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A Resampling Theory for Non-bandlimited Signals and Its Applications

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Abstract

Currently, digital signal processing systems typically assume that the signals are bandlimited. This is due to our knowledge based on the uniform sampling theorem for bandlimited signals which was established over 50 years ago by the works of Whittaker, Kotel'nikov and Shannon. However, in practice the digital signals are mostly of finite length. This kind of signals are not strictly bandlimited. Furthermore, advances in electronics have led to the use of very wide bandwidth signals and systems, such as Ultra-Wide Band (UWB) communication systems with signal bandwidths of several giga-hertz. This kind of signals can effectively be viewed as having infinite bandwidth. Thus there is a need to extend existing theory and techniques for signals of finite bandwidths to that for non-bandlimited signals.

Two recent approaches to a more general sampling theory for non-bandlimited signals have been published. One is for signals with finite rate of innovation. The other introduced the concept of consistent sampling. It views sampling and reconstruction as projections of signals onto subspaces spanned by the sampling (acquisition) and reconstruction (synthesis) functions. Consistent sampling is achieved if the same discrete signal is obtained when the reconstructed continuous signal is sampled. However, it has been shown that when this generalized theory is applied to the de-interlacing of video signals, incorrect results are obtained. This is because de-interlacing is essentially a resampling problem rather than a sampling problem because both the input and output are discrete. While the theory for the resampling for bandlimited signals is well established, the problem of resampling without bandlimited constraints is largely unexplored.

The aim of this thesis is to develop a resampling theory for non-bandlimited discrete signals and explore some of its potential applications. The first major contribution is the theory and techniques for designing an optimal resampling system for signals in the general Hilbert Space when noise is not present. The system is optimal in the sense that the input of the system can always be obtained from the output. The theory is based on the concept of consistent resampling which means that the same continuous signal will be obtained when either the original or the resampled discrete signal is presented to the reconstruction filter.

While comparing the input and output of a sampling/reconstruction system is relatively simple since both are continuous signals, comparing the discrete input and output of a resampling system is not. The second major contribution of this thesis is the proposal of a metric that allows us to evaluate the performance of a resampling system. The performance is analyzed in the Fourier domain as well. This performance metric also provides a way by which different resampling algorithms can be compared effectively. It therefore facilitates the process of choosing proper resampling schemes for a particular purpose.

Unfortunately consistent resampling cannot always be achieved if noise is present in the signal or the system. Based on the performance metric proposed, the third major contribution of this thesis is the development of procedures for designing resampling systems in the presence of noise which is optimal in the mean squared error (MSE) sense. Both discrete and continuous noise are considered. The problem is formulated as a semi-definite program which can be solved efficiently by existing techniques.

The usefulness and correctness of the consistent resampling theory is demonstrated by its application to the video de-interlacing problem, image processing, the demodulation of ultra-wideband communication signals and mobile channel detection. The results show that the proposed resampling system has many advantages over existing approaches, including lower computational and time complexities, more accurate prediction of system performances, as well as robustness against noise.

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