Assessing Tail-Related Risk for Heteroscedastic Return Series of Asian Emerging Equity Markets

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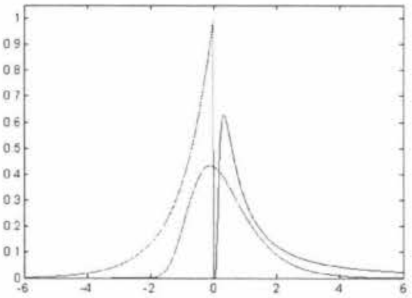

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Abstract

High degrees of leptokurtosis, heteroscedasticity and asymmetries in return series are the common features of Asian emerging equity markets, especially during the financial crisis. Thus, strengthening risk management with improved risk measures becomes increasingly important. According to the Basle Committee on Banking Supervision, the value at risk (VaR) should be calculated at the 99% confidence level or above with daily data. In the context of Asian equity markets, the use of the estimated conditional variance of market returns as the sole measure of market risk may result in serious underestimation of the true risk caused by tail events. Therefore, this research focuses on the tail-related risk measure of nine Asian index returns within the framework of extreme value theory. It employs the generalized extreme value (GEV) and the generalized Pareto distribution (GPD) approaches combined with AR(1)-GARCH(m, s) filtering of the return data. The VaR performances under different distributions with different volatility filtering are compared, and the estimated conditional and unconditional expected shortfalls based on the GPD are reported. The important findings include the following. (1) The nine heteroscedastic index returns indeed follow heavy-tailed distributions rather than the normal distribution. (2) Both the GPD and GEV distributions of daily returns are asymmetric between local maxima (right tail) and local minima (left tail). (3) The results of the GEV approach are somewhat sensitive to the block length chosen, while the GPD approach, with the thresholds determined much less arbitrarily, can avoid equivocalness with the GEV method. (4) The reported results indicate that the VaR based on the extreme value theory at high quantiles (above 99%) is more accurate than the VaR based on the normal distribution.

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Chapter 1

Introduction

In the early 1990s, Asian emerging markets witnessed better long-term growth prospects than developed markets, and offered multinational investors superior diversification benefits. Although the Asian financial crisis in the late 1990s had a far-reaching detrimental effect on foreign capital inflows for a while, the booming sign has recently reoccurred in these markets following the successes of stabilizing their macro-economies and the progress made in restoring the health of their financial sectors. During the first quarter of 2002, Asian emerging equity markets were the best performing region with impressive gains by Korea (+28.4%), and solid gains by Malaysia (+11.9%), Taiwan (+8.6%) and India (+6.9%). Such exceptional performances by tech-heavy Asian markets stand in sharp contrast to the 5.4% decline in the Nasdaq (IMF (2002)). Further more, according to the latest news alert of EmergingPortfolio.com Fund Research (EPFR) [1], up until the third quarter of 2003, foreign equity fund managers had conglomerated about $7.8 billion in Asian emerging markets. Markets such as China, Korea, Malaysia, Taiwan and India now continue to enjoy heavy portfolio inflows.

However, it is an indubitable fact that the Asian emerging markets once experienced extreme movements in returns as well as high volatility. Since asset price volatility reflects the process of pricing and transfers systematic risk as underlying conditions change in the markets, measuring and managing market risk is thus a major concern of financial managers and regulators.

[1] Details are available at http://www.gipinc.com.

Table 1. Average Monthly Risk Ratings of Asian Emerging Markets
Over the Period April 2002 Through March 2003

Markets	Financial Risk		Economic Risk		Political Risk		Composite Risk	
	Rating	Rank	Rating	Rank	Rating	Rank	Rating	Rank
China	45.0	7	38.6	3	66.5	2	75.0	3
Hong Kong	44.5	6	45.1	8	79.0	8	84.3	8
Indonesia	34.3	1	34.4	1	48.5	1	58.6	1
Korea	40.2	4	43.3	7	76.5	6	80.0	6
Malaysia	41.5	5	39.7	4	71.9	4	76.5	5
Philippines	37.5	2	36.5	2	67.4	3	70.7	2
Singapore	45.5	8	46.4	9	88.5	9	90.2	9
Taiwan	46.0	9	41.8	6	77.3	7	82.6	7
Thailand	39.8	3	38.8	5	72.3	5	75.4	4

Note: 1. The average ratings are calculated using the original data from the International Country Risk Guide (ICRG) which are available at http://www.prsgroup.com.

2. The categorization scheme of ICRG is:

Very high	0.0 – 49.5
High	50.0 – 59.5
Moderate	60.0 – 69.5
Low	70.0 – 84.5
Very low	85.5 – 100.0

3. Risk is ranked in an ascending order, depending on its average ratings.

Table1 shows the degree of risk involved in the Asian emerging markets. Figures in the table are average monthly risk ratings based on the data of the International Country Risk Guide (ICRG). Risk ratings between 0.0 and 49.5 indicate that the country surfers very high risk. We can see from Table 1 that the financial risk ratings of nine Asian emerging markets all fall into this range, meaning that they were exposed to very high financial risk. However, such ratings do not answer the question of exactly how high the risk is, thus providing insufficient useful information for risk management in Asian emerging markets. Therefore, how risk can be accurately measured is the key issue here.

The traditional risk measurement tools include gap analysis, scenario analysis, portfolio theory and derivatives risk measures. Among these methodologies, portfolio theory is the centrum characterized by the mean-variance CAPM framework. It provides a guide for investors to choose between portfolios on the basis of their expected returns and standard deviations (or variances). Here it is assumed that investors do not need information about higher order moments of the returns probability density function (e.g. kurtosis or skewness). However, recent empirical studies on emerging markets indicate that such conventional methods are highly misleading when applied to pricing assets. Bekaert et al (1998) provide the fact that there are highly skewness and kurtosis prevalent in emerging markets. Harvey (1995) finds that significant pricing errors persist and the standard asset pricing models do not account for the predictability of emerging market returns. Further, Hwang and Satchell (1999) argue that modeling emerging market risk premium should better consider higher moments, such as co-skewness and co-kurtosis, than be based on the conventional mean-variance CAPM.

In the late 1970s and 1980s, a number of major financial institutions started working on internal models to measure and aggregate risks across the institution as a whole. The best known of these systems is the RiskMetrics system developed by JP Morgan. It looks at the expected maximum loss (or worst loss) over a target horizon within a given confidence interval. This approach is also referred to as value at risk (VaR). Since the returns of emerging markets have notoriously high leptokurtosis and asymmetry, traditional (regular) methodologies to calculate the VaR are questionable due to the fact that they cannot capture the increased uncertainty about future extreme price movements by assuming normality.

In recent years, a new approach, known as extreme value theory (EVT), has been proposed to estimate the VaR. This approach emphasizes on modeling the tail of the distribution that carries information about the extreme behavior of returns, rather than on getting the tail as an outcome of modeling the entire distribution function. Although EVT has its history in the study of natural phenomena and in the insurance field, it is now becoming increasingly popular in the area of risk management, as it can better describes leptokurtosis and asymmetry in financial returns than the methodologies under the normality assumption.

The main purpose of this research is to assess tail-related risk for heteroscedastic return series of Asian emerging equity markets, based on the generalized extreme value (GEV) theory and the generalized Pareto distribution (GPD) approach that are extended to incorporate heteroscedasticity in return series. Our research, or the approach adopted in our research, is motivated by the following aspects:

First, although EVT is superior to the traditional VaR, its assumption of independence in financial returns is too restrict to be true in a dynamic world such as Asian emerging markets where highly volatile returns often exhibit autocorrelation caused by, for example, conditional volatility.

Second, it is well documented that conditional volatility can be modeled by GARCH family. However, subject to the assumption of normal distribution, the GARCH-type models cannot deal with tail risk that occurs at high quantiles. Financial managers are mainly concerned with extreme price movements. Since extreme events (large loss/profit events) may occur more often at high qnantile than the normal distribution would suggest, the VaR based on the normal GARCH models at a lower quantile, say 95%, would far underestimate tail risk for the risk management purpose.

Third, EVT allows fat tails to be asymmetric while the normal GARCH models do not. Even if a Student-t distribution is considered for the GARCH-filtered residuals, it still assumes that the left and right tails are symmetric. But in the real world, markets may well respond very differently to upward and downward oscillations.

Fourth, to the best of our knowledge, previous studies have not considered heteroscedasticity in return series in measuring tail risk in Asian emerging markets. Thus, the present research attempts to fill this void by allowing the coexistence of heteroscedasticity and heavy tail while assessing tail-related risk in the markets. Our results may be useful to any investors or portfolio managers who wish to understand potential sources of market risk in highly volatile financial environments.

The rest of this thesis is organized as follows. Chapter 2 briefly reviews the existing literature on risk measurement for Asian emerging markets. In Chapter 3, we introduce the concepts of value at risk and expected shortfall and then outline the GEV and GPD methods. This is followed by preliminary data analysis in Chapter 4 and reporting empirical results in Chapter 5. Finally, conclusions and directions for further research are presented Chapter 6.

Chapter 2

Literature Review

High sample average returns, low correlations with developed market returns, return predictability and high volatility are the common distinguishing features of emerging market returns (Bekaert and Harvey (1997)). As a primary concept, Jorion (1997) defines risk as the volatility of unexpected outcomes. One possibility for assessing market risk is that it is related to the time varying variance. The econometric breakthrough in estimating volatility is attributed to Engle (1982) who showed how to model the autoregressive conditional heteroscedasticity (ARCH (s)) process. To avoid the violation of the non-negativity constraints as the order s increased, Bollerslev (1986) suggested a further extension to the generalized ARCH (GARCH (m, s)) process. Over the last decade, ARCH family modeling was widely used for risk measures in many asset pricing models. Some excellent literature reviews in this area are available in Bollerslev, Chou and, Kroner (1992), Bollerslev, Engle and Nelson (1994) and Poon and Granger (2003). Regarding empirical studies of emerging equity market volatility especially in Asian-Pacific region, Ng et al (1991), Lee and Ohk (1991), and Engle and Susmel (1993) document that some markets exhibit time-varying volatility. Koutmos (1997) provide evidences from six Asian emerging markets that the conditional and unconditional distributions of returns are leptokurtic and clustering is an all-pervasive phenomenon. The finding of Pan et al (1999) also suggests that volatility transmissions among Asian-Pacific equity markets exist not only in the short run (volatility spillover), but also in the long run (common time-varying volatility). In the mean time, some researchers focus on the behavior of volatility in a single Asian emerging market. For example, Pyun et al (2000) finds the stock return volatility and trading volume as a source of the ARCH effect in the Korean market. Lin and Yeh (2000) study the distribution and conditional heteroscedasticity in stock returns on the Taiwan stock market and find that the mixed-normal-GARCH model is the most adequate specification for Taiwan stock returns. Su (1998) and Lee et al (2001) employ various GARCH models for the Chinese stock market and provide strong evidence of time-varying volatility which is shown to be highly persistent and predictable.

Along another line, the VaR approach gains increasing popularity in financial management as it can be applied to a much broader range of risk problems as well as to market price risks. Both theoretical and empirical studies can be found in Hull and White (1998), Jorion (1996), Duffie and Pan (1997), Christoffersen et al (2001), and Brooks and Persand (2002) among others. Guermat and Harris (2002) make a significant improvement to the VaR-based forecasting by introducing the exponentially weighted maximum likelihood (EWML) method. Their results show that the EWML-based VaR forecasts are generally more accurate than those generated by both the exponentially weighted moving average (EWMA) and GARCH models, particularly at high VaR confidence levels. However, the VaR measures are also open to criticism from a very different perspective. It is still the question whether the VaR is the best tail-based risk measure. Artzner et al (1999) argue that risk measures must satisfy coherence properties. As an alternative, however, the expected shortfall method is demonstrably superior to the VaR one (We will come back this point with more detailed discussion in Section 3.4).

The observed extreme movements in stock market returns make people realize that markets can deviate significantly from the norm and generate "outliers". Some of the previous investigations of "outliers" were based on the assumption of either normal distribution (e.g., Tsay (1986)) or mixture normal distribution (e.g., Venkataraman (1997)). Such studies failed to notice those of the last three decades that show that most financial data are distributed with tails heavier than suggested by the normal distribution. In as early as the 1960s, Mandelbrot (1963) and Fama (1965) found the heavy-tailed, highly peaked nature of certain financial time series in speculative markets, a nature that favors the Paretian hypothesis. Subsequent works along this line appear in Mittnik and Rachev (1993) and Rachev, Kim and Mittnik (1997). However, since data usually contain relatively few extreme observations, there is only little information about tails. As a result, selecting a reasonable heavy-tailed parametric distribution and estimating the parameters that determine the thickness of the tails are inherently difficult tasks. In addition, the historical simulation method also provides imprecise estimates of the tails (Danielsson and de Vries (2000).

Although extreme value theory (EVT) has a long history of applications in hydrology, climatology and engineering, it has not, until recently, attracted a great deal of attention for financial studies. EVT offers a potential solution to the problem of estimating the tails, as it enables one to employ a parametric family of distributions regardless of what particular distribution generates the data. There are a number of EVT studies in empirical finance. Embrechts, Klüppelberg, and Mikosch (1997) is the first choice as a comprehensive reference. Detailed discussion on methodologies can also be found in Login (1996, 2000). A study on the exchange rate behavior by Koedijk et al (1990) uses EVT to estimate the tail index of EMS data and it is found that outliers are the rule rather than the exception. Based on the safety-first criterion, de Haan et al (1994), Jansen et al (2000) and Consigli (2002) show how EVT proves to be a useful tool for estimating VaR-efficient portfolios and for describing portfolio risk with the events far out in the tails of the distribution. Subsequent researches by Susmel (2001) and Jondeau and Rockinger (2003) implement the ideas of generalized extreme value theory in forming an optimal portfolio in emerging markets. The latest studies of Bali and Neftci (2003) and Bali (2003) are the first to examine the stochastic behavior of short-term interest rates in terms of extreme values. Meanwhile, financial turmoil as extreme movement naturally yields high degrees of leptokurtosis during the crisis periods. Pownall and Koedijk (1999) use a VaR-x approach allowing the Asia 50 index returns to be subject to Student-t distribution. They estimate the tail index by the Hill estimator and conclude that EVT improves upon the conventional methodology to capture the downside risk. Ho et al (2000) model the tail distributions of Asian emerging market returns within the generalized extreme value (GEV) framework and find that the VaR measures generated via EVT are substantially different from those generated by conventional methods. In addition, Jondeau and Rockinger (2003) also employ the GEV method to test differences in the tails of emerging and mature market returns. Finally, a new method to estimate tail risk of heteroscedastic financial returns by using the generalized Pareto distribution

(GPD) is proposed by McNeil and Frey (2000). This method generates better 1-day estimates of the conditional VaR and expected shortfall than methods that ignore tail-thickness.

Chapter 3

Methodology

3.1 The Essential of Risk

Definition 1 A random variable X as a function can be defined on the set Ω of all possible outcomes which reflect uncertainty such as variability of the future values of a position in financial markets. Formally, the random variable is a measurable mapping X: $\Omega \rightarrow \mathbb{R}$ ($\mathbb{R}$ is real line). In actuarial applications, a nonnegative random variable is frequently called *risk*.

According to the International Country Risk Guide (ICRG) risk rating system, the risk components can be grouped into three categories: political risk, economic risk and financial risk. Another narrower categorization of risk consists of business risks, market risks, credit risks, liquidity risks, operational risks and legal risks (Dowd (1998)).

The market risk defined by Dowd is the risk of losses arising from adverse movements in market prices (e.g., equity prices) or market rates (e.g., interest or exchange rates). In this research, we concentrate on *tail-related risk* which the risk measures of returns or portfolios are underestimated with fat-tailed properties and a high potential for large losses.

3.2 Coherent Risk Measure

In their papers, Artzner et al (1997, 1999) argued that if X and Y are the univariate random variables of two risky positions, any risk measure $\rho(\cdot)$: X and $Y \in \mathbb{R}$ is said to be *coherent risk measure* and has the following properties:

$$\rho(X) + \rho(Y) \geq \rho(X+Y) \qquad \text{(subadditivity)}$$

$$\rho(tX) = t\rho(X) \qquad \text{(homogeneity)}$$

$$\rho(X) \geq \rho(Y), \text{ if } X \leq Y \qquad \text{(monotonicity)}$$

$$\rho(X+n) = \rho(X) - n \qquad \text{(risk-free conditions)}$$

The first condition ensures that the risk measure of various positions taken within a firm since the sum of individual measures provides a conservative estimate of the measure of aggregate risk. The second and third indicate that the function $\rho(\cdot)$ is convex. The last condition implies that the additional amount n to the position will decrease the risk as it increases the value of the end of period portfolio.

3.3 Value at Risk

To compute the VaR of a portfolio, suppose that our initial wealth (investment) is W_0 and the future (random) wealth is, at the end of the time horizon,

$$W = W_0(1+r) \qquad (3.1)$$

where r is the random rate of return. We are interested in characterizing the potential loss

$$\Delta W = W - W_0 = W_0 r \qquad (3.2)$$

The VaR at confidence level cl is thus defined by the following condition:

$$\Pr[\Delta W \leq \text{VaR}] = 1 - cl \qquad (3.3)$$

In terms of the cumulative distribution function (cdf) F(·), the probability that the portfolio profit and loss (P&L) is less than a given value which is just the 1 – *cl* quantile of the P&L distribution:

$$F(\text{-VaR}) = 1 - cl \qquad (3.4)$$

Therefore, to calculate the VaR, we need to find the inverse of the cdf of the portfolio's change in value at the 1 – *cl* percentile:

$$\text{VaR}_{cl} = -F^{-1}(1 - cl) \qquad (3.5)$$

The typical values for the confidence level *cl* are set arbitrarily. The Bank of International Settlements (BIS) suggest that VaR should be calculated for an *cl* = 99% confidence level over a 10-day time horizon, but other *cl*s (between 95% and 99.5%) and time horizons (between 1 and 252 days) are also commonly used. The time horizon should be chosen to reflect the amount of time taken to liquidate all the portfolio positions.

Under the condition that asset returns *X* are subject to the standard normal distribution, the probability of *X* being less than or equal to *x* (the tolerance limit) can be calculated as

$$\Pr[X \le x] = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{1}{2}((X - \mu)/\sigma)^2\right] dX \qquad (3.6)$$

where μ and σ are the mean and the standard deviation of *X* respectively. Hence,

$$\begin{aligned} \alpha_{1-cl} &= \Phi(z_\alpha) = \frac{X_{cl} - \mu}{\sigma} \\ X_{cl} &= \mu + \alpha_{1-cl}\sigma \end{aligned} \qquad (3.7)$$

Here, α_{1-cl} is the standard normal variate for the confidence level (eg. $\alpha_{0.05}$ = -1.645). Fig. 1 shows the VaR of the left tail at the 95% confidence level for normally distributed data with $\mu = 0$ and $\sigma = 1$.

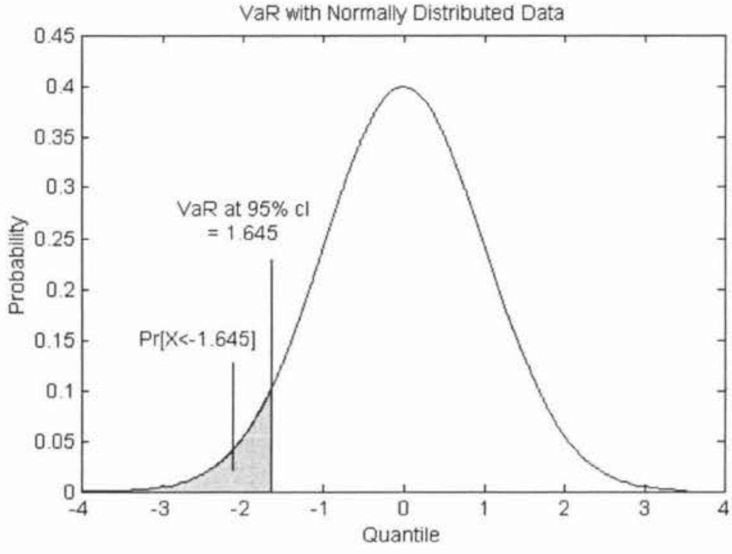

Figure 1

The variable X_t associated with the loss can be expressed as:

$$X_t = (P_t - P_{t-1}) / P_{t-1} = -Loss_t / P_{t-1}$$

$$= \text{-VaR}_{cl} \ / \ P_{t-1} \qquad (3.8)$$

Therefore,

$$\text{VaR}_{cl} = -X_t P_{t-1} = -(\mu + \alpha_{1-cl}\sigma) P_{t-1} \qquad (3.9)$$

If we define Δt as the time interval and substitute P_0 for P_{t-1}, then Eq. (3.9) becomes:

$$\text{VaR}_{cl} \text{ (absolute)} = -(\mu\Delta t + \alpha_{1-cl}\sigma\sqrt{\Delta t}) P_0 \qquad (3.10)$$

For the short time horizon, the drift term $\mu\Delta t \to 0$ and thus

$$\text{VaR}_{cl} \text{ (relative)} = -P_0 \times \alpha_{1-cl}\sigma\sqrt{\Delta t} \qquad (3.11)$$

The key result in Eq. (3.11) is that VaR is associated with the standard deviation (volatility) only. **Hereafter, we assume that the initial asset price P_0= 1 and the holding period Δt is 1 day.**

3.4 VaR and Expected Shortfall

VaR has a surprising property that $\text{VaR}_{cl}(X+Y) > \text{VaR}_{cl}(X) + \text{VaR}_{cl}(Y)$, [2] meaning that VaR is not subadditive. Furthermore, VaR does not tell anything about the size of the loss that exceeds it. For financial decision makers, what they are interested in is not only the tolerance limit, but also the question "how bad is bad?" Hence, VaR is incoherent. Artzner et al (1999) and Acerbi and Tasche (2002) propose a coherent risk measure known as expected shortfall (or conditional VaR or tail VaR). A formal definition of expected shortfall is as follows:

Definition 2 Let X be the random variable whose distribution function F_X describes the negative P&L distribution of the risky financial position at the specified time horizon (thus losses are positive). Then the expected shortfall for X is

$$\Im_{cl}(X) = \mathbb{E}\,(X \mid X > \text{VaR}_{cl}\,(X)) \qquad (3.12)$$

This equation tells us what we can expect to lose (or gain) if a tail event *does* occur, while the VaR only tells us the maximum we can expect to lose if a tail event does *not* occur. Figure 2 illustrates the standard normal VaR and expected shortfall of the right tail at the 95% confidence level. Therefore, the expected shortfall seems to be an effective way to estimate the tail relative to VaR. It is subadditive and coherent, puts fewer restrictions on the distribution of X, and requires only the first moment to be well defined. Nevertheless, both traditional VaR and expected shortfall assume that asset returns subject to the normal distribution. The reasons for this assumption may be because the normal distribution has several attractive properties. The first is that the normal distribution facilitates calculation and produces tractable results in many analytical studies. The second is that the normal distribution provides a link to mean-variance optimization theory through its first two moments. Finally, the normal distribution plays a central role in empirical modeling as it is the limiting distribution of a whole class of statistical tests.

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[2] In both Artzner et al (1999, P127) and Danielsson (2002, P1289), a same numerical example is quoted to explain why VaR is not subadditive. Meanwhile, for mathematical proof of this conclusion please see Tasche (2002).

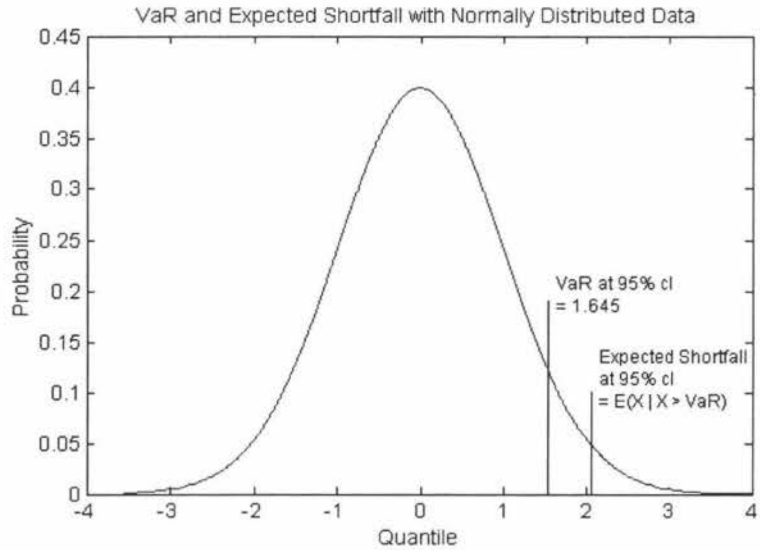

Figure 2

However, tail risk may result from various factors such as the tail index, the scale parameter, the tail probability, the confidence level and the dependence structure. The study by Yamai and Yoshiba (2002) shows that the traditional VaR and expected shortfall fail to capture the information of tail fatness and asymptotic dependence of extreme price movements. They conclude that the traditional VaR and expected shortfall should not dominate financial risk management. Meanwhile, Bao, Lee and Saltoğlu (2003) find that filtered VaR models (extreme-value-theory-based VaR models) provide better results than conventional (standard) VaR models. There is no wonder why extreme value theory (EVT) has increasingly become a popular tool for the risk measure.

3.5 Dynamic Financial Returns

It has now become a well-known fact that the variance of asset returns is not constant and independent. Some comprehensive surveys such as Bollerslev et al (1992) and Poon and Granger (2003) confirm that the behavior of conditional heteroscedasticity can be well described by GARCH (*m, s*) models. Hence, many researchers have used conditional variance to calculate dynamic VaR (See, for instance, Brooks and Persand (2003)). However, this methodology is still confined to the assumption of normal distribution and would bias the result of the tail risk measure. McNeil and Frey (2000) extract innovations from a AR(1)-GARCH(1, 1) model of matured financial market returns and find that the heavy-tailed pattern is clearly exhibited by the extracted error term. To deal with this problem, they propose that conditional VaR and conditional expected shortfall should be based on the estimation of the generalized Pareto distribution (GPD) with the GARCH process.

In this research, we follow the two-stage procedure of McNeil and Frey (2000). We first extract innovations from the regular AR(1)-GARCH (*m,s*) model via the maximum likelihood method:

$$X_t = D_t + \sqrt{h_t}\,\varepsilon_t \qquad \varepsilon_t \sim N(0,\,1)$$
$$\eta_t = \sqrt{h_t}\,\varepsilon_t$$

$$D_t = \varphi_0 + \delta X_{t-1}$$
$$h_t = \phi_0 + \sum_{i=1}^{s} \varphi_i \eta_{t-i}^2 + \sum_{j=1}^{m} \phi_j h_{t-j} \qquad (m, s = 1,2) \qquad (3.13)$$

Then $$\hat{\varepsilon}_t = \frac{X_t - \hat{D}_t}{\sqrt{\hat{h}_t}} \qquad (3.14)$$

In what follows, X_t represent the extracted innovations $\hat{\varepsilon}_t$ which are used to model the GEV and the GPD, and then to calculate the VaR and the expected shortfall.

3.6 Extreme Value Theory

In academic study, there are two broad classes of EVT: The classical extreme value theory represented by the generalized extreme value (GEV) distribution which is estimated using block (or per-period) maxima samples (assumed iid) of large observations; and the modern extreme value theory known as the peaks-over-threshold (POT) model which focuses on the realizations exceeding a given (high) threshold. Note that the generalized Pareto distribution (will be mentioned later) is based on POT. Fig. 3 gives a hint to understand the GEV and POT.

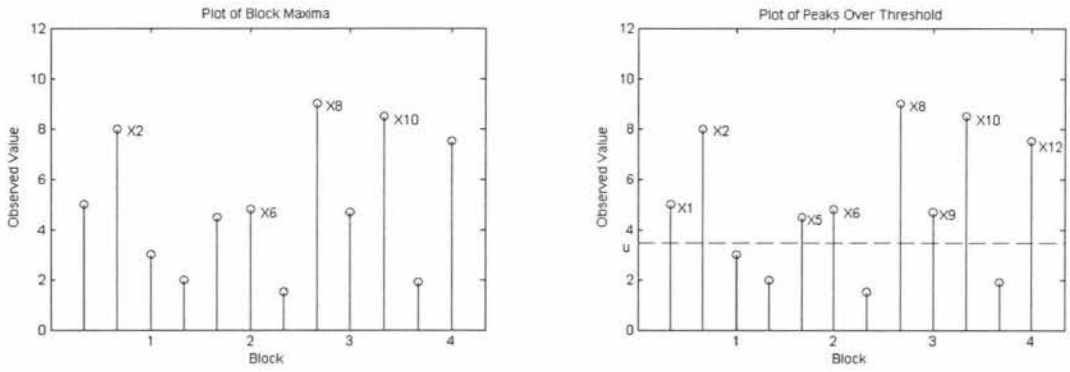

Figure 3

In the left panel of Fig. 3, only observations X_2, X_6, X_8 and X_{10} are the maxima values chosen from four blocked groups (each containing three observations) showing the basic idea of the GEV. Slightly different from the left panel, the right panel shows that, with the POT method, four more observations X_1, X_5, X_9 and X_{12} are treated as the maxima values as they are all above the given threshold u..

3.6.1 Generalized Extreme Value Theory

Let $X_1, X_2, \ldots\ldots, X_n$ be a sequence of non-degenerate random variables with unknown cumulative distribution function $F(x) = \Pr(X \le x)$, and we assume that the series X can be divided by N blocks and each block contains an identical number q of observations. In addition, let $M^{(n)} = (M^{(1)}, M^{(2)}, \ldots\ldots, M^{(N)})$ be a sequence of the N observed maximum values from N blocks:

$$\{X\} = \{ \underbrace{X_1, X_2, \ldots\ldots, X_q}_{M^{(1)}=\max(X_1, X_2, \ldots\ldots, X_q)}, \underbrace{X_{q+1}, X_{q+2}, \ldots\ldots, X_{q+q}}_{M^{(2)}=\max(X_{q+1}, X_{q+2}, \ldots\ldots, X_{q+q})}, \ldots\ldots, \underbrace{X_{n-q+1}, X_{n-q+2}, \ldots\ldots X_n}_{M^{(N)}=\max(X_{n-q+1}, X_{n-q+2}, \ldots\ldots, X_n)} \} \quad (3.15)$$

where sample size $n = q \cdot N$ ($q \ge 2$). The corresponding results for minima can be obtained from those for maxima by using the identity:

$$\min(X_1, X_2, \ldots\ldots, X_n) = -\max(-X_1, -X_2, \ldots\ldots, -X_n). \qquad (3.16)$$

The limit law for the block maxima is given by the following theorem:

Theorem 1. (Fisher and Tippett (1928)) Let $\{X_1, X_2, \ldots\ldots, X_n\}$ be a sequence of random variables. If there exists normalizing constants $\sigma_N > 0$ and centering constants $\mu_N \in \mathbb{R}$ ($\mathbb{R}$ is real line) and some non-degenerate distribution function H such that

$$\frac{M^{(N)} - \mu_N}{\sigma_N} \xrightarrow{d} H \qquad (3.17)$$

then H belongs to one of the three standard extreme value distributions:

$$\text{Gumbel: } \Lambda(x) = \exp(-\exp^{-x}), \qquad x \in \mathbb{R}$$

$$\text{Fréchet: } \Phi_\alpha(x) = \begin{cases} 0, x \le 0 \\ \exp(-x^{-\alpha}), x > 0 \end{cases} \qquad \alpha > 0 \qquad (3.18)$$

$$\text{Weibull: } \Psi_\alpha(x) = \begin{cases} \exp[-(-x^{-\alpha})], x \le 0 \\ 1, x > 0 \end{cases} \qquad \alpha < 0$$

where α is the tail index.

The shapes of the probability density functions for the standard Fréchet, Weibull and Gumbel densities are shown in Fig. 4. Here we set $\alpha = 1$ for the Weibull and Fréchet distributions. It is clear that the Weibull distribution starts from zero and has a finite right tail indicating thin-tailed distribution. The density of the Fréchet distribution also starts from zero but has a noticeably longer right tail showing heavy-tailed distribution. This means that the Fréchet distribution has considerably higher probabilities for very large values of X. The Gumbel distribution just has a tail that lies between the Weibull and Fréchet distributions. Thus, the tail behaviors of all possible distributions can be summarized as follows: thin-tailed distributions are characterized by finite tails (if $\alpha < 0$) or tails that decline exponentially (if $\alpha = \infty$); and fat-tailed distributions exhibit persistent tails that decline with a power (if $\alpha > 0$). Fig 5 gives a three-dimensional plot of Fréchet distribution with $\alpha = 2$.

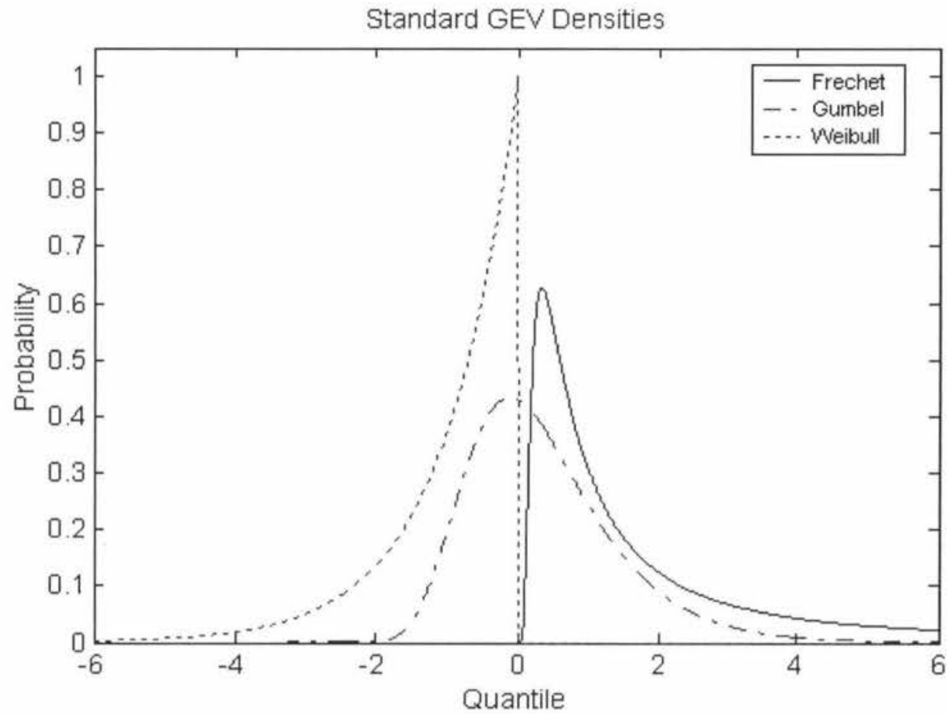

Figure 4

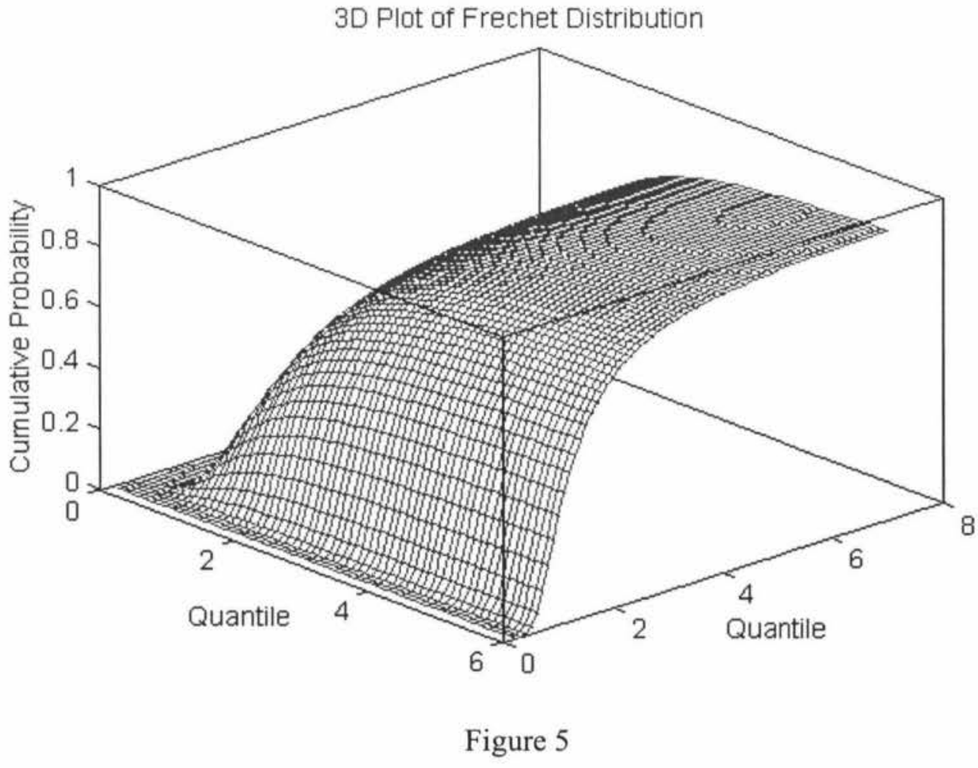

Figure 5

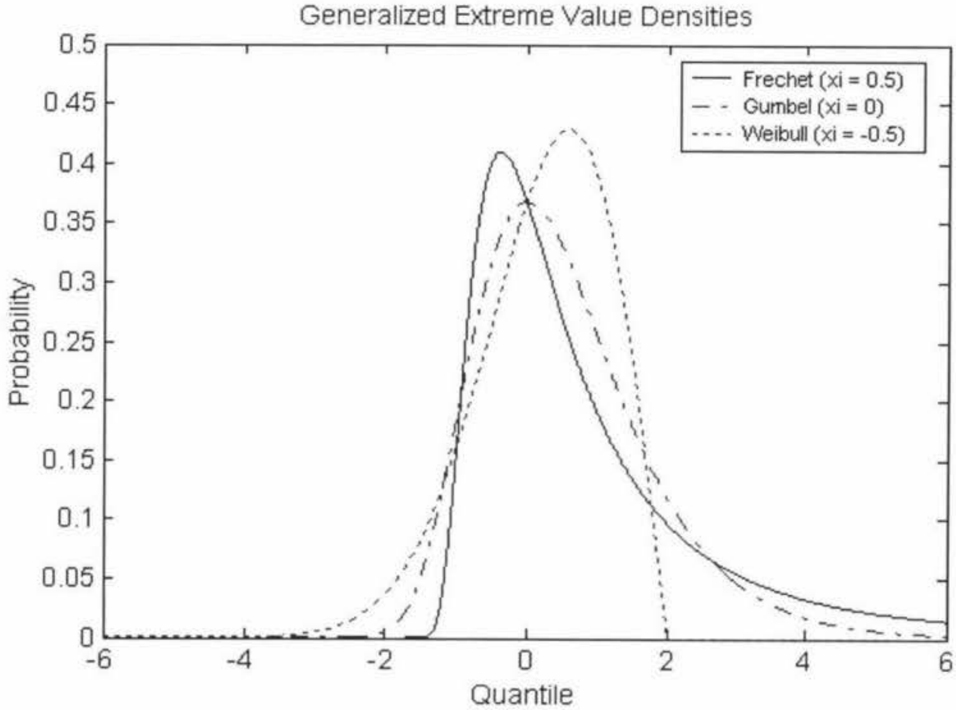

Figure 6

By taking the reparameterization $\xi = 1/\alpha$, Jenkinson (1955) proposed a generalized extreme value (GEV) distribution:

$$H_{\xi,\mu,\sigma}(x) = \begin{cases} \exp[-(1+\xi(x-\mu)/\sigma)^{-1/\xi}] & \text{if } \xi \neq 0 \\ \exp[-\exp(-(x-\mu)/\sigma] & \xi = 0 \end{cases} \qquad (3.19)$$

with x being such that $1+\xi(x-\mu)/\sigma > 0$. The location parameter μ and the scale parameter σ are measures of central tendency and dispersion respectively. When μ = 0 and σ = 1, $H_{\xi,\mu,\sigma}(x)$ becomes the standard GEV. The shape parameter ξ gives an indication of the shape (or fatness) of the tail. The plots in Fig. 6 show how different values of ξ alter the shape given $\mu = 0$ and $\sigma = 1$. If $\xi > 0$, the GEV becomes the heavy-tailed Fréchet distribution which in general holds for financial time series. The class of distributions with heavy-tails is large and includes Pareto, Student-t, Cauchy and mixture distributions. If ξ = 0, the distributions in the domain of attraction of the Gumbel distribution (including normal, exponential, gamma and lognormal distributions) correspond to the case where $F(x)$ has normal kurtosis. In the case of $\xi < 0$, the thin-tailed distributions such as uniform and beta distributions are dominated by the Weibull distribution and do not have much power in explaining financial time series.

3.6.2 Generalized Pareto Distribution

The GEV as classical EVT is sometimes applied directly by, for instance, fitting one of the extreme value limit laws to the maxima, and many empirical works have been done to contribute to this approach. However, the GEV method suffers some drawbacks when being applied to a more complicated problem such as highly volatile financial data. First, the block maxima approach involves efficiency loss because some extreme observations are overlooked by the block maxima. Second, there is no clear-cut guide regarding how long a block period should be determined and this begs a new bandwidth problem. The finding of Bali (2003) indicates that an alternative approach, the generalized Pareto distribution (GPD), provides more accurate estimates of the actual VaR than the GEV.

Let a sample of observations X_t, t = 1, 2,……, n follow an unknown distribution function $F(x) = \Pr(X_t \leq x)$. What we are interested in here is to estimate the distribution function F_u for those values of x above a certain threshold u. An excess over u is defined by $y_i = X_i - u$ (i = 1, 2,……, k; $k < n$ is the number of exceedance). (See Fig. 7 where the excess parts are bolded.)

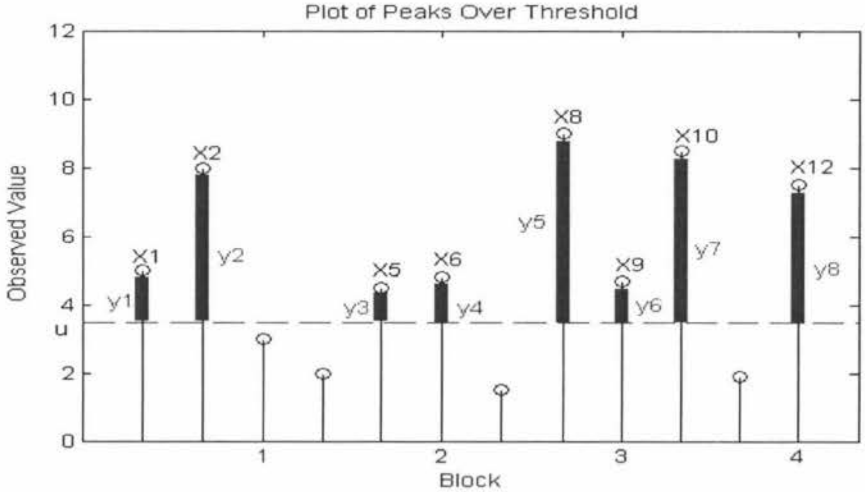

Figure 7

The distribution function F_u is also known as the conditional excess distribution function (cedf) and is formally defined as

$$F_u(y) = \Pr(X - u \leq y \mid X > u) \qquad (3.20)$$

This can be interpreted as the probability that the value of X exceeds the threshold u by an amount up to y given that X exceeds the threshold u. This conditional probability F_u can be expressed in terms of F, as follows:

$$F_u(y) = \frac{\Pr(X - u \leq y \mid X > u)}{\Pr(X > u)} = \frac{F(y+u) - F(u)}{1 - F(u)} = \frac{F(x) - F(u)}{1 - F(u)} \qquad (3.21)$$

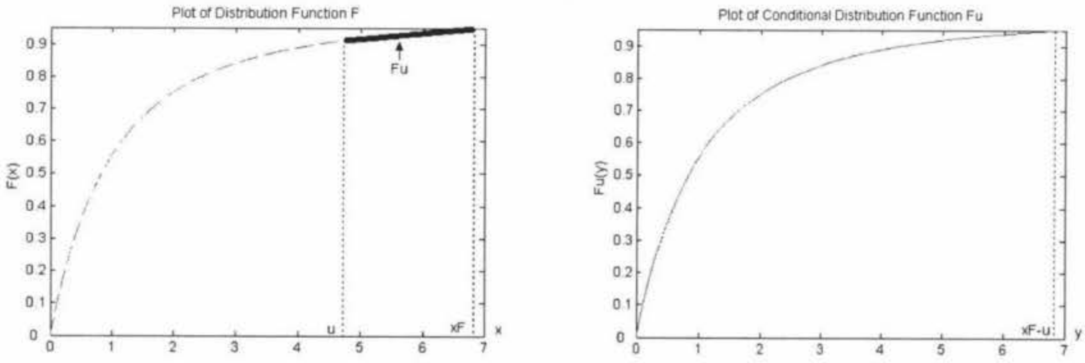

Figure 8

The left panel in Fig. 8 illustrates that the realizations of the random variable X mainly lie between 0 and u, and therefore the estimation of F in this interval can be easily handled. However, the estimation of the portion F_u (also displayed as $F_u(y)$ in right panel) within the interval $[u,\ x_F]$ ($x_F \le +\infty$ is the right endpoint of F) might be difficult as we have relatively few exceedance observations from this interval. A theorem by Balkema and de Haan (1974) and Pickands (1975) shows that as the threshold is sufficiently high, the excess distribution $F_u(y)$ converges to the GPD:

Theorem 2. (Balkema and de Haan (1974), Pickands (1975)) For a large class of underlying distribution functions F above a sufficiently high threshold u, the conditional excess distribution function $F_u(y)$ is well approximated by

$$F_u(y) \approx G_{\xi,\sigma}(y), \qquad \text{as } u \to \infty \tag{3.22}$$

where

$$G_{\xi,\sigma}(y) = \begin{cases} 1-(1+\xi\dfrac{y}{\sigma})^{-1/\xi} & \text{if } \xi \neq 0 \\ 1-\exp[-y/\sigma] & \phantom{\text{if }} \xi = 0 \end{cases} \tag{3.23}$$

If x is defined as $u + y$, then the GPD can also be expressed as a function of x:

$$G_{\xi,\sigma}(y) = \begin{cases} 1-(1+\xi\dfrac{x-u}{\sigma})^{-1/\xi} & \text{if } \xi \neq 0 \\ 1-\exp[-(x-u)/\sigma] & \phantom{\text{if }} \xi = 0 \end{cases} \tag{3.24}$$

with

$$x \in \begin{cases} [u, +\infty] & \text{if } \xi \geq 0 \\ [u, u - \sigma / \xi] & \text{if } \xi < 0 \end{cases}.$$

Figure 9 displays the GPD with different values of ξ.

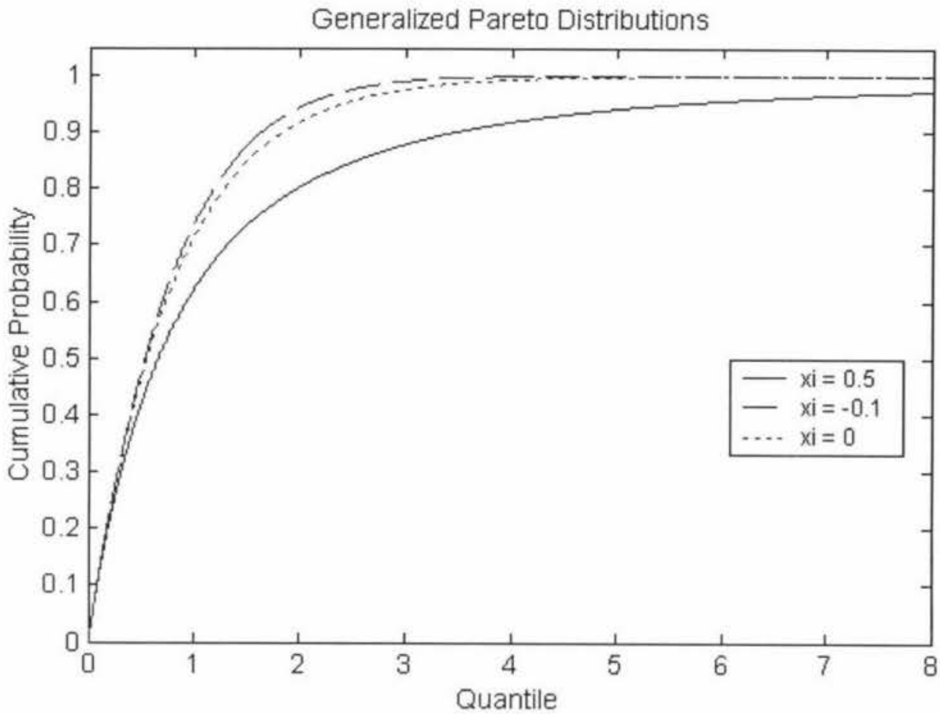

Figure 9

Again, $\xi = 1/\alpha$ is the shape parameter where α is the tail index, and σ is the scale parameter. Unlike the GEV distribution, the location parameter μ here is replaced by the threshold u. When $u = 0$ and $\sigma = 1$, the representation is known as the standard GPD. The relationship between the standard GPD $G_\xi(x)$ and standard GEV $H_\xi(x)$ is given by $G_\xi(x) = 1 + \ln H_\xi(x)$ if $\ln H_\xi(x) > -1$. The GPD embraces several distributions depending on parameter settings. When $\xi > 0$, it takes the form of the ordinary Pareto distribution. This particular case is the most relevant for financial time series analysis since the Pareto distribution is heavy-tailed. When $\xi = 0$, the GPD corresponds to the exponential (thin-tailed) distribution. The GPD becomes a Pareto II type (short-tailed) distribution, if $\xi < 0$. In addition, to answer the question – which one, the GEV or the

GPD, is better – might be difficult. But in practice, the GPD method involves only two parameters to be estimated, with the threshold u predetermined. This suggests that the maximum likelihood estimation of the GPD is easier to converge than that of the GEV. Further, the determination of the threshold u is less arbitrary than the determination of the block length, the results of the GPD may be less equivocal that those of the GEV. These will become clearer when we come to the empirical results.

Fig. 10 plots three different situations where the shape parameter ξ takes three different values: $\xi > 0$, $\xi = 0$ and $\xi < 0$. Clearly, the smaller the shape parameter (as it is going from a positive to a negative value), the thinner the tail of the distribution. Fig. 11 illustrates that, as the shape parameter ξ takes a larger positive value, the tail of the distribution becomes thicker.

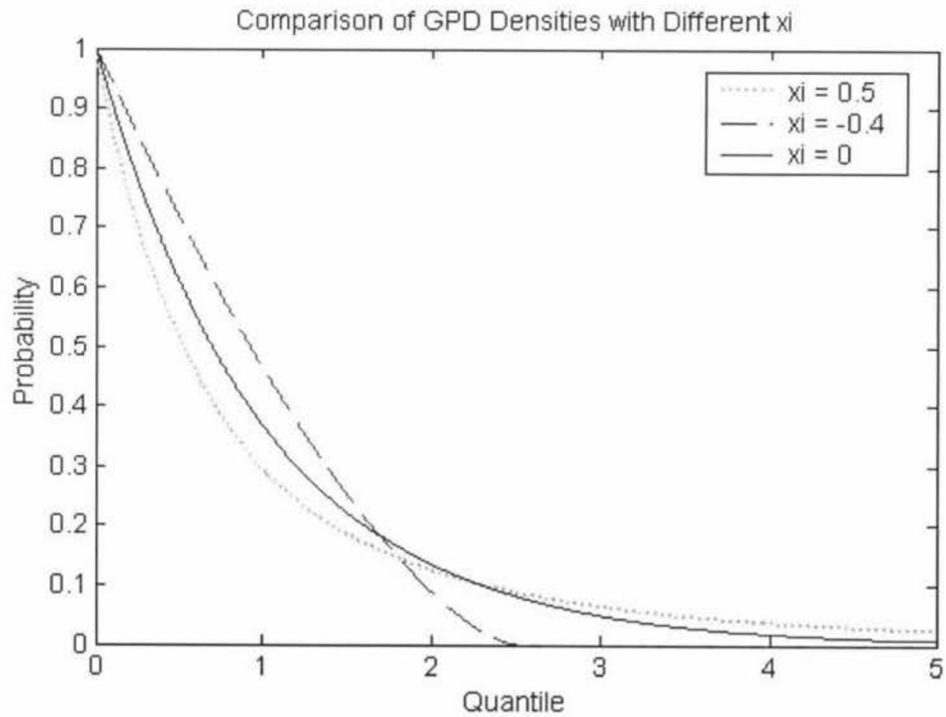

Figure 10

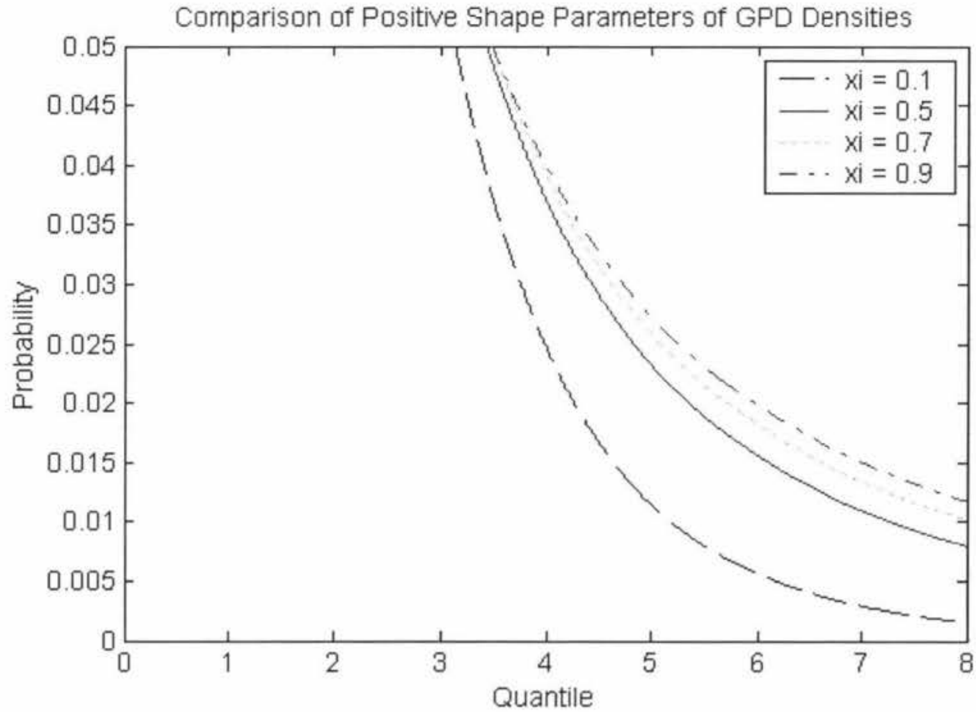

Figure 11

3.6.2.1 The Choice of Threshold

The key result of Balkema and de Haan (1974) and Pickands (1975) is that the distribution of exceedances may be approximated by the GPD through choosing ξ and setting a threshold u. The advantage of this POT method is that, since each exceedance is associated with a specific event, it is possible to make the parameters σ and ξ depend on covariates. But choosing an appropriate threshold is analogous to choosing an appropriate block length, in the sense that a balance between bias and inefficiency is needed: Too low a threshold is likely to violate the asymptotic basis of the model, leading to bias; too high a threshold, on the other hand, will generate few exceedances with which the model can be estimated, leading to a larger variance. Accordingly, Embrechts, Klüppelberg, and Mikosch (1997) advise that researchers should never expect a unique choice of u to appear. They recommend using graphical methods to reinforce judgment and common sense and comparing estimated results across a variety of thresholds. One useful tool to implement these ideas is the mean excess (or mean residual life) plot which is a plot of the mean of all excess values over a threshold u against u itself. This depends on the following assumption: if X is a random variable with a GPD, provided $\xi < 1$ (mean is finite), then for $u > 0$

$$\mathbb{E}(X-u \mid X>u)=\frac{\sigma+\xi \cdot u}{1-\xi}. \qquad (3.25)$$

Thus, a sample plot of mean excess against threshold should be approximately a straight line with the slope equal to ξ / (1 – ξ). The purpose of its implementation is to detect significant shifts in shape at a lower threshold. In practice, threshold should be chosen as low as possible subject to the limit model providing a reasonable approximation (See Davison and Smith (1990) for more details).

Another parallel efficient tool, the plot of maximum likelihood estimation (MLE), can be used to show how the estimate of the shape parameter varies with the number of extremes. The basic idea of this tool is to plot ξ (estimated by MLE) with a certain asymptotic confidence interval (e.g. 95%) against a range of exceedances. Once the confidence interval turns to be stable at the lowest value of exceedances, the corresponding valid threshold can be determined. In addition, one may also plot both the excess distribution and the tail of the underlying distribution to judge whether the selected threshold is valid. If the curve of the empirical excess distribution is closely around the curve of the theoretical distribution, this demonstrates the robustness of the chosen threshold value (Embrechts, Klüppelberg, and Mikosch (1997) PP360-363).

3.6.3 Parametric Modeling

There are several ways to estimate parameters and such ways are generally classified into two groups: parametric and nonparametric. The nonparametric method does not assume that extreme observations are drawn exactly from the asymptotic distribution. Hence, the tail estimation only relies on the parent variable X, as in the Hill estimator (Hill (1975)). However, McNeil and Frey (2000) point out that the Hill estimator is suitable only for the heavy-tailed case ($\xi > 0$), while during some periods financial returns may be characterized by light-tails ($\xi = 0$) or even short tails ($\xi < 0$) rather than heavy tails. Furthermore, Kearns and Pagan (1997) conclude that the Hill estimator can be very misleading due to its assumption of independent observations. To be prudent, we adopt a popular parametric method – maximum likelihood estimation – to estimate parameters. For the GEV, we first differentiate Eq. (3.19) to get the following density function:

$$f_{GEV}(x) = \frac{1}{\sigma}\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1+\xi}{\xi}} \times \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]\right\}^{-\frac{1}{\xi}} \qquad (3.26)$$

which yields the log-likelihood function:

$$\ln L_{GEV} = -k\ln\sigma - \left(\frac{1+\xi}{\xi}\right)\sum_{i=1}^{k}\ln\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right] - \sum_{i=1}^{k}\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}} \qquad (3.27)$$

where k is the number of extremal events.

Similarly, we also obtain the density function for the GPD from Eq. (3.24):

$$f_{GPD}(x) = \frac{1}{\sigma}\left[1+\xi\left(\frac{x-u}{\sigma}\right)\right]^{-\frac{1+\xi}{\xi}} \qquad (3.28)$$

with log-likelihood function reading as:

$$\ln L_{GPD} = -k\ln\sigma - \left(\frac{1+\xi}{\xi}\right)\sum_{i=1}^{k}\ln\left[1+\xi\left(\frac{x-u}{\sigma}\right)\right] \qquad (3.29)$$

Once the log-likelihood function has been determined, one can maximize it to obtain the estimated parameters. [3] Note that for both cases, $1+\xi(x-\mu)/\sigma$ and $1+\xi(x-u)/\sigma$ must be positive.

[3] Another parametric method is known as the regression method which is based on order statistics of extremes from a uniform parent (see Gumbel (1958)). For the empirical computation, three software packages can be considered: (1) Xtremes, (2) EVIS (Extreme Values in S-plus) and (3) EVIM (Extreme Values Analysis in Matlab). For more details, see McNeil (2000) and Gençay et al (2001). In this research, we use EVIM with Matlab 5.3.

3.6.4 Calculating VaR and Expected Shortfall

The VaR calculated here can be regarded as the filtered VaR based on extreme value theory. We start with the GPD case whose *cdf* $F(x)$ may be derived from Eq. (3.21) in the following way:

$$F_u(y) = \frac{F(x) - F(u)}{1 - F(u)} \Rightarrow F(x) = [1 - F(u)]F_u(y) + F(u) \qquad (3.30).$$

Replace $F_u(y)$ by $G_{\xi,\sigma}(y)$ and $F(u)$ by $(n - k) / n$ where n is the sample size and k is the number of exceedances. Then Eq. (3.30) becomes:

$$F(x) = \left(1 - 1 + \frac{k}{n}\right)\left\{1 - \left[1 + \xi\left(\frac{x - u}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\} + \left(1 - \frac{k}{n}\right)$$

$$= 1 - \frac{k}{n}\left[1 + \xi\left(\frac{x - u}{\sigma}\right)\right]^{-\frac{1}{\xi}} \qquad x \in \{X\} \qquad (3.31)$$

Inversing the above equation for a given probability $p = 1 - cl$ yields

$$\text{VaR}_{cl}^{GPD} = u + \frac{\sigma}{\xi}\left[\left(\frac{n}{k} \cdot p\right)^{-\xi} - 1\right] \qquad (3.32)$$

Given the relationship $G_\xi(x) = 1 + \ln H_\xi(x)$, the VaR of GEV can be easily obtained as

$$\text{VaR}_{cl}^{GEV} = \mu + \left(\frac{\sigma}{\xi}\right)\left\{\left[-\ln\left(1 - \frac{n}{k} \cdot p\right)\right]^{-\xi} - 1\right\} \qquad (3.33)$$

The estimation of expected shortfall for the GEV may have some difficulties due to the drawbacks of the GEV aforementioned above. One is that the block maxima approach involves efficiency loss because some extreme observations are excluded. Another one is that there is no clear-cut guide about how long the block periods should be and this will result a new bandwidth problem. Therefore, we only consider the unconditional expected shortfall for the GPD in this research.

The unconditional expected shortfall of the GPD is computed as

$$\Im_{cl}^{U} = \text{VaR}_{cl}^{GPD} + \mathbb{E}\,(X - \text{VaR}_{cl}^{GPD} \mid X > \text{VaR}_{cl}^{GPD}) \quad (3.34)$$

where the second term on the right-hand side is the mean of the excess distribution $F_{VaR_{cl}}(y)$ over the higher threshold VaR_{cl}^{GPD}. We can reconstruct it by

$$X - \text{VaR}_{cl}^{GPD} \mid X > \text{VaR}_{cl}^{GPD} = (X - u) - (\text{VaR}_{cl}^{GPD} - u) \mid (\text{X} - u) > (\text{VaR}_{cl}^{GPD} - u) \quad (3.35)$$

According to Eq. (3.22), $F_{VaR_{cl}}(y)$ is still subject to the GPD with the same parameters but a different scaling. Hence from Eq. (3.25), u is replaced by the $\text{VaR}_{cl}^{GPD} - u$ and the mean of the re-scaled GPD is

$$\mathbb{E}\,(X - \text{VaR}_{cl}^{GPD} \mid X > \text{VaR}_{cl}^{GPD}) = \frac{\sigma + \xi(VaR_{cl}^{GPD} - u)}{1 - \xi} \quad (3.36)$$

(For more details, see McNeil (2000) P9 and McNeil and Frey (2000) P292).
We thus have

$$\begin{aligned} \Im_{cl}^{U} &= VaR_{cl}^{GPD} + \frac{\sigma + \xi(VaR_{cl}^{GPD} - u)}{1 - \xi} \\ &= \frac{VaR_{cl}^{GPD}}{1 - \xi} + \frac{\sigma - \xi \cdot u}{1 - \xi} \end{aligned} \quad (3.37)$$

The unconditional expected shortfall becomes especially useful when forecasting expected shortfall over a *long horizon period*. However, financial managers often wish to apply extreme value theory to predict what the actual loss or gain of dynamic returns is beyond the VaR within a *short horizon period*. Because of this, we can model a conditional expected shortfall based on the one-step-ahead predicted conditional mean and volatility where returns have a dynamic structure (such as an AR(1)-GARCH (m, s) process):

$$\Im_{cl}^{C} = \hat{D}_{t+1} + \sqrt{\hat{h}_{t+1}} \left(\frac{VaR_{GARCH(m,s)}^{GPD}}{1 - \xi} + \frac{\sigma - \xi \cdot u}{1 - \xi} \right) \quad (3.38).$$

(See McNeil and Frey (2000) P293).

Chapter 4

Data Description

4.1 Basic Statistics

Investigated in this study are daily price indices from nine Asian emerging equity markets: China (CHI), Hong Kong (HON), Indonesia (IDS), South Korea (KOR), Malaysia (MAL), the Philippines (PHI), Singapore (SIN), Taiwan (TAI) and Thailand (THA). All data are collected from Datastream, and for each market the longest possible sample period is chosen since extreme value theory requires the sample period to be sufficient long. As a result, the starting date of the data varies across different markets depending on data availability, but all data end at February 26, 2003. Consequently, the range of the nine sample sizes is between 3002 and 6001, all covering such big events as financial liberalization and the Asian financial crisis. The daily return rate is computed as:

$$X_t = 100 \times \log(P_t / P_{t-1}) \qquad (4.1)$$

where P_t is the price index. Figure 12 shows the price index and return rates for each market.

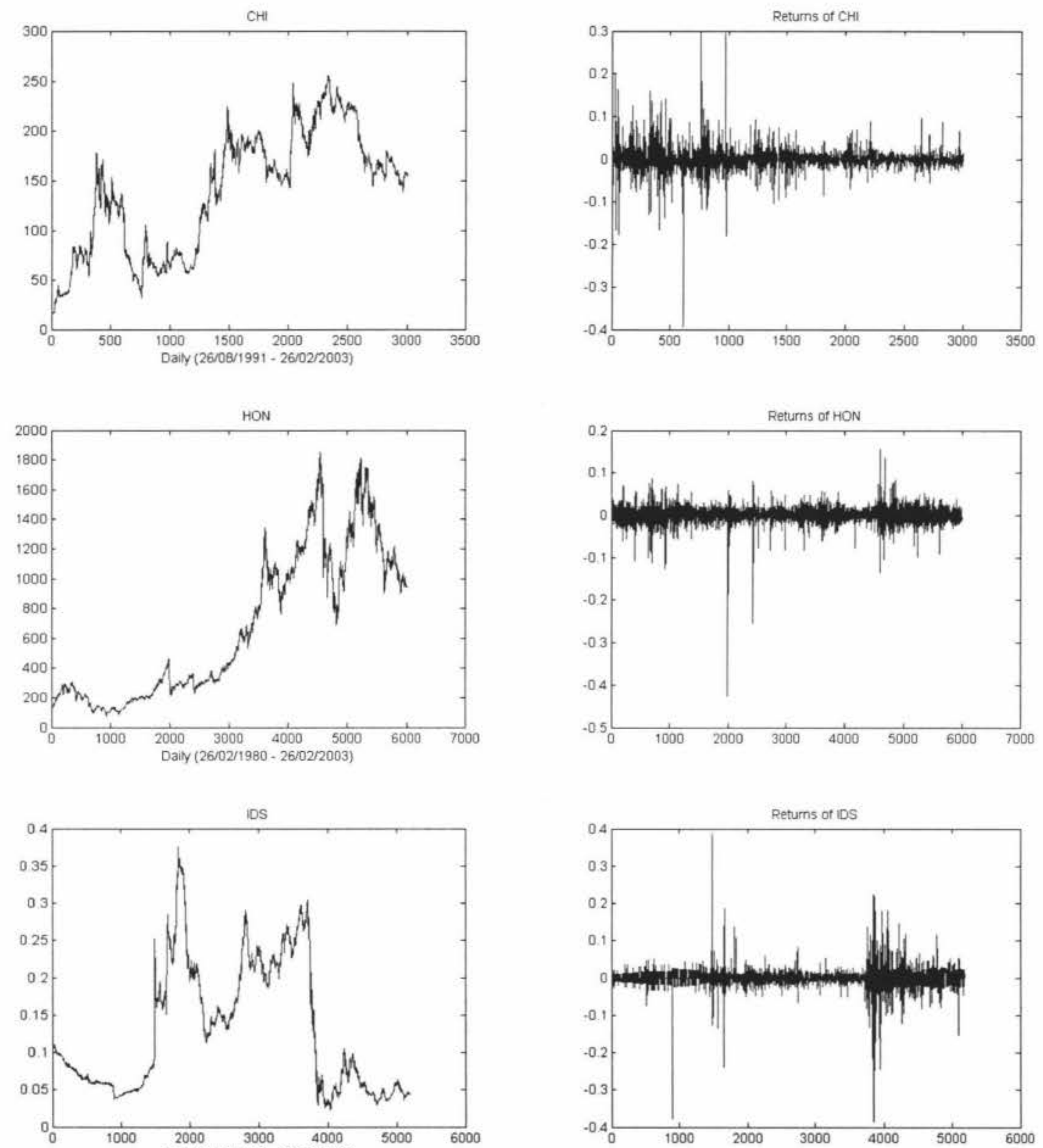

Figure 12(1). Plots of Price Indices and Index Returns

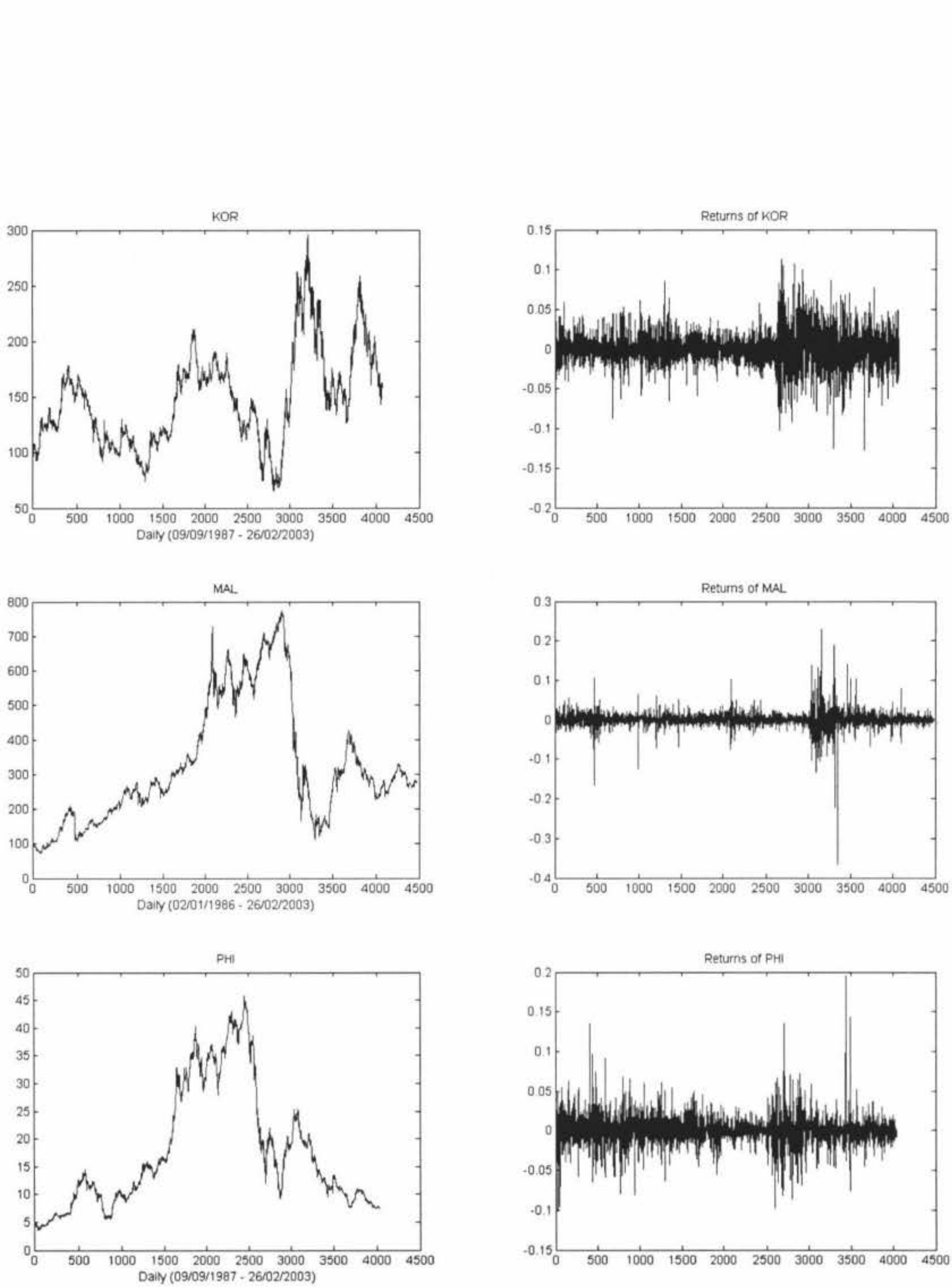

Figure 12(2). Plots of Price Indices and Index Returns

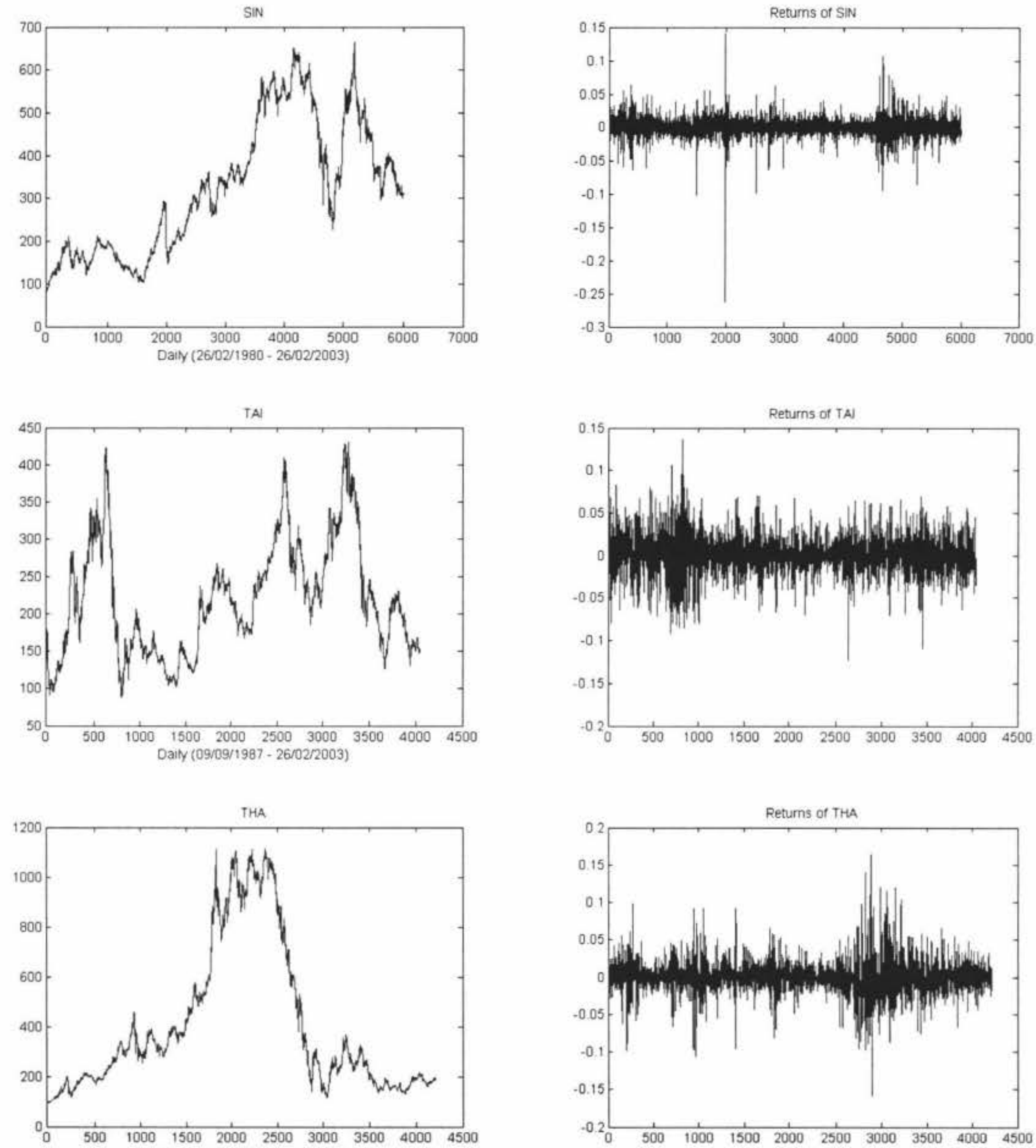

Figure 12(3). Plots of Price Indices and Index Returns

The descriptive statistics are represented in Table 2. The unconditional means of all index returns are close to zero. IDS has the lowest mean daily return (-0.01632%) but a relatively high standard deviations (2.504%). And the largest difference between the maximum and minimum returns is also exhibited by IDS. The highest mean daily return is found for CHI (0.07155%) whose standard deviation is the highest (2.836%). Like developed financial markets, the excess kurtosis statistics for the nine index returns are all considerably higher than 3, indicating thick tails relative to the normal distribution. Moreover, both the positive and negative skewness parameters imply that all the nine return series are asymmetrically distributed, rendering symmetric measures inappropriate for the total risk of assets. Pertaining to the Ljung-Box-Pierce $Q_x(50)$ tests, their highly significant statistics demonstrate that the nine return series are all positively autocorrelated. The squared $Q_{xx}(50)$ statistics are extremely significant, suggesting a higher-order relationship in volatility clustering as an autoregressive conditional heteroscedastisity process. For normality checking, the Kolmogorov-Smirnov (K-S) test provides a comparison of the empirical distribution with the standard normal distribution. The null hypothesis for the test is that the set of data has the standard normal distribution. According to the results of Monte Carlo simulations conducted by Lilliefors (1967), the critical value at the 1% level is $1.031/\sqrt{n}$ where n is the sample size. Using this formula, we can compute the critical values for each return series as follows: 0.0188 for CHI, 0.0133 for HON, 0.0143 for IDS, 0.0162 for KOR, 0.0154 for MAL, 0.0162 for PHI, 0.0133 for SIN, 0.0162 for TAI and 0.0159 for THA. Against these critical values, the statistics of the Kolmogorov-Smirnov test allow us to decisively reject the normality hypothesis for all the nine return series.

Table 2 Basic Statistics of Index Returns

Statistics	CHI	HON	IDS	KOR	MAL	PHI	SIN	TAI	THA
Length	3002	6001	5192	4067	4474	4035	6001	4035	4213
Maximum	29.83	15.56	38.88	11.34	22.99	19.55	14.03	13.73	16.35
Minimum	-39.53	-42.65	-38.57	-12.69	-36.77	-10.09	-26.28	-12.30	-15.89
Mean	0.07155	0.02933	-0.01632	0.01180	0.02246	0.00965	0.01970	0.00960	0.01417
Std.	2.836	1.831	2.504	2.019	1.879	1.781	1.372	2.227	2.117
Kurtosis	30.5911	63.8128	53.4108	6.7595	56.7604	12.7439	34.1263	5.2631	9.6147
Skewness	0.2939	-2.9650	-0.7856	0.1381	-1.4548	0.6177	-1.3763	0.0152	0.2797
Ljung-Box-Pierce Test									
$Q_x(50)$	170.0178*	96.7673*	362.4946*	93.1945*	235.5918*	168.7490*	147.1776*	111.7042*	191.9556*
$Q_{xx}(50)$	270.0329*	105.7403*	1671.1000*	3286.4000*	893.3220*	517.0287*	1239.2000*	4429.5000*	3044.2000*
K − S Test	0.1064**	0.0785**	0.1945**	0.1044**	0.0449**	0.0742**	0.0283**	0.1373**	0.0991**

Note: 1. Index return rate = $100 \times \log(P_t / P_{t-1})$.

2. * indicates significance at the 5% level.
** indicates significance at the 1% level.

3. The critical value of Ljung-Box-Pierce test is 67.5048 at 5%.

4. The critical values of Kolmogorov-Smirnov (K-S) test are based on Lilliefors (1967) where is as $1.031/\sqrt{N}$ at 1%

4.2 QQ Plots

A visual tool named QQ-plot (quantile to quantile plot) allows us to determine whether observations come from a specified distribution. The general idea of the QQ-plot is that, if our data are realizations from a specified distribution, then we should expect the resulting graph to show a straight-line pattern. Thus, the form of the deviations from linearity in the QQ-plot may be informative. If the data come from a distribution with a heavier tail than the reference distribution (for instance, the standard normal or exponential distribution with $\xi = 0$), then we should observe a concave pattern of the graph distinct from the straight line. A convex pattern, if observed, would suggest that the data have a thinner tail than the reference distribution.

Fig. 13 presents QQ plots making reference to the standard normal distribution (left panel) and to the exponential distribution (right panel) for each index return series. In the left panels, each empirical distribution clearly departures from the standard normal distribution: All the nine lower tails significantly bend down and all the nine upper tails up, implying a high degree of tail fatness. Turning now to the right panels, we can see that all show a concave pattern, which confirms the tail fatness of the nine empirical distributions.

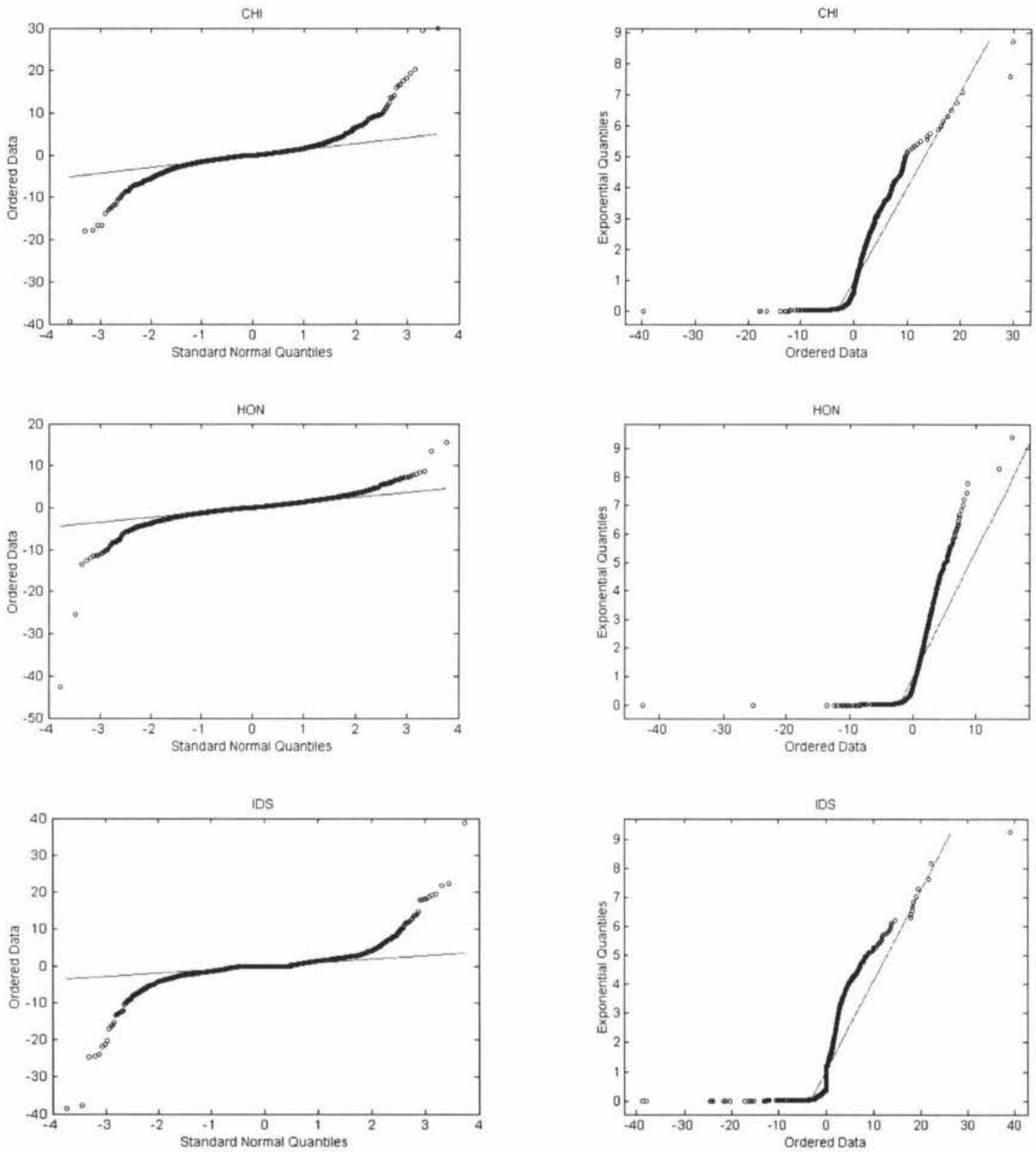

Figure 13(1). QQ Plots of Index Returns

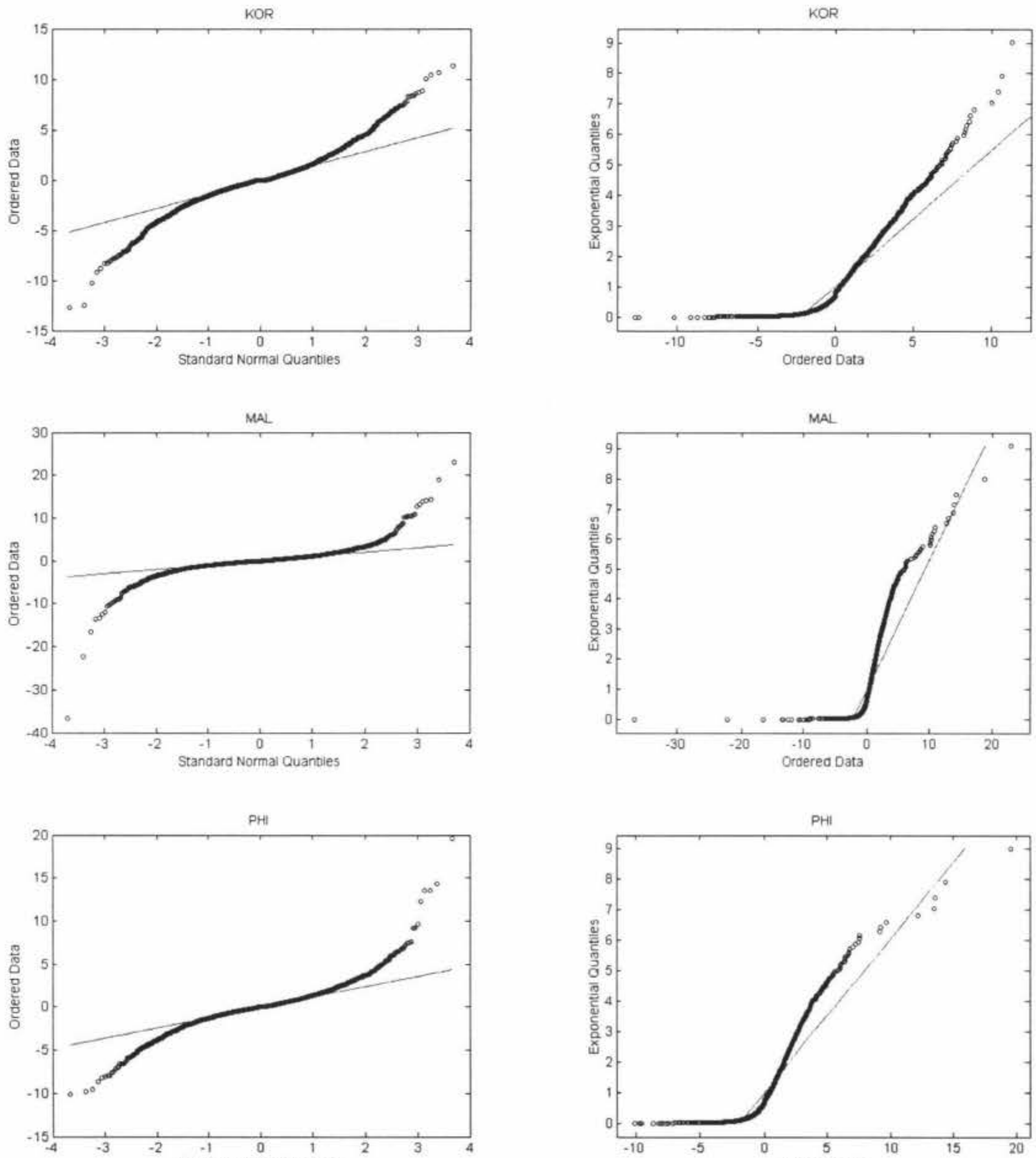

Figure 13(2). QQ Plots of Index Returns

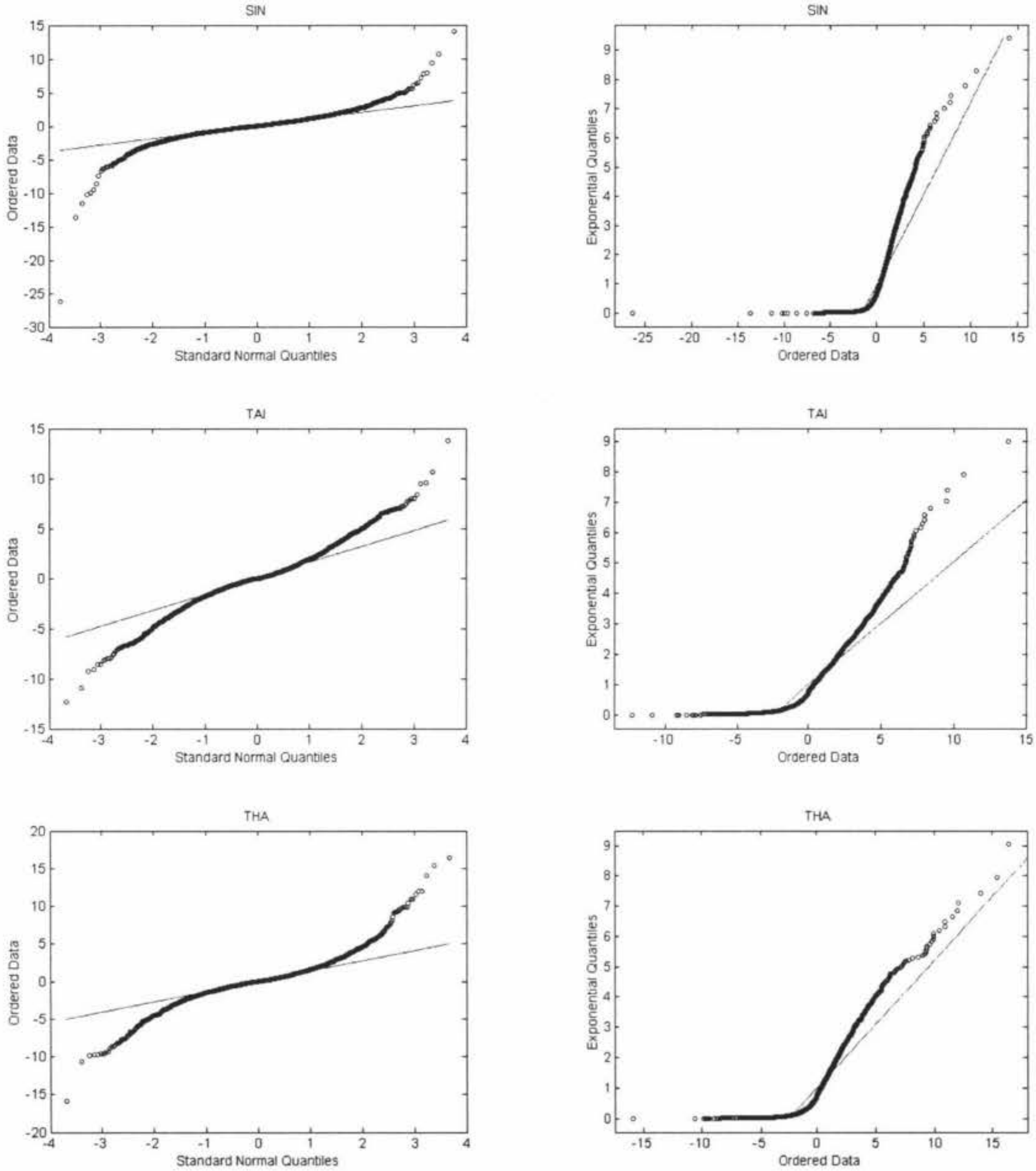

Figure 13(3). QQ Plots of Index Returns

Chapter 5

Empirical Results

5.1 The AR(1)-GARCH(m, s) Process

Following McNeil and Frey (2000), we first employ the maximum likelihood estimation method to gage the AR(1)-GARCH(m, s) effect for the dynamics of the conditional mean and the volatility that may characterize each index returns. Typically, m = 1, 2 and s = 1, 2 are specified. As reported in Table 3, the estimated GARCH(m, s) parameters are statistically significant, which implies that the GARCH model successfully captures the temporal dependence of volatility. Except CHI, all markets have their persistence $\sum_{i=1}^{s} \varphi_i + \sum_{j=1}^{m} \phi_j$ of volatility positive and less than 1, indicating that the impact of a shock to the volatility dies out over time. This information tells us that the unconditional variance of each series is finite but the conditional variance of each series is dependent and clustering. These results also suggest that the risk measure by the conventional EVT under the iid assumption is dubious. The persistence of the CHI volatility equal to 1 makes the integrated GARCH (IGARCH) model of Engle and Bollerslev (1986) appropriate for China where a shock to the market has its impact persistent over time. Moreover, we employed the likelihood ratio test to help choose a value for m and s between 1 and 2. The test statistic is asymptotically Chi-square distributed with degrees of freedom equal to the number of restrictions imposed on the parameters. A value of the statistic greater than the critical value of 3.8415 at the 5% level allows us to reject the unrestricted model in favor of the restricted model. Consequently, we chose AR(1)-GARCH(1, 1) model for IDS and PHI, AR(1)-GARCH(2, 1) model for MAL and SIN, and AR(1)-GARCH(2, 2) model for CHI, HON, KOR, TAI, and THA. The filtered innovations and conditional volatilities are plotted in Fig. 14. Again, following McNeil and Frey (2000), in Fig. 15 we plot correlograms for index returns and their absolute values as well as for the filtered innovations and absolute innovations to diagnose autocorrelation. It is obvious that after AR(1) – GARCH (m, s) filtering, iid assumption is tenable for the innovations but not for the index returns. Next, the basic statistics of innovations are given in Table 4. Although the mean and the standard deviation of each filtered innovation approximate to 0 and 1 respectively, the Kolmogorov – Smirnov test still rejects the normality hypothesis for all series. Further, from Fig. 16 one can see that the filtered innovations still exhibit a heavy-tailed pattern. Thus, as we mentioned before, the VaR calculated under the normality assumption is still dubious even after autocorrelation and volatility filtering. In what follows, therefore, we use these filtered innovations to estimate the parameters of the GEV and GPD models, and then calculate the VaR and the expected shortfall.

Table 3. Maximum Likelihood Estimated Parameters of AR (1)-GARCH (*m*, *s*) Models

Market	(*m*, *s*)	Estimated Parameters									
		φ_0	δ	ϕ_0	φ_1	φ_2	ϕ_1	ϕ_2	$\varphi_i + \phi_j$	Loglik	L Test
CHI	(1, 1)	0.0089	0.0437	0.0443	0.0804	-	0.9196	-	1.0000	-6668.2	-
		(0.0310)	(0.0148)	(0.0045)	(0.0022)	-	(0.0019)	-			
	(1, 2)	0.0089	0.0437	0.0443	0.0804	0	0.9196	-	1.0000	-6668.2	0
		(0.0311)	(0.0150)	(0.0048)	(0.0085)	(0.0082)	(0.0021)	-			
	(2, 1)	0.0086	0.0436	0.0485	0.0900	-	0.7975	0.1136	1.0011	-6667.7	0.9906
		(0.0311)	(0.0159)	(0.0058)	(0.0090)	-	(0.1033)	(0.0955)			
	(2, 2)	0.0113	0.0451	0.1040	0.0676	0.1028	0	0.8296	1.0000	-6664.8	6.9028*
		(0.0316)	(0.0152)	(0.0105)	(0.0040)	(0.0036)	(0.0092)	(0.0095)			
HON	(1, 1)	0.1001	0.0602	0.0771	0.1388	-	0.8484	-	0.9872	-11088	-
		(0.0172)	(0.0139)	(0.0049)	(0.0042)	-	(0.0051)	-			
	(1, 2)	0.1024	0.0637	0.0839	0.1094	0.0393	0.8373	-	0.9860	-11086	4.7147*
		(0.0173)	(0.0139)	(0.0059)	(0.0080)	(0.0086)	(0.0061)	-			
	(2, 1)	0.1001	0.0602	0.0771	0.1388	-	0.8484	0	0.9872	-11088	0
		(0.0174)	(0.0143)	(0.0055)	(0.0092)	-	(0.0641)	(0.0564)			
	(2, 2)	0.0972	0.0581	0.1439	0.1541	0.0897	0	0.7287	0.9725	-11083	9.2862*
		(0.0174)	(0.0138)	(0.0113)	(0.0069)	(0.0160)	(0.0564)	(0.0459)			
IDS	(1, 1)	-0.0055	0.0754	0.1260	0.1345	-	0.8600	-	0.9945	-10437	-
		(0.0210)	(0.0135)	(0.0066)	(0.0040)	-	(0.0041)	-			
	(1, 2)	-0.0055	0.0754	0.1260	0.1345	0	0.8600	-	0.9945	-10437	0
		(0.0212)	(0.0135)	(0.0067)	(0.0095)	(0.0099)	(0.0044)	-			
	(2, 1)	-0.0057	0.0760	0.1304	0.1390	-	0.8115	0.0436	0.9941	-10437	0.2530
		(0.0213)	(0.0135)	(0.0107)	(0.0098)	-	(0.0696)	(0.0608)			
	(2, 2)	-0.0010	0.0765	0.2184	0.1480	0.0887	0.0934	0.6615	0.9916	-10436	1.3510
		(0.0214)	(0.0136)	(0.0228)	(0.0090)	(0.0213)	(0.1356)	(0.1160)			

KOR	(1, 1)	0.0105	0.0416	0.0357	0.0759	-	0.9173	-	0.9932	-8075.8	-
		(0.0239)	(0.0161)	(0.0057)	(0.0054)	-	(0.0055)	-			
	(1, 2)	0.0105	0.0416	0.0357	0.0759	0	0.9173	-	0.9932	-8075.8	0
		(0.0240)	(0.0161)	(0.0058)	(0.0126)	(0.0138)	(0.0059)	-			
	(2, 1)	0.0087	0.0431	0.0456	0.1033	-	0.4191	0.4690	0.9914	-8073.3	4.8652*
		(0.0239)	(0.0170)	(0.0080)	(0.0092)	-	(0.1094)	(0.1031)			
	(2, 2)	0.0092	0.0428	0.0506	0.0993	0.0141	0.3044	0.5726	0.9904	-8073.2	5.0746*
		(0.0239)	(0.0170)	(0.0106)	(0.0119)	(0.0229)	(0.1896)	(0.1738)			
MAL	(1, 1)	0.0483	0.1467	0.0208	0.0677	-	0.9291	-	0.9968	-7771.1	-
		(0.0144)	(0.0144)	(0.0010)	(0.0027)	-	(0.0015)	-			
	(1, 2)	0.0483	0.1467	0.0208	0.0677	0	0.9291	-	0.9968	-7771.1	0
		(0.0145)	(0.0147)	(0.0010)	(0.0071)	(0.0082)	(0.0019)	-			
	(2, 1)	0.0465	0.1458	0.0290	0.0945	-	0.4670	0.4336	0.9951	-7763.8	14.6552*
		(0.0144)	(0.0153)	(0.0019)	(0.0068)	-	(0.0893)	(0.0835)			
	(2, 2)	0.0465	0.1458	0.0290	0.0945	0	0.4670	0.4336	0.9951	-7763.8	14.6552*
		(0.0145)	(0.0155)	(0.0043)	(0.0099)	(0.0230)	(0.2332)	(0.2163)			
PHI	(1, 1)	0.0273	0.1075	0.0711	0.1335	-	0.8571	-	0.9906	-7538.2	-
		(0.0190)	(0.0157)	(0.0089)	(0.0075)	-	(0.0072)	-			
	(1, 2)	0.0273	0.1075	0.0711	0.1335	0	0.8571	-	0.9906	-7538.2	0
		(0.0192)	(0.0162)	(0.0091)	(0.0141)	(0.0155)	(0.0082)	-			
	(2, 1)	0.0281	0.1062	0.0760	0.1441	-	0.7391	0.1066	0.9898	-7537.8	0.7351
		(0.0191)	(0.0162)	(0.0113)	(0.0146)	-	(0.1087)	(0.0960)			
	(2, 2)	0.0260	0.1086	0.1330	0.1307	0.1189	0	0.7330	0.9826	-7538.1	0.1731
		(0.0197)	(0.0162)	(0.0298)	(0.0107)	(0.0465)	(0.3833)	(0.3301)			
	(1, 1)	0.0460	0.0971	0.0679	0.1274	-	0.8395	-	0.9669	-9507.3	-
		(0.0134)	(0.0137)	(0.0052)	(0.0053)	-	(0.0063)	-			
	(1, 2)	0.0460	0.0971	0.0679	0.1274	0	0.8395	-	0.9669	-9507.3	0

SIN		(0.0135)	(0.0138)	(0.0054)	(0.0056)	(0.0058)	(0.0073)	-			
	(2, 1)	0.0442	0.0947	0.0835	0.1606	-	0.4487	0.3495	0.9588	-9498.6	17.3775*
		(0.0136)	(0.0143)	(0.0068)	(0.0069)	-	(0.0494)	(0.0449)			
	(2, 2)	0.0442	0.0947	0.0835	0.1606	0	0.4487	0.3495	0.9588	-9498.6	17.3775*
		(0.0136)	(0.0143)	(0.0117)	(0.0069)	(0.0200)	(0.1487)	(0.1251)			
TAI	(1, 1)	0.0405	0.0309	0.0867	0.0711	-	0.9105	-	0.9816	-8549.0	-
		(0.0292)	(0.0172)	(0.0125)	(0.0058)	-	(0.0072)	-			
	(1, 2)	0.0418	0.0293	0.0997	0.0550	0.0234	0.9006	-	0.9790	-8548.1	1.6350
		(0.0292)	(0.0167)	(0.0151)	(0.0108)	(0.0135)	(0.0092)	-			
	(2, 1)	0.0405	0.0309	0.0867	0.0711	-	0.9105	0	0.9816	-8549.0	0
		(0.0292)	(0.0173)	(0.0171)	(0.0120)	-	(0.1918)	(0.1785)			
	(2, 2)	0.0405	0.0287	0.1995	0.0518	0.0969	0	0.8087	0.9574	-8541.4	15.0769*
		(0.0293)	(0.0164)	(0.0276)	(0.0077)	(0.0101)	(0.0301)	(0.0306)			
THA	(1, 1)	0.0528	0.1061	0.0531	0.0918	-	0.8984	-	0.9902	-8384.6	-
		(0.0237)	(0.0157)	(0.0066)	(0.0049)	-	(0.0048)	-			
	(1, 2)	0.0528	0.1061	0.0531	0.0918	0	0.8984	-	0.9902	-8384.6	0
		(0.0237)	(0.0165)	(0.0069)	(0.0114)	(0.0127)	(0.0055)	-			
	(2, 1)	0.0525	0.1035	0.0596	0.1097	-	0.5988	0.2806	0.9891	-8382.7	3.7566
		(0.0239)	(0.0165)	(0.0089)	(0.0100)	-	(0.1143)	(0.1052)			
	(2, 2)	0.0533	0.1019	0.0964	0.1131	0.0537	0.0247	0.7904	0.9819	-8379.6	10.0240*
		(0.0237)	(0.0163)	(0.0122)	(0.0083)	(0.0110)	(0.0476)	(0.0423)			

Note: 1. Parameters via maximum likelihood estimation are based on the following functions:

$$X_t = D_t + \eta_t$$

$$D_t = \varphi_0 + \delta X_{t-1}$$

$$h_t = \phi_0 + \sum_{i=1}^{s} \varphi_i \eta_{t-i}^2 + \sum_{j=1}^{m} \phi_j h_{t-j}$$

2. L Test is the likelihood ratio test.
3. * indicates significance at the 5% level with the critical value being 3.8415.
4. Standard deviations are in parentheses.

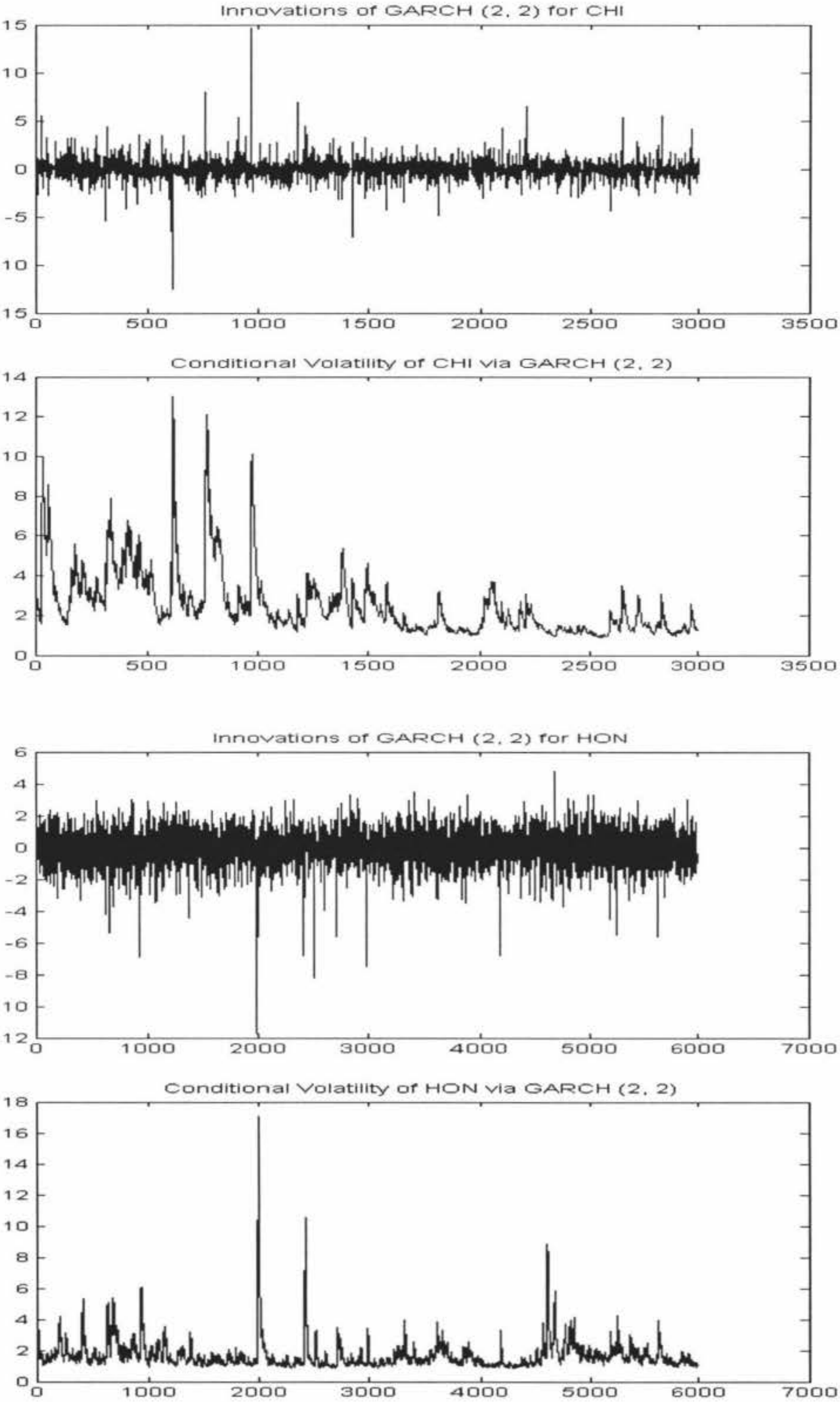

Figure 14(1). Innovations and Conditional Volatility of AR (1)-GARCH (*m*, *s*) Model

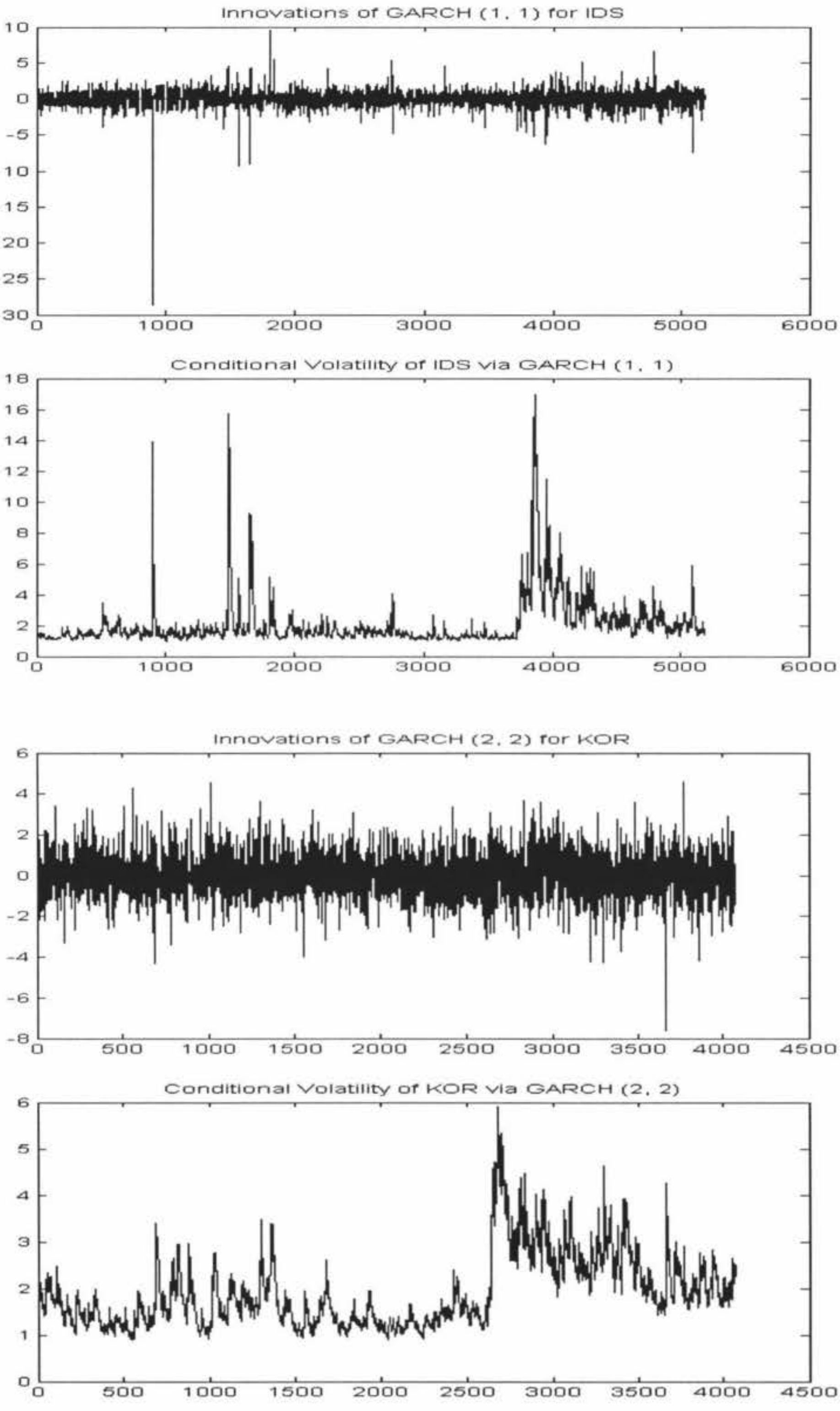

Figure 14(2). Innovations and Conditional Volatility of AR (1)-GARCH (*m*, *s*) Model

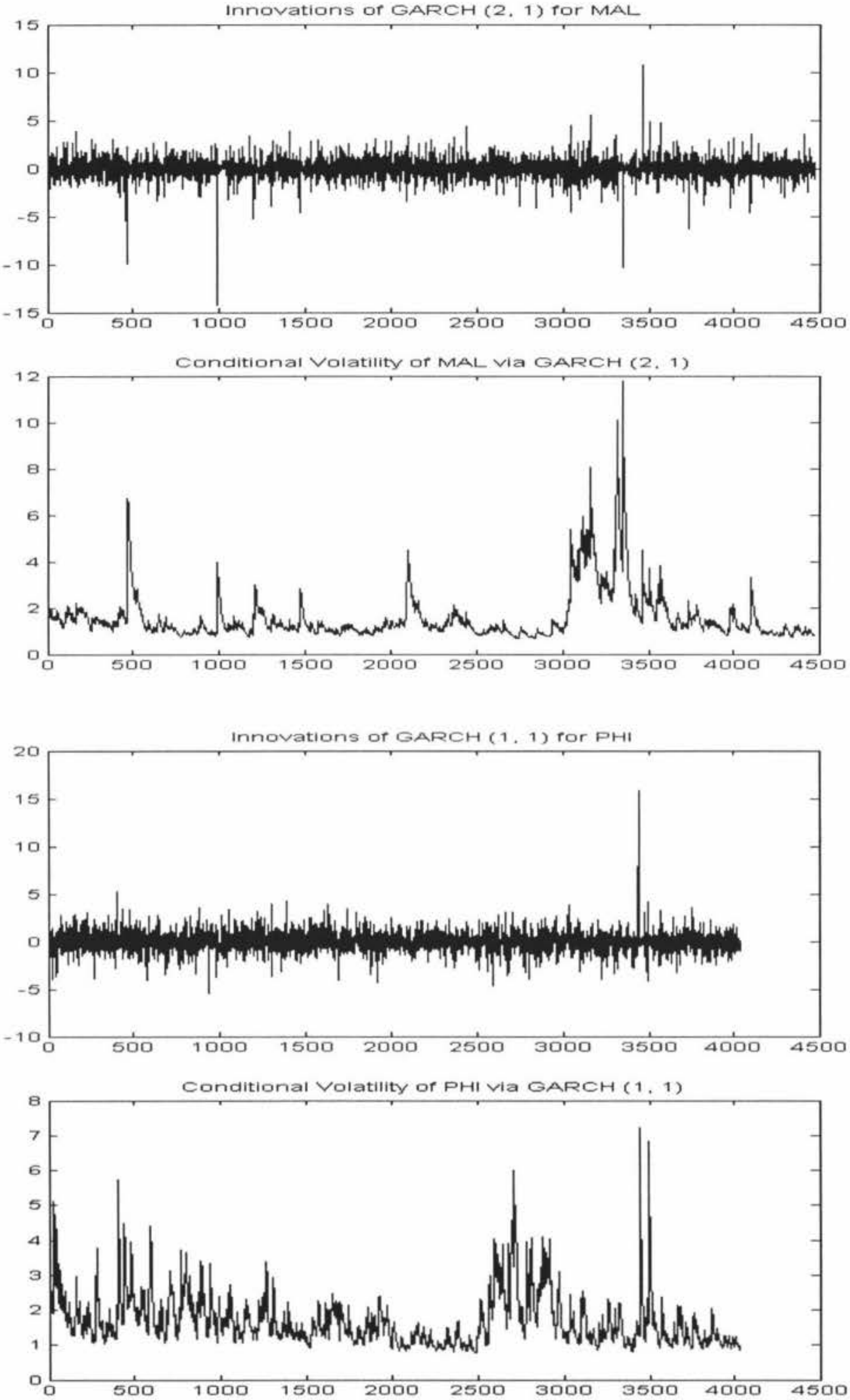

Figure 14(3). Innovations and Conditional Volatility of AR (1)-GARCH (*m*, *s*) Model

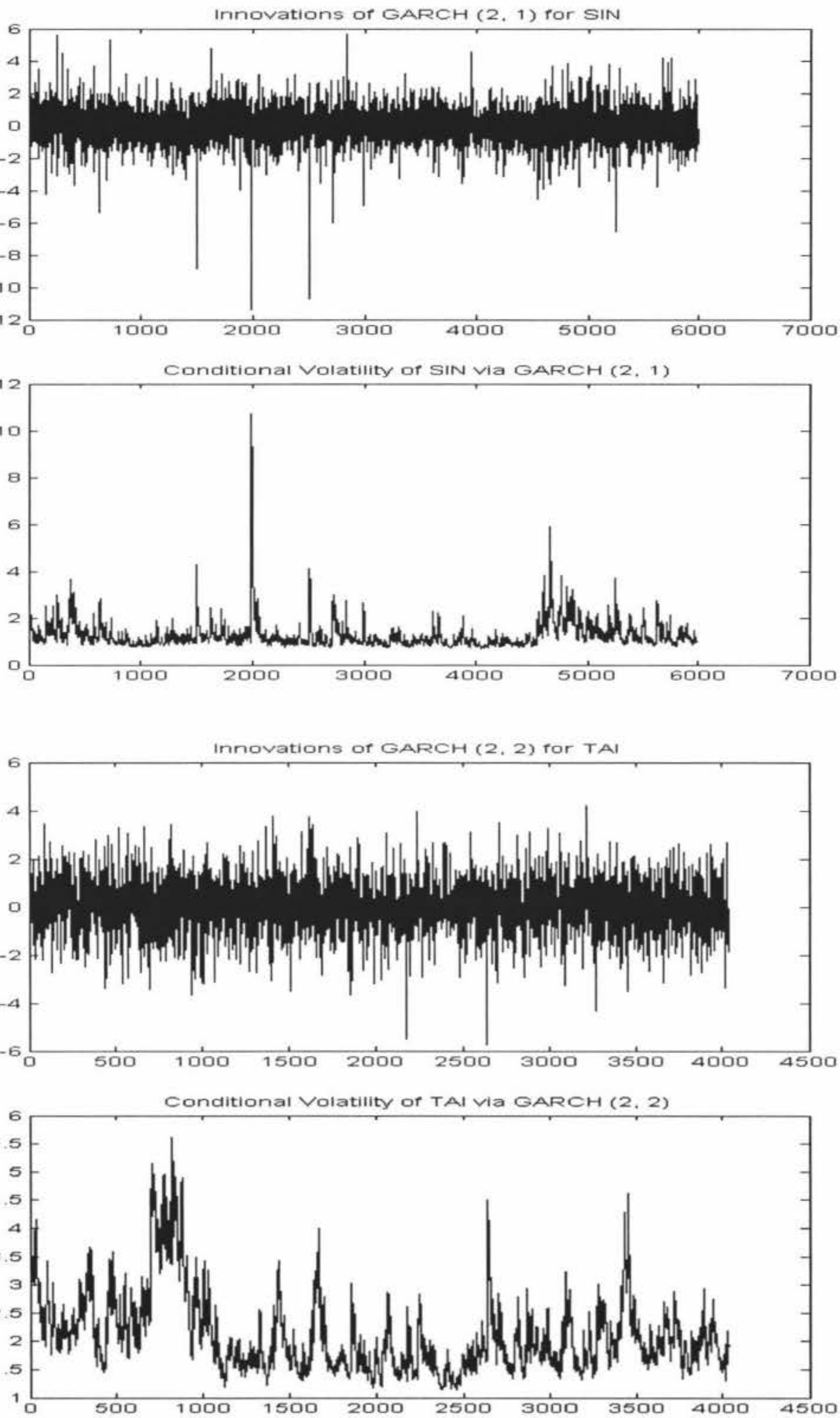

Figure 14(4). Innovations and Conditional Volatility of AR (1)-GARCH (*m*, *s*) Model

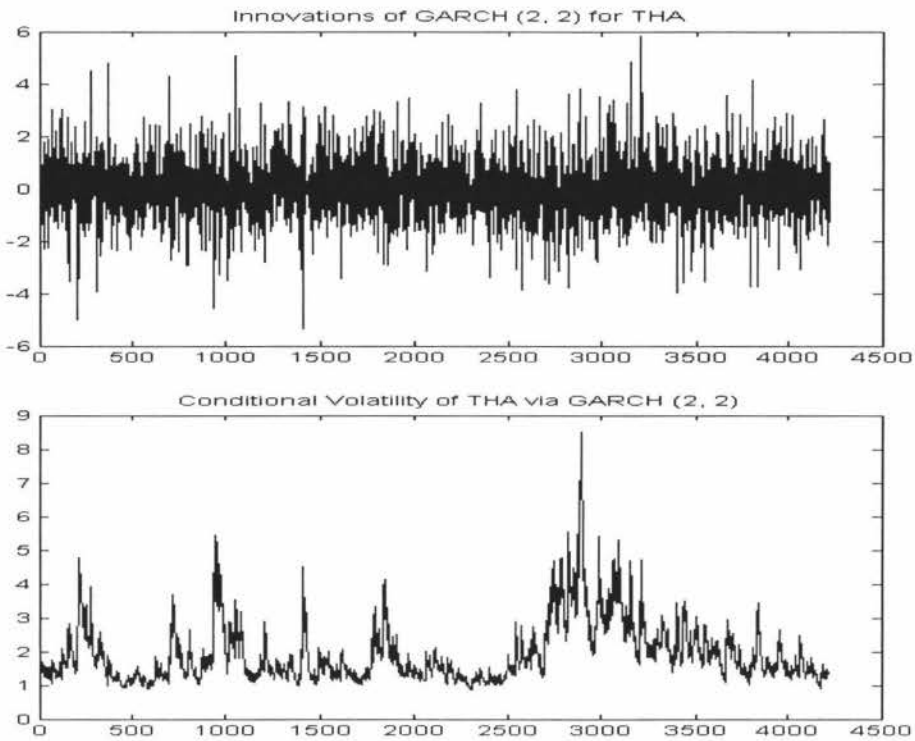

Figure 14(5). Innovations and Conditional Volatility of AR (1)-GARCH (*m*, *s*) Model

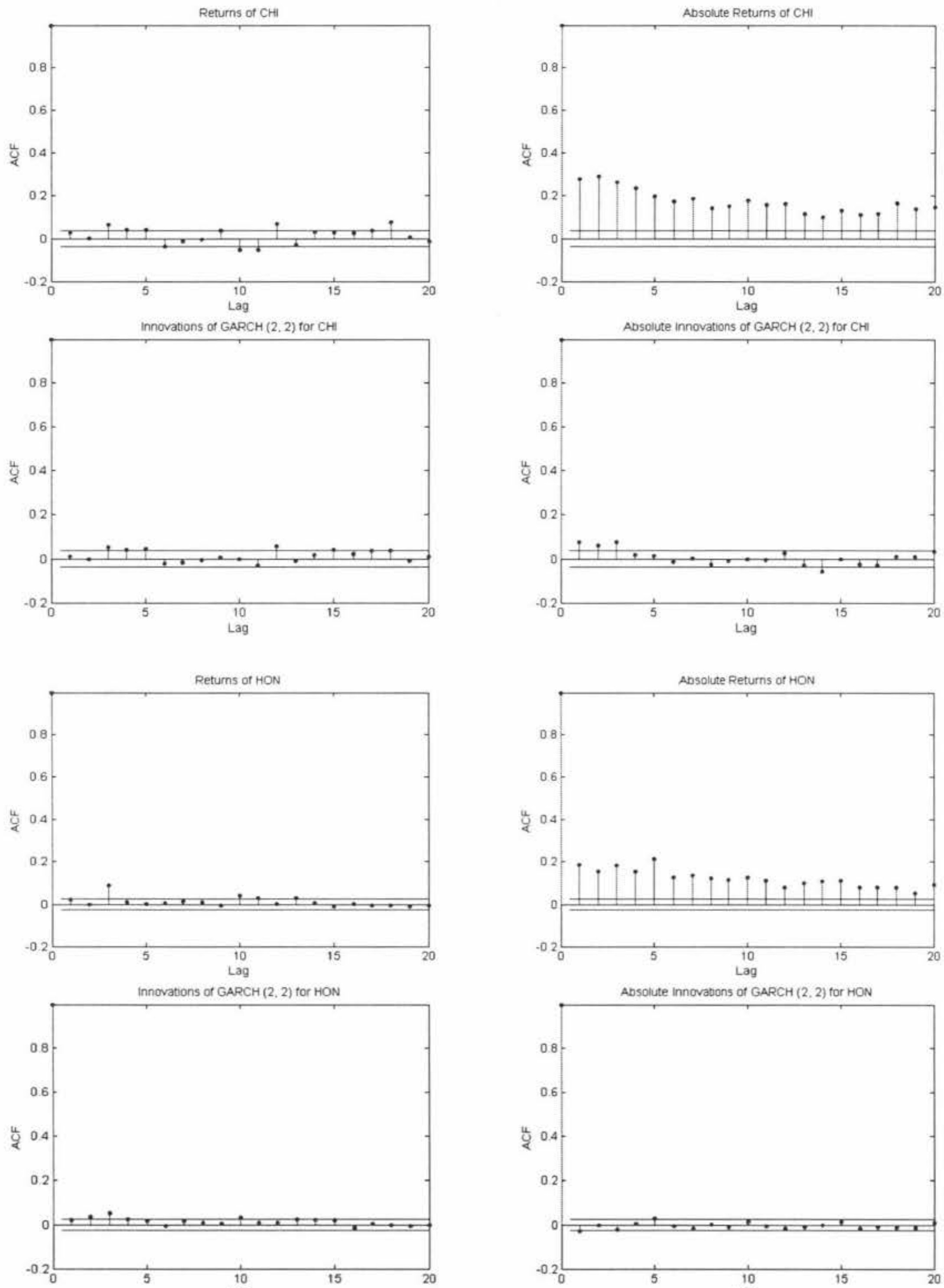

Figure 15(1).

First row: Correlograms for returns and absolute returns.

Second row: Correlograms for innovations and absolute innovations.

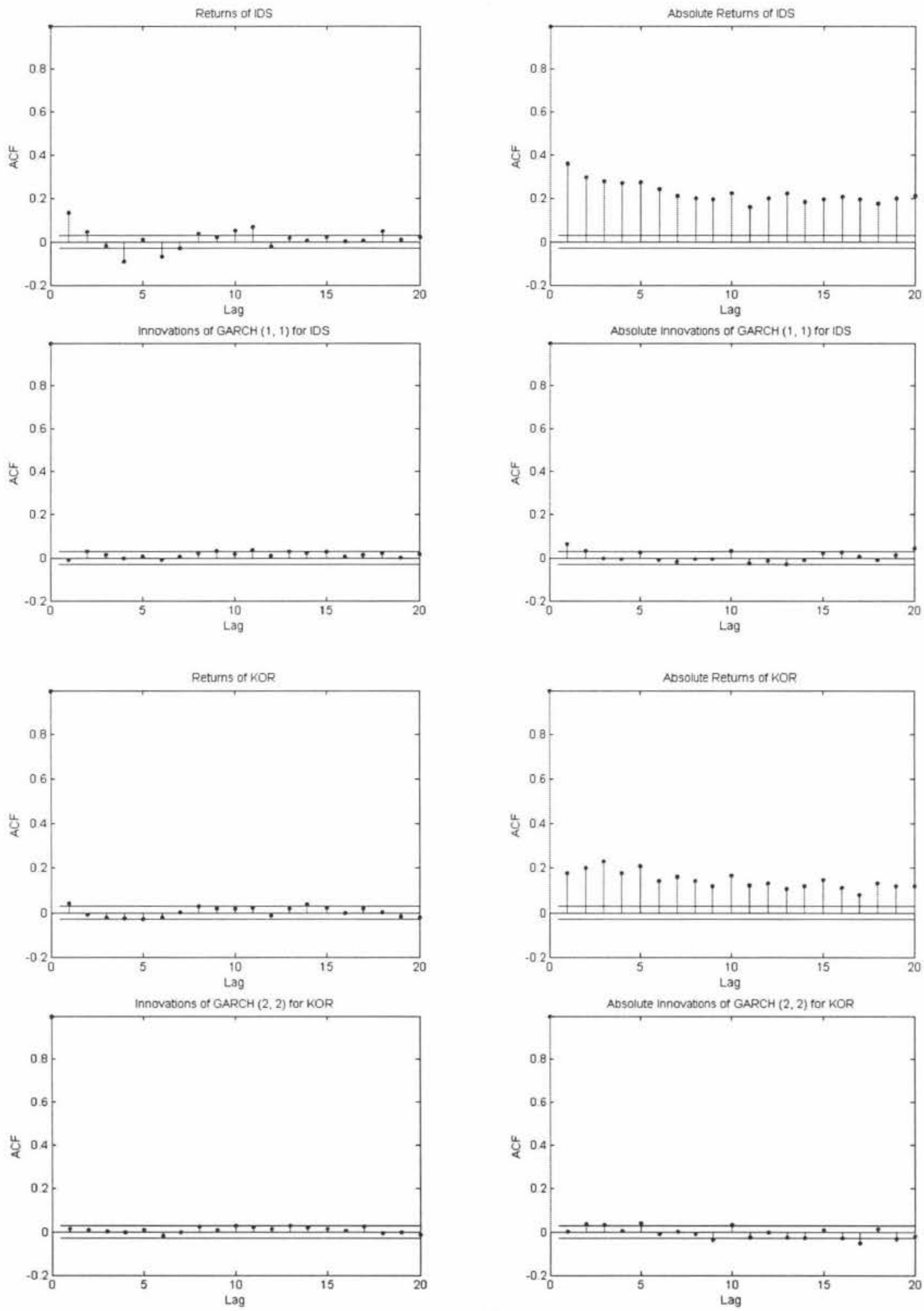

Figure 15(2).

First row: Correlograms for returns and absolute returns.

Second row: Correlograms for innovations and absolute innovations.

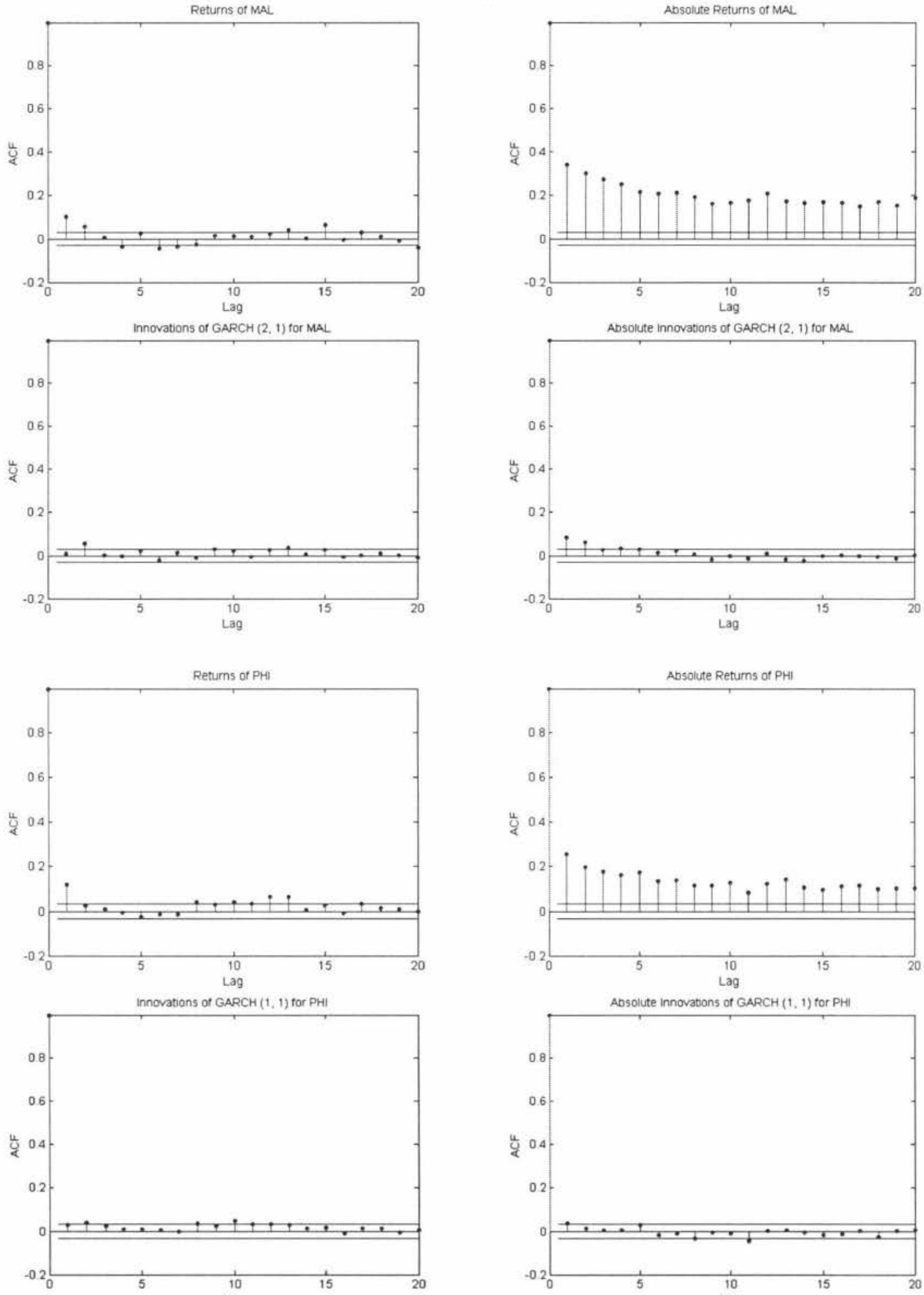

Figure 15(3).

First row: Correlograms for returns and absolute returns.

Second row: Correlograms for innovations and absolute innovations.

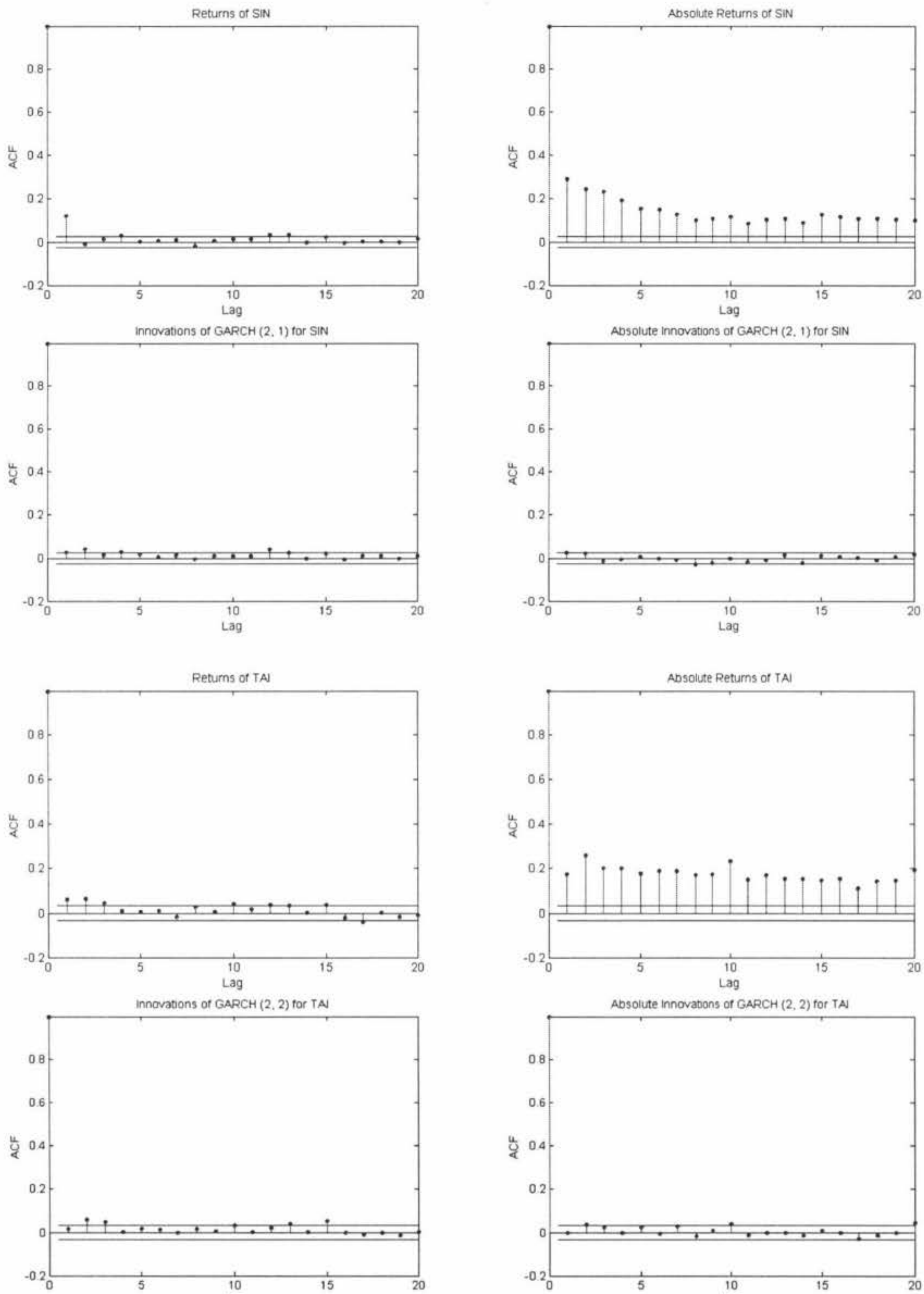

Figure 15(4).

First row: Correlograms for returns and absolute returns.

Second row: Correlograms for innovations and absolute innovations.

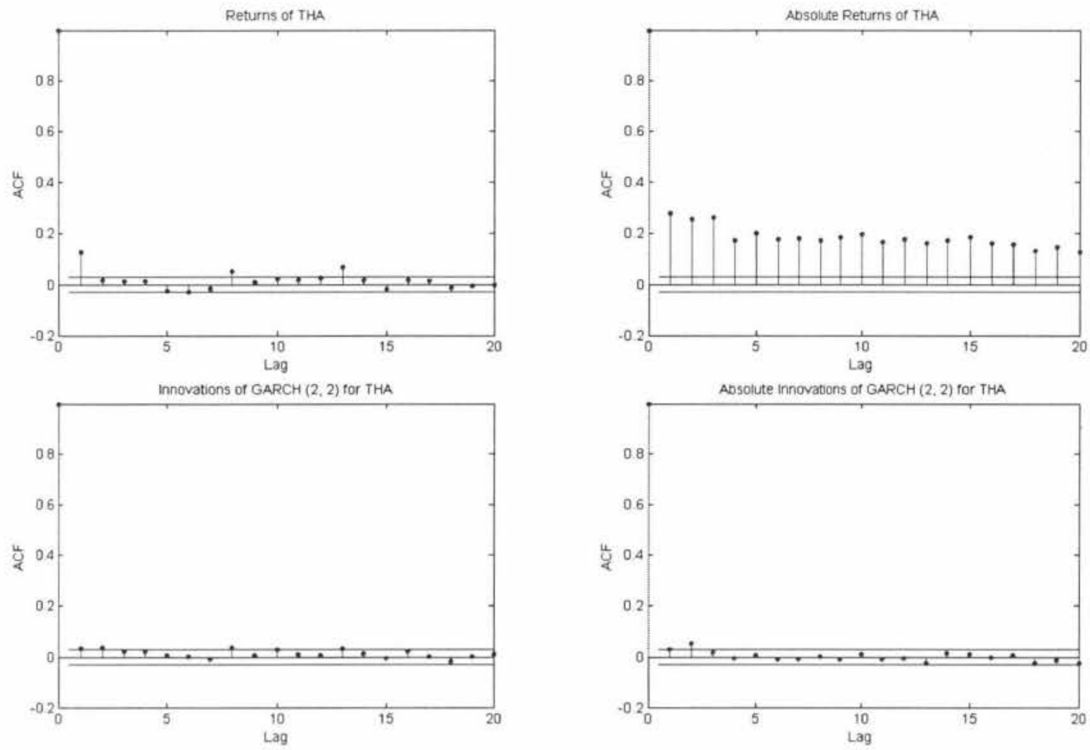

Figure 15(5).
First row: Correlograms for returns and absolute returns.
Second row: Correlograms for innovations and absolute innovations.

Table 4. Basic Statistics of Filtered Innovations

Market	Model	Statistic						
		Maximum	Minimum	Mean	Std.	Kurtosis	Skewness	K-S Test
CHI	(2, 2)	14.6538	-12.5248	-0.0010	1.3066	28.7741	0.7196	0.0810*
HON	(2, 2)	4.8146	-11.7329	-0.0438	0.9993	10.5738	-0.9455	0.0482*
IDS	(1, 1)	9.5864	-28.5862	-0.0092	0.9993	138.6120	-4.6603	0.1446*
KOR	(2, 2)	4.6192	-7.6169	-0.0098	0.9997	4.9473	0.0505	0.0596*
MAL	(2, 1)	10.7807	-14.2267	-0.0163	0.9997	21.5896	-0.8424	0.0644*
PHI	(1, 1)	15.8364	-5.4173	-0.0188	1.0002	20.2971	0.9180	0.0559*
SIN	(2, 1)	5.6904	-11.3654	-0.0242	1.0000	10.9487	-0.5600	0.0427*
TAI	(2, 2)	4.1972	-5.7366	-0.0175	1.0008	4.4475	-0.0197	0.0553*
THA	(2, 2)	5.8119	-5.3522	-0.0186	0.9991	5.3180	0.1806	0.0602*

Note: 1. The filtered innovations $\hat{\varepsilon}_t = \frac{X_t - \hat{D}_t}{\sqrt{\hat{h}_t}}$.

2. The critical values of Kolmogorov-Smirnov (K-S) test are based on Lilliefors (1967) where is as $1.031/\sqrt{N}$ at 1%.

3. * indicates significant at 1% level.

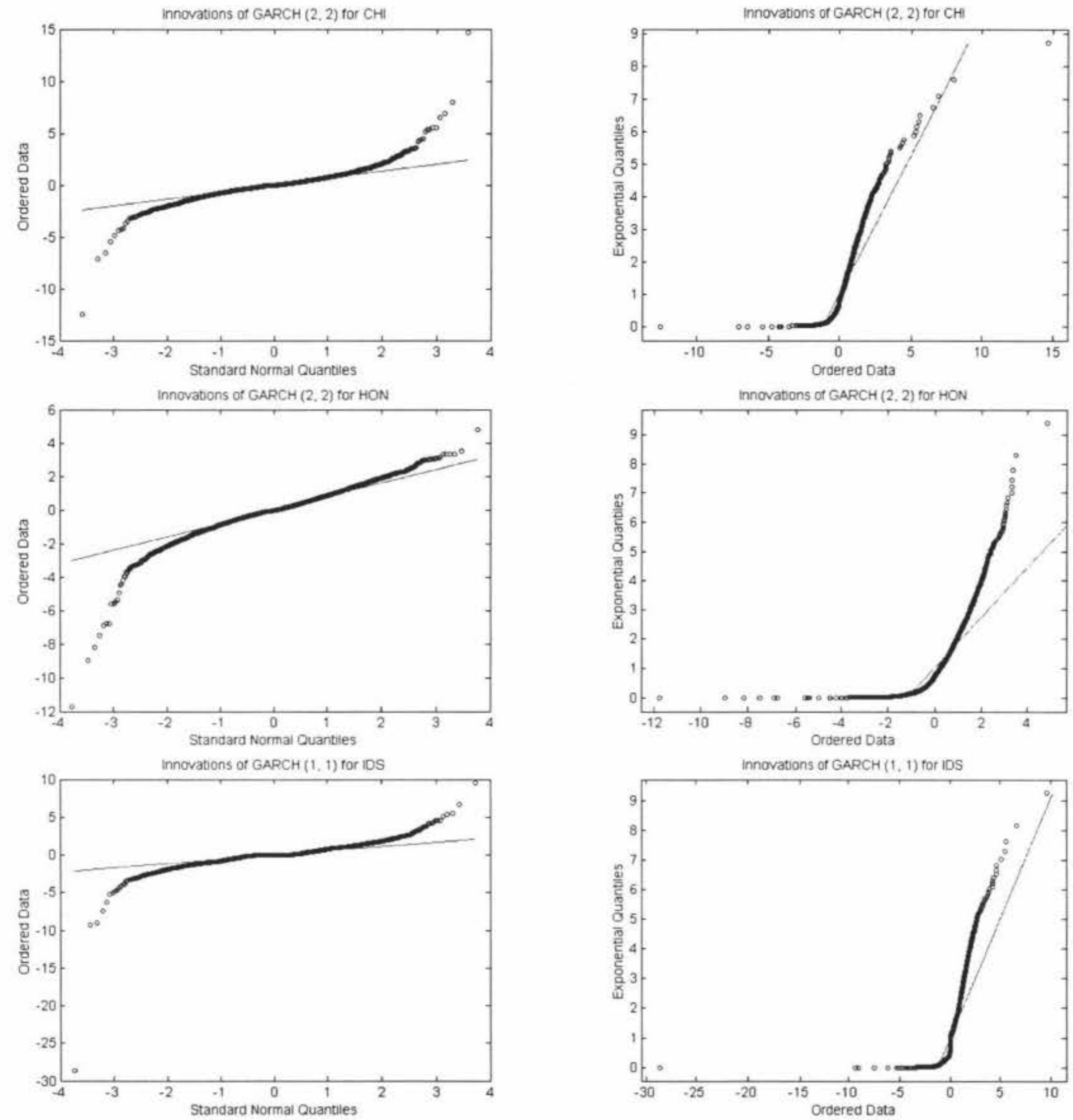

Figure 16(1). QQ Plots of Innovations of AR (1)-GARCH (m, s) Model

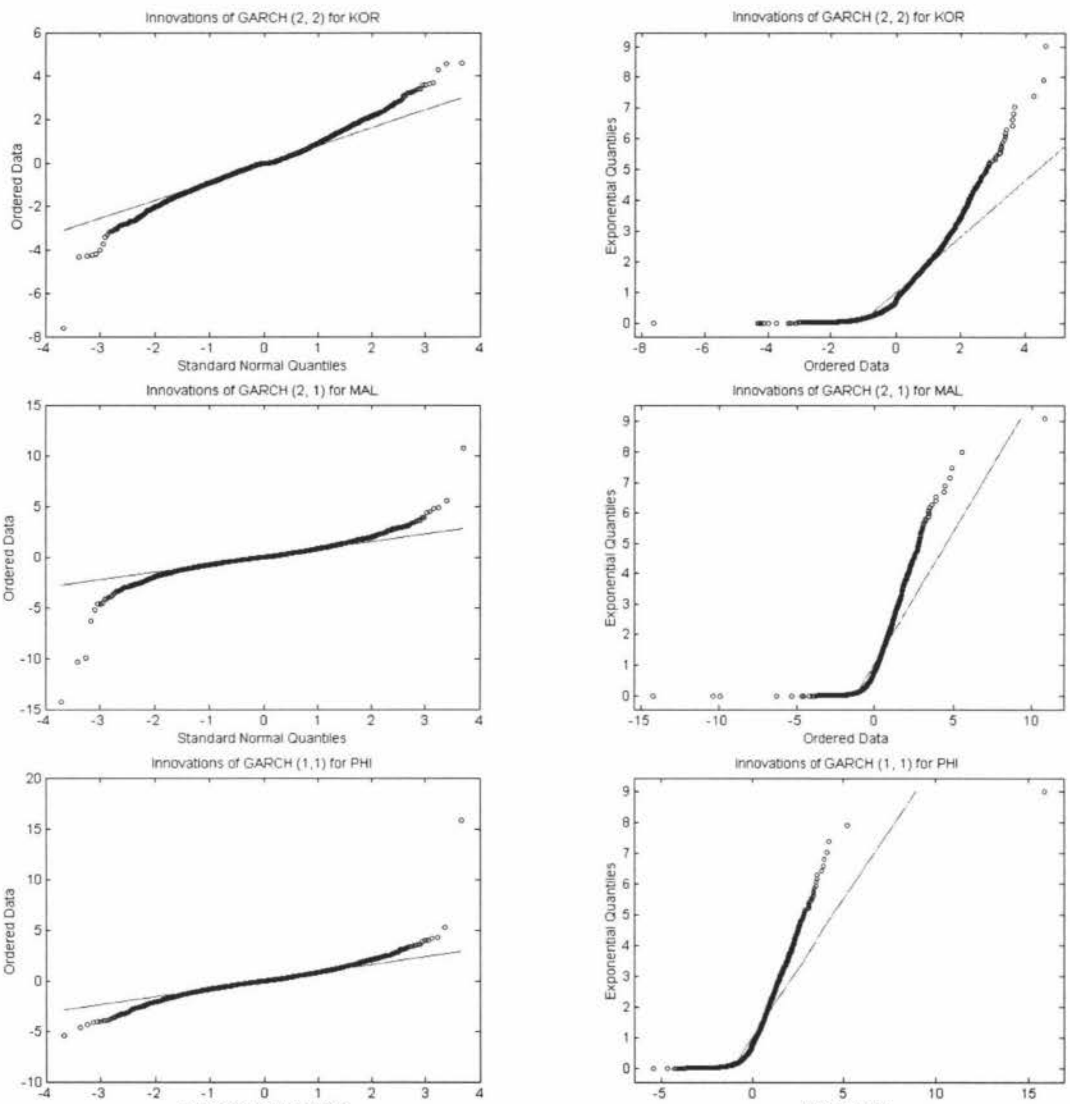

Figure 16(2). QQ Plots of Innovations of AR (1)-GARCH (*m*, *s*) Model

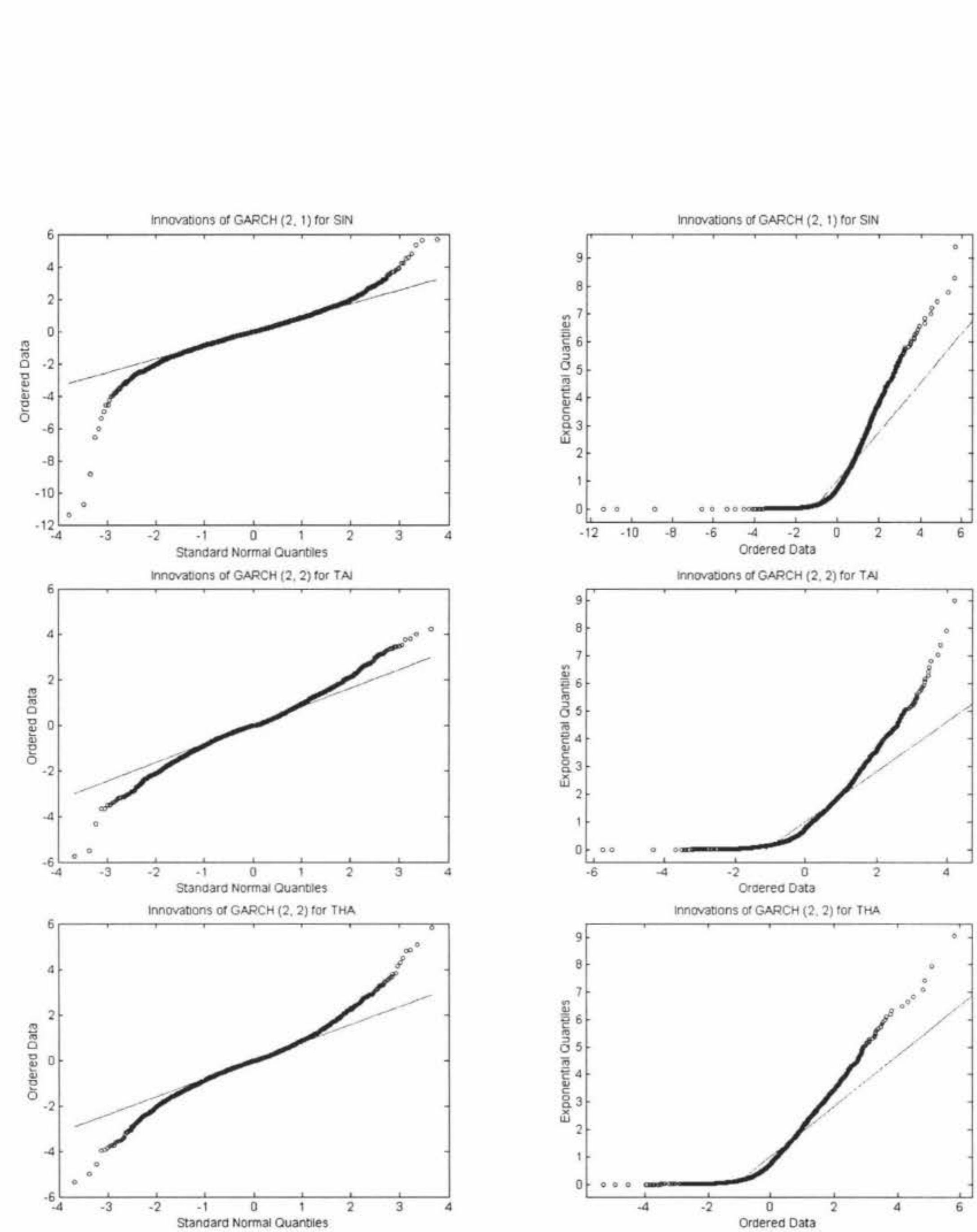

Figure 16(3). QQ Plots of Innovations of AR (1)-GARCH (m, s) Model

5.2 The GEV with AR(1)-GARCH(m, s) Filtering

5.2.1 Determining the Block Length of the GEV

The block length is the number of trading days contained in each identical, non-overlapping sub-sample from which the maximum (or minimum) of returns is collected. So the block length varies inversely with the number of maxima (or minima).[4] In application, the choice of the block length may face a trade-off between bias and inefficiency and so is often guided by practical considerations. Longin (2000) studied the S&P 500 Index returns and chose 1 week (5 days), 1 month (21, days), 1 quarter (63 days) and 1 semester (125 days) as four different block lengths to compare the results. Our sample sizes are unfortunately shorter, and so we choose three block lengths of 1 week (5 days), 2 weeks (10 days) and 1 month (21 days) respectively for the GEV approach.

5.2.2 Empirical Results of the GEV

The parameters estimated by the maximum likelihood method are reported in Table 5 with standard deviations given in parentheses. One can see that as the block length increases, the value of log-likelihood decreases. More importantly, in both panel A and panel B of markets CHI, IDS, MAL, PHI, SIN and THA, an inspection of the estimates of the shape parameter ξ associated with small standard errors reveals that the values of ξ are all greater than zero implying heavy-tailed behavior. This is consistent with our findings about leptokurtosis of these markets in Table 4. Meanwhile, the heavy tails can also be found in the left tails (panel B) of markets HON and KOR.

However, the results of the GEV model in Table 5 seem to convey ambiguous messages. Two points are worth noting here. First, the six markets HON, IDS, KOR, MAL, SIN and TAI show that the estimated shape parameter ξ for local minima is greater than that for local maxima regardless of the block length. This indicates that large losses are more likely to occur than positive gains of a similar magnitude in these markets. The similar results also apply to PHI for the 5-day and 10-day block lengths, to THA for the 10-day

[4] Some studies use the terms lower tail and upper tail or left tail and right tail to describe extreme returns. On the consideration that we study index returns, we prefer to use the terms maxima and minima hereafter.

and 21-day block lengths. Accordingly, the comparisons of ξ between local maxima and minima for markets KOR, PHI, and THA yield results that seem to contradict to their positive skewness as reported in Table 4. Meanwhile, it is also worth to note the uncertainty of shape parameter estimates along with the change of block length. For instance, for the local maxima, the shape parameter ξ exhibits negative value only in 5-day block length of SIN, and only in 21-day block length of THA with unknown reasons. Second, the variance of maxima or minima (related to the scale parameters σ) should increase as the block length rises (corresponding to fewer exceedances). Although the scale parameter σ of each filtered series is generally around 0.7% and less than its standard deviation as shown in Table 4, the values of σ suddenly declining for unknown reasons are shown in KOR and SIN for both local maxima and local minima, in HON for local maxima, and in THA for local minima. Likewise, the declined values of σ can be found in 10-day block length of PHI for both tails, in 21-day block length of TAI for local minima, and in 10-day block length of MAL for local maxima.

The estimated location parameter μ rises when the larger block length is chosen, demonstrating that the fewer the exceedances, the higher values the means will have for maximal and minimal returns.

Table 6 also reports the parameters of the GEV model estimated without using the AR(1)-GARCH(m, s) filters. The similar symptoms to those in Table 5 are also observed here. Referring back to Section 3.6.3, the shape parameter ξ, the scale parameter σ and the location parameter μ are the three key elements for the VaR calculation based on the GEV. Since ξ and σ estimated by the method of the GEV vary irregularly following the changes in the block length, the results are equivocal and thus less informative for financial decision.

Table 5. Maximum Likelihood Estimated Parameters of GEV with AR (1)-GARCH (m, s) Filtering

Market	Model (m, s)	N_{Block}	Panel A: Maxima					Panel B: Minima				
			No. of Exceedance	ξ	$\sigma\times100$	$\mu\times100$	Loglik	No. of Exceedance	ξ	$\sigma\times100$	$\mu\times100$	Loglik
CHI	(2, 2)	5	601	0.1434	0.6046	0.5838	693.1272	601	0.0902	0.5934	-0.6153	662.9250
				(0.0255)	(0.0206)	(0.0271)			(0.0222)	(0.0194)	(0.0263)	
		10	301	0.2194	0.6359	0.9858	376.3361	301	0.1305	0.6479	-0.9810	365.3687
				(0.0426)	(0.0326)	(0.0409)			(0.0394)	(0.0316)	(0.0416)	
		21	143	0.2978	0.7221	1.4273	203.4866	143	0.1674	0.6800	-1.4753	183.5951
				(0.0729)	(0.0578)	(0.0687)			(0.0550)	(0.0484)	(0.0629)	
HON	(2, 2)	5	1201	-0.0677	0.5748	0.7521	1183.5	1201	0.1073	0.6126	-0.7273	1375.7
				(0.0195)	(0.0133)	(0.0185)			(0.0200)	(0.0149)	(0.0197)	
		10	601	-0.0959	0.5576	1.1743	560.4861	601	0.1286	0.6269	-1.1465	709.5840
				(0.0214)	(0.0174)	(0.0248)			(0.0281)	(0.0216)	(0.0285)	
		21	286	-0.0692	0.4953	1.5613	236.8848	286	0.1881	0.6752	-1.6137	368.9375
				(0.0323)	(0.0224)	(0.0321)			(0.0438)	(0.0350)	(0.0448)	
IDS	(1, 1)	5	1039	0.0581	0.5896	0.5721	1119.9	1039	0.0870	0.6245	-0.5493	1189.6
				(0.0204)	(0.0150)	(0.0204)			(0.0160)	(0.0155)	(0.0211)	
		10	520	0.0491	0.6274	0.9696	587.2689	520	0.1209	0.6627	-0.9462	635.4110
				(0.0242)	(0.0216)	(0.0301)			(0.0249)	(0.0237)	(0.0319)	
		21	248	0.0742	0.6769	1.3804	303.3026	248	0.1343	0.7470	-1.3650	334.0518
				(0.0342)	(0.0338)	(0.0468)			(0.0307)	(0.0377)	(0.0513)	

KOR	(2, 2)	5	814	-0.0713	0.6872	0.7903	946.2869	814	-0.0472	0.6520	-0.7676	900.5650
				(0.0241)	(0.0192)	(0.0269)			(0.0144)	(0.0171)	(0.0246)	
		10	407	-0.0787	0.6467	1.2702	445.7865	407	0.0158	0.5787	-1.1649	420.2721
				(0.0323)	(0.0253)	(0.0357)			(0.0281)	(0.0225)	(0.0315)	
		21	194	-0.0796	0.6003	1.7623	197.5720	194	0.0612	0.5731	-1.5421	204.0930
				(0.0439)	(0.0333)	(0.0476)			(0.0452)	(0.0333)	(0.0455)	
MAL	(2, 1)	5	895	0.0229	0.6124	0.6992	976.9464	895	0.1178	0.5524	-0.6512	936.0695
				(0.0175)	(0.0160)	(0.0223)			(0.0206)	(0.0152)	(0.0203)	
		10	448	0.0757	0.6015	1.0940	496.3335	448	0.1375	0.5926	-1.0116	504.9491
				(0.0315)	(0.0235)	(0.0317)			(0.0271)	(0.0230)	(0.0306)	
		21	214	0.0093	0.7323	1.5405	266.9078	214	0.1972	0.6516	-1.4019	269.8339
				(0.0257)	(0.0367)	(0.0536)			(0.0425)	(0.0380)	(0.0488)	
PHI	(1, 1)	5	807	0.0315	0.6367	0.6912	911.8416	807	0.0329	0.6019	-0.7223	878.0417
				(0.0159)	(0.0173)	(0.0242)			(0.0234)	(0.0171)	(0.0235)	
		10	404	0.0870	0.6244	1.0891	463.9355	404	0.0940	0.5864	-1.0950	444.2573
				(0.0308)	(0.0256)	(0.0344)			(0.0392)	(0.0249)	(0.0330)	
		21	193	0.0687	0.6846	1.5499	234.5498	193	0.0544	0.6586	-1.5157	230.4303
				(0.0311)	(0.0379)	(0.0532)			(0.0582)	(0.0402)	(0.0539)	
SIN	(2, 1)	5	1201	-0.0115	0.6056	0.7497	1277.4	1201	0.0569	0.5900	-0.7359	1287.4
				(0.0166)	(0.0136)	(0.0192)			(0.0160)	(0.0135)	(0.0187)	
		10	601	0.0124	0.6044	1.1421	647.4810	601	0.1146	0.5715	-1.1098	647.2060
				(0.0255)	(0.0196)	(0.0273)			(0.0248)	(0.0191)	(0.0257)	
		21	286	0.0914	0.5735	1.5565	307.3607	286	0.2211	0.5697	-1.4862	326.5769
				(0.0442)	(0.0286)	(0.0382)			(0.0420)	(0.0299)	(0.0374)	
TAI	(2, 2)	5	807	-0.0938	0.6761	0.8027	913.1924	807	-0.0371	0.6443	-0.7658	897.9587
				(0.0223)	(0.0185)	(0.0263)			(0.0192)	(0.0173)	(0.0248)	
		10	404	-0.0837	0.6285	1.2572	430.6944	404	-0.0146	0.6456	-1.1468	456.6716
				(0.0339)	(0.0246)	(0.0349)			(0.0333)	(0.0258)	(0.0358)	
		21	193	-0.0584	0.5610	1.7255	187.2004	193	-0.0295	0.6425	-1.6521	214.8018
				(0.0543)	(0.0327)	(0.0455)			(0.0435)	(0.0363)	(0.0511)	

		5	843	0.0024	0.6657	0.7408	987.3929	843	0.0009	0.6019	-0.7433	900.0248
				(0.0222)	(0.0182)	(0.0254)			(0.0217)	(0.0165)	(0.0230)	
THA	(2, 2)	10	422	0.0174	0.6757	1.1797	505.0107	422	0.0481	0.5875	-1.0992	452.9367
				(0.0367)	(0.0273)	(0.0371)			(0.0343)	(0.0234)	(0.0320)	
		21	201	-0.0329	0.7158	1.7143	245.7049	201	0.0961	0.5864	-1.5272	221.3728
				(0.0465)	(0.0401)	(0.0562)			(0.0581)	(0.0357)	(0.0471)	

Note: 1. Parameters ξ, σ and μ represent shape parameter, scale parameter and location parameter respectively.

2. Loglik indicates log-likelihood.

3. Standard deviations are reported in parentheses.

Table 6. Maximum Likelihood Estimated Parameters of GEV without AR (1)-GARCH (m, s) filtering

Market	N_{Block}	Panel A: Maxima					Panel B: Minima				
		No. of Exceedance	ξ	$\sigma \times 100$	$\mu \times 100$	Loglik	No. of Exceedance	ξ	$\sigma \times 100$	$\mu \times 100$	Loglik
CHI	5	601	0.3166	1.3573	1.1556	1243.70	601	0.1161	1.4949	-1.2511	1227.90
			(0.0312)	(0.0524)	(0.0614)			(0.0183)	(0.0478)	(0.0652)	
	10	301	0.4572	1.4836	1.8483	672.08	301	0.3115	1.4606	-1.8795	643.14
			(0.0580)	(0.0921)	(0.0984)			(0.0483)	(0.0805)	(0.0950)	
	21	143	0.6173	1.7307	2.5686	352.47	143	0.4125	1.7607	-2.7069	340.04
			(0.1119)	(0.1808)	(0.1770)			(0.0859)	(0.1551)	(0.1713)	
HON	5	1201	0.1021	0.9215	1.1407	1864.90	1201	0.2116	0.9506	-0.9101	1978.40
			(0.0202)	(0.0223)	(0.0297)			(0.0213)	(0.0244)	(0.0307)	
	10	601	0.1219	0.9634	1.7451	967.21	601	0.2520	0.9990	-1.4857	1033.90
			(0.0273)	(0.0329)	(0.0435)			(0.0316)	(0.0372)	(0.0457)	
	21	286	0.2133	0.9510	2.2594	471.94	286	0.3242	1.1225	-2.0437	537.73
			(0.0480)	(0.0509)	(0.0636)			(0.0486)	(0.0636)	(0.0746)	
IDS	5	1039	0.2433	1.1050	0.8680	1887.50	1039	0.0684	1.4644	-0.9532	2051.70
			(0.0219)	(0.0307)	(0.0380)			(0.0099)	(0.0330)	(0.0480)	
	10	520	0.2715	1.2541	1.5080	1020.80	520	0.2981	1.2587	-1.4755	1029.30
			(0.0295)	(0.0494)	(0.0604)			(0.0339)	(0.0514)	(0.0617)	
	21	248	0.3551	1.4414	2.1278	535.10	248	0.3078	1.4982	-2.1260	536.89
			(0.0445)	(0.0874)	(0.0999)			(0.0430)	(0.0874)	(0.1041)	
KOR	5	814	0.1121	1.2630	1.2776	1528.40	814	0.0372	1.2625	-1.2485	1488.80
			(0.0280)	(0.0382)	(0.0502)			(0.0200)	(0.0346)	(0.0484)	
	10	407	0.1234	1.2964	2.0688	777.51	407	0.1736	1.1465	-1.8726	739.03
			(0.0441)	(0.0573)	(0.0741)			(0.0390)	(0.0501)	(0.0641)	

	21	194	0.0953	1.3825	2.8690	379.90	194	0.2016	1.2886	-2.4819	378.10
			(0.0616)	(0.0867)	(0.1140)			(0.0635)	(0.0849)	(0.1061)	
MAL	5	895	0.0864	1.0460	0.9211	1485.80	895	0.2621	0.8094	-0.7244	1358.40
			(0.0138)	(0.0266)	(0.0373)			(0.0242)	(0.0245)	(0.0300)	
	10	448	0.3085	0.9212	1.3850	749.71	448	0.3063	0.9023	-1.1874	742.24
			(0.0390)	(0.0414)	(0.0490)			(0.0325)	(0.0394)	(0.0467)	
	21	214	0.2453	1.1816	1.9491	404.70	214	0.4199	0.9847	-1.6253	387.31
			(0.0403)	(0.0705)	(0.0874)			(0.0559)	(0.0683)	(0.0748)	
PHI	5	807	0.0298	1.1761	1.0652	1406.40	807	0.0901	1.0340	-1.0064	1342.20
			(0.0138)	(0.0307)	(0.0442)			(0.0215)	(0.0295)	(0.0399)	
	10	404	0.1975	1.0855	1.5972	715.84	404	0.2176	1.0051	-1.5294	690.52
			(0.0404)	(0.0486)	(0.0613)			(0.0442)	(0.0463)	(0.0574)	
	21	193	0.1766	1.3151	2.2602	376.67	193	0.2243	1.1451	-2.0845	355.68
			(0.0503)	(0.0817)	(0.1052)			(0.0734)	(0.0793)	(0.0972)	
SIN	5	1201	0.0613	0.7880	0.8595	1643.30	1201	0.0766	0.7789	-0.7635	1635.10
			(0.0154)	(0.0178)	(0.0247)			(0.0125)	(0.0173)	(0.0241)	
	10	601	0.1432	0.7869	1.2851	852.70	601	0.2174	0.7200	-1.1475	825.38
			(0.0288)	(0.0274)	(0.0358)			(0.0297)	(0.0260)	(0.0327)	
	21	286	0.2048	0.8305	1.7551	431.68	286	0.3273	0.7757	-1.5398	433.42
			(0.0494)	(0.0446)	(0.0559)			(0.0453)	(0.0437)	(0.0509)	
TAI	5	807	-0.0641	1.5355	1.6119	1583.10	807	-0.0361	1.4577	-1.4648	1556.80
			(0.0157)	(0.0397)	(0.0582)			(0.0158)	(0.0376)	(0.0551)	
	10	404	0.0149	1.4021	2.4462	777.46	404	0.0906	1.3689	-2.1284	786.47
			(0.0362)	(0.0574)	(0.0785)			(0.0404)	(0.0586)	(0.0774)	
	21	193	0.0188	1.3985	3.3552	371.25	193	0.0278	1.4642	-3.0558	381.45
			(0.0527)	(0.0833)	(0.1134)			(0.0559)	(0.0881)	(0.1195)	

THA	5	843	0.0315	1.4127	1.2693	1628.90	843	0.1155	1.1876	-1.1329	1531.10
			(0.0145)	(0.0362)	(0.0520)			(0.0219)	(0.0337)	(0.0450)	
	10	422	0.1846	1.3310	1.9349	831.53	422	0.2532	1.1435	1.6826	784.43
			(0.0364)	(0.0569)	(0.0725)			(0.0422)	(0.0520)	(0.0634)	
	21	201	0.1856	1.4938	2.7718	419.44	201	0.3492	1.1966	2.3173	393.64
			(0.0569)	(0.0939)	(0.1192)			(0.0656)	(0.0845)	(0.0965)	

Note: 1. Parameters ξ, σ and μ represent shape parameter, scale parameter and location parameter respectively.

2. Loglik indicates log-likelihood.

3. Standard deviations are reported in parentheses.

5.3 The GPD with AR(1)-GARCH(m, s) Filtering

5.3.1 Searching the Threshold for the GPD

We first draw mean excess plots for both the maximal and minimal returns in Figure 17. Smith (2000) proposes that an apparent 'kink' near a low value (threshold) is the method to detect the threshold through the mean excess plot. However, all lines are significantly kinked at a high threshold. This is hard to interpret, because for a high threshold, there are only few exceedances leading to very high variability in the mean. Despite such a drawback of the mean excess plot, however, it can still tell us that "if the empirical plot seems to follow a reasonably straight line with positive gradient above a certain value of u, then this is an indication that the excesses over this threshold follow a generalized Pareto distribution with a positive shape parameter." (McNeil and Saladin (1997)). Accordingly, we can roughly say at this stage that, except KOR, TAI and the right tail of HON, all the maximal and minimal innovations of each index are subject to heavy-tailed distribution.

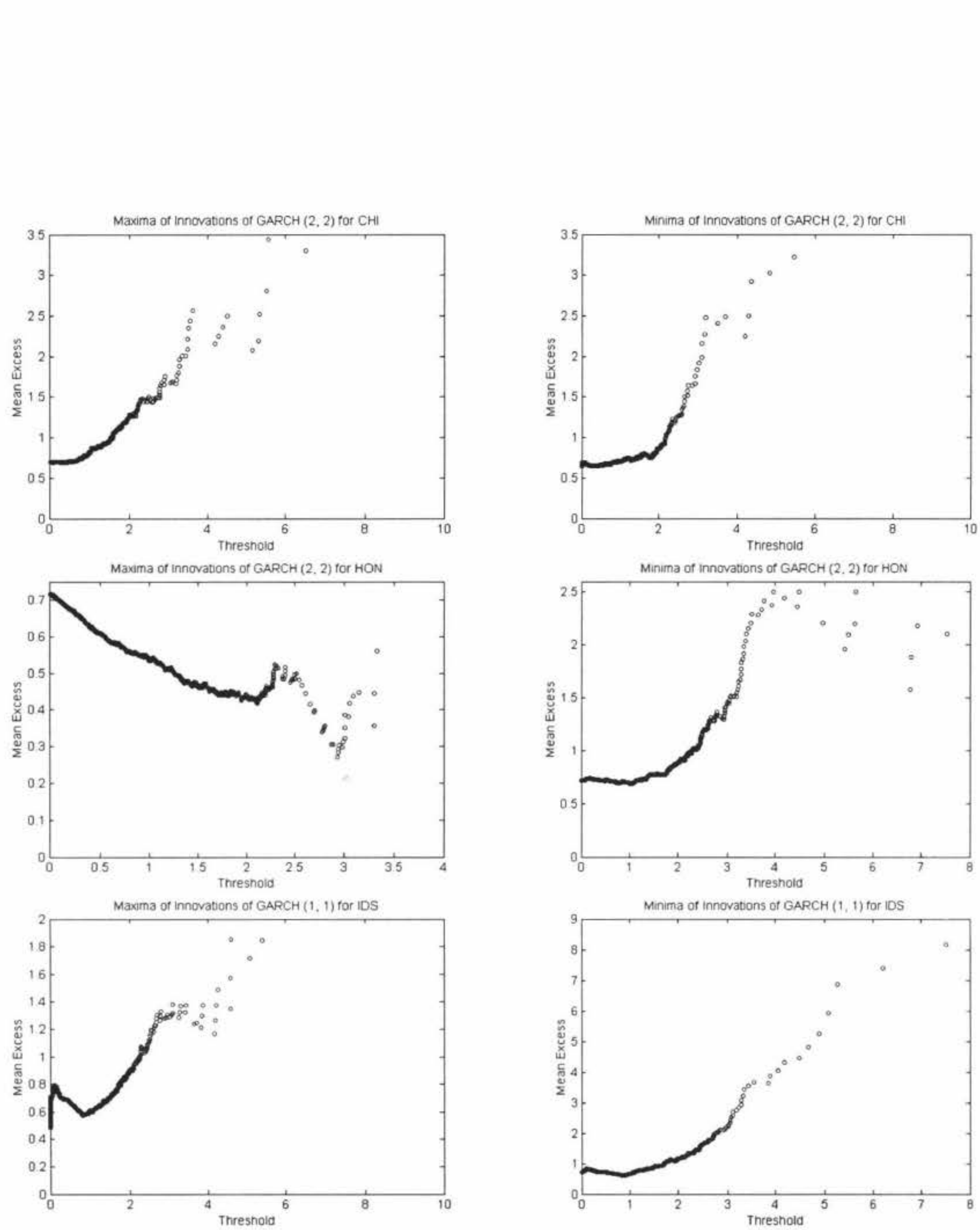

Figure 17(1). Mean Excess Plots of Innovations of AR (1)-GARCH (m, s) Model

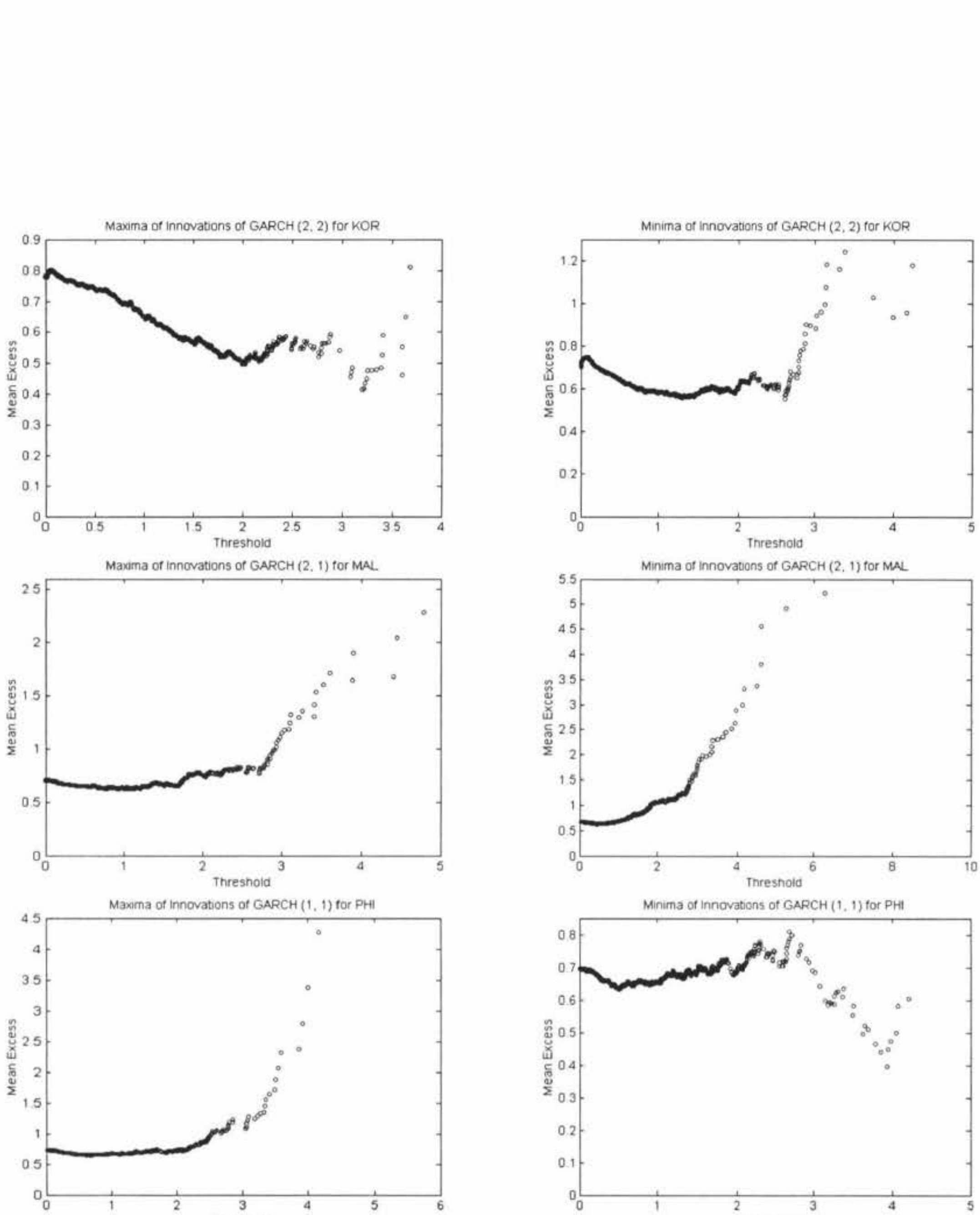

Figure 17(2). Mean Excess Plots of Innovations of AR (1)-GARCH (*m*, *s*) Model

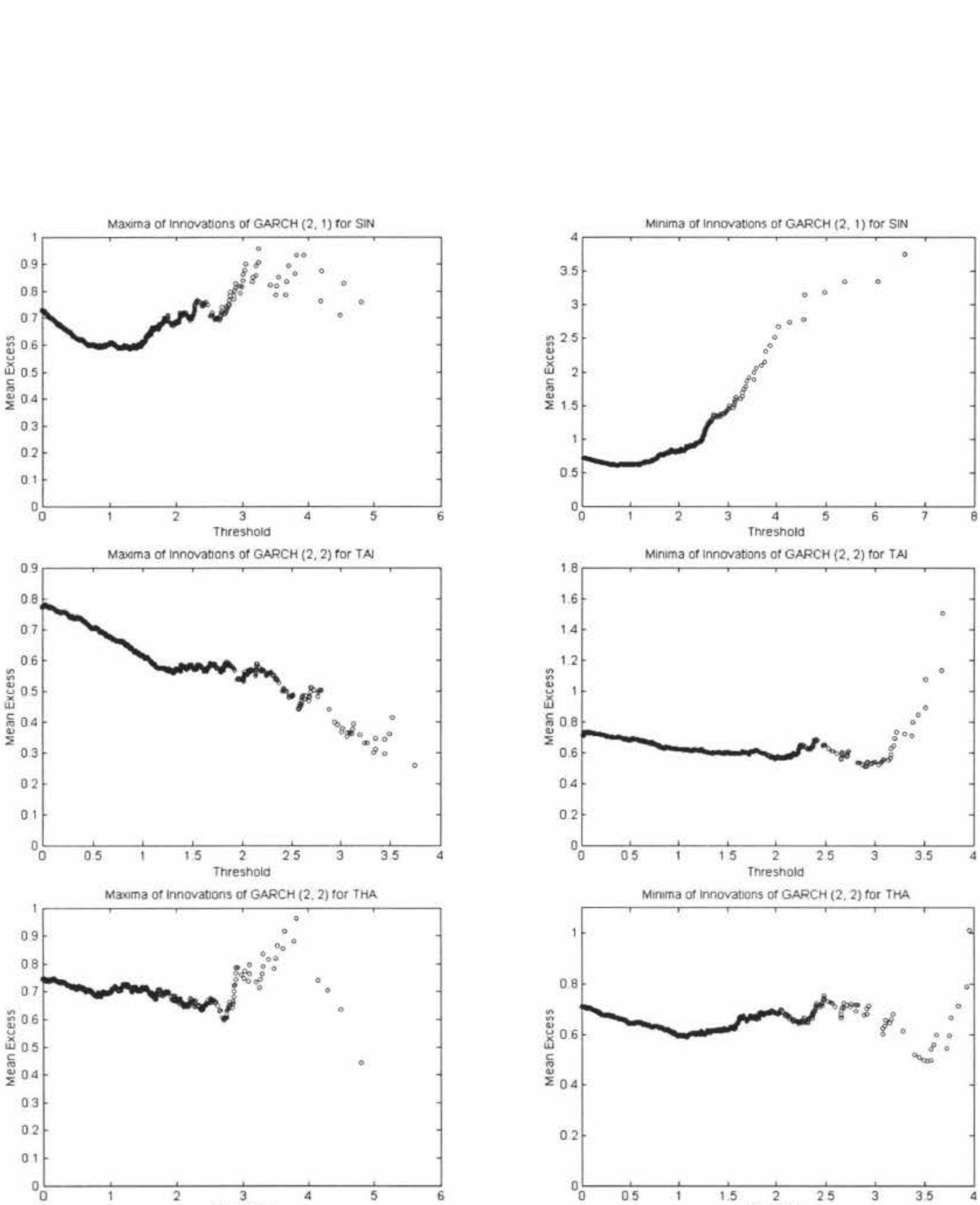

Figure 17(3). Mean Excess Plots of Innovations of AR (1)-GARCH (*m*, *s*) Model

We now turn to the plot of MLE of ξ as a function of exceedances with the asymptotic 95% confidence band in each first row of Fig. 18. From the plots of MLE of both the maximal and minimal innovations for each index return series, we can see that the MLE curve has a larger confidence band at lower values of exceedances (corresponding to higher thresholds). As the number of exceedances increases, the MLE curve becomes stable. Then the value of threshold is determined such that it corresponds the number of exceedances staring from which the MLE curve becomes stable, and Table 7 reports the threshold chosen in this way for each market under investigation. The diagnostic plots of the excess distribution and of the tail of its underlying distribution are presented in the second and third rows of Fig. 18. It can be seen that the empirical curve keeps close to the theoretical curve up to the number of exceedances corresponding to the chosen value of threshold, implying the robustness of the determined threshold.

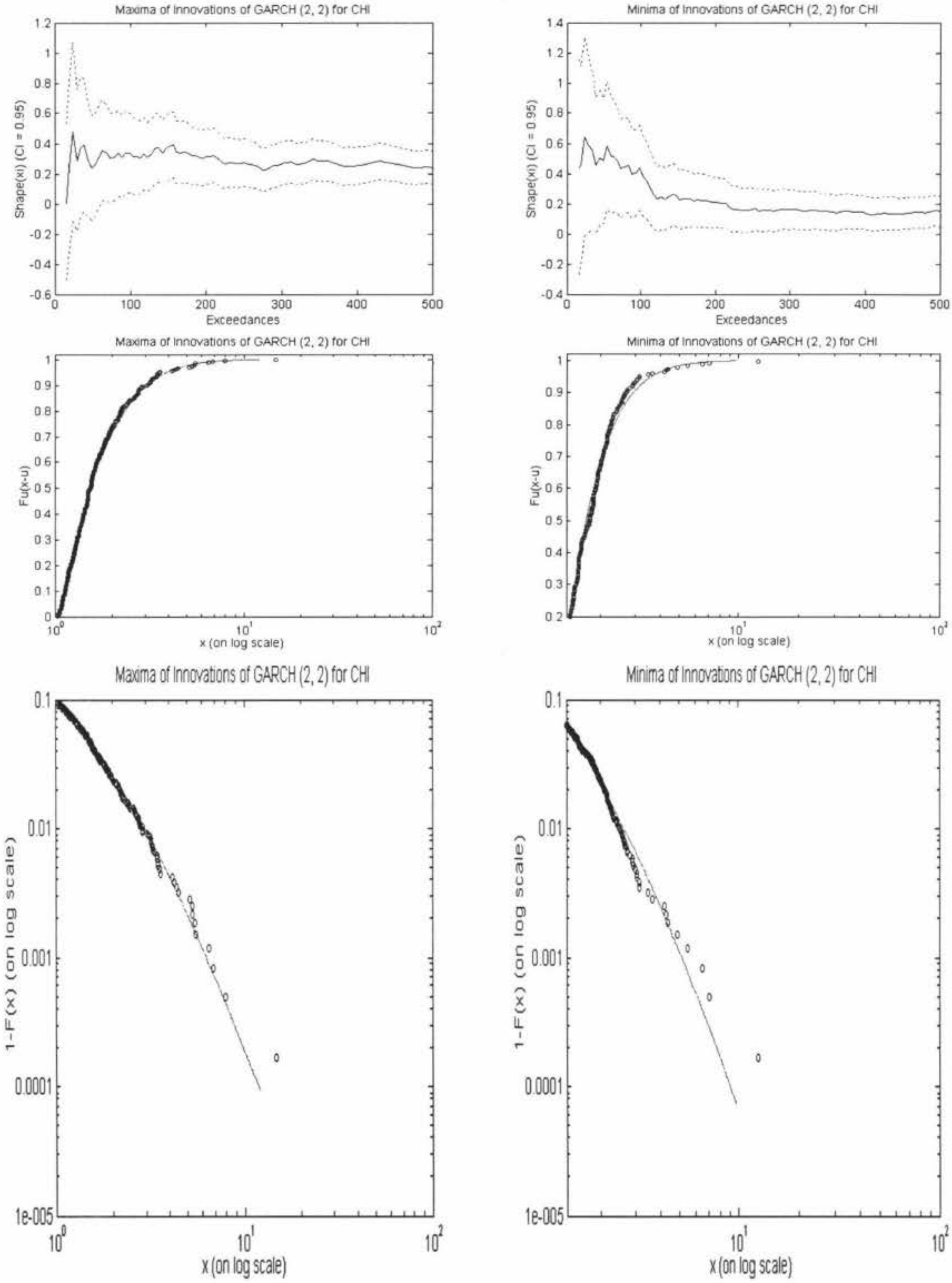

Figure 18(1)

First row: MLE of shape parameters ξ of the GPD vary with number of exceedances.

Second row: Fitted excess distributions of the GPD.

Third row: The underlying distributions of the GPD tails.

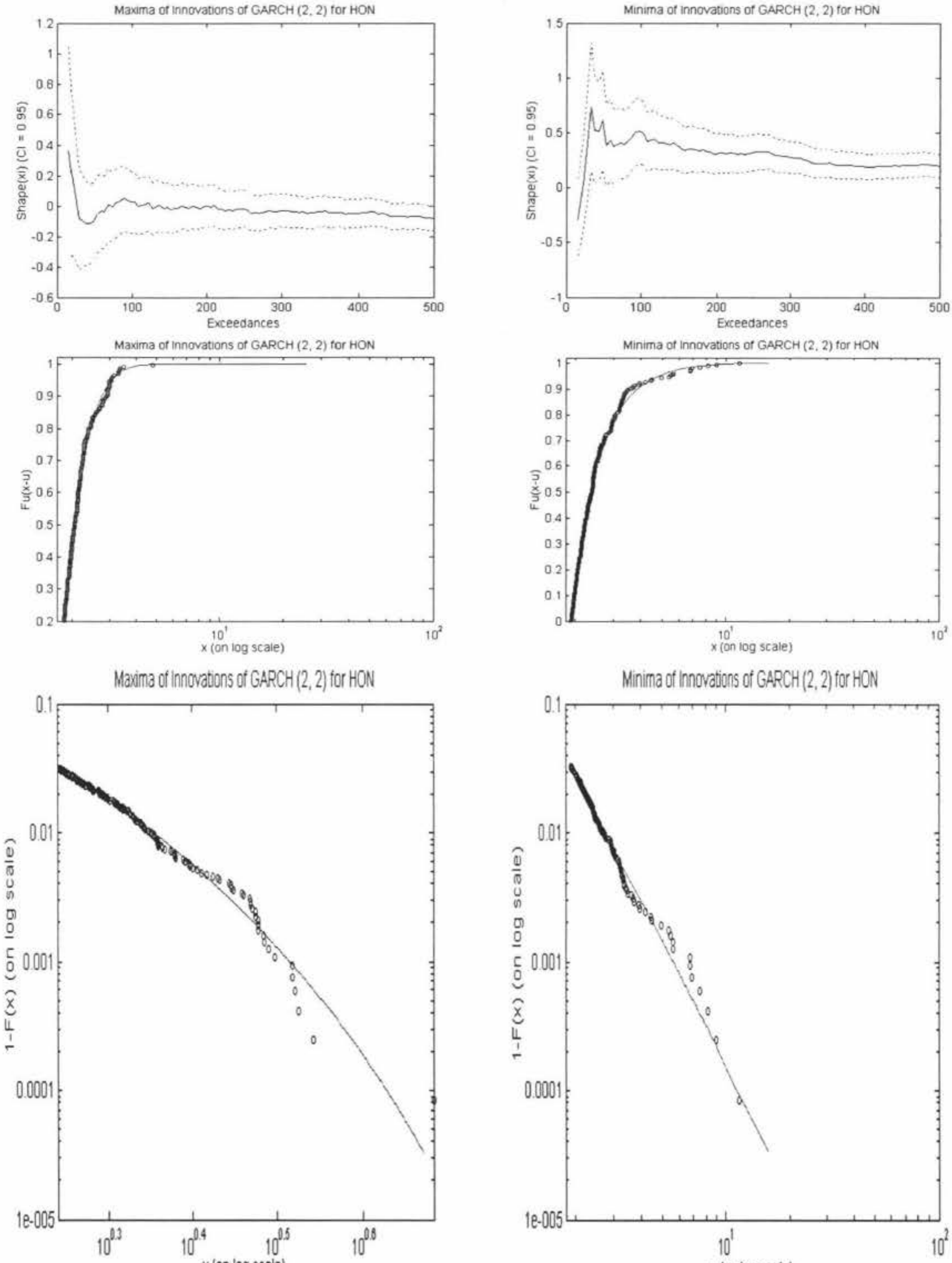

Figure 18(2)

First row: MLE of shape parameters ξ of the GPD vary with number of exceedances.

Second row: Fitted excess distributions of the GPD.

Third row: The underlying distributions of the GPD tails.

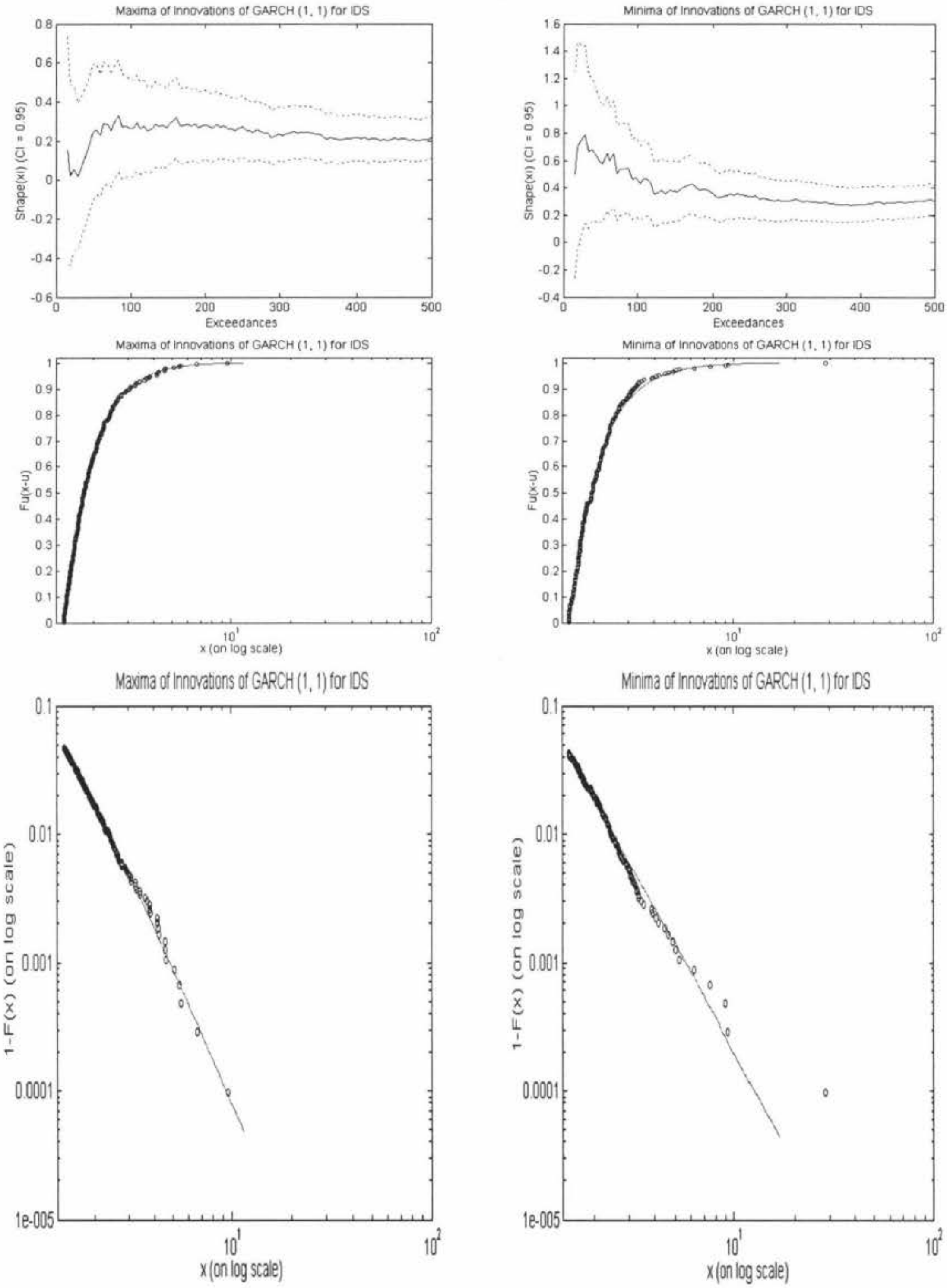

Figure 18(3)

First row: MLE of shape parameters ξ of the GPD vary with number of exceedances.

Second row: Fitted excess distributions of the GPD.

Third row: The underlying distributions of the GPD tails.

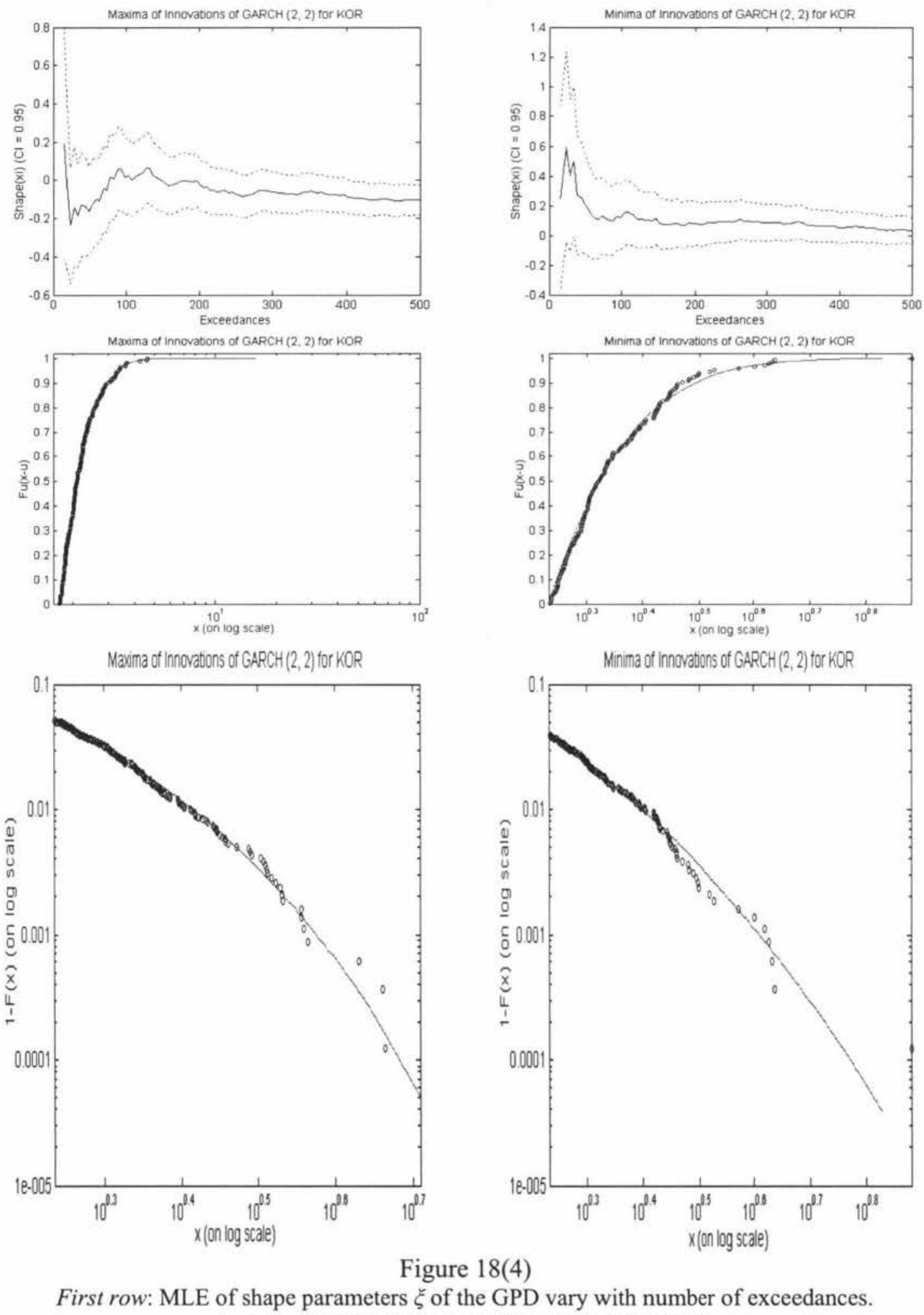

Figure 18(4)

First row: MLE of shape parameters ξ of the GPD vary with number of exceedances.

Second row: Fitted excess distributions of the GPD.

Third row: The underlying distributions of the GPD tails.

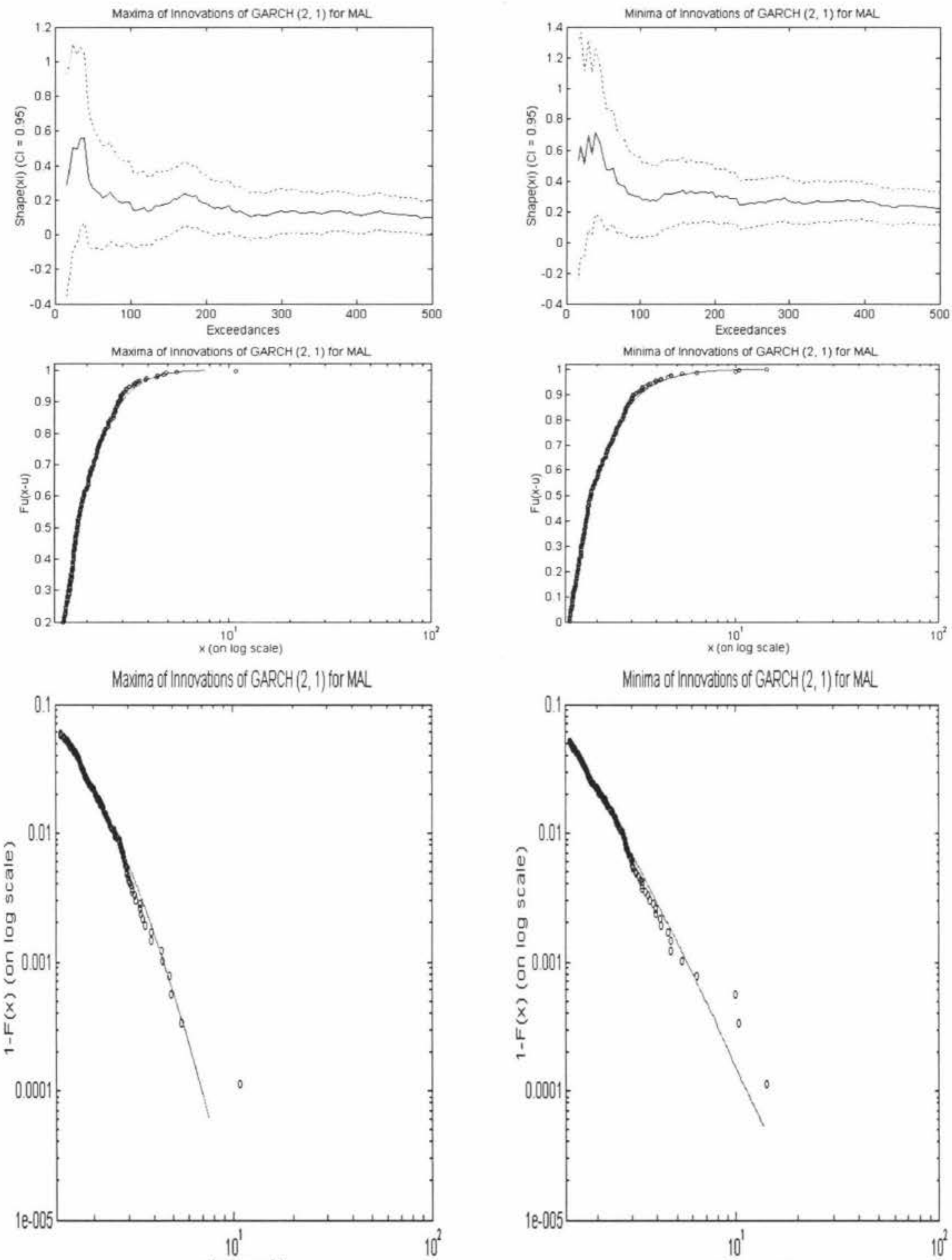

Figure 18(5)

First row: MLE of shape parameters ξ of the GPD vary with number of exceedances.

Second row: Fitted excess distributions of the GPD.

Third row: The underlying distributions of the GPD tails.

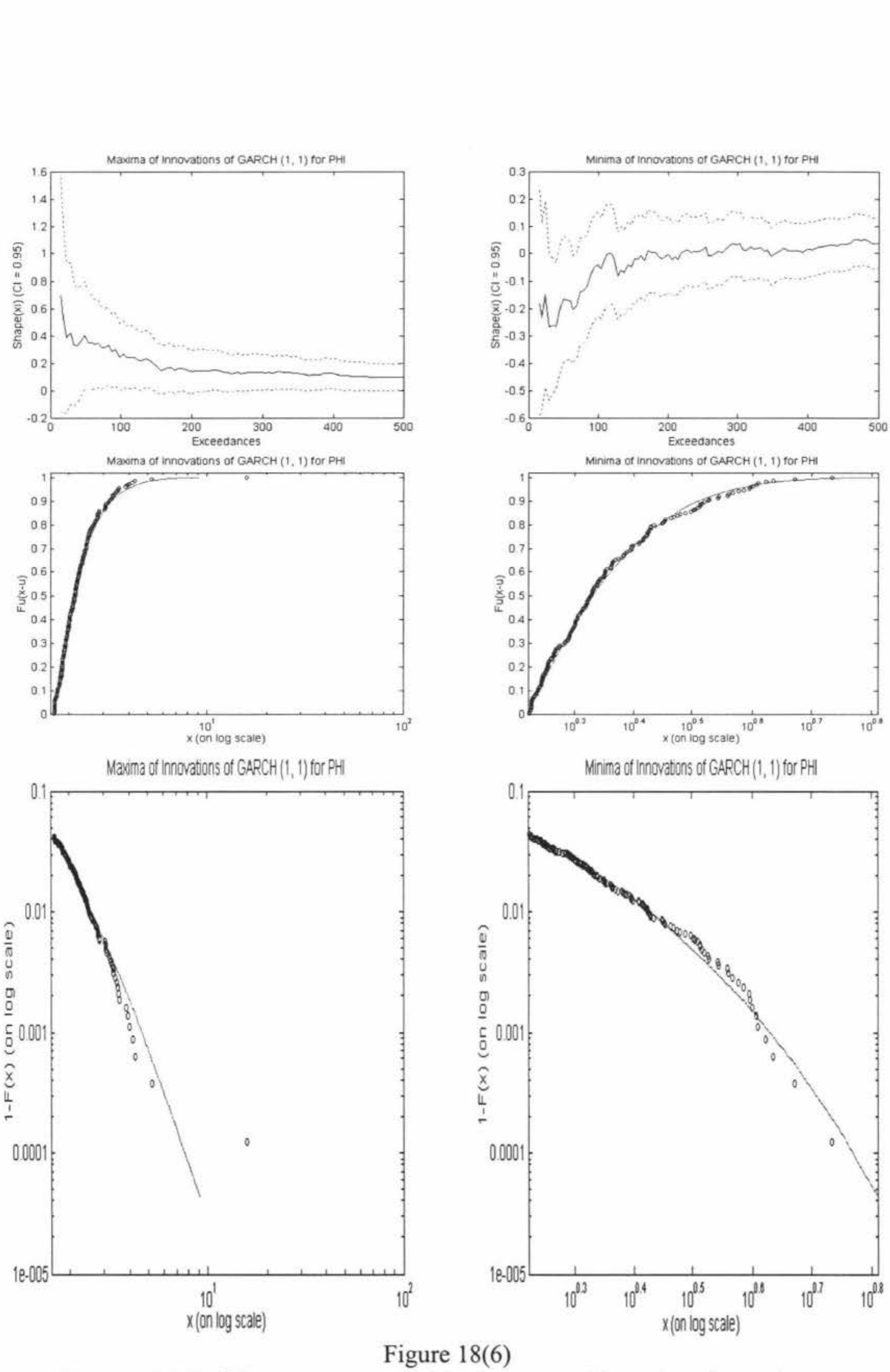

Figure 18(6)

First row: MLE of shape parameters ξ of the GPD vary with number of exceedances.

Second row: Fitted excess distributions of the GPD.

Third row: The underlying distributions of the GPD tails.

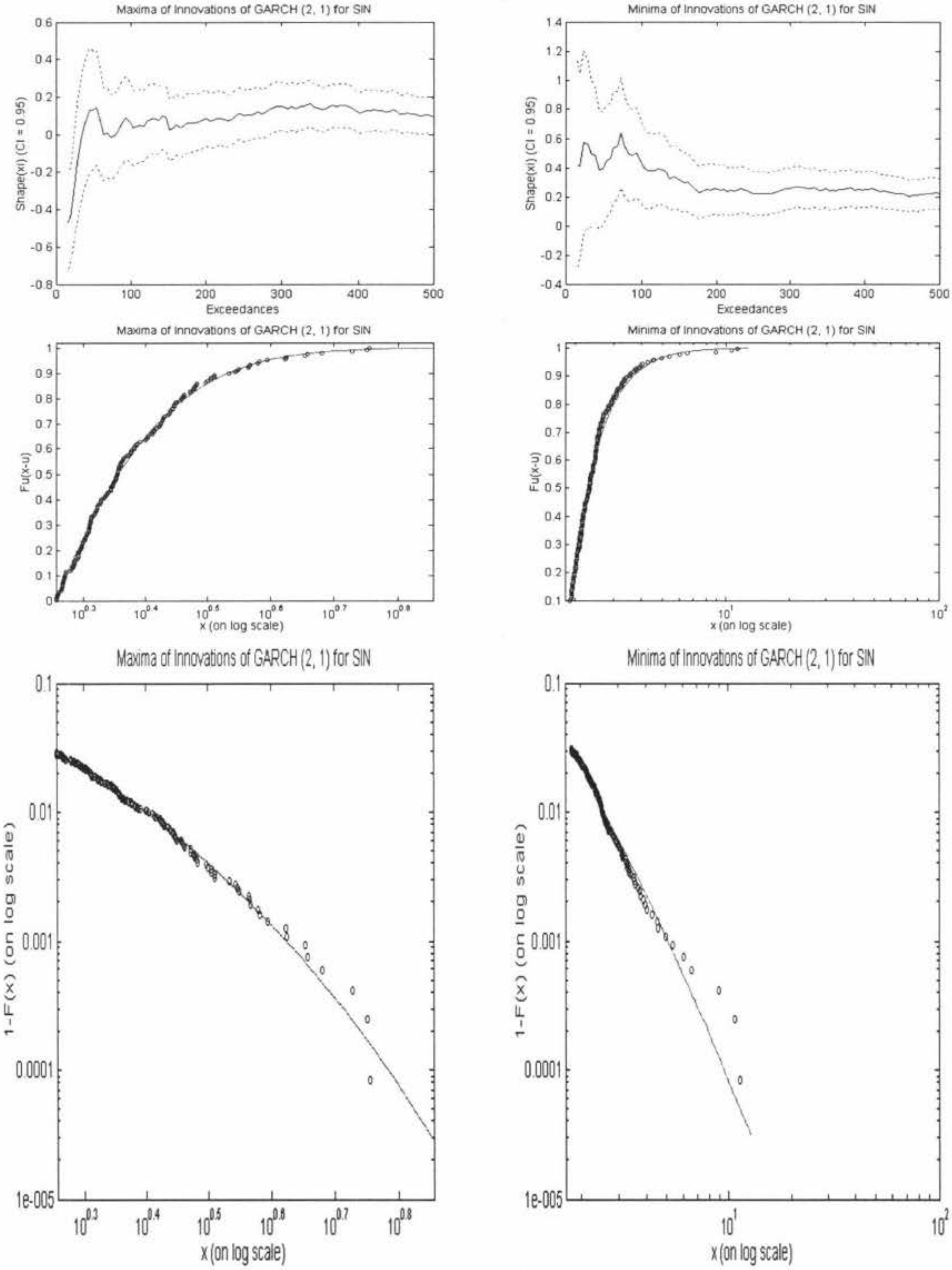

Figure 18(7)

First row: MLE of shape parameters ξ of the GPD vary with number of exceedances.

Second row: Fitted excess distributions of the GPD.

Third row: The underlying distributions of the GPD tails.

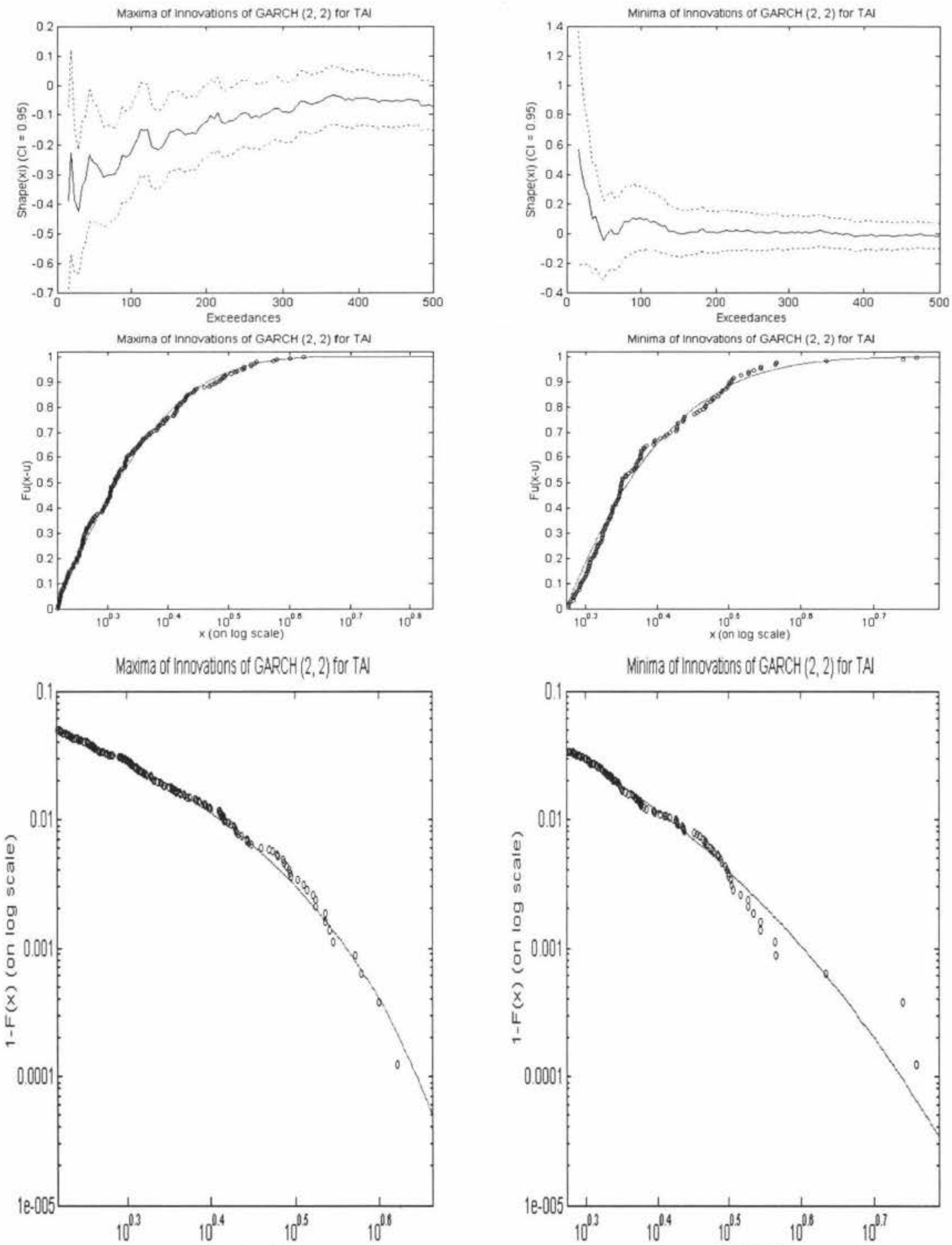

Figure 18(8)

First row: MLE of shape parameters ξ of the GPD vary with number of exceedances.

Second row: Fitted excess distributions of the GPD.

Third row: The underlying distributions of the GPD tails.

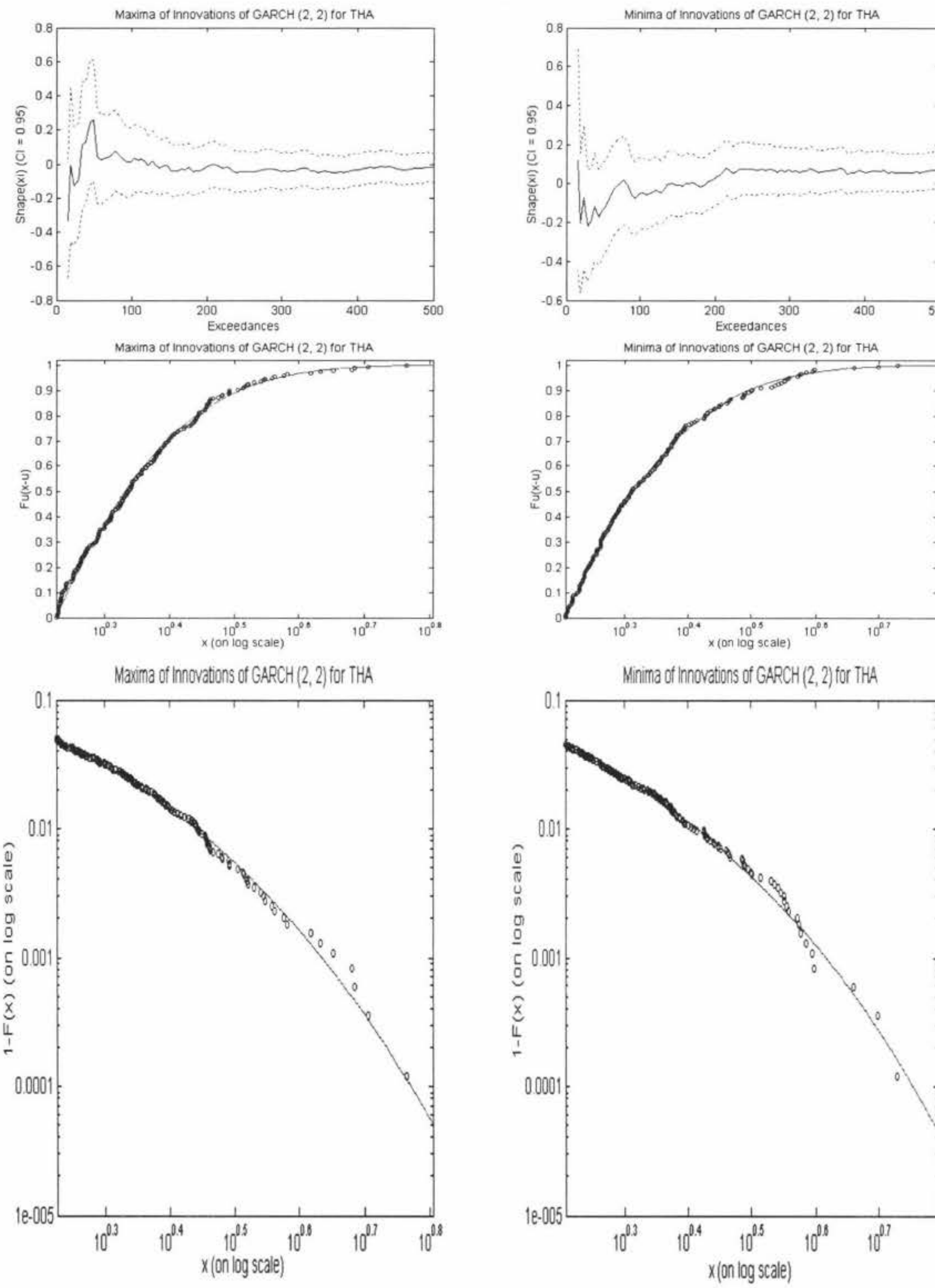

Figure 18(9)

First row: MLE of shape parameters ξ of the GPD vary with number of exceedances.

Second row: Fitted excess distributions of the GPD.

Third row: The underlying distributions of the GPD tails.

5.3.2 Empirical Results of the GPD

In Table 7, the parameters of the GPD with an AR (1)-GARCH (*m*, *s*) filter estimated by the maximum likelihood method are much more unambiguous than those of the GEV. Except local maxima of HON, KOR and TAI, all the shape parameters ξ are positive indicating the thickness in both left and right tails for each series. The range of threshold u for local maxima is between 1.0361 and 1.8078 (in percentage) and for local minima between −1.2750 and −1.9083 (in percentage), corresponding to their numbers of exceedances. Significantly, higher values of the shape parameter ξ exhibit in the local maxima and local minima of markets CHI, IDS and MAL as well as the local minima of markets HON and SIN. These higher values of ξ exactly match their high kurtosis presented in Table 4. Unlike the results of the GEV, the shape parameters ξ of local maxima for PHI and THA are greater than their counterparts of local minima. In the mean time, five other markets HON, IDS, MAL, SIN and TAI show their small values of ξ for local maxima relative to local minima. These are consistent with the information of skewness provided by Table 4. The unusual evidence is found for KOR, in that its higher value of ξ for local minima than for local maxima does not conform to positive skewness as reported in Table 4.

The evidence of lower kurtosis for markets KOR, TAI and THA is also supported here by their lower shape parameters. For KOR and TAI, like market HON, the values of their shape parameters for the local maxima are negative exposing short-tailed distribution. The scale parameter σ changes slightly around an average of 0.59% for the local maxima and of 0.60% for the local minima. Note that figures in parentheses are standard deviations.

The estimated parameters of the GPD without an AR (1)-GARCH (*m*, *s*) filter are given in Table 8. The values of threshold, shape parameter and scale parameter are higher than the case with AR(1)-GARCH(*m*, *s*) filtering corresponding to their counterparts in Table 7. Except KOR, the results of Table 8 exactly coincide with the information provided by Table 2.

Table 7. Maximum Likelihood Estimated Parameters of GPD with AR (1)-GARCH (m, s) Filtering

Market	Model (m, s)	Panel A: Maxima					Panel B: Minima				
		$u\times100$	No. of Exceedance	ξ	$\sigma\times100$	Loglik	$u\times100$	No. of Exceedance	ξ	$\sigma\times100$	Loglik
CHI	(2, 2)	1.0361	274	0.2240 (0.0670)	0.6720 (0.0600)	226.4650	-1.2750	215	0.2020 (0.0706)	0.5679 (0.0552)	136.7604
HON	(2, 2)	1.7397	191	-0.0190 (0.0680)	0.4569 (0.0454)	37.7764	-1.9083	196	0.3047 (0.0865)	0.6009 (0.0661)	155.8789
IDS	(1, 1)	1.4219	245	0.2688 (0.0813)	0.5033 (0.0513)	142.6663	-1.4989	220	0.3444 (0.0818)	0.5436 (0.0562)	161.6799
KOR	(2, 2)	1.7172	206	-0.0401 (0.0673)	0.5643 (0.0546)	79.8616	-1.7051	157	0.0738 (0.0722)	0.5623 (0.0604)	78.1953
MAL	(2, 1)	1.3870	264	0.1121 (0.0576)	0.5990 (0.0503)	158.3149	-1.4538	230	0.2846 (0.0790)	0.5708 (0.0578)	166.4934
PHI	(1, 1)	1.6758	166	0.1671 (0.0691)	0.5764 (0.0594)	102.2750	-1.6680	176	0.0045 (0.0893)	0.6821 (0.0797)	109.4605
SIN	(2, 1)	1.8078	171	0.0467 (0.0825)	0.6597 (0.0742)	107.8504	-1.8164	185	0.2481 (0.0796)	0.6043 (0.0645)	137.7209

TAI	(2, 2)	1.6402	201	-0.1228	0.6436	87.7482	-1.8751	137	0.0137	0.5902	66.6493
				(0.0761)	(0.0666)				(0.0778)	(0.0682)	
THA	(2, 2)	1.6828	209	0.0051	0.6683	125.8307	-1.6071	190	0.0045	0.6568	110.9862
				(0.0709)	(0.0662)				(0.0797)	(0.0708)	

Note: 1. Parameters ξ, σ and u represent shape parameter, scale parameter and threshold respectively.

2. Loglik indicates log-likelihood.

3. Standard deviations are reported in parentheses.

Table 8. Maximum Likelihood Estimated Parameters of GPD without AR (1)-GARCH (m, s) Filtering

Market	Panel A: Maxima					Panel B: Minima				
	$u\times100$	No. of Exceedance	ξ	$\sigma\times100$	Loglik	$u\times100$	No. of Exceedance	ξ	$\sigma\times100$	Loglik
CHI	3.5863	186	0.2400 (0.0959)	2.3714 (0.2832)	391.2579	-2.9563	196	0.2068 (0.0794)	2.0528 (0.2172)	377.4964
HON	2.9729	206	0.2014 (0.0889)	1.0334 (0.1158)	254.2726	-3.0846	210	0.4015 (0.0959)	1.1931 (0.1371)	331.4011
IDS	3.0032	196	0.3411 (0.1042)	2.2875 (0.2824)	435.8801	-3.1449	220	0.4964 (0.1039)	1.6570 (0.1976)	440.3259
KOR	3.7446	166	-0.0293 (0.0886)	1.5879 (0.1871)	237.8947	-3.1615	195	0.0755 (0.0846)	1.4097 (0.1560)	276.6745
MAL	2.1974	250	0.3587 (0.0828)	1.1713 (0.1191)	379.2019	-2.5330	196	0.3916 (0.1020)	1.3048 (0.1583)	324.9017
PHI	2.3442	269	0.2272 (0.0711)	1.1438 (0.1059)	366.2362	-2.7102	210	0.0689 (0.0850)	1.2760 (0.1395)	275.6536
SIN	2.3582	200	0.1743 (0.0780)	0.9124 (0.0954)	216.5100	-2.1055	250	0.3090 (0.0784)	0.8866 (0.0876)	297.1532

TAI	4.4764	132	0.0105 (0.0783)	1.2834 (0.1503)	166.3288	-3.8346	180	-0.1030 (0.0596)	1.6826 (0.1600)	255.1272
THA	3.3180	201	0.1512 (0.0849)	1.6676 (0.1831)	334.1821	-3.4647	186	0.0179 (0.0777)	1.7460 (0.1865)	292.9950

Note: 1. Parameters ξ, σ and u represent shape parameter, scale parameter and threshold respectively.

2. Loglik indicates log-likelihood.

3. Standard deviations are reported in parentheses.

5.4 VaR and Expected Shortfall

Table 9 makes comparisons of the VaR performances under different approaches for each filtered innovations. On the consideration that the 10-day and 21-day block lengths are more plausible than the 5-day block length in terms of the number of exceedance, we only present the results of $\mathrm{VAR}_{GARCH(m,s)}^{GEV}$ obtained with these two block lengths used. Four confidence levels 99.5%, 99%, 97.5% and 95% are chosen to measure risks and rewards. Bold figures indicate that the multiplicative factors, $\mathrm{VaR}_{GARCH(m,s)}^{GPD}/\mathrm{VaR}_{GARCH(m,s)}^{Normal}$ and $\mathrm{VaR}_{GARCH(m,s)}^{GEV}/\mathrm{VaR}_{GARCH(m,s)}^{Normal}$, are greater than 1 where the calculation of the $\mathrm{VaR}_{GARCH(m,s)}^{Normal}$ is based on normally distributed innovations.

Two messages emerge from Table 9. First, for all distributions, the VaR declines as the confidence level decreases. Using the GPD-based VaR as an example for illustrative purpose, a three-dimensional plot of VaR_{cl}^{GPD} in Fig. 19 gives a graphical description of this situation.

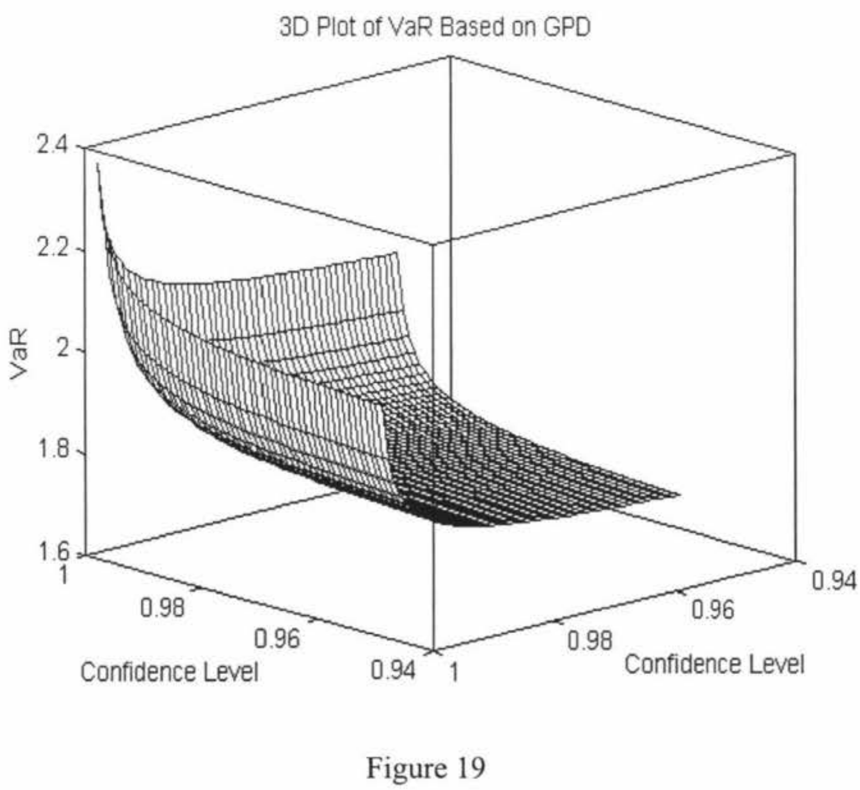

Figure 19

Second, consistently for all nine markets, the multiplicative factors under the GPD and the GEV approaches are greater than 1 at the two high confidence levels, 99.5% and 99%, for both right and left tails (except the local maxima of HON). These results indicate that extreme events can be better captured by the GPD and the GEV than by the normal distribution at high quantiles. In other words, the tail-related risk would be underestimated by the normal distribution at high confidence levels.

Table 9. Comparison of VaR Performance with AR (1)-GARCH (m, s) Filtering

Market	Model	Confidence Level of Maxima				Confidence Level of Minima			
		99.5%	99%	97.5%	95%	99.5%	99%	97.5%	95%
CHI	$GPD_{GARCH(2,2)}$	**4.0561**	**3.2292**	**2.3157**	**1.7391**	**3.2769**	**2.6480**	1.9410	1.4867
		[1.5167]	[1.3370]	[1.1398]	[1.0168]	[1.2253]	[1.0964]	[0.9554]	[0.8692]
	$GEV^{10-day}_{GARCH(2,2)}$	**3.6519**	**2.8391**	1.8995	1.2311	**3.3342**	**2.6781**	1.8599	1.2268
		[1.3656]	[1.1755]	[0.9350]	[0.7198]	[1.2467]	[1.1089]	[0.9155]	[0.7173]
	$GEV^{21-day}_{GARCH(2,2)}$	**3.6702**	**2.7318**	1.6504	0.8960	**3.2832**	**2.5873**	1.6814	0.9889
		[1.3724]	[1.1311]	[0.8124]	[0.5238]	[1.2277]	[1.0713]	[0.8276]	[0.5782]
	$Normal_{GARCH(2,2)}$	2.6743	2.4152	2.0317	1.7103	2.6743	2.4152	2.0317	1.7103
HON	$GPD_{GARCH(2,2)}$	2.5707	2.2629	1.8498	1.5324	**3.4298**	**2.7647**	**2.0757**	**1.6683**
		[0.9971]	[0.9719]	[0.9444]	[0.9294]	[1.3303]	[1.1874]	[1.0597]	[1.0118]
	$GEV^{10-day}_{GARCH(2,2)}$	**2.6161**	2.3037	1.8300	1.3763	**3.4155**	**2.7839**	1.9949	1.3832
		[1.0147]	[0.9894]	[0.9343]	[0.8347]	[1.3248]	[1.1956]	[1.0185]	[0.8389]
	$GEV^{21-day}_{GARCH(2,2)}$	2.5720	2.2429	1.7066	1.2486	**3.4534**	**2.7358**	1.8195	1.1261
		[0.9976]	[0.9633]	[0.8713]	[0.7573]	[1.3394]	[1.1750]	[0.9290]	[0.6829]
	$Normal_{GARCH(2,2)}$	2.5782	2.3284	1.9586	1.6489	2.5782	2.3284	1.9586	1.6489
	$GPD_{GARCH(1,1)}$	**2.9728**	**2.3908**	1.7705	1.3930	**3.2155**	**2.5158**	1.8134	1.4114
		[1.1531]	[1.0269]	[0.9040]	[0.8448]	[1.2472]	[1.0805]	[0.9259]	[0.8560]

IDS	$GEV^{10-day}_{GARCH(1,1)}$	**2.9770**	**2.4636**	1.7769	1.2031	**3.3159**	**2.6615**	1.8387	1.1961
		[1.1547]	[1.0581]	[0.9072]	[0.7296]	[1.2861]	[1.1431]	[0.9388]	[0.7254]
	$GEV^{21-day}_{GARCH(1,1)}$	**2.9997**	**2.4156**	1.5855	0.9019	**3.2791**	**2.5595**	1.5934	0.8249
		[1.1635]	[1.0375]	[0.8095]	[0.5471]	[1.2719]	[1.0993]	[0.8136]	[0.5003]
	$Normal_{GARCH(1,1)}$	2.5781	2.3283	1.9586	1.6488	2.5781	2.3283	1.9586	1.6488
KOR	$GPD_{GARCH(2,2)}$	**2.9650**	**2.6036**	**2.1101**	**1.7245**	**2.9456**	**2.5038**	1.9534	1.5610
		[1.1496]	[1.1177]	[1.0769]	[1.0454]	[1.1420]	[1.0749]	[0.9969]	[0.9463]
	$GEV^{10-day}_{GARCH(2,2)}$	**2.9834**	**2.6043**	**2.0382**	1.5045	**2.9252**	**2.4911**	1.8935	1.3782
		[1.1567]	[1.1181]	[1.0402]	[0.9121]	[1.1341]	[1.0694]	[0.9664]	[0.8355]
	$GEV^{21-day}_{GARCH(2,2)}$	**2.9741**	**2.5828**	1.9389	1.3825	**2.8922**	**2.4092**	1.7143	1.1444
		[1.1531]	[1.1088]	[0.9895]	[0.8381]	[1.1213]	[1.0343]	[0.8749]	[0.6937]
	$Normal_{GARCH(2,2)}$	2.5793	2.3294	1.9595	1.6495	2.5793	2.3294	1.9595	1.6495
MAL	$GPD_{GARCH(2,1)}$	**3.0903**	**2.5634**	1.9270	1.4871	**3.3412**	**2.6442**	1.9106	1.4697
		[1.1982]	[1.1006]	[0.9835]	[0.9016]	[1.2955]	[1.1352]	[0.9751]	[0.8910]
	$GEV^{10-day}_{GARCH(2,1)}$	**3.0984**	**2.5707**	1.8809	1.3187	**3.1867**	**2.5757**	1.8180	1.2356
		[1.2013]	[1.1037]	[0.9600]	[0.7995]	[1.2356]	[1.1058]	[0.9279]	[0.7491]
	$GEV^{21-day}_{GARCH(2,1)}$	**3.1708**	**2.6096**	1.7618	1.0321	**3.2003**	**2.4958**	1.6045	0.9177
		[1.2294]	[1.1204]	[0.8992]	[0.6258]	[1.2408]	[1.0715]	[0.8189]	[0.5564]
	$Normal_{GARCH(2,1)}$	2.5791	2.3292	1.9593	1.6494	2.5791	2.3292	1.9593	1.6494

PHI	$GPD_{GARCH(1,1)}$	**3.1320**	**2.5955**	**1.9752**	1.5652		**3.1527**	**2.6760**	**2.0481**	1.5749
		[1.2137]	[1.1137]	[1.0075]	[0.9484]		[1.2217]	[1.1482]	[1.0447]	[0.9543]
	$GEV^{10-day}_{GARCH(1,1)}$	**3.2064**	**2.6422**	1.9118	1.3228		**3.1052**	**2.5655**	1.8710	1.3148
		[1.2425]	[1.1337]	[0.9752]	[0.8015]		[1.2033]	[1.1008]	[0.9544]	[0.7966]
	$GEV^{21-day}_{GARCH(1,1)}$	**3.1786**	**2.5938**	1.7586	1.0610		**3.0575**	**2.5094**	1.7161	1.0483
		[1.2318]	[1.1130]	[0.8971]	[0.6429]		[1.1848]	[1.0768]	[0.8753]	[0.6352]
	$Normal_{GARCH(1,1)}$	2.5806	2.3305	1.9604	1.6504		2.5806	2.3305	1.9604	1.6504
SIN	$GPD_{GARCH(2,1)}$	**3.0038**	**2.5158**	1.8944	1.4417		**3.1956**	**2.5913**	1.9364	1.5310
		[1.1643]	[1.0797]	[0.9665]	[0.8737]		[1.2386]	[1.1121]	[0.9879]	[0.9279]
	$GEV^{10-day}_{GARCH(2,1)}$	**2.9717**	**2.5224**	1.9020	1.3654		**3.1332**	**2.5781**	1.8763	1.3250
		[1.1518]	[1.0825]	[0.9704]	[0.8275]		[1.2144]	[1.1065]	[0.9573]	[0.8030]
	$GEV^{21-day}_{GARCH(2,1)}$	**2.9538**	**2.4431**	1.7288	1.1561		**3.1002**	**2.4569**	1.6607	1.0714
		[1.1449]	[1.0485]	[0.8820]	[0.7006]		[1.2016]	[1.0545]	[0.8473]	[0.6493]
	$Normal_{GARCH(2,1)}$	2.5800	2.3300	1.9600	1.6500		2.5800	2.3300	1.9600	1.6500
TAI	$GPD_{GARCH(2,2)}$	**2.9292**	**2.5781**	**2.0656**	1.6378		**3.0206**	**2.6026**	**2.0561**	1.6473
		[1.1345]	[1.1056]	[1.0531]	[0.9918]		[1.1698]	[1.1161]	[1.0482]	[0.9975]
	$GEV^{10-day}_{GARCH(2,2)}$	**2.9106**	**2.5470**	**2.0016**	1.4851		**3.0242**	**2.5768**	1.9448	1.3839
		[1.1273]	[1.0923]	[1.0204]	[0.8994]		[1.1712]	[1.1051]	[0.9914]	[0.8381]
	$GEV^{21-day}_{GARCH(2,2)}$	**2.8855**	**2.5055**	1.8933	1.3523		**3.0228**	**2.5642**	1.8451	1.2162
		[1.1175]	[1.0745]	[0.9652]	[0.8189]		[1.1707]	[1.0996]	[0.9406]	[0.7366]

	$\text{Normal}_{GARCH(2,2)}$	2.5821	2.3319	1.9616	1.6513		2.5821	2.3319	1.9616	1.6513
THA	$\text{GPD}_{GARCH(2,2)}$	**3.2254**	**2.7575**	**2.1416**	**1.6775**		**3.0588**	**2.5998**	**1.9951**	1.5394
		[1.2513]	[1.1845]	[1.0936]	[1.0176]		[1.1867]	[1.1168]	[1.0188]	[0.9338]
	$\text{GEV}^{10-day}_{GARCH(2,2)}$	**3.2406**	**2.7317**	**2.0321**	1.4298		**2.9761**	**2.4966**	1.8547	1.3179
		[1.2572]	[1.1734]	[1.0377]	[0.8673]		[1.1546]	[1.0725]	[0.9471]	[0.7994]
	$\text{GEV}^{21-day}_{GARCH(2,2)}$	**3.2340**	**2.7260**	1.9265	1.2436		**2.9644**	**2.4377**	1.7044	1.1133
		[1.2546]	[1.1710]	[0.9838]	[0.7544]		[1.1500]	[1.0472]	[0.8704]	[0.6753]
	$\text{Normal}_{GARCH(2,2)}$	2.5777	2.3279	1.9582	1.6485		2.5777	2.3279	1.9582	1.6485

Note: 1. Calculations of VaR are based on the following functions:

$$\text{VaR}^{GPD}_{GARCH(m,s)} = u + \left(\frac{\sigma}{\xi}\right)\left[\left(\frac{(1-cl)\cdot n}{n_{Exceedance}}\right)^{-\xi} - 1\right]$$

$$\text{VaR}^{GEV}_{GARCH(m,s)} = \mu + \left(\frac{\sigma}{\xi}\right)\left\{\left[-\ln\left(1 - \frac{(1-cl)\cdot n}{n_{Exceedance}}\right)\right]^{-\xi} - 1\right\}$$

$$\text{VaR}^{Normal}_{GARCH(m,s)} = \alpha_{1-cl} \cdot \sigma^{Normal}_{GARCH(m,s)}$$

where u, μ, σ and ξ are parameters inferred via maximum likelihood estimation with GARCH (m,s) innovations.

cl is confidence level, n is sample size, $n_{Exceedance}$ is number of exceedances.

α_{1-cl} is the standard normal variate at the confidence level cl.

2. The actual value of each VaR should be divided by 100.
3. Figures in brackets are multiplication factors, ie. $\text{VaR}^{GPD}_{GARCH(m,s)} / \text{VaR}^{Normal}_{GARCH(m,s)}$, $\text{VaR}^{GEV}_{GARCH(m,s)} / \text{VaR}^{Normal}_{GARCH(m,s)}$.
4. Figures in bold indicate that multiplication factors are greater than 1.

The VaRs based on the GPD and the GEV but without AR(1)-GARCH(m, s) filtering are reported in Table 10. Unlike the results in Table 9, the VaR^{GEV} without GARCH filtering are quite sensitive and inconsistent to the block length chosen. For instance, the multiplicative factors corresponding to the 10-day block length are greater than 1 at both the 99.5% and 99% confidence levels for CHI, KOR, PHI, SIN, TAI and THA, but only at the 99.5% level for HON, IDS and MAL, as far as local maxima or local minima concerned. To the 21-day block length, the significant multiplicative factors at 99.5% and 99% levels for maximal and minimal returns are only shown in market TAI, and only for local maxima are shown in KOR, PHI and THA. In contrast, as we can see, the performances of the VaR based on the GPD are more consistent than the GEV at 99.5% and 99% confidence levels. Hence, the VaR estimates of the GPD approach are more robust and less ambiguous than those of the GEV approach. Since different models may yield different results, financial practitioners do need to know such different results from different models whose risk implications would become too large to negligible if a huge amount of assets was involved. That is, financial practitioners are likely to face not only market risk but also model risk.

Table 10. Comparison of VaR Performance without AR (1)-GARCH (m, s) Filtering

Market	Model	Confidence Level of Maxima				Confidence Level of Minima			
		99.5%	99%	97.5%	95%	99.5%	99%	97.5%	95%
CHI	GPD	**11.7833**	**9.0127**	**5.9910**	4.1081	**9.9171**	**7.6619**	5.1361	3.5194
		[1.6102]	[1.3637]	[1.0776]	[0.8778]	[1.3551]	[1.1593]	[0.9239]	[0.7520]
	GEV $^{10\text{-}day}$	**11.2365**	**7.6941**	4.3472	2.4470	**9.0280**	**6.6505**	4.1095	2.4529
		[1.5355]	[1.1642]	[0.7820]	[0.5228]	[1.2337]	[1.0063]	[0.7392]	[0.5241]
	GEV $^{21\text{-}day}$	**10.6620**	6.6079	3.1299	1.3005	**9.0125**	6.1870	3.2605	1.4006
		[1.4569]	[0.9998]	[0.5630]	[0.2779]	[1.2315]	[0.9361]	[0.5865]	[0.2993]
	Normal	7.3169	6.6079	5.5586	4.6794	7.3169	6.6079	5.5586	4.6794
HON	GPD	**5.4052**	**4.4197**	3.3113	2.5986	**6.6034**	**5.0267**	3.5142	2.6879
		[1.1445]	[1.0362]	[0.9229]	[0.8603]	[1.3982]	[1.1785]	[0.9795]	[0.8899]
	GEV $^{10\text{-}day}$	**5.1954**	4.2416	3.0433	2.1084	**5.9044**	**4.5138**	2.9503	1.8717
		[1.1000]	[0.9945]	[0.8482]	[0.6980]	[1.2502]	[1.0583]	[0.8223]	[0.6197]
	GEV $^{21\text{-}day}$	**4.9288**	3.8703	2.5503	1.5682	**5.6460**	4.1145	2.3929	1.2129
		[1.0436]	[0.9074]	[0.7108]	[0.5192]	[1.1955]	[0.9647]	[0.6669]	[0.4016]
	Normal	4.7240	4.2662	3.5888	3.0212	4.7240	4.2662	3.5888	3.0212
IDS	GPD	**9.6613**	**6.8473**	4.0154	2.3902	**9.4498**	**6.6425**	4.1444	2.8816
		[1.4958]	[1.1739]	[0.8183]	[0.5786]	[1.4630]	[1.1387]	[0.8446]	[0.6976]
	GEV $^{10\text{-}day}$	**7.2271**	5.3937	3.3664	1.9933	**7.4930**	5.5156	3.3779	1.9661
		[1.1189]	[0.9247]	[0.6860]	[0.4826]	[1.1601]	[0.9455]	[0.6884]	[0.4760]
	GEV $^{21\text{-}day}$	**6.9410**	4.8580	2.5836	1.0453	**6.8452**	4.8608	2.5963	1.0045
		[1.0746]	[0.8328]	[0.5625]	[0.2530]	[1.0598]	[0.8333]	[0.5291]	[0.2432]
	Normal	6.4603	5.8343	4.9078	4.1316	6.4603	5.8343	4.9078	4.1316
	GPD	**6.9781**	**5.9326**	**4.5174**	**3.4214**	**6.6365**	**5.5073**	**4.1025**	3.1025
		[1.3393]	[1.2608]	[1.1413]	[1.0268]	[1.2737]	[1.1704]	[1.0365]	[0.9311]

KOR	GEV^{10-day}	**6.7212**	**5.4328**	3.8161	2.5563	**6.3299**	**5.0305**	3.4685	2.3078
		[1.2900]	[1.1546]	[0.9641]	[0.7672]	[1.2149]	[1.0691]	[0.8763]	[0.6926]
	GEV^{21-day}	**6.2540**	**5.0140**	3.2864	1.8948	**6.0511**	4.6470	2.8772	1.5422
		[1.2003]	[1.0656]	[0.8303]	[0.5686]	[1.1614]	[0.9876]	[0.7269]	[0.4628]
	Normal	5.2090	4.7043	3.9572	3.3314	5.2090	4.7043	3.9572	3.3314
MAL	GPD	**6.6936**	**4.9850**	3.2894	2.3302	**6.9961**	**5.1431**	3.3516	2.3649
		[1.3806]	[1.1385]	[0.8931]	[0.7515]	[1.4430]	[1.1746]	[0.9100]	[0.7627]
	GEV^{10-day}	**5.8674**	**4.3802**	2.7866	1.7445	**5.5613**	4.1131	2.5582	1.5393
		[1.2102]	[1.0004]	[0.7566]	[0.5626]	[1.1471]	[0.9394]	[0.6946]	[0.4965]
	GEV^{21-day}	**5.4025**	4.0069	2.3192	1.0635	**5.1958**	3.5915	1.9421	0.8800
		[1.1143]	[0.9151]	[0.6297]	[0.3430]	[1.0717]	[0.8202]	[0.5273]	[0.2838]
	Normal	4.8478	4.3781	3.6828	3.1004	4.8478	4.3781	3.6828	3.1004
PHI	GPD	**6.3782**	**5.0569**	**3.6009**	2.6842	**5.9542**	**4.9393**	**3.6698**	2.7614
		[1.3882]	[1.2187]	[1.0317]	[0.9135]	[1.2960]	[1.1904]	[1.0514]	[0.9398]
	GEV^{10-day}	**5.9850**	**4.6752**	3.1325	2.0119	**5.7282**	**4.4498**	2.9697	1.9148
		[1.3027]	[1.1268]	[0.8975]	[0.6847]	[1.2468]	[1.0724]	[0.8508]	[0.6517]
	GEV^{21-day}	**5.8028**	**4.4337**	2.6678	1.2876	**5.3482**	4.0469	2.4420	1.2293
		[1.2630]	[1.0686]	[0.7643]	[0.4382]	[1.1640]	[0.9753]	[0.6996]	[0.4183]
	Normal	4.5950	4.1497	3.4908	2.9387	4.5950	4.1497	3.4908	2.9387
SIN	GPD	**4.4094**	**3.5803**	2.6272	2.0009	**4.7606**	**3.6955**	2.5959	1.9482
		[1.2461]	[1.1203]	[0.9773]	[0.8841]	[1.3453]	[1.1564]	[0.9656]	[0.8608]
	GEV^{10-day}	**4.1999**	**3.3762**	2.3600	1.5830	**4.1547**	**3.2393**	2.1794	1.4239
		[1.1869]	[1.0565]	[0.8779]	[0.6995]	[1.1741]	[1.0136]	[0.8107]	[0.6292]
	GEV^{21-day}	**4.0629**	3.1528	2.0088	1.1527	**4.0387**	2.9743	1.7812	0.9655
		[1.1482]	[0.9866]	[0.7473]	[0.5093]	[1.1413]	[0.9307]	[0.6626]	[0.4266]
	Normal	3.5398	3.1968	2.6891	2.2638	3.5398	3.1968	2.6891	2.2638

TAI	GPD	**6.9110**	**6.0070**	**4.8220**	**3.9332**	**7.1315**	**6.1665**	**4.7805**	3.6415
		[1.2031]	[1.1579]	[1.1049]	[1.0706]	[1.2414]	[1.1886]	[1.0954]	[0.9912]
	GEV^{10-day}	**6.7061**	**5.6568**	4.2114	2.9640	**6.7962**	**5.5476**	3.9361	2.6411
		[1.1674]	[1.0904]	[0.9650]	[0.8068]	[1.1831]	[1.0693]	[0.9019]	[0.7189]
	GEV^{21-day}	**6.5016**	**5.4111**	3.7784	2.3795	**6.3831**	**5.2224**	3.4994	2.0295
		[1.1318]	[1.0430]	[0.8658]	[0.6477]	[1.1112]	[1.0067]	[0.8019]	[0.5524]
	Normal	5.7457	5.1889	4.3649	3.6746	5.7457	5.1889	4.3649	3.6746
THA	GPD	**7.8007**	**6.2572**	**4.4501**	3.2401	**7.3428**	**6.0923**	**4.4627**	3.2476
		[1.4285]	[1.2688]	[1.0727]	[0.9278]	[1.3446]	[1.2353]	[1.0757]	[0.9299]
	GEV^{10-day}	**7.2045**	**5.6517**	3.8026	2.4430	**6.7510**	**5.1541**	3.3606	2.1248
		[1.3193]	[1.1460]	[0.9166]	[0.6995]	[1.2363]	[1.0451]	[0.8101]	[0.6084]
	GEV^{21-day}	**6.8325**	**5.2519**	3.2294	1.6853	**6.2806**	4.5707	2.6930	1.4247
		[1.2512]	[1.0649]	[0.7784]	[0.4826]	[1.1501]	[0.9268]	[0.6492]	[0.4079]
	Normal	5.4619	4.9326	4.1493	3.4931	5.4619	4.9326	4.1493	3.4931

Note: 1. Calculations of VaR are based on the following functions:

$$\text{VaR}^{GPD} = u + \left(\frac{\sigma}{\xi}\right)\left[\left(\frac{(1-cl)\cdot n}{n_{Exceedance}}\right)^{-\xi} - 1\right]$$

$$\text{VaR}^{GEV} = \mu + \left(\frac{\sigma}{\xi}\right)\left\{\left[-\ln\left(1-\frac{(1-cl)\cdot n}{n_{Exceedance}}\right)\right]^{-\xi} - 1\right\}$$

$$\text{VaR}^{Normal} = \alpha_{1-cl}\cdot\sigma^{Normal}$$

where u, μ, σ and ξ are parameters inferred via maximum likelihood estimation.

cl is confidence level, n is sample size, $n_{Exceedance}$ is number of exceedances, α_{1-cl} is the standard normal variate at the confidence level cl.

2. The actual value of each VaR should be divided by 100.

3. Figures in brackets are multiplication factors, ie. $\text{VaR}^{GPD} / \text{VaR}^{Normal}$, $\text{VaR}^{GEV} / \text{VaR}^{Normal}$.

4. Figures in bold indicate that multiplication factors are greater than 1.

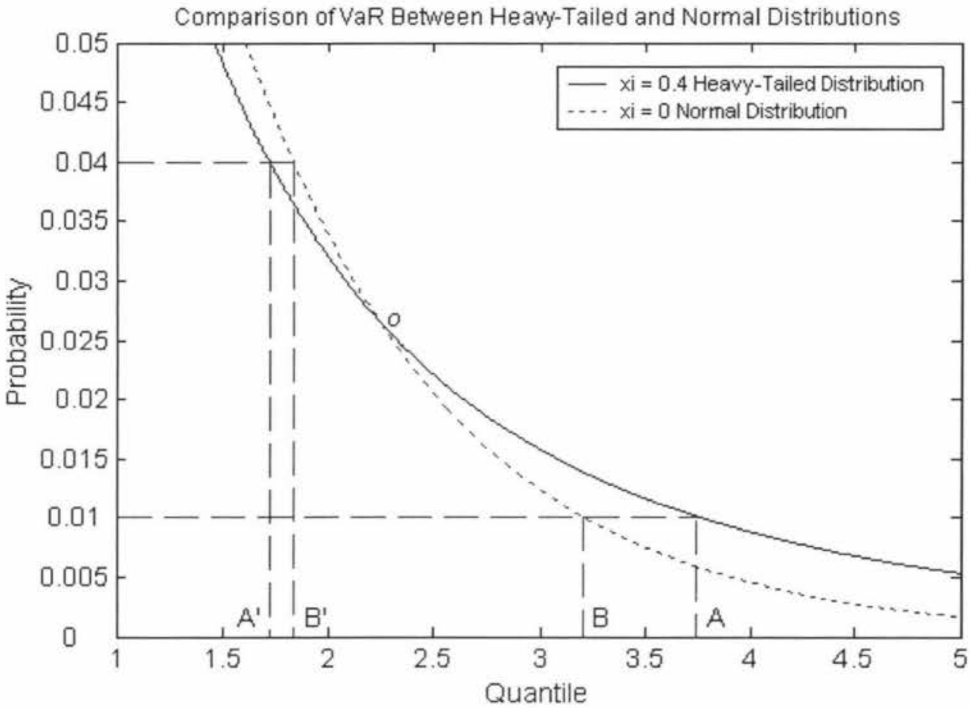

Figure 20

It is also interesting to notice that the $\mathrm{VaR}_{GARCH(m,s)}^{GPD}$ (or VaR^{GPD}) and the $\mathrm{VaR}_{GARCH(m,s)}^{GEV}$ (or VaR^{GEV}) are greater than the $\mathrm{VaR}_{GARCH(m,s)}^{Normal}$ (or VaR^{Normal}) at higher confidence levels, but sometimes smaller at lower confidence levels. Fig. 20 shows that, to the right of point o (where the $\mathrm{VaR}_{cl}^{Heavy-Tailed}$ = the $\mathrm{VaR}_{cl}^{Normal}$), the $\mathrm{VaR}_{cl}^{Heavy-Tailed}$ becomes greater than the $\mathrm{VaR}_{cl}^{Normal}$ for the same probability (See, for example, point A and point B both corresponding to the 99% confidence level). To the left of point o, on the other hand, the $\mathrm{VaR}_{cl}^{Heavy-Tailed}$ becomes smaller than the $\mathrm{VaR}_{cl}^{Normal}$ for the same probability (See, for example point A' and point B' both corresponding to the 96% confidence level).

Tables 11 and 12 set out unconditional and conditional expected shortfalls respectively. They are GPD-based only and calculated only at the 99.5% and 99% confidence levels. This is because the VaR_{cl}^{GPD} is more appropriate than the VaR_{cl}^{GEV} and performs well at such high quantiles as 99.5% and 99%. One can make cross-references to the VaR and expected-shortfall results as reported in Tables 9 through 12. The messages contained in these results are straightforward to comprehend by just looking at these numbers. For example, in Table 9 the VaR_{cl}^{GPD} with AR(1)-GARCH (1, 1) filtering at the 99% level for IDS's local minima inform investors that the loss will not exceed 2.5158 percent in the current period (day) if tail events *do not* occur. But over a long time horizon, IDS is expected to lose 3.8791 percent (see Table 11), or in the next period (day) it is expected to lose 4.8679 percent (see Table 12) if tail events *do* occur, with a 1% chance (corresponding to the 99% confidence level). Tables 11 and 12 also draw our attention to such emerging markets as HON, IDS, MAL, SIN and TAI, as they have higher unconditional and conditional expected shortfalls for minimal returns than for maximal returns at high quantiles. This is not surprising, if we recall that these markets suffer negative skewness as shown in Table 4.

Table 11. Unconditional Expected Shortfall

Market	Model	Confidence Level of Maxima		Confidence Level of Minima	
		99.5%	99%	99.5%	99%
CHI	GPD $_{GARCH(2,2)}$	5.7159	4.6503	4.4953	3.7073
	GPD	17.4920	13.8466	14.3198	11.4767
HON	GPD $_{GARCH(2,2)}$	3.0036	2.7015	4.9609	4.0042
	GPD	7.3126	6.0786	10.9574	8.3230
IDS	GPD $_{GARCH(1,1)}$	4.2312	3.4354	4.9465	3.8791
	GPD	16.5798	12.3090	18.9548	13.3804
KOR	GPD $_{GARCH(2,2)}$	3.4595	3.1119	3.6515	3.1745
	GPD	8.4288	7.4130	8.4451	7.2237
MAL	GPD $_{GARCH(2,1)}$	3.9799	3.3866	4.8899	3.9156
	GPD	11.0350	8.3707	12.0134	8.9677
PHI	GPD $_{GARCH(1,1)}$	4.1162	3.4720	3.8446	3.3657
	GPD	9.0443	7.3345	7.5647	6.4746
SIN	GPD $_{GARCH(2,1)}$	3.7544	3.2425	4.4577	3.6539
	GPD	5.9474	4.9433	7.2309	5.6896
TAI	GPD $_{GARCH(2,2)}$	3.3615	3.0488	3.6349	3.2111
	GPD	8.2339	7.3203	8.3491	7.4742
THA	GPD $_{GARCH(2,2)}$	3.9050	3.4348	3.7252	3.2640
	GPD	10.5638	8.7455	9.1913	7.9180

Note: 1. GPD $_{GARCH(m,s)}$ indicates the GPD with AR (1)-GARCH (m, s) filtering.

2. GPD indicates the GPD without AR (1)-GARCH (m, s) filtering.

3. The calculation of unconditional expected shortfall is based on $\Im_{cl} = \frac{VaR_{cl}^{GPD}}{1-\xi} + \frac{\sigma - \xi u}{1-\xi}$.

4. The actual value of each unconditional expected shortfall should be divided by 100.

Table 12. Conditional Expected Shortfall

Market	Model	$\hat{D}_{t+1}\times 100$	$\sqrt{\hat{h}_{t+1}}\times 100$	Confidence Level of Maxima 99.5%	Confidence Level of Maxima 99%	Confidence Level of Minima 99.5%	Confidence Level of Minima 99%
CHI	GPD $_{GARCH(2,2)}$	0.0147	1.3941	7.9832	6.4977	6.2816	5.1830
HON	GPD $_{GARCH(2,2)}$	0.0786	1.0459	3.2200	2.9041	5.2672	4.2666
IDS	GPD $_{GARCH(1,1)}$	-0.0055	1.2563	5.3102	4.3104	6.2088	4.8679
KOR	GPD $_{GARCH(2,2)}$	0.0470	2.3276	8.0992	7.2903	8.5463	7.4360
MAL	GPD $_{GARCH(2,1)}$	0.0066	0.7716	3.0775	2.6197	3.7796	3.0279
PHI	GPD $_{GARCH(1,1)}$	-0.0750	0.8989	3.6250	3.0460	3.3809	2.9505
SIN	GPD $_{GARCH(2,1)}$	0.0230	0.9595	3.6254	3.1342	4.4801	3.7090
TAI	GPD $_{GARCH(2,2)}$	0.0371	2.0960	7.0828	6.4273	7.6559	6.7676
THA	GPD $_{GARCH(2,2)}$	0.0404	1.3309	5.2376	4.6117	4.9982	4.3845

Note: 1. GPD $_{GARCH(m,s)}$ indicates the GPD with AR (1)-GARCH (m, s) filtering.

2. The calculation of conditional expected shortfall is based on

$$\Im_{cl}^{Conditional} = \hat{D}_{t+1} + \sqrt{\hat{h}_{t+1}} \times \left(\frac{VaR_{GARCH(M,s)}^{GPD}}{1-\xi} + \frac{\sigma - \xi u}{1-\xi}\right).$$

3. The actual value of each conditional expected shortfall should be divided by 100.

Chapter 6

Conclusion and Further Research

Asian emerging equity markets have played and will continue to play an important role in international financial markets, but the returns on these markets depart considerably from normality and possess significant leptokurtosis. The usefulness of market risk measure is that it can trace the loss arising from adverse movements in market prices. Since most Asian financial markets experienced some big events that can take place at a higher than expected frequency, the characteristics of these returns are thus featured as heavy-tailed and asymmetric. Thus, a tailed-related risk measure can provide us with richer information about extreme price movements in the uncertain future. However, the traditional risk measures are unable to capture the tail events, under the normality assumption. And the iid assumption underlying the extreme value theory fails to conform to highly volatile environments such as Asian emerging markets. In view of these, we decide to employ a two-stage approach proposed by McNeil and Frey (2000) to assess the tail risk inherent in the heteroscedastic return series of nine Asian emerging equity markets. Our main findings can be summarized as follows:

First, all the nine index returns do contain heavy tails, as evidenced by the positive shape parameter ξ in both the GEV and the GPD distributions. This result is consistent with those in previous studies on Asian emerging markets. But unlike these studies that attribute risk or tail fatness to the presence of volatility clustering only, we suspect that the innovations after volatility filtering may still remain heavy-tailed and thus still involve tail-related risk that cannot be captured by GARCH-type models only. This problem, we believe, can be dealt with more appropriately within the blend of the GARCH and EVT frameworks.

Second, the blend of the GARCH and EVT frameworks has enabled us to find that the distributions of daily heteroscedastic returns have different characteristics between local maxima and local minima, a sign of asymmetry. More specifically, while the heteroscedastic returns on China, Indonesia and Malaysia exhibit heavy tails in both maxima and minima, the left tails of six stock markets, Hong Kong, Indonesia, Korea, Malaysia Singapore and Taiwan, are fatter than their right tails. This suggests that, for these markets, negative returns are more likely to happen than positive returns.

Third, the reported results indicate that the VaR based on the extreme value theory, especially on the GPD approach, is more accurate than the VaR based on the normal distribution at high quantiles (above the 99% confidence level). This finding is important given the requirement of the Basle Committee on Banking Supervision that the VaR calculation must be made at the 99% confidence level with daily data. Meanwhile, the results of the GPD-based unconditional and conditional expected shortfalls provide richer information about potential losses (gains) beyond the VaR for financial practitioners and policy-makers.

Finally, we compare the GEV and the GPD approaches, and find that the results of the former are somewhat sensitive to the block length chosen, while the GPD approach, with the threshold determined less arbitrarily, can avoid the equivocalness with the GEV method. This work has not been done in the previous studies on Asian emerging markets.

Two issues are worth further exploring. One involves the determination of threshold u. Although the plot of maximum likelihood estimation (MLE) offers an easy way to find an appropriate threshold, the optimal level of threshold needs to be determined by some other methods. Monte Carlo simulation (McNeil and Frey (2000)) or Bayesian inference (Smith and Goodman (2000)) may be an ideal alternative. Another direction points to the use of multivariate extreme value theory. In the present study, we have only examined the behavior of tails for single index return series. For VaR calculation, it is of more interest to look at the joint distribution of changes in multiple market factors. However, when we go beyond the normal distribution, the conventional wisdom that involves only variance and correlation is not sufficient for us to cope with the problems of multivariate dependence. Some recently developed techniques using *copula* will enable us to deal with the dependence structure by modeling the links between the marginal distributions of individual variables (Embrechts, McNeil and Straumann (2000, 2002)). These are the two lines along which further research can be pursued.

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