also not used for missing values to prevent marginalised conclusions being drawn from the results

## Hypotheses

In this research the following hypotheses will be tested:

### Volume Hypotheses

**Hypothesis 1:**

$H_0$: That the September 11 attacks will not cause an increase in the total volume of options traded.

**Hypothesis 2:**

$H_0$: That the September 11 attacks will not cause an increase in the total volume of the options close-to-the money to increase

### Volatility Hypotheses

**Hypothesis 3:**

$H_0$: That implied volatility will not increase after the September 11 attacks.

**Hypothesis 4:**

$H_0$: Implied volatility does not contain any information about future realised volatility.

**Hypothesis 5:**

$H_0$: Implied volatility is not an unbiased forecaster of future realised volatility.

**Hypothesis 6:**

$H_0$: Implied volatility is not an efficient forecaster of future realised volatility.

Hypothesis 4,5 and 6 are tested for the pre September 11 attacks period, post September 11 attacks period and the two periods combined.

## *Volume*

To analyse the impact that the September 11 terrorist attacks had on trading volume of the S&P 100 options, two groups are created. The first group contained volume of all available options contracts traded on a specific date with sub groups comprising of call volume, put volume and the sum of call and put volumes creating total volume. The second group contains the volume of four call option contracts plus four put option contracts that are comprised of:

- The two nearest to expiration put (call) options that have strike prices that straddle the current index price;
- The two next to nearest to expiration put (call) options that have strike prices that straddle the current index price.

Total volume of this group is the sum of the four-call option and put option contracts.

Two sample T-test's are run to test the hypothesis that trading volume of options does not increase after the September 11 terrorist attacks. This is also combined with a regression specified by equation (1) on which a Chow (1960) test is conducted. The regression is further expanded to include a dummy variable in equation (2) to also test for a regime change post the September 11 terrorist attacks.

$$V_t = \alpha_0 + \alpha_m d_{t=m} + \alpha_{ts} d_{t=ts} + \alpha_{th} d_{t=th} + \alpha_f d_{t=f} + \alpha_{vxo} VXO_t + e_t \tag{1}$$

$$V_t = \alpha_0 + \alpha_m d_{t=m} + \alpha_{ts} d_{t=ts} + \alpha_{th} d_{t=th} + \alpha_f d_{t=f} + \alpha_I d_{t=p} + \alpha_{vxo} VXO_t + e_t \tag{2}$$

where $\alpha_0$ denotes the constant which is Wednesday, $d_{t=x}$ is a dummy variable the equals 1 for a respective day of the week, $VXO_t$ denotes the implied volatility index value and $d_{t=p}$ denotes a dummy variable that is equal to one in the post September 11 attack period.

### *Day of the Week Volume Effect*

There is an established day of the week volume trading effect in the US markets with the least trading occurring on Monday and increasing throughout the week to be at its greatest on Fridays. To test if this effect changed after the September 11 attacks a standardised measure of volume is created. This standardised measure, measures what proportion of the weeks trading volume occur on each individual day of the week. It is calculated as follows:

$$f_{\chi t \kappa} = \frac{\text{\# of daily options trades for option type } \chi \text{ for day t during time interval } \kappa}{\text{Mean \# of daily option trades for option type } \chi \text{ during time interval } \kappa}$$

where $\chi$ is the type of option contract, t is a day of the week and the time interval $\kappa$ is either pre or post September 11$^{th}$ time interval. Values greater (less) than 1 indicate above (below) average trading occurring on that day of the week. T-test's are run to test the hypothesis that the day of the week trading volume does not change after the September 11 attacks.

## *Volatility*

### *Calculating Implied Volatility Indexes*

Two implied volatility indexes will be created and used to measure the effects on implied volatility of the September 11 terrorist attacks. To create the indexes individual implied volatilities that form the index must be first be calculated. To do this an appropriate option valuation model is required otherwise the calculated implied volatilities will be miscalculated. Therefore, an American Binomial tree option valuation model that specifically accounts for discrete dividends and early exercise is chosen to determine implied volatilities as the options in the data set that is used are of the American form.

Individually calculated implied volatilities are subjected to bias. These biases may occur due to infrequent trading of stocks in an index (Jorion 1995), and different closing times of the respective stock and options market.

To negate the aforementioned problems two separate implied volatility indexes are created. The first index follows the methodology as set out in Whaley (1993) and this methodology is used to form the original volatility index, VXO that is quoted by the Chicago Board Options Exchange (CBOE). This index is derived from options based on the S&P 100 index. It is a weighted index of eight American style options which consists of:

- The two nearest-to-expiration call options with strike prices that bracket the current index level;
- The two next-nearest-to-expiration call options with strike prices that bracket the current index level;
- The two nearest-to-expiration put options with strike prices that bracket the current index level;
- The two next-nearest-to-expiration put options with strike prices that bracket the current index level.

The method of construction eliminates miscalculation and smile effects thus making it a more accurate measure of market implied volatility. In addition, both puts and calls are used to increase the amount of information content of the volatility index and to eliminate put/call clientele effects (Blair et al 2001).

The volatility index that is constructed is weighted accordingly to form a hypothetical at the money option with 22 trading days or 30 calendar days to expiration. This is done to eliminate the problem documented by Fleming et al (1993) that short-term options have a higher implied volatility than their long-term counter parts, therefore to minimize this effect a constant length to expiration is maintained.

Only individual option quotes with a bid-ask spread greater than zero and a maturity of greater than 7 calendar days will be included in the multiple quote.

The 7-day maturity requirement is because some contracts with less than 7 days to maturity exhibit excessively noisy implied volatility.

The inputs into the binominal option-pricing model are the current index value, the options exercise price, the time to expiration and the risk free interest rate. The options price is derived from the mid-point of the bid-ask spread for each of the options in question. This negotiates the problem of bid-ask bounce and as Chan, Chung and Johnson (1995) point out, it also solves the problems associated with the infrequent trading hypothesis. The risk free interest rate is the United States 13-week Treasury Bill rate whose maturity most closely matches the expiration date of the option. A detailed explanation of its formation is included in the appendix.

The second implied volatility index (VIX) used is the new version of the implied volatility index used by the CBOE. VIX still contains the essential characteristics underlying the formation of VXO. The essential differences between the two come from their method of calculation. VIX uses a wide range of options to formulate expected volatility opposed to VXO, which uses only eight nearest the money options. VXO was also calculated from the Black Scholes (1973) option-pricing model opposed to VIX, which is independent of any model. VIX uses a newly developed formula that uses weighted prices of out-of-the-money puts and calls to derive the implied volatility.

The other essential difference arises from the change in the index upon which the options are derived. The VXO used options on the S&P 100 index where as VIX uses options on the S&P 500 index. These changes were to reflect the latest changes in academic research and the fact that the S&P 500 index derivatives have now become the most active index derivatives, moving ahead of the S&P 100. A detailed explanation of the calculation of VIX is provided in the appendix.

### *The relationship between implied volatility and realised volatility*

To test whether implied volatility remains an adequate and superior forecast of future volatility opposed to historical realised volatility after the September 11 attacks, a regression model is used, modelled from the framework in Christensen and Prabhala (1998).

The Christensen and Prabhala (1998) methodology differs from previous research in that they use nonoverlapping data with one implied volatility matched with one realised volatility measure covering each time period. The results they obtain differ from previous research and they attribute to, that previous research being based upon overlapping data. They acknowledge that the use of overlapping data produces highly autocorrelated errors, which in turn produces imprecise and inconsistent regression estimates. This marginalises previous research results conducted in the area of implied volatility as a forecast of future volatility.

The sampling procedure that this study follows is similar to Christensen and Prabhala (1998) where nonoverlapping data is used. They used monthly-realised volatilities paired with the implied volatility that is determined at the beginning of the period to form a nonoverlapping observation pair. The implied volatility is calculated from the option that is closest to being at-the-money and expires the month following. The realised volatility is calculated over the remaining life of that option. They next constructed an entire sequence using this process to form their data set.

In this study, the calculated implied volatility indexes VXO and VIX are used as the measure of implied volatility. The implied volatility is then paired with the realised volatility of the following 22 trading days in a rolling fashion to determine the efficiency of implied volatility as a future forecast of realised volatility. Then realised volatility is then estimated using equation (3).

$$\sigma_{ht} = \sqrt{\frac{1}{\tau_t}\sum_{k=1}^{\tau t}\left(r_{t,k} - \bar{r}_t\right)^2} \tag{3}$$

where $\tau_t$ is the number of days to expiration, $\bar{r}_t = \tau_t^{-1}\sum_{k=1}^{\tau t} r_{t.k}$, and $r_{t,r}$ is the index return on day k of month t. This is calculated over a 22 trading day period to also mitigate the overlapping sample problem. Both realised and implied volatilities are converted to annual rates to assist with interpretation. It also must be noted that the volatility series are both in natural log form for the conduction of the empirical work. Implied volatility will be denoted by $i_t$=log $\sigma_{it}$ and realised volatility will be denoted by $h_t$=log $\sigma_{ht}$.

## *The September 11 terrorist attacks*

### *Implied Volatility*

The September 11 terrorist attacks induced a major negative response by the financial markets. The October 1987 crash also induced a major negative response by the financial markets and Christensen and Prabhala (1998) document a significant regime change following the crash with implied volatility becoming less biased following the crash. To test for a regime change following the September 11 attacks, the same regressions will be run with the Chow (1960) test being applied to the regressions to test for a regime change in the use of implied volatility as an ex-ante forecast for future realised volatility. This will test whether the information content of implied volatility changed post the September 11 attacks. An unequal variance t-test is used to test if there were changes in the level of realised volatility for each respective index and their respective implied volatility index measures.

### *Interrelationship between implied volatility and volume*

To assess if the interrelations between implied volatility and volume change post the September 11 attacks bivariate VAR's are utilised using data from OEX option contracts. Both variables in the VAR model are in their natural logarithm form. Appropriate lag lengths are selected through the use of Akaike information criterion as suggested by Kilian (2001). VAR models assume that all variables are endogenous to each other.

The variance/co-variance matrix is identified through a Choleski decomposition in which the variable higher in the ordering has a contemporaneous effect on the variable lower in the ordering. The variable lower in the ordering affects the variable higher in the ordering only with a lag.

Koch (1993) acknowledges that standard VAR models omit the contemporaneous interactions among the variables that is the VAR model does not allow for the variables being determined simultaneously. Koch (1993) then promotes an alternative method to capture contemporaneous interactions through the use of a simultaneous equation to estimate the parameters involved This methodology was adopted by Kyriacou and Sarno (1999) to investigate the dynamic relationships between spot market volatility, future trading and options trading. Sarwar (2002) also used the methodology to investigate the relationship between implied volatility and trading volume of currency options.

To control for this without the use of a simultaneous equation model the Choleski ordering is alternated between the two variables. In this study, the use of a one-standard deviation structural shocks are given to both the variables in the VAR model and one-standard deviation confidence intervals are obtained from 10000 Monte Carlo draws.

# CHAPTER 4

# Discussion of Results

## *Volume*

**Table 1:**

**OEX Total Volume**

| | **Call** | | **Put** | | **Total Volume** | |
|---|---|---|---|---|---|---|
| | Pre 9/11 ($\mu_0$) | Post 9/11 ($\mu_1$) | Pre 9/11 ($\mu_0$) | Post 9/11 ($\mu_1$) | Pre 9/11 ($\mu_0$) | Post 9/11 ($\mu_1$) |
| Mean | 2186631 | 2630437 | 2485963 | 2868661 | 4672594 | 5499098 |
| p-value | 0.1476 | | 0.3163 | | 0.213 | |

**OEX Eight Contract Volume**

| | **Call** | | **Put** | | **Total Volume** | |
|---|---|---|---|---|---|---|
| | Pre 9/11 ($\mu_0$) | Post 9/11 ($\mu_1$) | Pre 9/11 ($\mu_0$) | Post 9/11 ($\mu_1$) | Pre 9/11 ($\mu_0$) | Post 9/11 ($\mu_1$) |
| Mean | 117871.5 | 213954.2 | 198631.5 | 401524.1 | 315503 | 615478.4 |
| p-value | $0.003^{***}$ | | $0.000^{***}$ | | $0.000^{***}$ | |

Hypothesis Tested: $H_0=\mu_0-\mu_1=0$

*** p-value significant at one percent level

The results in the table 1 show that the change in trading volume behaviour was different for the two sub-groups. The total volume group results show that volume increased for both types of contracts but neither contract types nor total volume had a significant change in volume post the September 11 attacks. The put call ratio increased in the post September 11 period, however this change was not significant.

For the eight-contract group there were significant changes in trading volume post September 11. Both call and put contract volume and total volume had significant changes in volume post September 11 at the 1% significance level. These results are also confirmed when a regression model is run using a dummy variable and a Chow test. The only anomaly exists with the Chow test p-value for calls in the eight-contract sub-group. A reason for this anomaly could be attributed to the robustness of Chow tests being compromised when heteroscedasticity is present. Also the put call ratio change was significant at the ten percent level with the ratio increasing in the post September 11 period above it's pre September 11 period value.

**Table 2:**

**OEX Total Volume**

| | **Call** | **Put** | **Total Volume** |
|---|---|---|---|
| Dummy Variable p-value | 0.258 | 0.378 | 0.311 |
| Chow Test p-value | 0.743 | 0.788 | 0.979 |
| Put Call Ratio p-value | 0.136 | | |

Hypothesis Tested: $H_0 = \mu_0 - \mu_1 = 0$

**OEX Eight Contract Volume**

| | **Call** | **Put** | **Total Volume** |
|---|---|---|---|
| Dummy Variable p-value | 0.0020*** | 0.0010*** | 0.0000*** |
| Chow Test p-value | 0.1724 | 0.00598*** | 0.02205** |
| Put Call Ratio p-value | 0.098* | | |

Hypothesis Tested: $H_0 = \mu_0 - \mu_1 = 0$

*** p-value significant at one percent level

** p-value significant at five percent level

* p-value significant at ten percent level

The results for the eight-contract sub-group are consistent with Donders et al (2000) who found that when surprise economic announcements were made the volume of put contracts increased. This is also consistent with Sarwar (2003) who proposes that increased volatility will lead to an increase in volume of option trading due to their hedging functions. Sears (2000a, b) and Tan (2001) also report evidence of a positive relationship between higher price volatility and trading volume of stock options. However, these conclusions do not seem to hold for the total volume group.

An explanation could be that the September 11 attacks and the resulting dramatic decline in the stock prices, which suppressed the equity indexes, caused an increase in demand for options for hedging purposes as uncertainty in the markets increased. As demand increased for options this would result in an increase in the price premium of options. Options prices will also have incorporated the changes in index levels into their price, coupling this with the dramatic increase in the price premium, option prices should be changing faster than their underlying index in a time of great uncertainty. This leads to an increase in the delta of options, which causes delta neutral portfolios to become unbalanced. To rebalance delta neutral portfolios options must be traded and it is likely that the options that form the eight-contract sub-group are used for such measures due to their high liquidity. This implies that the volume of such option contracts will increase in times of increased uncertainty.

Another possible explanation to these contrary results is as follows. In the post September 11 attacks, the economic environment was highly volatile and unstable which should have encouraged more hedging and speculation to take place. This does not appear the case due to there not being a significant increase in traded volume of options. Although the attacks may have provoked more activity on the options market by those left participating the overall level of investors participating may have declined, cancelling out the positive effect on volume by the remaining participants.

### *Day of the week effect*

**Table 3:**

**OEX Total Volume**

**Pre September 11**

| | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| Call | 0.4153 | 0.5746 | 0.8347 | 1.1778 | 1.9075 |
| Put | 0.4387 | 0.6013 | 0.8273 | 0.9359 | 2.1027 |
| Total Volm | 0.4277 | 0.5888 | 0.8308 | 1.0491 | 2.0113 |
| **Post September 11** | | | | | |
| Call | 0.4906 | 0.7217 | 0.9191 | 1.0681 | 1.7681 |
| Put | 0.5822 | 0.7323 | 0.9017 | 1.0208 | 1.7357 |
| Total Volm | 0.5383 | 0.7272 | 0.9100 | 1.0434 | 1.7512 |

**OEX Eight Contract Volume**

**Pre September 11**

| | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| Call | 0.4969 | 0.6444 | 0.9310 | 1.2173 | 1.6397 |
| Put | 0.6017 | 0.7457 | 1.3622 | 1.0849 | 1.1732 |
| Total Volm | 0.5626 | 0.7079 | 1.2012 | 1.1344 | 1.3476 |
| **Post September 11** | | | | | |
| Call | 0.5351 | 0.6840 | 1.1279 | 1.1309 | 1.5184 |
| Put | 0.7925 | 1.0359 | 0.7011 | 1.1292 | 1.3625 |
| Total Volm | 0.7030 | 0.9136 | 0.8495 | 1.1298 | 1.4167 |

Table 3 reports the standardised measure's values for each day of the week for the two sub-groups. The table illustrates the day of the week trading volume effect with traded volume of options increasing as the week progresses for the total volume sub-group. This trend is generally transferable to the eight-contract sub-group, however there are a few differences. Put volume on Thursday's in the pre-September 11 period for the eight-contract volume sub-group is less than its Wednesday's counterpart. This affects the total volume measure for Thursday's as well, producing the same result. In addition, Wednesday's trading in the post

September 11 period for put volume is well below all other days of the week, which is contrary to the day of the week effect illustrated in the total volume group.

A trend observable from the table is that the trading volume spread over the week has become smaller for both groups, i.e. the amount of trading on Mondays has increased with respect to Fridays trading. This indicates market participants are now more willing to hedge positions not just over the weekend but also over the whole week.

**Table 4:**

| **OEX Total Volume** | | | | | |
|---|---|---|---|---|---|
| | **Monday** | **Tuesday** | **Wednesday** | **Thursday** | **Friday** |
| Call p-value | 0.4197 | 0.2417 | 0.6523 | 0.6981 | 0.7784 |
| Put p-value | 0.1934 | 0.2862 | 0.6865 | 0.5767 | 0.5544 |
| Total Volm p-value | 0.2526 | 0.2456 | 0.6522 | 0.9529 | 0.6310 |
| **OEX Eight Contract Volume** | | | | | |
| | **Monday** | **Tuesday** | **Wednesday** | **Thursday** | **Friday** |
| Call t-value | 0.8539 | 0.8636 | 0.4674 | 0.7992 | 0.8179 |
| Put t-value | 0.4872 | 0.2728 | 0.0353** | 0.8734 | 0.6139 |
| Total Volm t-value | 0.4856 | 0.3395 | 0.1546 | 0.9850 | 0.8442 |
| Hypothesis Tested: $H_0 = \mu_0 - \mu_1 = 0$ | | | | | |

** p-value significant at five percent level

The results presented in table 4 are for the T-tests for the hypotheses that the day of the week trading volume does not change after the September 11 attacks. The results indicate that there was no significant change in the day of the week effect after the September 11 attacks. That is the behaviour of the traders with respect to the day of the week remains the same. The only significant result is a decrease in the proportion of put contracts trading volume in the eight-contract sub-group on Wednesday's with respect to the weeks trading. This implies that market participants' delay buying put contracts to later in the week for close to the money contracts. Nofsinger and Prucyk (2002) acknowledge that most macroeconomic announcements occur on Thursdays and Fridays so a logical explanation for this phenomenon may be attributed to market participants now deferring trading on Wednesday's until prior to or once these announcements have been made. This increases market participants' ability to glean as much information as possible before they make a trade, thereby reducing their risk. This implies that market participants have become more risk adverse.

## Implied Volatility

### Descriptive Statistics

Presented in table 5 are the descriptive statistics for the realised and implied volatility series along with their log counterparts. The statistics presented have been divided up into two sub periods, pre September 11 attacks and post September 11 attacks.

Firstly, we must acknowledge the difference in means between the implied volatility and the realised volatility series in both periods. The implied volatility that is used is derived from the calculated volatility index to avoid the previously mentioned mis-measurement problems associated with standard implied volatilities. Although robust to mis-measurement, it still contains some bias as a forecast for future realised volatility. Blair et al (2001) acknowledge the source of this bias to come from the conversion of conventional implied volatility to a trading day measurement from a calendar day measurement. The approximation of this bias should be equal to 1.2[2] with Blair et al (2001) finding the scaling factor to be 1.23 from the period of 1987-1999. Therefore, an adjusted implied

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[2] VIX multiplies conventional implied volatility by $\sqrt{N_c/N_t}$, with $N_c$ being calendar days and $N_t$ being trading days until expiration.

volatility series has been created to correct for this bias by using the scaling factor of 1.204[3].

**Table 5:**

| | Realised Volatility | Implied Volatility | Adj. implied volatility | Log realised Volatility | Log implied Volatility | Log Adj. implied volatility |
|---|---|---|---|---|---|---|
| ***S&P 100 VXO*** | | | | | | |
| *Pre 9/11* | | | | | | |
| Mean | 0.2027 | 0.2599 | 0.2159 | -1.6486 | -1.3636 | -1.5489 |
| 100×Var | 0.4608 | 0.2442 | 0.1686 | 10.4764 | 3.1213 | 3.1213 |
| *Post 9/11* | | | | | | |
| Mean | 0.2208 | 0.2882 | 0.2395 | -1.5679 | -1.2770 | -1.4622 |
| 100×Var | 0.6235 | 0.5741 | 0.3964 | 11.1234 | 6.4324 | 6.4324 |
| ***S&P 500 VIX*** | | | | | | |
| *Pre 9/11* | | | | | | |
| Mean | 0.1917 | 0.2439 | 0.2027 | -1.7025 | -1.4267 | -1.6120 |
| 100×Var | 0.4008 | 0.2137 | 0.1475 | 10.0586 | 3.0402 | 3.0402 |
| *Post 9/11* | | | | | | |
| Mean | 0.2108 | 0.2602 | 0.2162 | -1.6133 | -1.3761 | -1.5613 |
| 100×Var | 0.5596 | 0.4194 | 0.2896 | 10.887 | 5.8095 | 5.8095 |

The data from above table shows that the means for realised volatility and adjusted implied volatility are relatively close to each for both the VXO and VIX index series. Adjusted implied volatility is slightly higher than its realised counterpart in both the sample periods with each measure experiencing an increase in the post September 11 period. This would be as expected. The notion that implied volatility is a smoothed expectation of realised volatility also holds for both sample periods with variability of implied volatility being lower than the

3 $\sqrt{365/252} = 1.204$

realised volatilities variability. Also to be noted is that the realised volatility of S&P 100 index and its implied volatility index, VXO, is lower than the respective volatility measures of the S&P 500 index and its implied volatility index, VIX.

The time series properties of the two volatility series are assessed to see whether they contain similar properties to series used in previous research. An ARIMA (p,d,q), model is fitted in the form of

$$\Phi(B)(\Delta^{d}\chi_{t}-\mu)=\Theta(B)\varepsilon_{t} \tag{4}$$

where $\chi_t$ represents one of the two log-volatility series the parameter $\mu$ is the mean, $\varepsilon_t$ is white noise, $\Phi$ and $\Theta$ are polynomials of order $p$ and $q$ in $B$, the backshift operator defined by $B\chi_t = \chi_{t-1}$, and $\Delta = 1-B$ is the first difference operator. French et al (1987) and Schwert (1989) identified that log-volatility series conform more closely to normality than do non log-volatility series so the time series models are fitted to the log-volatility series.

Estimates of the ARIMA (p,d,q) models from equation(4) of the form of

$$\Phi(B)(\Delta^{d}\chi_{t}-\mu)=\Theta(B)\varepsilon_{t} \tag{4}$$

fitted to the time series $\{\chi_t\}$, with $\chi_t = i_t$ or $\chi_t = i_t$, where $i_t$ denotes the natural logarithm of the implied volatility index value, $h_t$ denotes the natural logarithm of the ex-post daily return volatility of the applicable index, $\varepsilon_t$ is white noise, $\Phi(B)$denotes the AR polynomial $1-\phi_1 B-\phi_2 B^2$, $\Theta(B)$ denotes the MA polynomial $1-\theta_1 B$, $B$ denotes the backshift operator, and $\Delta=1-B$ denotes the first difference operator.

The results are displayed in table 6. Results in this table indicate the series are integrated ARIMA(1,1,1) for all volatility time series. This conclusion uses the Akaike information criterion which, is lowest for the ARIMA(1,1,1) model. These results are different from previous research. Christensen and Prabhala (1998) used monthly volatility series from the S&P 100 and found an ARMA(1,1) model best described the time series properties of volatility, which, they state was consistent with other research. However, it must be noted their next best choice was an ARIMA(1,1,1) model to model the time series volatility properties.

**Table 6:**

| **Fitted Model** | $\mu$ | $\phi_1$ | $\phi_2$ | $\theta_1$ | **AIC** |
|---|---|---|---|---|---|
| *Implied Volatility Index: VXO $\{i_t\}$* | | | | | |
| AR(1) | -1.537*** | 0.5790*** | | | 0.7146 |
| ARMA(1,1) | -1.529*** | 0.4674*** | | 0.2137 | 0.7060 |
| AR(2) | -1.516*** | 0.5867*** | -0.0436 | | -0.5765 |
| ARIMA(1,1,1,) | -1.519*** | 0.5577*** | | 0.6732*** | -1.7902 |
| *S&P 100 Realised Volatility $\{h_t\}$* | | | | | |
| AR(1) | -1.619*** | 0.4536*** | | | 0.4358 |
| ARMA(1,1) | -1.596*** | 0.2146** | | -0.0819 | 0.4389 |
| AR(2) | -1.597*** | 0.4302*** | 0.0228 | | 0.4040 |
| ARIMA(1,1,1,) | -1.567*** | 0.3889*** | | 0.9974*** | -0.8228 |
| *Implied Volatility Index: VIX $\{i_t\}$* | | | | | |
| AR(1) | -1.614*** | 0.6425*** | | | -0.7557 |
| ARMA(1,1) | -1.600*** | 0.5120*** | | 0.1219 | -0.7141 |
| AR(2) | -1.612*** | 0.6575*** | -0.1239 | | -0.7527 |
| ARIMA(1,1,1,) | -1.618*** | 0.0954*** | | 0.9852*** | -2.0767 |
| *S&P 500 Realised Volatility $\{h_t\}$* | | | | | |
| AR(1) | -1.669*** | 0.4084*** | | | 0.4083 |
| ARMA(1,1) | -1.648*** | 0.5556*** | | -0.2434 | 0.4184 |
| AR(2) | -1.669*** | 0.4060*** | 0.0974 | | 0.4149 |
| ARIMA(1,1,1,) | -1.635*** | 0.3886*** | | 0.9972*** | -0.8087 |

*** p-value significant at one percent level

** p-value significant at five percent level

A reason for the difference could stem from the choice of a different sample period. Christensen and Prabhala (1998) used data from the period of November 1983 to May 1995 and consisted of 139 non-overlapping monthly observations.

The sample period for this study is January 1998 to October 2003 and consists of 66 non-overlapping monthly observations.

Another difference could be attributed to the sample used by Christensen and Prabhala (1998) contained data pre the stock market crash of October 1987 and the stock market crash itself. They documented a significant change in the bias of implied volatility as a forecast for future realised volatility post the 1987 stock market crash. They reasoned this occurred because the OEX options market was still in its infancy and the market was still determining how it priced options and therefore, how the market determined its implied volatility. They state that in the post crash period, the market had matured and determined how it priced and therefore, how it calculated implied volatiles of OEX options.

Another reason they present is related to the stochastic process followed by index returns, which changes after the crash. Therefore, the reason for different results of time series properties of volatility in this study could be attributed to the sample in this study which, does not contain the infancy period of the index options market and nor does it contain the regime change that occurred in volatility as a result of the 1987 crash.

The implication of an ARIMA (1,1,1) model is that the time series exhibits first-order autocorrelation. This means the series may require a first difference to be taken to make the series stationary before it can be used in analysis. However, an Augmented Dickey-Fuller Unit root test was preformed and found that a unit root did exist at the one percent significance level and therefore the series can be deemed stationary without a first difference being taken.

## *The relationship between implied and realised volatility.*

In this section, the relationship between implied volatility and realised volatility is investigated. The information content of implied volatility will be analysed from each respective implied volatility measure and the effects of the September 11 attacks will be investigated for each measure.

### *Conventional Analysis*

Christensen and Prabhala (1998) acknowledge that the information content in previous literature is measured by estimating a regression in the form

$$h_t = \alpha_0 + \alpha_1 i_t + e_t, \tag{5}$$

where $h_t$ denotes the realised volatility for period $t$ and $i_t$ denotes the implied volatility index at the beginning of period $t$.

Christensen and Prabhala (1998) state equation (5) can be used to test three hypotheses. The first hypothesis tested is if the implied volatility index contains some information about future volatility then $\alpha_i$ should not be equal to zero. The second hypothesis tested is if the implied volatility index is an unbiased forecast of future realised volatility then $\alpha_0$ should equal zero and $\alpha_i$ should equal 1. The final hypothesis tested is if the implied volatility index is efficient, the residuals $e_t$ should exhibit white noise. Also to compare the information content of the implied volatility index to that of the past-realised volatility a multiple regression is estimated using equation 6. To test the information content of past-realised volatility a univarite regression is estimated using equation 7.

$$h_t = \alpha_0 + \alpha_i i_t + \alpha_h h_{t-1} + e_t, \tag{6}$$

$$h_t = \alpha_0 + \alpha_h h_{t-1} + e_t \tag{7}$$

It must be noted that the Durbin Watson statistics for rejecting autocorrelation in the preformed regressions without a first difference were borderline thus further analysis was preformed. This is consistent with an ARIMA (1,1,1) model being found to best describe the time series properties of the respective volatility measures. However, when the residuals were tested it was found that they were stationary and therefore exhibited white noise so indicating autocorrelation was not significant. Also when the errors were adjust using the Newey and West

(1987) procedure the results gained did not change from the previous results which, also suggested autocorrelation was not a problem

**Table 7:**

| Dependent variable: Log realised volatility of S&P 100 $h_t$ | | | | |
|---|---|---|---|---|
| Independent variables | | | Adj. $R^2$ | White noise $e_t$[4] |
| Intercept | $i_t$ | $h_{t-1}$ | | |
| -0.0709 | 1.0210*** | | 0.3528 | Yes |
| -0.8782*** | | 0.4585*** | 0.1962 | Yes |
| -0.0814 | 0.9227*** | 0.0859 | 0.3458 | Yes |
| Dependent variable: Log realised volatility of S&P 500 $h_t$ | | | | |
| -0.06257 | 1.0084*** | | 0.3402 | Yes |
| -0.9600*** | | 0.4259*** | 0.1670 | Yes |
| -0.0642 | 0.9121*** | 0.0912 | 0.3345 | Yes |

*** p-value significant at one percent level

Table 7 report the regression estimates for $\alpha_i$ for the VXO and VIX series are 1.021 and 1.0084 respectively and for both the null hypothesis that $\alpha_i$=0 can be rejected at the one percent level. Therefore, we can conclude that both implied volatility indexes contain information about future realised volatility. We can also conclude that both indexes are unbiased. Although the null hypothesis $\alpha_i$=1 and $\alpha_o$=0 can be rejected when they are tested jointly for both series at he five percent level for VXO and ten percent level for VIX. When the null hypothesis $\alpha_i$=1 and $\alpha_o$=0 are tested individually both hypotheses cannot be rejected for both VXO and VIX series. The difference in results between the joint tests and the testing the coefficients individually can be attributed to aggregation problems associated with joint hypothesis testing. The residuals also for both series are white noise and therefore implied volatility can be concluded as efficient.

[4] To test if the residuals had white noise an Augmented Dickey-Fuller Unit Root Test was preformed to check for stationary of the residuals.

These results differ from the results reported in Christensen and Prabhala (1998). They found $\alpha_0$ to be negative and significantly different from zero and $\alpha_i$ to be less than and not significant to unitary. This contrasts to the results in this study where $\alpha_0$ is negative but not statistically significant from zero and $\alpha_i$ is statistically significantly unitary. Christensen and Prabhala (1998) attribute the negative intercept to the use of the log-volatility series opposed to the level series and also due to the consequence of errors-in-variable problem that is a result of using implied volatility. The errors-in-variable problem is also a cause for their less than unitary value for $\alpha_i$. The negative value for $\alpha_0$ in this study can also be attributed to the log-volatility series but the reason for it not been significantly different from zero and $\alpha_i$ being unitary is because of the use of the implied volatility index which eliminates the errors-in-variable problem.

Also tested was the information content of historical realised volatility as the sole determinant of future realised volatility. The results indicate that historical realised volatility does contain some information about future realised volatility but less information than its implied index counterparts do because the values of the coefficients were smaller. The coefficients for past-realised volatility of both series were significant at the one percent level. In addition, the adjusted $R^2$ is lower in the regressions that use past realised volatility as the only explanatory variable compared to the regressions that include an implied volatility index measure as an explanatory variable. This result is comparable to Christensen and Prabhala (1998).

When an implied volatility index and historical realised volatility are combined in a multiple regression model, equation (6), the coefficient for historical realised volatility ($\alpha_h$) drops dramatically from 0.4585 to 0.0859 and 0.4259 to 0.0912 for the VXO and VIX series respectively and is no longer significant. This suggests that the implied volatility indexes are efficient in forecasting future realised volatility.

The joint hypothesis of $\alpha_0=0$, $\alpha_i=1$ and $\alpha_h=0$ is however only rejected for the VXO series at the five percent level but is not rejected for the VIX series. When the hypotheses $\alpha_0=0$, $\alpha_i=1$ and $\alpha_h=0$ are tested individually for the VXO series they can be rejected at the one percent level indicating that VXO is also an unbiased forecaster of future realised volatility.

Also seen is that implied volatility coefficient ($\alpha_i$) has been reduced to 0.9227 and 0.9121 for the S&P 100 and S&P 500 indexes respectively. Despite both coefficients falling below unitary the null hypothesis that $\alpha_i = 1$ cannot be rejected and they also remain significant at the one percent level. These results indicate that historical realised volatility does not contribute any new information other than what is imputed into the implied volatility index. These results once again differ from Christensen and Prabhala (1998) as they found historical realised volatility still contributes some information towards future realised volatility. Therefore, this implies that the use of an implied volatility index opposed to the use of a single implied volatility provides a more accurate and robust proxy as an ex-ante forecaster of future realised volatility. This occurs as both implied volatility indexes already incorporate all information contained in historical realised volatility, but single implied volatilities do not.

## *Comparison between the VIX and VXO implied volatility measures*

On initial observation, the results between the two implied volatility index measures appear to provide very similar results. When the only explanatory variable that is used to estimate realised volatility is the implied volatility measure, both measures provide unitary slope coefficients and an intercept that is not significantly different from zero. However, the adjusted $R^2$ is slightly higher for the VXO methodology in table 7. Similarly, the results derived from the multiple regression models also exhibit coefficient values that are almost the same. However, once again, the adjusted $R^2$ is slightly higher for the VXO methodology.

Using the adjusted $R^2$ figure in determining which index measure provides the most accurate and superior in forecast of future realised volatility leads to the conclusion that the VXO methodology is superior to VIX methodology. However, using the results from the joint hypothesis tests, results are more favourable for the VIX methodology, particularly in the joint hypothesises test used for the multiple regression, which cannot be rejected. This indicates the VIX methodology produces a less biased forecast of future realised volatility than the VXO methodology.

### *The September 11 terrorist attacks*

**Table 8:**

| | **VXO** | **VIX** |
|---|---|---|
| Realised Volatility | $0.0021^{***}$ | $0.0002^{***}$ |
| Adj. Implied Volatility Index | $0.0000^{***}$ | $0.0000^{***}$ |
| Hypothesis Tested: $H_0 = \mu_0 - \mu_1 = 0$ | | |

*** p-value significant at one percent level

The results for the t-tests for a change in the levels of the implied volatility indexes post the September 11 attack are presented in table (7). The results indicate that there was a significant change in volatility. We cannot reject the null hypothesis that there was no change in the levels of volatility in both its realised and implied indexed measures for both the VXO and VIX series. This implies that the September 11 terrorist attacks caused increased uncertainty and therefore risk in the US equity markets in the US.

**Table 9:**

| **S&P 100: VXO** | | | | | |
|---|---|---|---|---|---|
| *Pre-September 11 Attacks* | | | *Post-September 11 Attacks* | | |
| Dependent Variable: $h_t$ | | | Dependent Variable: $h_t$ | | |
| Independent Variables: | | Adj. $R^2$ | Independent Variables: | | Adj. $R^2$ |
| Intercept | $i_t$ | | Intercept | $i_t$ | |
| -0.0480 | 1.0340*** | 26.86% | -0.1650 | 0.9696*** | 43.86% |
| Chow Break Point test statistic: | | | 0.8570 | | |
| **S&P 500: VIX** | | | | | |
| *Pre-September 11 Attacks* | | | *Post-September 11 Attacks* | | |
| Dependent Variable: $h_t$ | | | Dependent Variable: $h_t$ | | |
| Independent Variables: | | Adj. $R^2$ | Independent Variables: | | Adj. $R^2$ |
| Intercept | $i_t$ | | Intercept | $i_t$ | |
| -0.0579 | 1.0144*** | 27.26% | 0.0741 | 0.9950*** | 42.04% |
| Chow Break Point test statistic: | | | 0.9772 | | |

*** p-value significant at one percent level

** p-value significant at five percent level

Tables (9), (10) and (11) depict the results of the various regressions when the sample period is divided into pre and post September 11 periods. The first point to note is that the adjusted $R^2$ values improved for all models in the post period when compared to the pre period. This implies that the models explain more variance in the independent variable, realised volatility than their counterparts in the pre-period.

The results using the univariate regression specification, equation (5) are illustrated in table (9) and show that the coefficient for the implied volatility index drops for both implied volatility index measures below unitary. However, the hypothesis that $\alpha_i$=1 and $\alpha_0$=0 when tested individually still cannot be rejected for both implied volatility index measures in both periods. This indicates both implied volatility indexes are unbiased forecasters in the pre and post periods and therefore, the September 11 attacks did not affect the characteristics of using implied volatility indexes as an ex-ante forecast for future realised volatility.

**Table 10:**

| **S&P 100: VXO** | | | | | |
|---|---|---|---|---|---|
| *Pre-September 11 Attacks* | | | *Post-September 11 Attacks* | | |
| Dependent Variable: $h_t$ | | | Dependent Variable: $h_t$ | | |
| Independent Variables: | | Adj. R2 | Independent Variables: | | Adj. R2 |
| Intercept | $h_{t-1}$ | | Intercept | $h_{t-1}$ | |
| -0.9674*** | 0.4044** | 13.27% | 0.7446** | 0.5438*** | 27.93% |
| Chow Break Point test statistic: | | | 0.8399 | | |
| **S&P 500: VIX** | | | | | |
| *Pre-September 11 Attacks* | | | *Post-September 11 Attacks* | | |
| Dependent Variable: $h_t$ | | | Dependent Variable: $h_t$ | | |
| Independent Variables: | | Adj. R2 | Independent Variables: | | Adj. R2 |
| Intercept | $h_{t-1}$ | | Intercept | $h_{t-1}$ | |
| -1.1081*** | 0.3409** | | -0.7494** | 0.5529*** | 32.05% |
| Chow Break Point test statistic: | | | 0.9587 | | |

*** p-value significant at one percent level

** p-value significant at five percent level

Table 10 illustrates the regression results using equation (7). The coefficient of historical realised volatility when used as the single explanatory variable increases for both equity indexes in the post period. This implies more information about future volatility in the post-period is held than in the pre-period. The adjusted $R^2$ also increases for both equity indexes in the post period.

Table 11 illustrates the regression results using the multiple regression specification, equation (6), the coefficients for the implied volatility index drops from the pre-period to the post-period for both index measures. However, the coefficients for historical realised volatility have increased from the pre-period to the post period for both index measures. The changes in the values of the coefficients indicate that implied volatility loses some explanatory power for determining future realised volatility whilst historical realised volatility gains explanatory power.

**Table 11.**

| **S&P 100: VXO** | | | | | | | |
|---|---|---|---|---|---|---|---|
| *Pre-September 11 Attacks* | | | | *Post-September 11 Attacks* | | | |
| Dependent Variable: $h_t$ | | | | Dependent Variable: $h_t$ | | | |
| Independent Variables: | | | Adj. $R^2$ | Independent Variables: | | | Adj. $R^2$ |
| Intercept | $i_t$ | $h_{t-1}$ | | Intercept | $i_t$ | $h_{t-1}$ | |
| 0.0482 | 1.0211*** | 0.0664 | 27.38% | -0.1682 | 0.8405** | 0.1197 | 41.81% |
| Chow Break Point test statistic: | | | | 0.9587 | | | |
| **S&P 500: VIX** | | | | | | | |
| *Pre-September 11 Attacks* | | | | *Post-September 11 Attacks* | | | |
| Dependent Variable: $h_t$ | | | | Dependent Variable: $h_t$ | | | |
| Independent Variables: | | | Adj. $R^2$ | Independent Variables: | | | Adj. $R^2$ |
| Intercept | $i_t$ | $h_{t-1}$ | | Intercept | $i_t$ | $h_{t-1}$ | |
| -0.0520 | 0.9603*** | 0.0549 | 25.55% | -0.0953 | 0.8081** | 0.1697 | 40.73% |
| Chow Break Point test statistic: | | | | 0.9808 | | | |

*** p-value significant at one percent level

** p-value significant at five percent level

An explanation of this phenomenon could be that the September 11 attack introduced an increased amount of uncertainty in the financial markets as illustrated by the significant increase in volatility. The increased uncertainty of market participants could be a result of their uncertainty of how fellow market participants would behave and how the economy would react in light of the unprecedented event of the September 11 attacks. As a result of this uncertainty, market participants could be unsure of the level of implied volatility they should price into options; therefore, they look at past-realised volatility as a guide to the reactions of fellow market participants. Because of this, efficiency of implied volatility indexes as an ex-ante forecast for future realised volatility diminishes.

This is consistent with Barberis et al (1998) whose behavioural model implies that a bad news surprise leads to increased uncertainty for the future as it is expected more bad news will follow.

However, it must be noted that these changes were not significant according to the Chow break point test. Therefore, we can conclude that no regime change after the September 11 attacks involving the implied volatility indexes as ex-ante forecasts for future realised volatility occurred. Also the hypothesis of $\alpha_0=0$, $\alpha_i=1$ and $\alpha_h=0$ when tested individually cannot be rejected for both implied volatility index measures in both periods, indicating that the September 11 attacks did not alter the measures ability to be unbiased and efficient forecasters of future realised volatility. It also provides evidence that the increase in the information component of historical volatility, used to predict future realised volatility in the multiple regression specification, is not significant.

When comparing the robustness of the two implied volatility index methodologies in light of the September 11 attacks, it is found that the VIX methodology is more robust in the univariate regression specification. This conclusion is derived by examining the implied volatility index coefficients. The implied volatility index coefficient for the VIX methodology decreases below unitary by less than its VXO counterpart.

In the multivariate regression specification the opposite occurs. The VXO appears to be more robust as its implied volatility index coefficient decreases below unitary by a smaller amount than its VIX counterparts coefficient. In addition, the historical realised volatility coefficient is smaller in the multivariate regression for the VXO methodology. This indicates that the VXO captures more information about future realised volatility than VIX does in the multivariate regression. This is in conflict with the results gained from the comparison of the univariate regression results.

To conclude which implied volatility index methodology was a more robust forecaster of future realised volatility in light of the September 11 attacks is not clear. The adjusted $R^2$ results indicate that VXO methodology explains more variation in future realised volatility than VIX, but this difference is only of a nominal amount. VXO also appears to be more robust when used in the multivariate regression specification. However, VIX appears to be a more robust measure in the univariate regression specification. Since there is no clear pattern on which implied volatility index methodology provids the more robust results, with the differences between the two measures being minimal, it can be concluded that one methodology is neither superior nor inferior to the other.

## *Interrelationship between implied volatility and volume*

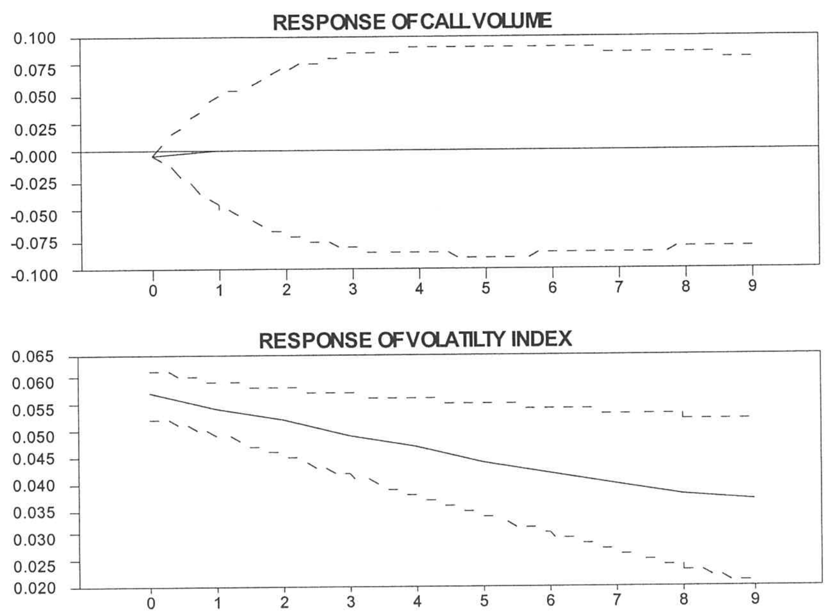
In the pre September 11-attack period, call volume does not significantly respond to an increase in implied volatility. However, in the September 11 period an increase in implied volatility induces a significant persistent increase of call volume by approximately 1.27% for every one percent increase in implied volatility. The results are obtained through an impulse response function for a one standard deviation shock to implied volatility and are illustrated in figure 4.1.

The interrelationship between put volume and implied volatility is similar to the relationship between call volume and implied volatility. In the pre September 11 period, an increase in implied volatility did not lead to a significant change in put volume. However, post September 11 an increase in implied volatility leads to a significant increase in put volume with a lag of one day. An increase of implied volatility of one percent leads to an approximate increase in put volume of 2.18%. The results are obtained through an impulse response function for a one standard deviation shock to implied volatility and are illustrated in figure 4.2.

When implied volatility increased in the pre September 11 period, it had no significant impact on total volume of option contracts traded. In the post September 11 period however, an increase in implied volatility induced a persistent significant increase total volume. When implied volatility increases by one percent it causes total volume to increase by approximately 1.45%. The results are obtained through an impulse response function for a one standard deviation shock to implied volatility and are illustrated in figure 4.3.

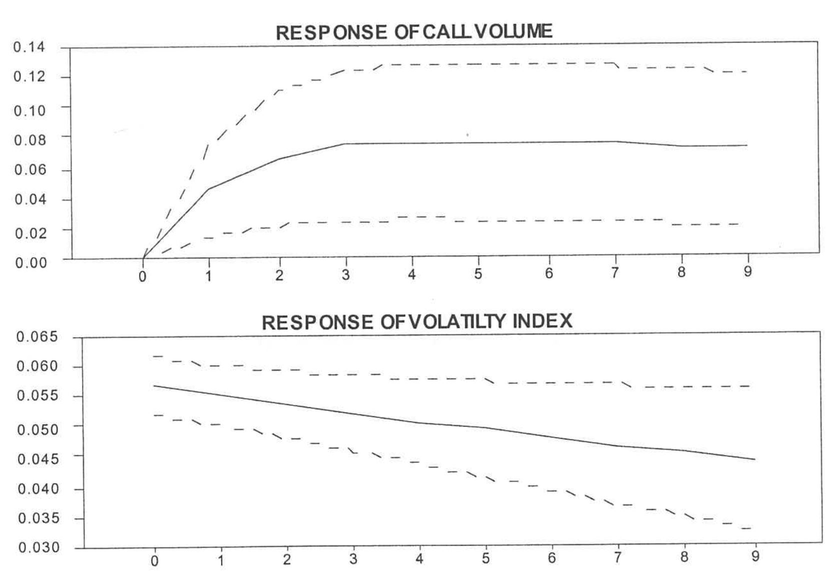
## Figure 4.1: The effects of a one-standard deviation shock to implied volatility on call volume

### Pre the September 11 attacks




Approximated with a lag length of 1

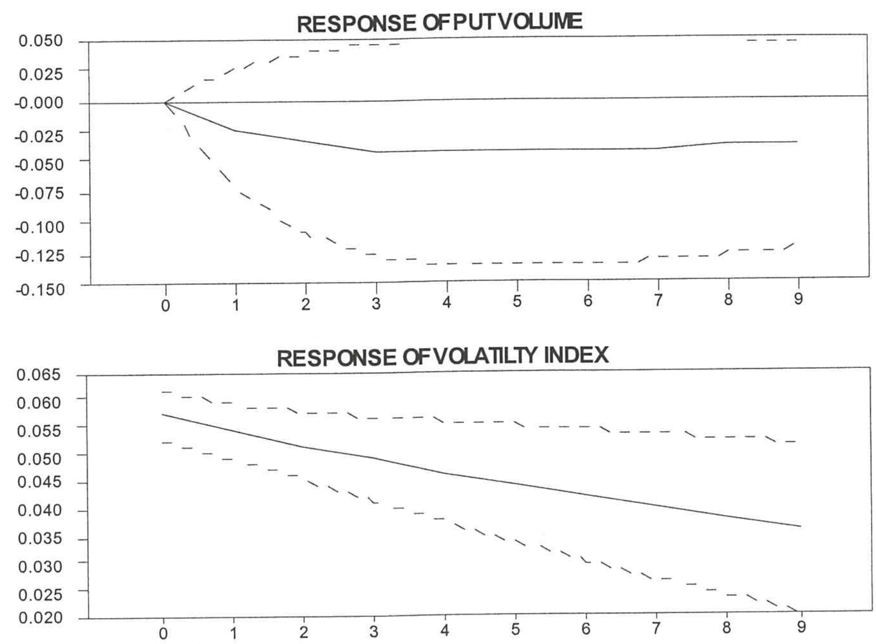
### Post the September 11 attacks




Approximated with a lag length of 1

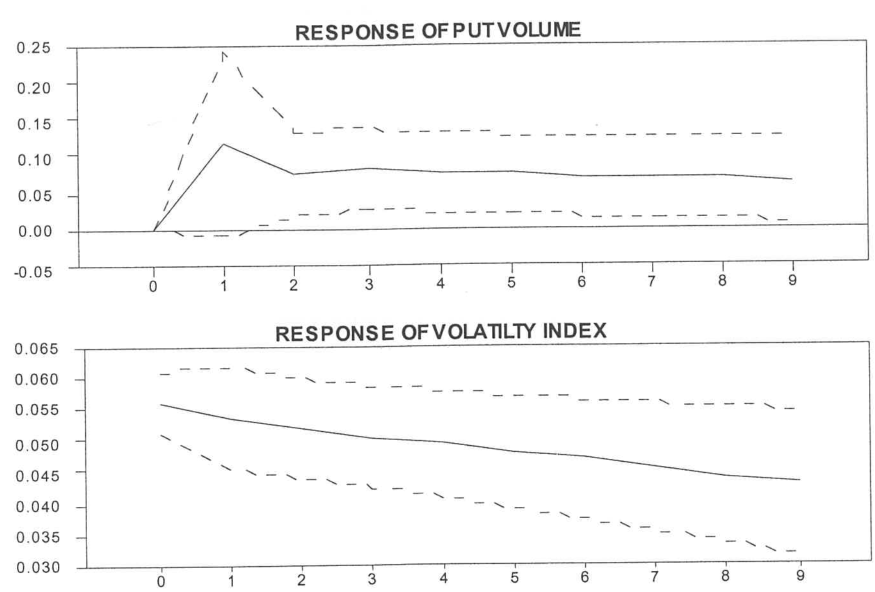
# Figure 4.2: The effects of a one-standard deviation shock to implied volatility on put volume

## Pre the September 11 attacks




Approximated with a lag length of 1

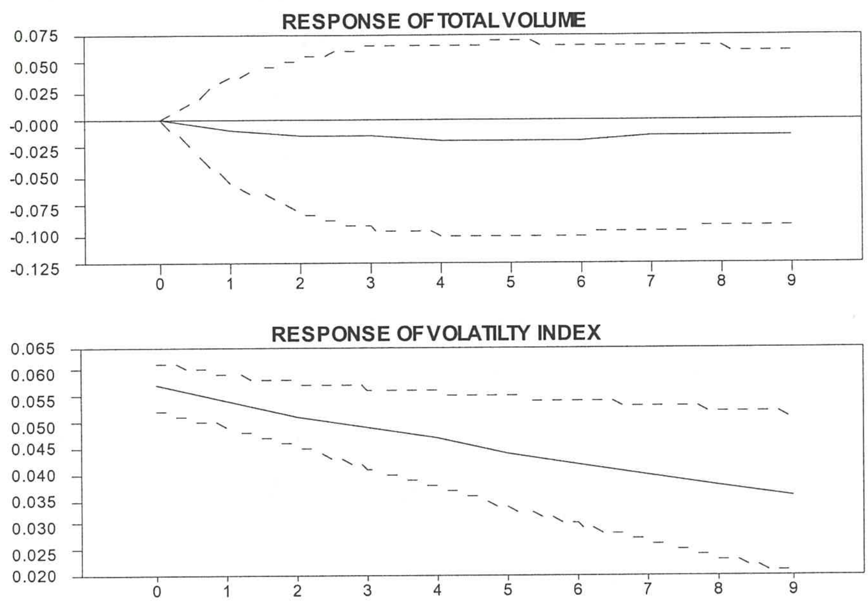
## Post the September 11 attacks




Approximated with a lag length of 1

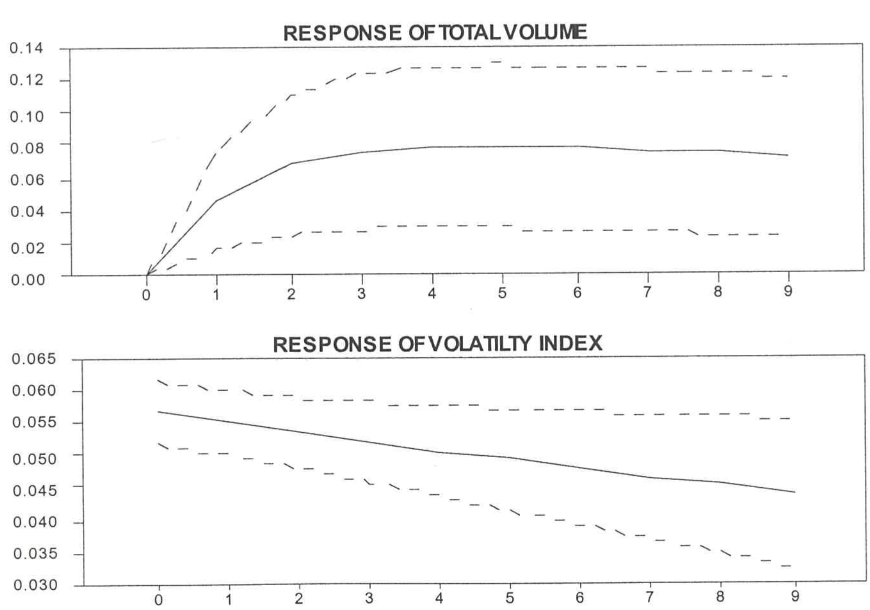
## Figure 4.3: The effects of a one standard deviation shock to implied volatility on total volume

### Pre the September 11 attacks




Approximated with a lag length of 1

### Post the September 11 attacks




Approximated with a lag length of 2

The results of this analysis imply that market participants have become more risk adverse in the post September 11 period. This is because in the pre September 11 period, changes in implied volatility did not induce a significant response in trading volume of option contracts. However, in the post September 11 period changes in implied volatility induced a significant increase in trading volume of option contracts. Option contracts are financial derivatives that can be used as hedging instruments and as illustrated here when future perceived risk as measured by implied volatility increases, volume in option trading increases. This means that as future perceived risk increases market participants become more willing to hedge away this risk through the purchase of options. This behaviour did not occur in the pre September 11 period illustrating that market participants were not as sensitive to risk as they are in the post September 11 period.

Also to be noted is that similar results were obtained when the Choleski ordering is alternated between the two variables. These results are available upon request.

# CHAPTER 5

# CONCLUSIONS AND FURTHER REASERCH AREAS

## *Conclusions*

### *Volume*

After analysing the effects that the September 11 terrorist attacks had upon option trading volume, we cannot reject the null hypothesis that the attacks lead to a significant increase in total trading volume. There was a significant increase in trading volume of both call and put options that are close-to-the money. This could be possibly attributed to portfolio managers rebalancing portfolios to maintain delta neutral positions. Another possible reason is that the increased volatility of economic climate caused many market participants to quit the market. Therefore, although those still participating in the market had increased their trading, this increase was offset by a decrease in the number of market participants.

Day of the week trading patterns that were established in the pre-September 11 terrorist attack period did not change significantly. This indicates trading pattern behaviour was not altered by the attacks.

### *Implied Volatility*

The hypothesis that implied volatility did not change post the September 11 attacks can be rejected for both implied volatility index measures and realised volatility.

Both implied volatility index measures, VXO and VIX provide nearly identical results as forecasters for future realised volatility. Both are efficient forecasters of future realised volatility but the null hypotheses that they are unbiased forecasts cannot be rejected. It also appears that the use of an implied volatility index as a proxy for implied volatility rather than an individual measure of implied volatility improves the performance of implied volatility as an ex-ante forecast for future realised volatility. However, given the evidence comparing of the two volatility index methodologies, VXO and VIX, one cannot be deemed more superior than the other.

When the implied volatility index measures are analysed in pre and post September 11 attack periods to determine if they remain robust in light of these attacks, results indicate that they do. Both measures remain robust and once again determining which is superior and more robust after the September 11 attacks is inconclusive.

### *Interrelationship between implied volatility and volume.*

It also can be concluded that market participants became more risk adverse. This is because results from the VAR's showed that market participants increased the volume of options traded which are hedging instruments when there were shocks to implied volatility in the post September 11 period. This did not occur on the pre-September 11 period, with implied volatility shocks having no effect upon the volume of options traded.

## *Further Research*

Further research could be conducted by applying each respective index construction methodology to the alternative indexes. For example, the VIX methodology could be used upon S&P 100 OEX options and the VXO methodology could be used upon S&P 500 SPX options. This would provide a more robust comparison, as in this research VXO methodology was used to determine future realised volatility on the S&P 100 index and the VIX methodology was used to determine future realised volatility on the S&P 500 index. Since they were not forecasting for the same index the resulting comparisons between the two, provide weekend results despite the two indexes being highly correlated meaning their realised volatilities should be approximately the same. The results of the extended analysis would provide a more robust and accurate comparison in determining which methodology was best and whether the CBOE was justified in making the changes in the methodology used for the construction of their implied volatility index.

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# Appendix

## *VXO Implied Volatility Index Construction*[5]

The VXO is constructed from the eight implied volatilities of the two nearest-to-expiration call options that have strike prices that straddle the current index price and the two next-to-nearest-to-expiration call options that have strike prices that straddle the current index price. The two nearest-to-expiration put options that have strike prices that straddle the current index price and the two next-to-nearest-to-expiration call options that have strike prices that straddle the current index price. The nearest contracts are those that have the least time to expiration but a maturity of greater than seven calendar days. The nest-nearest-to-expiration contracts are those of the following contract month.

The first step requires the conversion of implied volatility from a calendar day basis to a trading day basis to eliminate the weekend effect on variance calculation as illustrated by French and Roll (1985). The number of trading days is determined by the following equation,

$$N_t = N_c - 2 \times \text{int}\left(N_c/7\right) \tag{1}$$

where $N_c$ denotes the number of calendar days to expiration and $N_t$ denotes the number of trading days to expiration.

The next step is to convert the calendar day implied volatility into trading day volatility by multiplying the calendar day implied volatility by the ratio of the square root of calendar days until expiration to the square root of trading days until expiration, more formally,

[5] Information on the VXO index contruction was obtained from Whaley (1993)

$$\sigma_t = \sigma_c \left( \frac{\sqrt{N_c}}{\sqrt{N_t}} \right) \tag{2}$$

where $\sigma_t$ is the trading day implied volatility and $\sigma_c$ is the calendar day implied volatility.

S denotes the current S&P 100 index level; $X_b$ denotes the OEX option with an exercise price just below the current index level whilst $X_u$ denotes the OEX with an exercise price just above the current index level. The implied volatilities of the nearest and next-to-nearest OEX options are:

| | **Nearest Contract (1)** | | **Next Nearest Contract (2)** | |
|---|---|---|---|---|
| | **Call** | **Put** | **Call** | **Put** |
| $X_b(<S)$ | $\sigma_{c,1}^{x_b}$ | $\sigma_{p,1}^{x_b}$ | $\sigma_{c,2}^{x_b}$ | $\sigma_{p,2}^{x_b}$ |
| $X_u(\leq S)$ | $\sigma_{c,1}^{X_u}$ | $\sigma_{p,1}^{X_u}$ | $\sigma_{c,2}^{X_u}$ | $\sigma_{p,2}^{X_u}$ |

First the average of put and call volatilities must be calculated for the four categories;

$$\sigma_1^{x_b} = \left(\sigma_{c,1}^{x_b} + \sigma_{p,1}^{x_b}\right)/2 \tag{3A}$$

$$\sigma_2^{x_b} = \left(\sigma_{c,2}^{x_b} + \sigma_{p,2}^{x_b}\right)/2 \tag{3B}$$

$$\sigma_1^{x_u} = \left(\sigma_{c,1}^{x_u} + \sigma_{p,1}^{x_u}\right)/2 \tag{3C}$$

$$\sigma_2^{x_u} = \left(\sigma_{c,2}^{x_u} + \sigma_{p,2}^{x_u}\right)/2 \tag{3D}$$

Next step is to create at-the-money implied volatilities by interpolating between the nearest and next-nearest implied volatilities.

$$\sigma_1 = \sigma_1^{x_b}\left(\frac{x_u - s}{x_u - x_t}\right) + \sigma_1^{x_u}\left(\frac{s - x_b}{x_u - x_t}\right) \tag{4A}$$

$$\sigma_2 = \sigma_2^{x_b}\left(\frac{x_u - s}{x_u - x_t}\right) + \sigma_2^{x_u}\left(\frac{s - x_b}{x_u - x_t}\right) \tag{4B}$$

The last step is to create an option with 30 calendar days or 22 trading days until expiration. To do so, interpolate between the nearest and the next-nearest-to-expiration options, more formally,

$$VXO = \sigma_1\left(\frac{N_{t_2} - 22}{N_{t_2} - N_{t_1}}\right) + \sigma_2\left(\frac{22 - N_{t_1}}{N_{t_2} - N_{t_1}}\right) \tag{5}$$

where $N_{t_1}$ denotes the number of trading days to expiration of the nearest option and $N_{t_2}$ denotes the number of trading days to expiration of the next-nearest option.

## *VIX implied volatility index construction.*[6]

In calculation of VIX the generalised formula used is as follows,

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 \tag{6}$$

where $\sigma$ denotes *VIX/100* so *VIX* = $\sigma$ *x 100*, $T$ denotes time to expiration, $F$ denotes forward index level derived from index option prices, $K_i$ denotes the strike price of $i^{th}$ out-of-the-money option, $K_0$ denotes the first strike price below the forward index level, $F$, $R$ denotes the risk-free interest rate until expiration, $Q(K_i)$ denotes the mid-point of the bid ask spread for each option with the strike price $K_i$ and $\Delta K_i$ denotes the interval between strike prices – half the distance between the strike on either side of $K_i$,

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2} \tag{7}$$

Note: $\Delta K$ for the lowest strike price is the difference between the lowest strike price and the next strike price above the lowest. $\Delta K$ for the highest strike price is the difference between the highest strike price and the next strike price below the highest.

In calculating VIX the put and call options that are used are of the two nearest-to-expiration options in order to form a bracket around a 30-day calendar period. However, like the VXO only options with expiration of greater than seven calendar days to expiration are used to avoid the excessive volatility of options that are near to expiry.

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[6] Information on the construction of the VIX index was sourced from http://www.cboe.com/micro/vix/vixwhite.pdf

VIX measures the time to expiration, *T,* in minutes rather than in days as VXO does. This is done in order to gain complete accuracy of implied volatility that is often used by volatility traders. The time to expiration is expressed by,

$$T = \{M_{current\ day} + M_{settlement\ day} + M_{other\ days}\}/Minutes\ in\ a\ year$$

where $M_{\text{current day}}$ denotes the number of minutes remaining until midnight of the current day, $M_{\text{settlement day}}$ denotes the number of minutes from midnight until 8:30 a.m. on the SPX settlement day and $M_{\text{other days}}$ denotes the total number of minutes in the days between the current day and the settlement day.

The first step is for each contract month to determine the forward index level, *F,* which is derived from at the money-option prices. The at-the-money strike price is determined by the smallest difference between the strike price and call and put prices. The formula for calculating the forward index level is,

$$F = Strike\ Price + e^{RT} \times (Call\ Price - Put\ Price)$$

From this determine $K_0$ and select call options that have strike prices greater than $K_0$ and a non-zero bid price. Stop selecting call options once two consecutive call options with a bid price of zero are encountered. Next, select put options that have strike prices less than $K_0$ and a non-zero bid price. Stop selecting put options once two consecutive put options with a bid price of zero are encountered. Match the put and calls that have the same $K_0$ and average their bid-ask mid-point prices.

The second step involves calculating the implied volatility for both the nearest-to-expiration and the next-nearest-to expiration options by applying equation (6)

$$\sigma_1^2 = \frac{2}{T_1}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT_1} Q(K_i) - \frac{1}{T_1}\left[\frac{F_1}{K_0} - 1\right]^2 \tag{8}$$

$$\sigma_2^2 = \frac{2}{T_2}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT_2} Q(K_i) - \frac{1}{T_2}\left[\frac{F_2}{K_0} - 1\right]^2 \tag{9}$$

where $\sigma_1^2$ denotes implied volatility for the nearest-to-expiration options, $T_1$ denotes the time until expiration of the nearest-to-expiration options, $\sigma_2^2$ denotes implied volatility for the next-nearest-to-expiration options and $T_2$ denotes the time until expiration of the next-nearest-to-expiration options.

VIX is an aggregation of the information that the price of each option contains that is used in VIX's formation. The contribution of an option to VIX is proportional to the options price and inversely proportional to its strike price.

The next step is to calculate,

$$\frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

for the nearest term $(T_1)$ and the next nearest term $(T_2)$:

The following step is to calculate $\sigma_1^2$ and $\sigma_2^2$ and to interpolate these to form a single value with a constant maturity of 30 calendar days until expiration. Following this take the square root of the interpolated value and multiply it by 100 to get VIX.

$$\sigma = \sqrt{\left\{T_1\sigma_1^2\left[\frac{N_{T_2}-N_{30}}{N_{T_2}-N_{T_1}}\right]+T_2\sigma_2^2\left[\frac{N_{30}-N_{T_1}}{N_{T_2}-N_{T_1}}\right]\right\}\times\frac{N_{365}}{N_{30}}} \quad (10)$$

where $N_{T1}$ denotes the number of minutes to expiration of the nearest-to-expiration options, $N_{T2}$ denotes the number of minutes to expiration of the next-to-nearest-expiration options, $N_{30}$ denotes the number of minutes in 30 days and $N_{365}$ denotes the number of minutes in a 365-day year.