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THE USE OF SIMULATION
IN THE STUDY OF
GRAZING MANAGEMENT PROBLEMS

by

PETER L. ARCUS

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CHAPTER I
INTRODUCTION

1.1 Definition of "A Grazing Management System"

Grazing Management is a large and integral part of farm management in New Zealand. It involves the making and implementing of all the decisions relating to the grazing of animals. This is a complex management function which necessitates the assessment and integration of a large number of factors.

Grazing Management involves decisions relating to all aspects of pasture production, its utilization by grazing animals, and the efficiency with which these animals convert the pasture ingested into useful livestock products. Many factors are involved. These include soil type, pasture composition, fertilizer, type, age and number of livestock, grazing method, and the influence of season. Many of these factors interact with each other. Pasture production for instance, is influenced by the number of stock present, and the grazing method. Equally, livestock intakes and productions depend on pasture availability and composition. These interactions are a major influence in grazing management.

The timing of events is also important in grazing management. Many decisions depend for their effectiveness on being made and implemented at the "right" time. Fertilizer applied in the spring or autumn, for example, is of more value in stimulating pasture growth and if applied in the slower growing periods of summer or winter.

Variability in most of the factors involved, is another characteristic of grazing management. Much of this is due to the influence of weather but the influences of variation in pasture and livestock factors apart from weather, are also significant. Frequently this variability is difficult to predict. This may be a consequence of lack of knowledge or the lack of

suitable methods for obtaining the measurements required. As a result uncertainty exists in grazing management about the possible outcome of any particular course of action. Risks, or alternatively insurance measures are therefore often associated with grazing management. Further when knowledge is limited, or objective methods for assessing the effects of particular actions are inadequate, (as in frequently the case in grazing management) reliance must be placed on subjective judgement of situations. This type of approach is typically that used in many practical grazing management situations.

Grazing management is thus a complex process involving decisions relating to all aspects of the grazing of livestock. As such, it is a difficult "factor of production" to measure. The value of grazing management as an input in an agricultural production situation is not measured by the number of decisions made. It is measured by the effectiveness of those that are made. This involves a consideration of the effectiveness of each decision in dealing with each of the features of grazing management described above. This is a very difficult task for which no satisfactory objective measures have yet been devised.

Against this background of the main characteristics of grazing management, a definition of a grazing management system can be introduced. Because of the nature of grazing management this takes the form of a strategy.^{1/}

A grazing management system is thus defined as

"A set of decision rules which indicate the action to be taken in every possible contingency which might arise in grazing livestock in a particular manner."

^{1/} See Williams, J.D. (44)

In this way a grazing management system is defined according to rules to be followed in making decisions on how, when, where and why to graze livestock. This is the concept of a grazing management system adopted as the basis for the research of this study.

1.2 Analysis and Synthesis in Research

Generally speaking research on any subject can be divided into two phases: analysis and synthesis. The first of these, analysis, is concerned with breaking down a problem into its constituents and the analysis of the effects of the individual elements in the system. Typically this involves holding most factors constant whilst allowing one or two elements to vary. Examples of this type of work are to be found in laboratory experiments and plot trials. Usually this work is accompanied by some degree of experimental design and statistical analysis. The methods and practice of this type of research are well known.

The processes of synthesis in research are by contrast less well known. This phase of research is concerned with the assimilation and integration of analytical research results. When, for instance it is found from analytical work, that pasture under certain conditions grows most rapidly if maintained at a height of between three and seven inches, then it is also the function of research to integrate this knowledge by synthesising it into a workable grazing management system. In doing this one or more new grazing management systems may be proposed and appropriate research must be initiated to evaluate these.

In the past, methods for this type of grazing management research have been confined to small farm experiments and grazing management trials. Both of these involve an outlay of large quantities of research resources (land,

livestock, labour and finance) and in consequence are expensive to operate. This has resulted in this type of research being restricted to a few trials or experiments each year. In view of this, new methods of research applicable to this phase of grazing management merit attention.

1.3 Motivation for this Study

With the development of high speed electronic computers and a new discipline known as Operations Research, increased attention has been given to management problems of the type encountered in grazing management. Operations Research, the study of the operations of businesses, industrial and defence organisation, has developed rapidly since World War II, and has provided a new philosophy and several new techniques for use in the study of business operations. ^{2/} One of these techniques is simulation.

Simulation, as applied to grazing management, involves mathematically programming, by means of equations and logical decision rules, the events involved in an actual grazing management system. Basic features of such a simulation are equations linking weather and pasture production, pasture production and animal intake, and animal intake and output. These equations can be loaded into an electronic computer together with the basic data for their solution. In this way an entire grazing management system can be studied at speed and in a manner which readily allows variation to be taken into account. Using this technique it is possible, at least conceptually, to test several grazing management systems over as many as 500 seasons in as little time as a few days.

Such a research technique offers considerable potential as a tool for synthesis in grazing management research. Compared with the existing methods (field trials and small farm experiments) simulation would appear to have

^{2/} For a review of Operations Research see R.L. Ackoff Progress in Operations Research (1)

several advantages. These include speed in producing results, control over all variables, and scope for the incorporation of variation in all the important parameters by varying the information fed into the computer. The large apparent potential of the simulation method has been the reason for initiating this introductory study.

1.4 Limitations of the Study

Simulation is a research method which properly demands a team of research scientists, and, when a problem as large as grazing management is to be studied, a considerable period of time in which to conduct the study. In view of this, the contribution which a single master's degree student can make in the course of eighteen months is restricted. For these reasons, the investigation accounted in this thesis is necessarily shallow. It has not been possible, in the time available to both, cover the subject, and to expose it in detail. In these circumstances, it was felt that greatest value would be obtained from a shallow study of the overall situation rather than a detailed consideration of a section of the subject.

Further, no attempt has been made to take economic considerations into account. An endeavour has been made to understand and apply the principles of simulation to the physical processes of grazing management only. Costs and prices, and the influence of economic criteria in decision making have not been taken into account.

This study has also been influenced by the lack of suitable data for use in a simulation. Strictly speaking, simulation involves only the actual processes of combining equations to represent a real life situation, and the the solution of these under certain conditions. The location of data and the formulation of such equations is therefore not part of the simulation

per se. However, as grazing management information had not previously been assembled in this form, the work of locating and preparing this data was a necessary part of this study. This additional work meant that less time was available for the study and development of simulation as such. The depth of the study has been influenced accordingly.

1.5 Guide to the Thesis

This thesis gives account of a study conducted to investigate the use of simulation in the study of grazing management problems. In Chapter 1, the ideas of a grazing management system and simulation have been introduced. These are expanded in subsequent chapters. Simulation as a general approach to problem solution is discussed in detail in Chapter 2. In this chapter simulation is first defined and specific characteristics of the technique are then identified. The discussion of these characteristics is followed by the introduction and definition of a special type of simulation, Monte Carlo Simulation. This is the particular type of simulation proposed for grazing management and its characteristics are elaborated in the latter sections of Chapter 2.

Following the introduction of Monte Carlo Simulation, consideration is given to usefulness of this method in comparison with alternative methods which might be used to study grazing management problems. This discussion includes small farm experiments, farm surveys, and the subsidisation of farm practices, and is presented in Chapter 3. Chapters 4 and 5 introduce and discuss certain technical problems associated with the use of monte carlo simulation methods.

Chapter 6 then considers a general monte carlo simulation model for grazing management. Attention is given to specific aspects of grazing

management, and the formulation of a overall model to represent the grazing management, is discussed. This is followed in Chapter 7 by the formulation and exposition of a special monte carlo model of grazing management. The process of operating this special model for a simulated season is then described in Chapter 8. The results of this, together with those from a further four "seasons", are then discussed in Chapter 9. Finally, the overall value of the method is considered and conclusions are drawn.

CHAPTER 2

SIMULATION AS A GENERAL APPROACH TO PROBLEM SOLUTION

2.1 Simulation Defined

Simulation is a word with a wide meaning. In general it is taken to indicate any process or activity which resembles, or bears similarity to, or is parallel to, another. Thus, many everyday events can be identified as simulations. Children playing "house", the game of monopoly, a stage play, and laboratory experiments are all simulations.

In research, the word simulation has a similar meaning to that in general use. (11) (30) (35)(41) In this case it may be defined as

"Any process or activity which represents, or parallels, or in any other way duplicates the essence of a real life situation."

This definition does not imply any set form for a simulation. Rather it allows each simulation to take that form which allows it to most adequately represent the real situation under study. Thus a mathematical model and a field experiment are equally simulations. Any process, activity, or operation which parallels an actual situation without achieving reality itself is a simulation within the terms of this definition.

2.2 The Dual Nature of Simulation

A simulation as defined above performs two functions. The first of these is the provision of a descriptive model of reality. The second is that of operating this model for the purpose of observing its activity and outcomes.

When a simulation of any kind is considered, it is first necessary to construct a model which represents reality. This may be a physical model, or it may be an abstract model set in terms of mathematical equations.

In both cases the formulation of a simulation necessitates a full and detailed description of the situation under study. This is the first function of simulation.

The model, once formulated, can then be operated so that observations on its working and results can be made. A specific set of circumstances which might be expected to occur in reality are superimposed on the model. The model is then allowed to run its course of action under these conditions. Observations on its action and its results are made. This is the active phase of simulation. The two aspects of description and action indicate the dual nature of simulation. This can be illustrated by reference to the game of monopoly.

In this game, the descriptive phase of simulation takes the form of a board on which play is conducted. Around the perimeter of the board are set out a series of rectangles each of which bears the name of an item of real estate, or an operating instruction. The board set up in this way describes a business situation. This is the descriptive aspect of simulation.

The active phase consists of the process of playing the game. Dice are thrown, and moves are made according to the outcome of this process. Properties are bought and sold and rents are paid. "Business" is conducted. This is the active phase of simulation. Together, the active and descriptive aspects of this game illustrate the dual nature of simulation.

2.3 Simulation as a Non Analytical Technique

A simulation typically consists of a model of reality which is set up and operated for the purpose of observing the operation and outcomes of a particular process or activity. As such, it is a model which is "run" rather than one which is solved. It evaluates only the consequences of decisions

and actions in a particular set of circumstances. Simulation does not yield a "best" answer selected from a range of possible outcomes. It does not, per se, attempt to optimize any particular objective function.

For example, a grazing management simulation indicates only the outcome which may be expected when a particular grazing management system is used. It does not indicate whether this outcome is better or worse than others which may have occurred if alternative grazing management systems had been used. ^{1/}

This contrasts with the results of analytical research methods. These produce results which are in some way "the best" for the given circumstances. One value is selected from a range of possible outcomes as being the optimum. For example, consider the technique of linear programming. This technique (12) selects from a range of possible outcomes a single activity or combination of activities which optimize a particular objective function (maximum profit or least cost). For the given conditions this is then the best plan. Simulation is a non analytical technique in the sense that it does not achieve this uniqueness of result.

2.4 Physical Experiments as Simulations

Within the broad definition of simulation given in section 2.1, physical experiments can be considered simulations. Physical experiments simulate reality by using real physical conditions on a reduced scale. A fertilizer plot trial can be used to illustrate this type of simulation. In this, a small area of ground is marked out, fertilizer is applied, and in due course the result of its action, is observed. The small area observed represents

^{1/} Though this can of course be assessed if the alternative grazing management systems are also simulated.

a larger area such as a complete soil type. The experiment thus illustrates the two characteristics which identify a simulation. These are a parallel or model of reality, and a process which involves the running of the system so defined. Physical experiments can therefore be considered as simulations.

2.5 Sources of Variation in Physical Relations

Most physical relationships exhibit some degree of variability. This must be taken into account when simulation of these processes is considered. Classification of such sources of variation is, to a degree, arbitrary. It is useful however, to make certain distinctions before proceeding to consider methods which allow this aspect of physical relations to be included in a simulation. The distinction is made between (a) inherent variability, (b) variability which is explainable but for which there is insufficient data to allow it to be explained, and, (c) variability which exists but which is not worth explaining.

(a) Inherent variability, the first of these involves that type of variation which appears to be entirely random; variation for which no logical explanation can be given. The variation in the genotype of animals can be cited as an example of this type of variability. In this case variation arises as a consequence of two purely random processes: the random interchange of genes between chromosomes during meiosis, and the random union of gametes to form a zygote. These processes jointly give rise to the variation in animal genotypes. This is inherent variability.

(b) The second type of variation found in natural systems is that which is "explained" but which is not explained because of insufficient information being currently available to allow this to be done. This

is the situation where cause and effect can be observed but where existing information does not allow the relationship to be identified specifically in a way which would allow the variation to be predicted. Animal production under grazing conditions provides an example of this type of variation. Animal output under grazing conditions depends on intake, feed quality, productive status (lactating or dry), animal health, body weight, maintenance requirements, activity and environmental conditions. Many of these factors are currently ill defined and difficult to measure. Little factual information, for example, exists on the influence of animal activity on production and in consequence the variation in animal output is "explainable" but at present unexplained.

- (c) The third type of variability is that which is "explainable" but not worth explaining. In this case, variation exists and may occur for one or more of a number of reasons, but because the magnitude of the variation is not very great or because the causal relationship is of little or no interest, it is not considered worth explaining. The daily variation of milk yields from a dairy herd may be cited as example of this type of variation. Each day, the milk yields vary. Provided the difference between days is not great, the farmer will not be concerned to find the cause of the variability. He could, if necessary provide some explanation (in terms of weather conditions or feed quality for instance) of the variation, but so long as the variation encountered is within his bound of expectation he does not bother to do this. The variation therefore remains and can be classed as "explainable" but not worth explaining.

These different sources of variability are not mutually exclusive. The concepts presented are all relative and it is quite possible that the examples

given in one category could, when viewed from a different point of view, be considered to fall into one of the other categories. What is important, however, is the recognition of variability in natural systems so that account can be taken of this in preparing and operating a simulation.

2.6 Numerical Simulation and Monte Carlo

One method of conducting a simulation is to use numbers and equations to represent reality. One way of allowing for variability in the system being simulated is to use random sampling. These are the processes of numerical simulation and monte carlo.

2.6.1. Numerical Simulation

A numerical simulation (30) of any process uses numbers and mathematical equations to describe and represent reality. This requires complete enumeration of every aspect of that process and the formulation of an equation to represent each phase. To do this, it is first necessary to break the activity under study down into its smallest components or elements. When this has been done, groups of elements and submodels of the whole can be identified. These are then expressed as mathematical functions which are taken to represent reality.

Once formulated, these equations can be placed in their correct places in a master plan or flow diagram of the whole system. This indicates how the elements and sub models fit together and interact. At the same time the flow diagram also presents a schematic representation of the whole process under study.

Time, in a time lapse sense, is usually associated with any real activity. Provision must therefore be made for this aspect of real life in a simulation. This is done by super imposing the appropriate time space

intervals on the overall model. If, for instance, the critical time interval for use in making grazing management decisions is one week, then each equation in the model must be adjusted to this time base. If necessary, a different equation must be provided for each time period. Alternatively, an allowance can be made for this by the inclusion of a time variable in each equation. A statement of the time interval to be used must be made together with the equations and the flow diagram. These three components, the individual equations, the flow diagram and a statement of the time intervals involved, together constitute the simulation "model".

Operation of the model then proceeds in the following manner:

Values for each of the elements are first selected. These values (numbers) are then inserted in the appropriate equations in the submodels. This allows individual submodel effects to be evaluated. These are then combined in the sequence indicated by the master flow diagram and the result of their combined interaction is calculated. The result is recorded. The process is then advanced one time unit, new parameter values are selected, and the calculations are repeated. The process continues until a pre-arranged stop signal is received. The final result is then recorded.

This pattern of events can be represented algebraically as in the following statement:-

- (a) Collection of elements $(X_i)_j$ (where X_i identifies the particular element and j indicates the time interval it lies in) whose movements in space time through the system are to be observed, are first enumerated.
- (b) Ordered submodels (M_k) are defined which determine the interaction of the $(X_i)_j$ with the total system and with each other.
- (c) Submodels (M_k) are connected in specific ways by input-output flow-lines including feedback where necessary.

- (d) At each time interval M_k receives inputs from outside the system, from within the system, from $(X_i)_j$ and or from other M_k submodel outputs.
- (e) Within each time interval, the sequence of submodels is applied in a specific order and subject to specific logical rules.
- (f) At the end of the cycle all submodel outputs are transferred to the next points of input, the (X_i) are moved ahead according to their system passage rules, the system registers are advanced or "updated" and the cycle is repeated. This continues until a pre-arranged stop signal is received. The final simulation output is then punched out.

2.6.2. Characteristics of Numerical Simulation

The process of numerical simulation described above has several distinctive characteristics. These are summarised below.

- (a) Firstly it is an enumerative technique. It "spells out" the situation and details its operation using numbers and mathematical equations to link the sections together.
- (b) The simulation model provides a complete description of all the elements and submodels involved in a system and also formulates an overall picture of the system under study. The model indicates how a system operates, and it also provides a description of its performance in operation.
- (c) The technique is a flexible one which allows a model to be drawn up in a manner which allows it to represent the activity being simulated in the most suitable manner. Thus, any system or activity can be simulated provided it can be broken down into a series of elements each of which can be described by a set of working rules. If however, these rules cannot be formulated even in a probabilistic sense, numerical simulation is not possible.

- (d) The method allows time intervals to be taken into account. The time space over which an activity takes place is subdivided into suitable intervals and simulation proceeds within these intervals. Combined effects taken over several time periods are assessed by summing the results of the short periods.
- (e) As the scale of digital simulations is frequently large and because even the simplest requires many mathematical calculations, electronic calculating machinery is a virtual necessity. This is one of the reasons why little was known or achieved with numerical simulation prior to the advent of electronic computers. With this facility now becoming more readily available the technique is being studied more carefully and its applications are becoming more widespread.
- (f) If necessary logical decisions can be incorporated in a numerical simulation. This can be achieved by building a decision sequence into the simulation programme. Simple decisions based on yes - no answers to simple questions, (or where computers are used, on positive or negative signs of numbers,) can be included in the simulation model.
- (g) Numerical simulation is not an efficient method of research when compared with analytical methods. It uses professional skill and computer time extensively to formulate and operate what is really a very simple and unsophisticated model of reality. Its use is therefore only justified when problems are beyond the scope of orthodox research methods and analysis.

Account has been given above, of the main characteristics of numerical simulation. From these it is apparent that the method is one of comparative simplicity. Numbers and equations are used to parallel reality and the insertion of specific values in these equations allows particular circumstances

to be studied. Because of its simplicity, the method is extremely flexible and is therefore suited for application to a wide range of subjects.

2.6.3 Monte Carlo

When variation in a parameter involved in a numerical simulation is encountered, one way to evaluate its effect is to take a series of random samples from the probability distribution function which defines the variation involved. This is Monte Carlo Sampling and an example of its use is found in the Random Walk quoted by Churchman (6). This example is quoted in detail here as it allows several features of monte carlo method to be clarified.

The Random Walk as an example of a monte carlo simulation, is accredited to a legendary mathematician who observed the perambulation of a saturated drunk. His observation led him to ask "How many steps, on average, would the drunk have to take to cover a specified distance from his starting point?" The mathematician, who presumably wished to avoid the circumstances of a practical experiment, devised a simple simulation to provide a solution to this problem. Details of this were as follows:-

It was first assumed that each of the drunk's steps had equal probability of going in any direction. A set of cartesian co-ordinates was then set up on graph paper and using this, the drunk's simulated course of perambulation was followed. Each of the drunk's steps was decided by choosing a 2 digit random number and moving according to the following rules. The first digit of the random number was taken to represent one unit along the X axis; positive if even or zero negative if odd. The second digit of the same number was taken to represent one unit along the Y axis, positive if even or zero, negative if odd. The drunk was assumed to start at the origin (0,0) of this system. (X_n, Y_n) then represented the drunk's position at the end of the nth phase and $\sqrt{X_n^2 + Y_n^2}$ was the distance of the drunk from the starting point after n steps.

Random numbers were selected and ~~and~~ using these rules, the drunks path was plotted until the specified distance from the starting point was reached. The number of steps to get this far was then recorded. This process is reproduced and illustrated in Figure 2.1.

In this two simulated random walks are shown. One takes five steps to cover the five unit distance, the other nine.

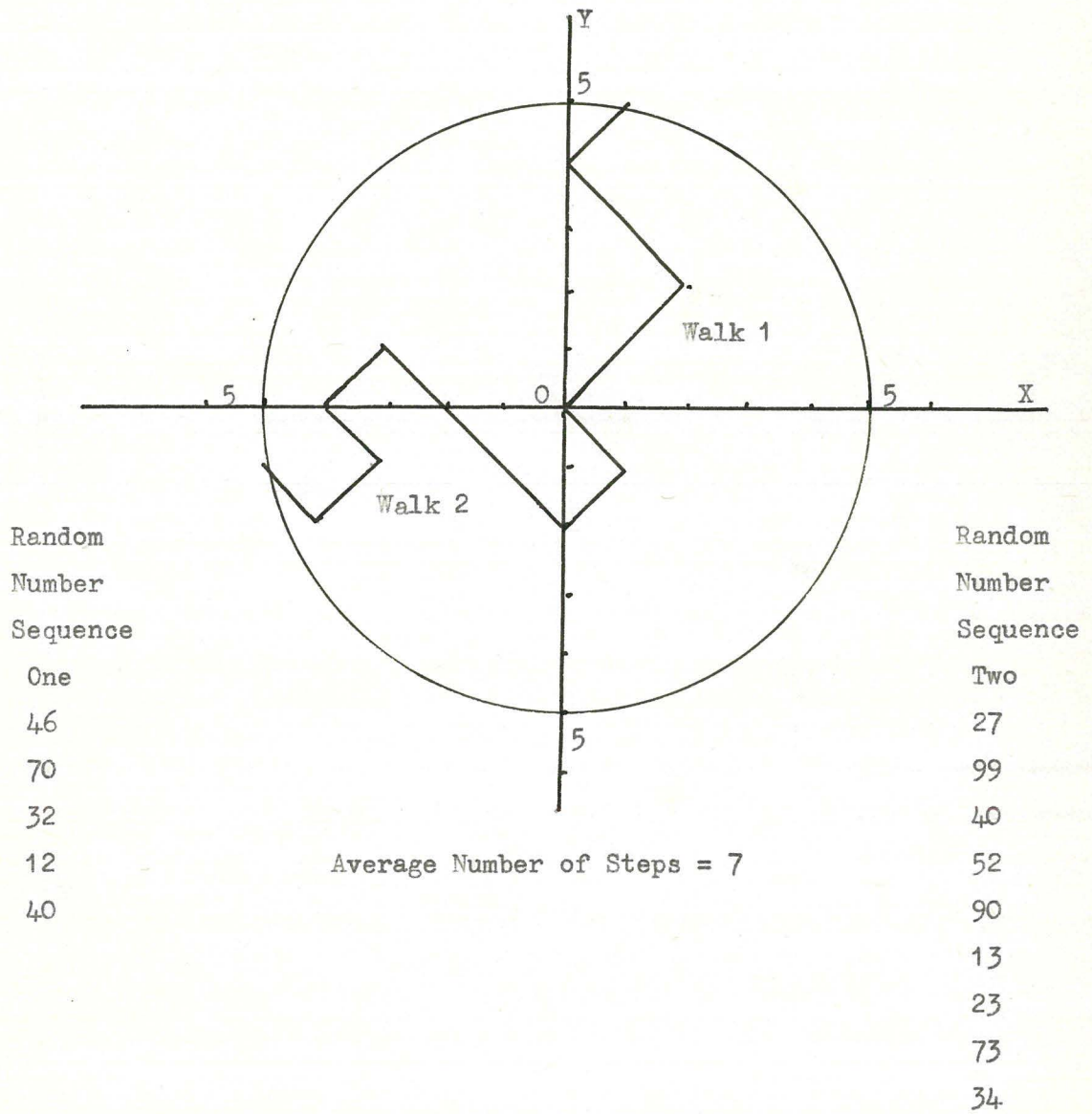


Figure 2.1 Random Walks

2.6.4. Characteristics of Monte Carlo

The random walk example allows the main features of the Monte Carlo method to be identified.

- (a) Firstly, the process is one involving random sampling. This means that a ready supply of random elements must be available to a user of Monte Carlo techniques. Also, as the process of sampling has to be repeated several times for each estimate in the simulation, the number of random elements required per simulation is large. The provision of these elements is discussed in section 4.1.
- (b) Samples are taken from a probability distribution function of the variable concerned. In the random walk case two discrete distributions, one for X and one for Y were sampled. Within each of these distributions only two feasible points ($X = -1$, and $X = +1$ and $Y = -1$, and $Y = +1$) existed. More generally a continuous distribution of some kind is involved, particularly where events of nature are being studied. In these cases it is frequently possible to use formal distribution functions (normal, poisson or binomial) for defining the variation to be sampled.
- (c) Single random samples are inadequate to represent the effects of variability in a simulation. Repetition of the sampling process is therefore necessary. In each case random sampling must be repeated a sufficient number of times to provide an accurate and representative sample of the variable involved.
- (d) Monte Carlo produces a representative answer. This is generally expressed as an average of the outcomes observed and constitutes a measure of the expected value of the outcomes of the various events which form the simulation. As well a variance for these estimates may be

calculated. This indicates the degree of variability in the answer and reflects the effect of variability within the simulation. Monte Carlo thus produces a mean, a range and a variance for the results, and can be considered to produce more information from which to draw a conclusion than do some other types of study.

- (e) Monte Carlo is essentially imprecise. Iterations give a mean value and possibly a confidence limit about it. There is no absolute measure of error and there is no way of testing for a departure from reality. As a rule only two significant digits are obtained from a reasonable number of trials (19) and it is uneconomic to improve precision by increasing the number of trials. Accuracy only increases in proportion to the square root of the number of trials.

Monte Carlo is thus an unsophisticated method of incorporating variability in numerical simulations. It uses the simple processes of random sampling to select values for use in a simulation.

2.7 Definition of a Monte Carlo Simulation

A Monte Carlo Simulation (19) (22) (23) (29) (32) is a numerical simulation which employs random sampling for the selection of parameter values. It thus combines the processes of numerical simulation and monte carlo. In consequence of this, it exhibits the characteristics of both these processes.

An example of this type of simulation is that developed for grazing management in this thesis. The simulation model in this case is based on three sets of equations. These relate weather and pasture growth, pasture growth and animal intake, and animal intake and output. Each equation is accompanied by a statement of error variance. Also included in the model are definitions of the populations to be sampled in selecting parameter values for use in these

equations. These are stated in terms of distribution of specific mean and variance.

The model is run by using random sampling and solving the equations. Random samples of each parameter subject to variation, are taken. These are inserted in the appropriate equations which are then solved. As each equation is solved, a random sample of its error variance is made. Equations are solved sequentially in the order in which the events which they represent, occur in reality. Repetition of these calculations is made to allow a representative sample of outcomes to be produced. The result is expressed as a mean butterfat production with a particular variance.

The essence of this type of simulation is given in a statement by King (19)

"The Monte Carlo method (of simulation) sets up a typical operation, programming the sequence of events by deterministic rules where they arise, and by tossing a coin (random sampling) when either, the determining factors are obscure, or subject to probabalistic processes"^{2/}

An evaluation of this method can conveniently be made in terms of the advantages and limitations which the technique has a research tool.

2.7.1 Advantages of Monte Carlo Simulation

- (a) A Monte Carlo Simulation provides a model and a method which deals with a problem in its entirety. Not only are the elements which compose the situation under study, studied in detail. Their interactions and the overall integration are taken into account too.
- (b) A descriptive model is provided. This enumerates the situation under study in detail. As well only a minimum of idealization of the problem is necessary to put the real situation into a suitable model

^{2/} The words in brackets have been added by the author.

for study. In fact, in most situations it would be more correct to say that the model is fitted to the situation.

- (c) The method is one which provides control over all variables. Changes in each of these can be made by the operator as and when required. Where variability is characteristic of a situation under study, this can be incorporated in the simulation by the process of monte carlo sampling.
- (d) When the simulation is operated (run) it provides, in addition to a specific answer, a display of how the system operates. The method is flexible and can incorporate time considerations and logical decisions.

2.7.2 Limitations of Monte Carlo Simulation

- (a) There are no formal tests for assessing the accuracy of a simulation model. This means that it is possible to inadvertently omit important variables from a simulation, and equally to include variables which are in fact redundant. In the absence of any formal tests of model adequacy (tests which in themselves would be extremely difficult to formulate because of the diversity of situations to which simulations may be applied), only improved knowledge of the real situation will alleviate this position. Simulation is thus only a best representation of reality, a guide to thinking - not reality itself.
- (b) Simulation is a comparatively inefficient means of research. Any cut and try enumerative procedure is necessarily so. Lack of sophistication in the model is a major reason for this and the need to iterate random sampling procedures also contributes to the low efficiency of the method.
- (c) Large numbers of people are involved in collecting data, designing submodels, fitting functions and programming the simulation. This is costly and to these costs must be added computer time. Financially

simulation is likely to be expensive.

2.7.3 Conclusion

Monte Carlo Simulation is a technique which offers considerable potential for use in the study of large scale problems. It is financially expensive and must therefore be used with discretion. When this is done it can be considered a most useful technique. It can be of real value as a check on other methods and invaluable when all else fails.

CHAPTER 3

THE PLACE OF MONTE CARLO SIMULATION IN THE STUDY OF GRAZING MANAGEMENT PROBLEMS

The study of grazing management problems necessitates an evaluation of grazing management systems in their entirety. Because grazing management involves decisions which integrate a series of productive inputs, an overall evaluation of their joint action is necessary. There are four ways of doing this. These are case Farm Surveys, Small Farm Experiments, Subsidised Farmer Research and Monte Carlo Simulation. These will now be discussed.

3.1 Case Farm Surveys

Where a grazing management system is already in use, a survey of a number of case farms can be used to study the grazing management system. A survey for this purpose is defined (4) as

"A series of interviews with farmers to gain information about some one (grazing) management practice" ^{1/}

In this type of survey, interviews are made by the research worker himself and particular attention is paid to the way the farm practice has fitted into the overall management system. This requires that the work be done by a trained technologist (farm management specialist) who is capable of discerning the influence of particular decisions on an overall management system.

The survey is conducted on a case farm basis. Each farm and farmer is treated as an individual and conclusions are drawn on the basis of case farm results and not from aggregated data.

Frequently problems are encountered in obtaining the type of data

^{1/} The word grazing has been added to this definition by the author.

required for an evaluation of a grazing management system. These stem mainly from a lack of farm records and a consequent reliance on memory. The professional skill of the interviewer in summing up both farm and farmer is therefore important. As well, objectives must be clearly defined beforehand. This is particularly so in the case where sampling of any records is required.

The financial cost of survey work, by comparison with other methods which might be used, is comparatively low. The time taken to conduct a survey depends on the scope of the survey and the manpower available to conduct it. Where a single research worker is employed to formulate a survey, conduct it, and collate the results, a period in excess of 12 months is typical.

The case farm survey method is a simple one and can easily be operated by trained personnel. Given this latter, practical results, capable of illustrating real differences, can be obtained. The technique is well suited to the study of existing grazing management systems.

3.2 Small Farm Experiments

Small Farm Experiments provide a situation where one or more farm management systems can be studied in detail on a small scale. The advantage of this type of research compared with the other methods which might be used, is that it uses actual physical conditions to study a grazing management system. Such a technique yields factual information based on real physical conditions. This type of information has a greater general acceptance than that based, for instance, on a Monte Carlo Simulation of the same situation. In this sense, small farms make good demonstration units for extension purposes.

The method is one which takes, in general, 12 months at least to yield

a set of results. Further these results are, strictly speaking, applicable only to the year and conditions under which they were observed. As a consequence of these two conditions, the method is very slow in producing estimates of variability. This contrasts sharply with the potential for producing results indicated for Monte Carlo Simulation.

Costs of small farm experiments are high. Cash costs are high because of the large amounts of physical resources involved. These are partly offset by the sale of produce from the experiment. Opportunity costs are high because of the large outlay of capital, land, and labour involved and the long time taken to obtain useful results. This factor restricts the use of small farm experiments severely. As well, the value and interpretation of replicates of these experiments is subject to doubt.

Small farms can be used to study grazing management problems in several ways. Firstly a single small farm can be used to study an existing management practice. This would however, seem an expensive and slow way of collecting information on existing practices. More could probably be gained in the same time, and for less cost, by using the survey method.

Where small farms do appear useful however, is in the testing of new farm management plans. In this case, no information exists elsewhere and the method can therefore be useful for producing the knowledge required. A single small farm is sufficient, but if adequate resources are available a comparison of two small farms set down under closely comparable conditions will yield more useful results. Because of the high costs and the time taken to obtain results in this way, it would seem logical to compare the two management systems which appear most likely to differ widely. To this end one farm should be set down as a base farm and represent average management practice for the district. The other should be defined in terms of "being

similar to the base form, except.....". Where the exceptions defined deviations from average practice which appear most likely to be profitable.

Also because most components of a grazing management system interact, there is little point in varying one component of the system at a time. Better purpose is served by varying all factors in the direction of maximum profit. The most worthwhile research use of small farms therefore appears to be in comparing (and thus evaluating) new management systems.

The use of small farms in this manner may be considered complimentary to Monte Carlo Simulation. Monte Carlo Simulation provides a means of creating and testing new management systems at speed. When a "most profitable management system" has been hypothesised in this way, it can then be verified by small farm experimentation.

3.3 Subsidisation of Farm Practices

A second method for evaluating new grazing management systems is that of subsidisation of farm practices. In this case subsidies, are offered to farmers to induce them to adopt new techniques of management. In this way the costly overheads of a small farm experiment or a simulation are avoided. The resources of the farmer are used instead.

This method of research into grazing management problems would appear to have a real advantage in that it yields "on farm" results.

These represent actual farmer experience under commercial farming conditions. This is the ultimate practical test for a new management system. Subsidies, either in the form of a guarantee of income or a payment for the extra resources required, or both, need only be offered to the extent required to induce farmers to adopt the new management system. In this way

the method could well be considerably less expensive than small farm experiments. This depends, of course, on the nature of the plan to be adopted and farmers' willingness to indulge in this type of research work.

The success of this method depends almost entirely on the extent to which the farmer is prepared to become a research worker. If farmers can be found who are prepared to respond to the incentives offered and follow accurately the definitions of the new grazing management system, the method can be used to provide useful information. In event of these conditions not being fulfilled, it is doubtful whether the method would be of any great value. The method is one which to the author's knowledge has not been widely used in practice. It has however been included in this section as it constitutes one of the few ways which allows a study to be made of a grazing management system as a whole.

3.4 Monte Carlo Simulation

The relative strengths and usefulness of each of the techniques which might be used for the study of grazing management systems, depends largely on the amount of information which is available. This is particularly true in the case of Monte Carlo Simulation. If there is no information available for the individual phases of Monte Carlo, then no simulation is possible. When however, at least some, how be it poor information is available, simulation provides a useful technique for the co-ordination and evaluation of this data. The advantage of monte carlo simulation in this case is that it synthesises pieces of information. It is a method which collects together a miscellany of research results and evaluates them in terms of their usefulness in grazing management systems.

Monte Carlo Simulation methods force attention onto the mechanism of

a system. Here the analogy of driving a car is useful. ^{2/} Simulation forces research workers to be like a mechanic driving a car. The mechanic understands and appreciates the mechanism of the car. The other methods (surveys, small farm experiments and subsidised farm practices) are, to a degree, more akin to the lady driver who does not necessarily appreciate the mechanism of the car she drives. The fact that Monte Carlo forces research to take the mechanic's point of view means that more attention is given to understanding the processes involved in grazing management.

Monte Carlo Simulations are capable of producing results at a high speed. New grazing management systems can be evaluated over a wide variety of simulated seasons in a short time using an electronic computer. In this respect, monte carlo simulation has an advantage over the other methods so far discussed. It can produce results and an indication of their variability at a rate which is incomparably faster than the other methods.

Disadvantages of Monte Carlo Simulation include the requirement of detailed input information. Little information of this type is currently available for grazing management systems, and the cost of obtaining this may be high. As well a monte carlo simulation of grazing management is likely to involve extensive use of computer services. These may also be expensive.

Where Monte Carlo methods are used there is no means of checking the results except by practical trials. The question of the accuracy with which a monte carlo simulation represents reality is one which is unanswered. Reference to the plausibility of its results is not a sufficient test of the value of the model. Errors of up to 50% could be encountered before this criterion would reject results as unsatisfactory. Thus, whilst it may be possible to reject results on the basis of their being implausible, it is

^{2/} This analogy is Lerner and is quoted by Candler and Sargent. (5)

not safe to accept the results of all plausible findings. The necessary condition in assessing the value of a monte carlo simulation is that it yield information which could not be obtained by any other method.

The ability of monte carlo simulations to produce new information, combined with the potential of the method to do this at speed, are the major advantages of using a simulation to study grazing management. In the absence of any further techniques for studying grazing management systems, Monte Carlo Simulation would appear to be worthy of further attention.

CHAPTER 4

SPECIAL PROBLEMS OF MONTE CARLO SIMULATION

Associated with the use of monte carlo simulation methods are certain technical problems. These are the provision of random numbers, the generation of random variates, and variance reduction. In this chapter, consideration is given to these aspects of monte carlo simulation.

4.1 The Generation of Random Elements

The generation of random elements involves consideration of the ways in which randomness can be achieved when there is a demand for a continuous supply of random elements to a computer conducting a Monte Carlo Simulation. Random numbers are by far the most extensively used supply of random elements, but other supplies such as pseudo-random numbers and random physical processes may also be employed. These three methods have been discussed by Tocher (40).

Three methods of providing random elements, for use with automatic computers, are suggested: external provision, internal provision by a random physical process, and internal generation of sequences of digits by a recurrence relation.

4.1.1 External Provision

One solution to the problem of supplying a series of random digits is to derive these from a source outside the computer. The main group in this category is tables of random numbers. These are generally inadequate, except for very small simulations being operated on desk machines. Where computers are involved such a method is inadequate and too slow. Further, repetitive extension of the tables soon becomes necessary and this is not readily acceptable method of generating additional random values.

4.1.2 Internal provision using random physical processes

A second solution to the problem of supplying random elements is the use of random physical processes. Such random physical processes as radioactive decay or thermal noise from an electronic valve circuit are suitable for this purpose.

The disadvantages of these methods is that they cannot be reproduced. Checking of calculations is therefore not possible. A record of the actual numbers generated can be made for this purpose. If however, the number of digits used is substantial it will be necessary to read the digits out of the machine and pass them back again when required. This is unsatisfactory.

4.1.3 Internal Provision by a recurrence relation

This third category of solutions contains methods of more widespread application. These utilize recurrence relations within a computer to generate random elements (numbers) as they are required. Both random and pseudo-random ^{1/} numbers can be generated in this way. Probably the best known method in this category is the Mid-Square method.

From the middle of the square of the preceding "random" four digit number b_i , a new four digit number b_{i+1} is extracted by the computer. This is used in the monte carlo trial to be decided at that instant and is then stored for the next application. For example, suppose the probability of a new born calf's survival is described by the discrete probability function

$$0 < p(\text{live}) \leq .95 < p(\text{die}) \leq 1$$

and it is desired that a sample of this distribution be taken for inclusion in a monte carlo simulation. Using the mid square method, b_i may for instance,

^{1/} Pseudo-random numbers are a set of numbers which conform to some, but not all the known tests of randomness.

be equal to 2444. .

$$b_i^2 = 5,973,136 \text{ and thus } b_{i+1} = 9731$$

Applying this random number, as a decimal to the calf survival probability function, it will be seen that the calf dies in this simulation.

Methods of the mid-square type can be used to produce long sequences of random numbers, but inevitably, since there is a finite number of digits in the original random number, any process of this type will repeat itself after some point. This is seldom of practical importance as methods now exist for producing sequences of up to 10^{10} random numbers. Nevertheless it is important that the user of any sequential random number producing technique be aware of the tendency of these methods to be periodic. Of greater importance by far is the fact that random numbers produced in this manner are the product of simple arithmetic calculations. They are therefore reproduceable. This allows checks, and accurate comparisons of two systems using the same sequence of random numbers, to be made.

The chief objection to this type of technique is philosophical. The theory relating to randomness asserts that the subsequent variation in "random" numbers all depends on the choice (even if arbitrary) of the first number. This can of course, be met with the pragmatic argument that if the figure produced satisfy the known statistical tests of randomness, they will do. This latter justification is commonly taken as sufficient and methods of this type find widespread application.

More sophisticated methods than mid-square method of producing random numbers using a recurrence relation do exist. Many of these are referred to in the Symposium on Monte Carlo Methods (29) and in the U.S. Department of Commerce National Bureau of Standards, series on Monte Carlo. (32)

Multiplicative and additive congruence methods have been devised to generate random number sequences of greater length than the mid square method. One method for instance, the residue class method, quoted by Foster, (9) is capable of producing 1.7×10^{10} numbers exactly uniformly distributed in the range 0-1 before repeating itself.

The process of generating Random elements can be summarised by saying that there exist many useful ways of generating random elements. None is perfect. For use in engineering and physical science applications (where a large part of the application of Monte Carlo has been in the past) absolute randomness is desirable. In Operations Research attention to this detail is not of such great importance for frequently there is much less accuracy in the simulation model, than in the random elements used.

4.2 The Generation of Random Variates

A random variate is an observation or sample value of a parameter selected from the range of all possible values by random sampling. The generation of these elements for a monte carlo simulation involves drawing sample values from a probability distribution, using random numbers. In each case a random sample is drawn from a sample space (population) described by a particular probability distribution function. This distribution function may be defined in one of two ways, depending on the amount and type of information available. Methods for generating random variates vary accordingly. Two methods are discussed.

4.2.1 The case of a well defined distribution function

Where there is sufficient information on a parameter to allow it to be stated in terms of a well defined distribution function (such as the normal distribution functions), the process of generating random variates is

comparatively simple. The simulation parameter for which a random variate is required, is first defined. The parameter under study may for instance be mean temperature for the month of January. This may be defined as being normally distributed with mean 65°F and variance 36°F (that is, mean temperature is $N(65,36)$). Knowing this, a random sample of this population can be made by using random normal numbers. ^{2/} A table of these numbers is consulted and a value is drawn. Such a value might be -0.752 . Multiplication of the standard deviation of the mean temperature population by this figure gives a random deviate of the temperature population. In this case, $\sqrt{36} \times -0.752 = -4.51$. This deviation is then applied to the mean temperature value (65°) and yields the random variate, $65^{\circ} - 4.51^{\circ} = 60.49^{\circ}$. This is a random variate for the case of a well defined distribution function.

4.2.2 The case of an empirical distribution function

When there is insufficient information to allow a formal distribution function to be used, or when empirical information does not fit one of these well known functional forms, an alternative method of generating random variates must be used. The method ^{3/} in this case proceeds as follows. To draw an item at random from a population described by the probability density function $f(x)$,

(a) First plot the cumulative probability function

$$y = F(x) = \int_{-\infty}^x f(u).du \quad (\text{Figure 4.1})$$

^{2/} Random normal numbers are a series of randomly selected deviations, from the mean of the standardised $N(0,1)$ distribution function. For a table of these see Massey and Dixon (28)

^{3/} See Sasieni (34) Chapter 3.

- (b) Choose a random decimal between 0-1 (to as many decimal places as desired) using, for instance, a table of random numbers.
- (c) Project horizontally, the point on the y axis corresponding to this random decimal value until the projection line intersects the curve $y = F(x)$.
- (d) Write down the value of x corresponding to the point of intersection.
This value is the sample value for x .

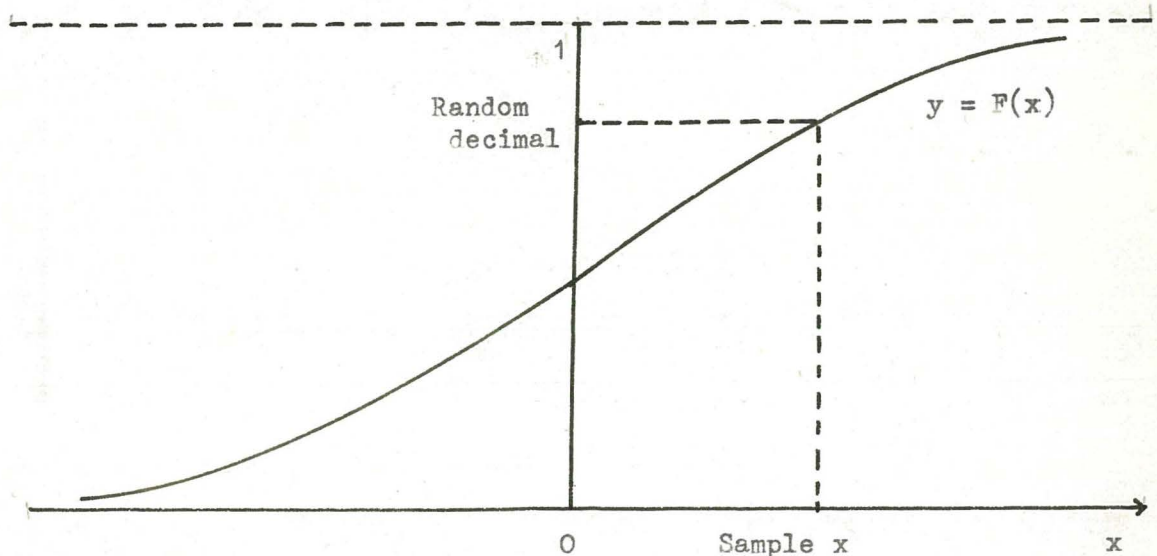


Fig. 4.1 Monte Carlo Sampling: "Drawing an item from a population with a cumulative distribution function $F(x)$ "

Proof of the validity of this procedure involves showing that any item in the population has equal probability of being measured, that is, that the probability of ending up with a sample value between x' and $x' + dx$ is proportional to the probability density function $f(x')$.

Reference to Figure 4.2 shows that

$$P \{ x' < x < x' + dx \} = dy \quad (4.1)$$

$$\begin{aligned} \text{and since } \frac{dy}{dx} &= F'(x) \\ &= f(x) \end{aligned}$$

$$dy = f(x) \cdot dx \quad (4.2)$$

Insertion of the value of dy from equation (4.2) in equation (4.1) gives

$$P \{ x' < x < x' + dx \} = f(x) \cdot dx$$

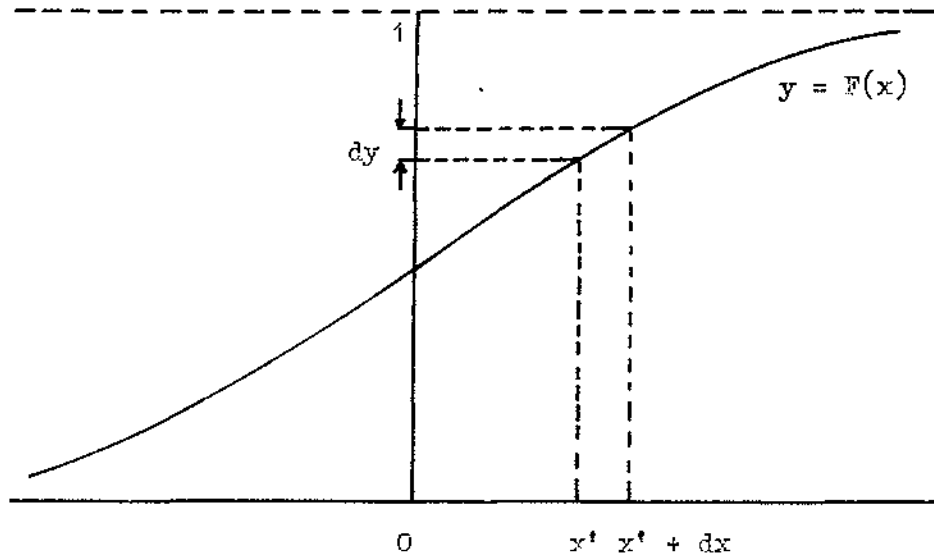


Fig. 4.2 Diagram for the justification of the Monte Carlo sampling routine

That is, the probability of getting any particular value of x is proportional to $f(x)$, as desired,

All types of empirical data can be sampled in this way. If however raw data is used in this way, it can be argued that all that is achieved in reality is a simulation of the past. It is a much stronger assumption that

the basic form of the distribution will remain unchanged with time.

Formalized or even smoothed empirical distributions are to be preferred for this reason.

Two apparently different sampling techniques have been described. The difference between them is however, superficial. Both methods in fact rest on the same foundation. The first is a specialized case of the second. It is only the formalization a particular distribution type in the first case that makes the two appear different. Tables of random normal numbers are in effect "ready reckoner" summaries of the random sampling process for the standardised normal distribution. The proof of the validity of the sampling process given above therefore applies to both sampling methods.

4.3 Variance Reduction

As may be expected the overall variance of a system involving several random sampling processes can become very large. This is substantiated by an examination of the rules of probability theory which cover this domain.

Where two random variables are added or subtracted, variances add in the following manner.

$$\text{Var}(aX_1 + bX_2) = a^2\text{Var } X_1 + b^2\text{Var } X_2 - 2ab\text{Covar}(X_1, X_2)$$

Where random variables are multiplied a complete function involving squares and expected values of squares evolves e.g.

$$\begin{aligned}\text{Var}(X_1 \cdot X_2) &= E \left[X_1 X_2^2 - 2 X_1 X_2 \cdot E(X_1 X_2) + E(X_1 X_2)^2 \right] \\ &= E(X_1 X_2)^2 - E(X_1 X_2)^2\end{aligned}$$

The variance of a quotient is defined as

$$\text{Var} \left(\frac{X_1}{X_2} \right) = \frac{E(X_1^2) - E(X_1)^2}{E(X_2^2) - E(X_2)^2} ; E(X_2) \neq 0$$

(E in all cases stands for Expected Value)

The increase in variance from addition or subtraction is clear. The increase resulting from multiplication is not so obvious, as the definitive formula does not lend itself to factorisation into parts which are clearly $\text{Var } X_1$ or $\text{Var } X_2$. In this case, it is the presence of squared terms which suggests an increase in variance. In the quotient case there is no a priori reason for assuming any particular direction in variance change. It is only included for completeness.

Large variances can thus be generated in the process of using Monte Carlo procedures. The extent of this in any one project is difficult to assess a priori, but the evidence of the equations above indicates that Monte Carlo methods tend to produce large variances. Actual evidence is only available by deduction from Kahn and Mann who cite examples of a reduction of variance by factors of up to 10^4 (17).

Several variance reducing techniques have been devised. Most of these involve a modified sampling procedure, (16) (17) (27) although suggestions have been made (35) that it may be possible to use a degree of experimental design in Monte Carlo. To date the main achievements in variance reduction have been made with modified sampling procedures. These include

1. Importance Sampling,
2. Russian Roulette and Splitting,
3. The Use of Expected Values,
4. The Correlation and Regression,
5. Systematic sampling and
6. Stratified Samples.

4.3.1 Importance Sampling

Importance sampling is, as the name suggests, a technique of deliberately distorting the natural probability distribution according to the importance

placed on samples from a particular part of the distribution. The distortion is subsequently discounted by appropriately weighting the samples from the unimportant part of the distribution. In this way the desired number of important sample values can be observed without including the variance associated with the inclusion of a proportion of "unimportant" samples. The method achieves variance reduction by the exclusion of unimportant samples.

4.3.2 Russian Roulette and Splitting

Russian Roulette and Splitting is a similar technique to Importance Sampling. Both Russian Roulette and Splitting, and Importance Sampling employ processes which dispense with certain groups of samples. In Importance Sampling "unimportant" samples are determined a priori. In Russian Roulette and Splitting the decision to dispense with some samples is withheld until sampling has been completed.

In this latter case, samples are drawn from the natural population. After sampling, the outcomes are split into two groups: those which are in some way "interesting" and of further use in the simulation, and those which are "uninteresting". Some percentage of the "uninteresting" samples is then killed off. The remainder are weighted to make up for the loss. Usually, the "killing off" is done by a supplementary game of chance. If the supplementary game is lost the sample is killed; if it is won the sample is counted with extra weight to make up for the fact that it ran some risk of being killed. The similarity of this procedure with the Russian game of chance played with revolvers and foreheads, has led to the method being entitled Russian Roulette. By means of splitting the samples into "interesting" and "uninteresting" groups and by "killing off" some of the "uninteresting" samples, fewer sample outcomes continue in the simulation. The variance of the overall simulation outcome is thereby reduced.

4.3.3 Use of Expected Values

An expected value is the sum of all the possible sample outcomes, each weighted by the frequency with which it occurs. This value summarises the long term balance of outcomes expected from continued sampling of a population. It is the mean of all the possible values which might occur from sampling a population and it takes the variance of the population into account. An expected value can thus be used to express the effect of a particular parameter without introducing the variance of that parameter into the simulation. In circumstances where the deliberate sampling of a population only adds to the variance whilst contributing no more than a mean value this method is valuable as a variance reducing technique. It is one of the more powerful techniques of variance reduction. Kahn and Mann (17) quote the use of expected values as having decreased effective variance by factors of the order of 10^4 - 10^6 in physics and engineering projects and by factors of 2 - 100 in Operations Research applications.

4.3.4 Correlation and Regression

Correlation or Regression may be employed when Monte Carlo Simulation is being used to make comparisons or calculate differences. In these circumstances elimination of irrelevant fluctuations which do not affect the comparison can be achieved by using either correlation or regression.

The variance reduction which can be achieved by the use of these techniques can be illustrated as follows. Consider for instance, pasture production is being simulated using monte carlo methods, and two types of pasture are to be compared over a series of "seasons". Equations are set up to represent the growth of each type of pasture. These may, for instance, relate pasture growth to weather. Random samples are then drawn for weather and the predicted growth of each area of pasture is calculated. The results

are then compared.

In this situation there are two ways of drawing the random samples. The first of these involves drawing separate random samples for each pasture production equation. The second is more subtle and uses the same random sample in both pasture equations. This means that whilst the random influence of weather is maintained in the simulation, both equations are affected alike. The comparison of their outcome, thus becomes more accurate because the irrelevant fluctuation associated with sampling the weather has been eliminated.

In this illustrative case a correlation of of plus one was assumed between the random numbers used for deciding the parameter values for each of the pasture production equations. This need not have been so. Nor need it have been the random numbers which were correlated. Any system of sampling which includes some degree of correlation or regression between two aspects of a sampling procedure is a variance reducing technique of the correlation and regression type. This method of variance reduction is of most use in cases of comparison.

4.3.5 Systematic Sampling

In certain multistage sampling problems it is frequently possible to conduct the first stage systematically. If for example, two dice are going to be tossed one at a time, there is no real value in actually tossing the first dice, provided the total number of samples to be taken is known. If, for example 600 samples were planned, 100 samples of each face of the dice would be expected. In this case no bias is introduced to it, instead of random sampling, 100 cases of the dice showing 1 spot are assumed to occur first, followed by 100 cases of the two spots showing etc. This is

systematic sampling.

The advantage of this method is that the error caused by fluctuations in the proportions of the different faces occurring with random sampling the first die outcomes is eliminated. The extent to which this technique achieves variance reduction is small. Factors of 5 - 30 percent are indicated (17) but as the method is likely to cost little or nothing to implement it is one which is readily used. The main application of Systematic Sampling is in multistage sampling problems where it is trivial to calculate the distribution of events at the first stage.

4.3.6 Stratified Sampling

Stratified Sampling involves a procedure which combines the principles of Importance Sampling and Systematic Sampling. As in Systematic Sampling, each group of outcomes is assigned a definite number of samples instead of a random amount. However instead of just taking this number equal to the expected number of outcomes for that region, it is chosen to minimise the variance of the estimate being made. In this latter respect Stratified Sampling is similar to Importance Sampling.

For example, suppose 600 tosses of a pair of dice were to be made and the only outcomes of interest are those which total three. In this case, systematic sampling can obviously be used in place of random sampling for the throw of the first die. In addition, Importance Sampling can be used. In the trial being considered; results of throwing the first die which yield values of three to six are of no importance, since under these circumstances, it will never be possible to get a total of three. The systematic sampling of throws of the first dice yielding three to six can therefore be declared "unimportant" and left out of the sampling. The combined practices of systematic and important sampling constitute Stratified Sampling.

Six variance reducing techniques have been discussed. Each has value in certain circumstances. Some methods find widespread application and achieve large reductions in variance. Others are more restricted in their usefulness. Overall, their use can contribute significantly to reducing variance in Monte Carlo Simulations.

In Operations Research applications of Monte Carlo Simulation however, use of variance reducing techniques has not been as great as may be anticipated from the discussion above. Marshall (27) has suggested possible reasons for this.

1. It is possible that to some extent users of the Monte Carlo method have been insufficiently aware of the resources at their disposal for this type of work.
2. In most Operations Research very accurate answers, are not required. Therefore sample size is not large even when random sampling is used. Further, it is often the case that model accuracy, no matter how complex the model, is only nominal. This may arise from difficulties in defining the variables involved, or from weaknesses in the empirical data used. It is also true that Operations Researchers are usually looking for, and only interested in large differences between, for example, existing and proposed systems.
3. Interest often centres on the working model itself as much as on the outcome of its operation. Study of the model leads to understanding and suggestions for improvements. "Importance" can be decided afterwards when it is seen "how the model goes", rather than a priori as is necessary for importance sampling. Realism promotes understanding. Distortion of the model for increased efficiency is not readily acceptable in this situation.

It is thus evident that the chances of achieving significant gains in variance reduction in Operations Research Monte Carlo Simulations are compromised by the emphasis on simulation. Nevertheless, useful variance reduction can be

achieved by using the techniques discussed to exploit specific details of Monte Carlo Simulations.

CHAPTER 5

SPECIAL PROBLEMS ASSOCIATED WITH A MONTE CARLO SIMULATION OF GRAZING MANAGEMENT

In this chapter two further problems associated with the use of monte carlo simulation are discussed. These are distinct from the problems of the previous chapter as they arise from the application of the technique of Monte Carlo Simulation to a specific situation: grazing management. The two problems are, the specification of simulation functions and the difficulty of obtaining appropriate data for the simulation.

5.1 Specification of Particular Simulation Functions

Before a monte carlo simulation can be formulated completely, each of the events in the grazing management system under study must be specified in detail. This involves identifying dependant and independant variables within the system, and expressing these in the form of mathematical equations. To the author's knowledge no equations of this type were available from previous grazing management research. The process of formulating equations to represent real events in grazing management therefore become part of this study.

Two problems were encountered. These were the specification of the functional form of relationships (equations) and, the amount of data required to make the equations truly representative of reality. In the absence of any a priori knowledge on this aspect of simulation, solution to these two problems was found by empirical procedures. In considering any one function the method was as follows:

Firstly, the particular function in question was framed in general terms. For example, dairy cow milk production was expressed as

$$\text{Milk production} = f(\text{Intake, food conversion efficiency, liveweight gain or loss, age and health of stock, length of lactation})$$

Secondly, an investigation of the data available for each of the parameters in the function was made. Where parameters had been recorded over a range of possible outcomes, these were included in a numerical estimate of the function. Where only one observation, or no observations at all were available, the parameters were omitted from the numerical estimate of the function. In this latter case the variation due to these parameters has been taken into account in one of two ways. The first of these involves making a statement of fact. For example, "The milk production equation applies only to 5 year old cows". This reduces the general applicability of the simulation equation to a particular group of animals. The second method involves formally disregarding the influence of the parameter. The variability due to the parameter, in this case, is assumed to appear in the error term for the equation deduced without it. For example, minor fluctuations in stock health has been dealt with in this way. There were no observations for this parameter so it could not be included in any formal equation. Its influence is nevertheless included as part of the error variance of a milk production equation.

Once the parameters to be included in the numerical estimate of the functional relationship had been identified, graphs of the dependant/independant variable relationships were made. From these, a functional form to fit the data, was proposed. This was then fitted to the data using regression analysis. The function so provided was used in the simulation.

Solution to the problem of the amount of data to use in the formulation of these equations, was given by the amount of data available. In each case

the amount of data currently available was found to be limited. The equations which it was possible to formulate are based on one or at the most two years' observations. These may or may not be representative observations of the real function being represented by the equation over longer periods of time.

The specification of simulation functions has thus been made using empirical methods. Problems including the form of functional relationships and the amount of data required for the enumeration of these, have been encountered. Nevertheless it has been possible to enumerate sufficient functions to allow a monte carlo simulation of grazing management to be formulated.

5.2 Difficulty in Obtaining Appropriate Data

Associated with the problem of specifying the functional relationships involved in grazing management is the problem of obtaining appropriate data.^{1/} The simulation of grazing management was found to require data fulfilling the following conditions.

- (a) Data should be directly applicable to grazing management situations. There is little or no value in trying to simulate reality with data which is not relevant to a real situation. For example, data for pasture growing in the absence of livestock is inappropriate for use in a simulation which includes grazing animals.

^{1/} It should be appreciated that this is not a problem of Monte Carlo per se. Monte Carlo Simulation in a strict sense involves only the formulation and operation of a model of reality from given data. The problem of providing appropriate data however, has been a practical problem in this study.

(b) Three relationships are fundamental to grazing management and to the simulation model. These are the weather-pasture growth relationships; the pasture growth-animal intake relationship; and the animal intake output relationship. In order to delineate these functions, observations are required on weather, pasture production, animal intake, and animal production. The minimum information required for a grazing management simulation includes at least some observations on each of these four parameters.

(c) Time is an important variable in grazing management and simulation is essentially atomistic. Observations on grazing management variables were therefore required at reasonably short intervals (1 - 2 weeks). In addition it was preferable that data extend over at least one season and if possible over a longer period.

(d) Data from a single farming situation is desirable. Apart from providing a co-ordinated factual base for simulation, a single farm provides a test site to which simulation results are properly applicable. If data is derived from a wide range of sources, the interpretation of the simulation and its results is difficult.

These then were the formal objectives in searching for and evaluating possible sources of simulation data. It was realized it may not have been possible to fulfil all these objectives simultaneously, but equally it was hoped that more data than just that fulfilling these requirements, might be located. Several sources of grazing management records and data were investigated. Five sources were studied in detail. These were -

1. The Massey College Small Farm Intensive Project
2. The Te Awa Hill Research Farm
3. The Wanganui Farm Improvement Club Sheep Farm Records
4. A commercial farm employing a Harvestore

5. The Ruakura No. 2 Dairy Farm Project

For each of these situations, weather information was available in the form of monthly summaries of observations made in the vicinity of the projects. Since, then, weather information was available for all five situations, evaluation of the sources of data was based mainly on their ability to supply information on the other three parameters pasture production, intake, and output, and on their ability to fulfill the other objectives set out above. Investigation proceeded in the order shown.

5.2.1 Massey Small Farm Project:

An intensive small farm project involving 30 acres of one of the Massey College Dairy farms at Palmerston North, New Zealand., stocked at 1 cow/acre was conducted in the years 1952-57 (31) (33). The object of this trial was to test the hypothesis that 500lb of Butterfat per acre could be attained with ryegrass white clover pastures on the heavy soils of the Manawatu.

This project was closely recorded and most of the original information was still available at the time the author made enquiries about it in March 1962. Initially it appeared that there was sufficient information recorded on this trial to allow some attempt at simulation to be made.

Detailed investigation of these records however revealed that there were no estimates of pasture production made at any time during the study. This meant that the first functional relationship in the simulation, weather - pasture growth, could not be estimated. Further, estimates of intake were based on Wallace's (1956) equation (42) which only provides a measure of intake over a full year and is not suitable for short term studies.

In view of those two deficiencies it was decided that this source of information was inadequate and that others should be investigated.

5.2.2 The Te Awa Research Farm

Te Awa is a hill country research station of 185 acres in the Pohangina Country of the Manawatu Province, New Zealand. It was set up by Grasslands Division of Department of Scientific and Industrial Research in conjunction with the Manawatu Catchment Board, to study hill country pastures and erosion control. Details of the farm and the projects there on have been recorded by Suckling (37) (38).

Recent research has involved study of pasture and livestock production under several stocking rates. Full records were available for, weather, fertilizer, pasture production, wool weights, stock liveweights, and lamb growth rates, for levels of stocking of 3, 4, 5, $6\frac{1}{2}$ ewes per acre with and without cattle.

The inadequacy of this situation was the lack of some measure of intake. There has been no measure of this made at Te Awa. In view of this this source of information had to be disregarded.

5.2.3 Wanganui Farm Improvement Club Data

Observation by one of the author's fellow students, that the Wanganui Farm Improvement Club was keeping some useful records, lead to the consideration of another source of sheep farming data.

Comprehensive records of production were available for farms in the Wanganui Farm Improvement Club. Livestock production had been closely recorded and pasture production had been estimated on the basis of ewe equivalent grazing days per acre. There was a wide range of information available.

The problem with this source of information was the pasture production data. The measurement used, ewe grazing days per acre, confounds three factors: pasture production, livestock intake and management. It measured

in effect, whichever is the least of the pasture production potential, the animal production potential and the management potential. For example, if the amount of pasture available is in excess of animal requirements, then the animals determine the number of grazing days per acre. If feed is short, the pasture determines the grazing day limit. If the manager decides to fatten stock, the stocking rate is kept low and the stock are well fed. Conversely, if the manager wishes to restrict livestock intakes he will increase the stocking rate. In each case, the number of grazing days measures something different. In no case does the measure record the amount of pasture produced effectively. The confounding of several influences in the records available from this source was the reason for disregarding this supply of data.

5.2.4 The Harvestore Farm

After investigating the three sources of information described above, no suitable source of simulation data had been found and both pasture and intake measurements under grazing conditions appeared to be deficiencies in the provision of this information. Consideration was therefore given to avoiding the difficult circumstances associated with the measurement of pasture growth and livestock intake under grazing conditions by studying a system which dissociates the animal from the pasture. This situation appeared to be adequately represented on a Waikato farm where a Harvestore had been installed. ^{2/} On this farm it was thought that records of pasture production and animal intake in terms of harvestore input and output (with known losses in between) may have been available, and using these, a simulation of "a pasture animal association" may have been formulated.

^{2/} The Harvestore is an airtight steel silo used for storing cut wilted pasture. Preservation in the silo is by carbon dioxide and a very nutritious and palatable feed is produced from this type of storage.

A visit to the property of Mr. E.R. Elliot, Te Kowhai, was therefore made to ascertain the potential of this source of information. From this the main features of the management of this system of Harvestore farming were appreciated. Severe limitations in the amount of data recorded were apparent. The commercial nature of the enterprise meant that measurements of Harvestore inputs and outputs were in terms of trailer loads of pasture harvested and "feed lot fulls" fed out. The farmer was not interested in conducting detailed research and in lieu of this no useful attempt at simulation of this situation could be made.

5.2.5 The Ruakura No. 2 Dairy Farm Project

The fifth and final grazing management situation investigated was that of the No. 2 Dairy Farm Project at Ruakura Animal Research Station, Hamilton, New Zealand. This project was originally set up to study two grazing management systems, set stocking and rotational grazing, on small farm scale (24). In 1957 this study was extended to take different stocking rates into account. The records of this latter stage of this trial included observations on all four parameters required for a simple monte carlo simulation of grazing management.

The No. 2 Dairy Farm project over the years 1957 - 61 at Ruakura has been fully accounted by Mc Keekan and Walshe (25) and only a summary is required here. Basically four small farm experiments were laid down. The aim of these was to evaluate controlled (rotationally grazed) and uncontrolled (set stocked) grazing practices under high and low stocking rates. The details of this trial are shown in table 5.1.

TABLE 5.1

SUMMARY OF THE RUAKURA NO. 2 DAIRY FARM PROJECT 1957-61

Treatment	Area	1957/58 1958/59	1959/60 1960/61	Stock Details
CL	42.6 ac			Standard Herd Composition and Age Distribution
CH	35.0 ac	40 cows only	42 cows only	
UL	45.0 ac	per treatment	per treatment	18% Replacement rate
UH	34.5 ac			7 2 yr olds/yr.

Where C stands for controlled grazing and where U stands for uncontrolled grazing

and where H stands for high stocking rate and where L stands for low stocking rate.

The controlled grazing treatments involved fifteen approximately equally sized paddocks. The uncontrolled grazing treatments used only two paddocks, a day and a night paddock. Paddocks for different treatments were intermingled to obtain a random and even distribution of soil and pasture types. Two hundred weight of serpentine superphosphate fertilizer was applied annually. Hay was made on the uncontrolled treatments and hay and silage on the controlled treatments. Pasture dry matter was measured on a pilot area of five acres, set up within the experimental area on a replicated basis. This duplicated both grazing method and stocking rate differences of the major experiment. Caged areas on the U treatments were cut at 14 = 2 day intervals and similar areas on the C treatments were cut at intervals determined by the rate or rotation of the herds about the

paddocks on these treatments.

Associated with, or adjacent to, these experimental areas the following information was recorded.

Weather: Temperature: Maximum (Stevenson Screen); °F
 ————— Minimum (Stevenson Screen); °F
 . ————— Grass Minimum, °F
 Soil at 2", 4", 6", 1', 3' depths; °F.

Relative Humidity %

Wind: Direction

Force

Run, miles per 24 hours.

Cloud.

Rain, Inches.

Bright Sunshine, hours.

Evaporation, inches.

On a daily basis all years.

Pasture Production:(pounds of dry matter)

Cutting Interval

Cow Grazing days

Net grown Total
 Period

Available Total
 Period

Consumed Total
 Period

% utilization Total
 Period

Cow days/ac. Total
 Period

For three seasons 1958/59, 1959/60, 1960/61.

Intake: Estimates of dry matter consumed by the stock on the pasture production sampling area.

Production:

Individual cow milk weights, fat percentage and liveweight by weeks for all cows in all years.

Summaries of Production/cow.

and Production/acre.

General: Calving Dates.

Lactation Lengths.

Herd Composition, wastage and disease.

Calf weights.

Hay and Silage made and fed and measurements of Autumn saved pasture.

The situation of No. 2 Dairy thus appeared to fulfil all the initial requirements for data for a monte carlo simulation of grazing management. It was therefore selected as the source of information for the simulation of grazing management in this study. Subsequent detailed investigation, however showed that even this situation was inadequate in some respects. In particular, the animal intake measured on the No. 2 Dairy Project proved unsatisfactory for use in a simulation, ^{3/} and data from another Ruakura project had to be substituted.

^{3/} See Chapter 6 section 5.1

CHAPTER 6

A GENERAL MONTE CARLO SIMULATION MODEL OF GRAZING MANAGEMENT

In this chapter a discussion of the problems encountered in formulating a grazing management simulation for dairy cattle, is presented. This is followed in the next chapter by the development of a numerical model for a specific grazing management system.

6.1 A Description of the Grazing Management Problem

The management of grazing livestock is a very complex function. It involves decisions relating to all aspects of pasture production, its utilization by grazing animals and the efficiency with which these animals convert the ingested pasture to a saleable livestock product. These factors are influenced by a series of other factors. The amount of pasture grown is influenced by type of soil on which it is growing, the pasture composition, the amount of fertilizer applied, and the weather. Pasture growth is also dependant on the degree of farm subdivision, drainage, grazing, treading and pugging by livestock. In addition, each of these factors has a different influence according to the time it is applied.

The utilization of the feed by livestock is similarly dependant on many factors. The amount of pasture consumed depends on whether the animals are dry or lactating, growing or mature. The feeding plan: set stocking, rotational grazing, strip grazing, the conservation of Autumn sowed pasture and the making and feeding of hay and silage, also plays an important part in determining the amount of grass eaten by livestock.

Once the pasture has been ingested by the animal, other factors operate

to influence the amount of intake actually converted to animal output. In this category genetic merit, stock health and thrift, and physiological productive status (stage of lactation, pregnancy etc.) operate to influence production.

Furthermore many of the factors involved in grazing management, interact. Adjustments to one factor can rarely be made without studying the changes required or induced in other factors. There are few truly independent variables in a grazing management system.

Continuity through time is another feature of a grazing system. Each month's production is influenced by that of the preceding month and that of the current month influences that which follows. The end of one season is the start of another. Within each season, grazing management decisions influence one another. Each new decision can only be made within the limits of the previous one. For example, if two thirds of the dairy herd is dry by the end of April, then decisions about drying off can only be applied to the remaining one third of the herd. The collation of decisions through time is therefore an important part of grazing management.

There is also the aspect of time which is concerned with the timing of events. This is frequently one of the most critical factors in grazing management. This can be illustrated by the case of the one farm manager, who moves his stock one or two days before another. In the same circumstances judiciously avoids a stock thrift problem which the second man encounters simply because he waits two days longer.

Uncertainty also influences grazing management practice. There is for instance no way of knowing in advance the type of growing season to expect in any one year. In consequence grazing managers must take risks. Alternatively they must make provision for the possible detrimental effects

of uncertainty by insuring against these. In both cases, the decision taken influences the rest of the grazing management system.

Many factors associated with grazing management require subjective evaluation. This occurs for one of two reasons. Either it is not possible to define particular influences in the grazing management system explicitly, or if it is, no suitable method for measuring the variation in the factor exists. Judgement, the art of evaluating situations subjectively, is therefore important in most grazing management situations.

Economic considerations are also involved in grazing management. Each of the many factors involved in grazing management can only be obtained at some cost. Against this, a price for the output produced, can be set. A profitable balance of these two economic factors is required.

The management of grazing stock thus calls for considerable skill and knowledge. It requires knowledge of the physical characteristics and responses of the large number of factors involved and knowledge of the economic implications of the introduction and adjustment to these. Overall the grazing manager's problem is one of optimising (in the sense of using resources effectively and efficiently for profit) the combination of pasture producing and livestock factors. A simulation model for studying the physical aspects (exclusive of economic considerations) of this problem is now considered.

6.2 An Overall Model

Before a grazing management simulation can be formulated in detail, it is first necessary to reduce the grazing management problem stated above to a grazing management problem involving a particular situation. In this study, the simulation of the grazing management of lactating dairy cows has

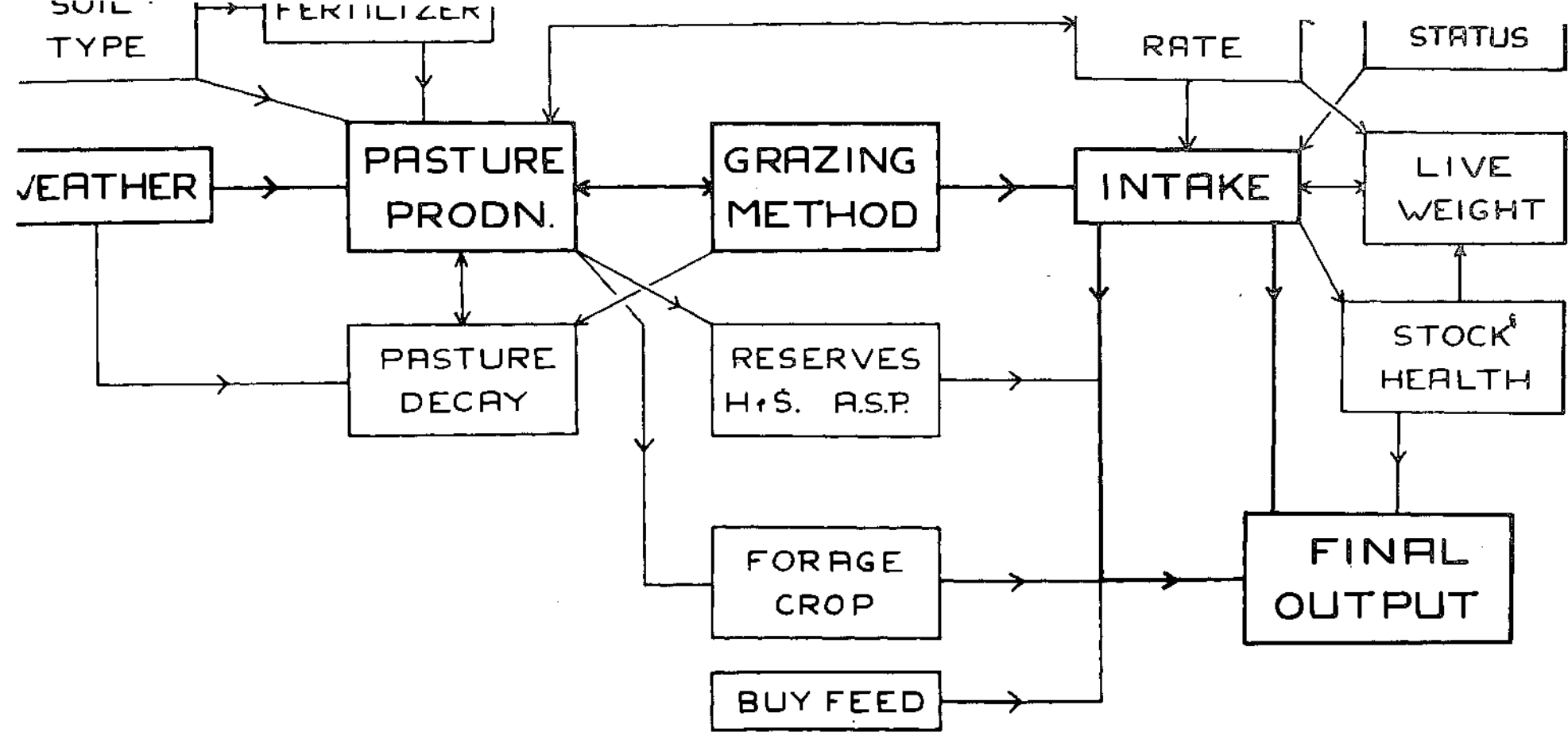


Figure 6.1 Simulation Flow Diagram.

been considered. A dairy cattle situation was chosen because this was the only grazing management situation for which sufficient information of the type required for simulation was available.

The first stage in the process of formulating of a monte carlo simulation of grazing management is the drawing up of a flow diagram or master plan. In this, all the known important variables are specified in a way which indicates the logical order of events and the interactions of these, in the simulation. Such a flow diagram for grazing management expressed is shown in general terms in Figure 6.1.^{1/} This is thought to contain all the essential features of a grazing management system. Actions and interactions are indicated by arrows in one or two directions respectively.

From this diagram submodels and dependant sets can be identified. Obvious submodels are pasture production, intake and livestock output. Each of these can be expressed as a function of several dependant, independant and interdependant variables. The following identities were made.

^{1/} In this diagram, the abbreviation Prodn. Status. means productive status and represents the influence of stock type (lactating or dry, pregnant or not pregnant) age and sex.
H and S and A.S.P. mean hay and silage and autumn saved pasture respectively.

TABLE 6.1

TABLE OF GRAZING MANAGEMENT SIMULATION SUBMODELS

Amount of Pasture Grown	=	f(Pasture species, soil type, weather, fertilizer, state and stage of growth after last grazing.)
Pasture Available:	=	f(Amount grown, decay losses, digestability and nutritive value)
Intake:	=	f(Amount pasture available, amount supplementary feed available, stocking rate, rationing system, productive state of animals, level of production, maintenance ration required, liveweight change, environmental conditions)
Production:	=	f(Intake, food conversion efficiency, liveweight gain or loss, age and health of stock, stage of lactation)

Each of these functions is now discussed. In each case consideration is given to the problems of formulating a simulation model from the data available.

6.3 The Pasture Production Function ^{2/}

The use of data from a particular source (in this instance the Ruakura No. 2 Dairy Project) means that certain restrictions are placed on the type of simulation function which can be formulated. In considering the pasture

^{2/} The pasture production function discussed in this section considers jointly the pasture grown and pasture available functions of Table 6.1.

grown function (as stated in Table 6.1) in particular, the use of data from Ruakura meant that the factors of soil type, fertilizer application and pasture composition were automatically determined and constant. As well the fact that pasture production estimates had been made at all meant that a stage and state of growth at each cutting was implicit in the data. Effectively then, the only real variable which could be included in a pasture production simulation function was weather.

In considering the pasture production function there are thus three points to be considered: the form and amount of weather information to be used, the amount, type, and usefulness of the pasture production information, and the determination of the availability of the pasture to the grazing animals.

6.3.1 Weather

Advice was sought from members of the Plant Physiology Division of New Zealand Department of Scientific and Industrial Research on the significance of the various climatic influences recorded on pasture growth. It was proposed that these should be used on the variables in a simulation function relating pasture weather conditions. Two points of view were expressed. Firstly, it was suggested that the function most likely to provide an accurate and comprehensive prediction of pasture growth using weather information, was one which employed as many weather parameters as possible. The following parameters were therefore recommended.

Mean Temperature

Maximum Temperature

Minimum Temperature

Evaporation

Wind Run

A measure of soil moisture status.

An index of the quantity of light.

Soil Temperature at 4" and 8".

Rainfall

Rain Days

In conflict with this point of view was a second which emphasized that each of these observations was only one index of the total environment in which the plants were growing. As such, each observation is related to the others. Each measures a different aspect of the same thing - weather. This lack of independence amongst the recorded weather variables is well evidenced in the case of the four temperature measurements suggested above. Each is related to the other. Equally, but perhaps not so obviously evaporation is correlated with wind run, soil moisture, rainfall and temperature.

This fact has two implications in Monte Carlo Simulation.

- (1) If a multiple regression model is to be used to represent the functional relationship involved between weather and pasture production, one of the assumptions of this model is violated by using this data. The assumption is that the independent variables of the regression are independent, and those suggested are not.
- (2) If a multiple regression model is fitted in spite of the assumptions of the model being violated, conditional probability functions would be required for random sampling the parameters involved. For the number of parameters suggested above the formulation of these would present a task of immense proportions. Even if the number of parameters was reduced to three; temperature moisture and light, the task would not be small. As well it is not clearly established that there is a

need to use as many parameters as suggested for prediction equations in a simulation. It is quite possible that a single parameter for weather may be sufficient. This will not explain all the variance involved in reality but in a monte carlo simulation this is not necessary. Equation error variances can be used to include "unexplained" variation. A measure of this is included in the simulation by random sampling the equation error variance.

In view of these considerations, it was decided that a single parameter, temperature, would be used to represent weather in the weather-pasture production function.

6.3.2 Pasture Production

Related closely to this problem of measuring and expressing the environment adequately is the problem of measuring pasture growth under grazing conditions. For this situation there is no fully satisfactory method and constructive suggestions are difficult to formulate. Discussion here relates mainly to the methods currently in use and value of these for simulation purposes.

Predominant amongst pasture measurement problems is the fact that the pasture is grown for grazing animals. This means that simple growth effects in pasture, which might quite adequately be measured in terms of pounds of dry matter, are confounded by the fact that the only pasture that is of any value is that which is palatable, digestible and preferably of high nutritive value to the animal. As yet none of these latter attributes of pasture have been satisfactorily measured without the introduction of the animal. These three attributes can, however, be assessed without actually applying the animal to the pasture. That is, by indoor feeding trials. This however eliminates important animal influences on the pasture. These are treading,

the influence of excreta, selective grazing, and grazing pressure (extent and severity of defoliation). These are known to have substantial effects in pasture growth. The measurement of pasture production in the absence of livestock is therefore not a true measure of growth for grazing management studies. Animals must be included and when this is done the pasture measurements recorded are a function of the animal factors: appetite, treading, return of dung and urine, selective grazing, and the management factors of stocking rate and length and frequency of grazing. Much as it is desirable to separate plant and animal effects this does not appear possible if one insists on measuring pasture grown under grazing conditions.

Several methods have been devised for measuring pasture growth under grazing conditions (21). Most of these involve the placement of protective cages within the grazing area. Pasture grown within the covered area is harvested at suitable intervals with a mower or other suitable cutting tool. Pretrimming of the covered area is practiced with some methods.

A method of this type was employed in obtaining the pasture production data recorded on the No. 2 Dairy Project at Ruakura (3). Two replicates of approximately one acre, each grazing one average cow from the herd on the particular treatment, were employed. Five cages were deployed randomly within each area. The enclosed areas were cut at 14 day intervals and the cages relocated randomly thereafter. There was no pretrimming of cage areas. Instead a second area outside each cage, but similar to the caged area was harvested at each sampling date. This method provides a means of measuring as nearly as possible the weight of dry matter produced in the areas in a particular time period. The method does however have the following limitations -

(a) The method gives a measure of net change in the amount of pasture

available over the period. If the rate of decay (due to slugs, worms parasite and saprophytic fungal growth) exceeds the rate of growth, as was found in some months, a negative net growth figure is registered. Whether such a figure is realistic measurement or not, is a point of conjecture. The pasture may decay in the cage, but outside it may well be eaten by the livestock present, particularly under conditions of continuous grazing (set stocking).

- (b) Strictly speaking, data obtained in this manner is only applicable to grazing situations of exactly the same type as those in the trial. This confines the applicability of the simulation equations based on this data to set stocked areas grazed at a stocking rate of about one milking cow/acre. Since one of the objectives of simulation is to investigate the influence of changes in such factors as stocking rate this data is inadequate for this purpose.
- (c) Grazing livestock are not concerned with pasture dry matter per se. Thirty pounds of dry matter as rank dried up summer "top" is not the same productive ration for a cow as thirty pounds of dry matter as lush spring or autumn growth. Therefore in order to simulate at all adequately, some measure of pasture quality must be included in the observations. There were no pasture quality observations recorded on the No. 2 Dairy Project. In lieu of this, pasture quality observations for the simulation of grazing management have been taken from another Ruakura trial.

6.4 The Animal Intake Function

The pasture intake of grazing dairy cattle is determined by a balance

of two groups of factors: the pasture available and the animal requirements. Factors influencing the pasture available include the amount of pasture grown, the rate of pasture decay, the feed rationing system, the stocking rate and the amount, if any, of supplementary feeding. The factors influencing animal requirements are the number of stock present, their productive status, their level of production, maintenance requirements, liveweight change and health.

6.4.1 Factors involved in the Intake Function

Consideration is given first to the factors influencing the feed available side of this balance. From a simulation point of view the amount of pasture grown is determined by the pasture production function discussed in the previous section. The other factors, (feed rationing system, stocking rate and the amount of supplementary feed fed) are determined by specific grazing management decisions. For any one grazing management system, these are factors which are made explicit by the definition of the system. For example, a grazing management system may be defined as

"Continuous grazing at a stocking rate of one cow per acre, making hay where possible, buying it if necessary, and feeding it when insufficient pasture is available". ^{3/}

This statement defines the factors other than pasture grown, involved in the pasture available side of the balance.

The problem of evaluating the animal requirements side of the balance is more difficult. This side of the balance contains a number of factors which are difficult to measure. In New Zealand attempts to estimate the

^{3/} This definition takes no account of economic considerations. This is in line with the formal assumption that no economic considerations would be taken into account in this study.

requirements of grazing dairy cattle have been made by Wallace (42)(43). The method used was one of partitioning the variance of measured intakes between milk production, liveweight change, and a factor (liveweight to the power 0.73) for body maintenance.

Consideration was given to using the results of this work together with liveweight and milk production data from cows on the No. 2 Dairy Project, for estimating the intake requirements of grazing dairy cattle in a simulation. This however, proved to be an unsatisfactory approach as the estimates available from Wallace's work exhibit considerable variability. In addition the estimates apply only to periods of time in excess of 3 months. For simulation, this was considered unsatisfactory and an alternative was sought.

A solution to the problem of estimating grazing cow feed requirements was found in the use of some empirical data collected by Hutton in a nutrition trial studying the full feeding of dairy cattle (13). The data available from this trial included measurements of intake for fully fed lactating dairy cows. These figures were taken as indicative of the requirements of milking cows and have been used in the formulation of a simulation of a grazing management system.

6.5 The Animal Production Function

Of the factors involved in the animal production function (intake, food conversion efficiency liveweight gain or loss, age and health of stock), intake is the factor which normally exerts the greatest influence on production. This is discussed first.

6.5.1 Intake

The intake of a grazing animal is predominantly pasture. This is grazed selectively by the animal as and when, and in such quantities (within

the limits of the amount available to it) as it may decide for itself. Intake under grazing conditions is thus characterised by the fact that it is an independent and selective animal function. Little is known of why livestock select pasture for ingestion in the manner in which they do. There is therefore a problem of specifying what to measure. Associated with this is a problem of how to measure animal intake, on pasture.

(a) Measures of Intake

Three measures of pasture intake have been proposed and used under New Zealand conditions. Each is subject to limitations and no one measure is fully adequate. The three measures are Dry Matter, Digestible Organic Matter and Energy measures (calories). Of these, dry matter is the most simple to measure but it lacks any measure of nutritive value. In view of this it finds little useful application. Digestible Organic Matter is better as it incorporates a measure of digestibility. This measure has been used extensively by Wallace (42) (43) in dairy cattle nutrition studies. It has a limitation. Digestible Organic Matter has been found by Hutton (13) to exhibit considerable seasonal variation. In view of this he has suggested and employed an energy measurement for intake. The measurement of energy ingested and digested provides detailed information of one nutrient of the ration eaten by dairy cows. As it is generally accepted that New Zealand pastures normally provide a well balanced ration in terms of nutrients, the fact that only one nutrient (energy) is measured is not a significant shortcoming. This measure does however fail to make any measurement of the physical bulk of the pasture ingested and this may be an important factor in the spring feeding of New Zealand dairy cattle (15).

(b) Measurement of Intake

Associated with the problem of what to measure for intake is how to measure the intake of grazing animals. So far only one method has been found at all satisfactory under New Zealand conditions. This is the chromic-oxide - nitrogen indicator method. In this, chromic oxide is fed as an external marker and is used to calculate a feed - faeces ratio. At the same time indigestibility is calculated from a measurement of the nitrogen in the faeces. (20) The two coefficients so obtained allow an estimate of intake under grazing conditions to be made. This is the method employed by Wallace (42) for estimating Digestible Organic Matter intakes of milking cows grazing pasture.

Energy measures of intake have not been made in the field. Indoor feeding is necessary. This requires much labour and extensive facilities and tends to become remote from grazing conditions.

A method has however been devised by Hutton (13) which allows a simulation of outdoor conditions to be made whilst these measurements are being taken. This can conveniently be termed indoor outdoor feeding. Stock are fed indoors, so that measurement of intake can be made, but are muzzled and kept out of doors at all other times. Care is taken to present fresh undamaged herbage to these animals, and sufficient quantity is always offered to allow animals to select their feed in the manner similar to that in which they employ in the field. This method is not ideal but does allow measurement of greater accuracy than the Digestible Organic Matter estimates, to be made.

(c) Intake Data

The intake output function of dairy cattle is composed basically

of two elements. Some measure of each is required for an enumeration of this function. The two elements are:

1. The production achieved on full feeding (to appetite capacity) and
2. The production achieved when intakes are depressed below this full feeding level.

The intake measurements recorded on the No. 2 Dairy Project were in terms of dry matter. These measurements were in the form of estimates of intake made from using a difference cutting technique on the pasture production sample areas. Estimates were made at 14 ± 2 day intervals. The basis of the pasture production trial under which these estimates of intake were made was such that one cow was grazing an area equivalent to the average area grazed by each cow in the main herd of the grazing method treatment. This meant that an individual animal could be identified and its particular input output function followed.

Several features of this information made it unsuitable for simulation. Firstly measurements were in terms of dry matter - not nutritive value, and as such bear little or no relation to the animal's requirement. Production is therefore unlikely to be highly correlated with this index of intake. Secondly estimates based on pasture production difference cutting techniques are subject to considerable limitation, not the least of which is the confounding influence of pasture decay over certain portions of the growing season. Only one animal's records were available in each season and this prevented any allowance of animal variability being included in the simulation.

Finally there was no way of telling from the information available what plane of nutrition this animal had been subject to throughout the season. It is possible that the cow providing the estimates of intake made, was fully fed over the spring and autumn periods, for instance, and severely restricted during a dry summer. Thus, whilst an animal production function may perhaps have been satisfactorily estimated from this data, the level of production to which it applied and also the effects of variation in this level would be unknown. In view of this and the other deficiencies in this data, other sources were sought.

The only trials which have been conducted for the specific purpose of studying production according to level of intake have been two conducted by Hutton. These trials involved full feeding a group of dairy cattle under his "indoor - outdoor" system (15) and a "depression trial" operated under similar conditions (14). The first of these trials provides information on intake, production and liveweight changes by weeks from calving, for fully feed milking dairy cows. The second trial assessed the degree of depression of intake possible before the production of milk was affected. ^{4/} In both cases intake was measured in energy terms. Data from these trials has been used in the simulation of dairy cow grazing management.

6.5.2 Food Conversion Efficiency and Age of Stock

There are two ways of taking these two factors into account in monte carlo simulations.

^{4/} This was found to be 7% on average over the season.

The first of these is by definition. In this case, the type of cows, their ages and food conversion efficiency, are defined at the outset of the simulation. The equations to be used and parameter populations to be sampled are defined as being specifically applicable to a particular group of cows.

The alternative to this is to neglect these factors and allow them to contribute to the error term of the equations used. This second method assumes however that the error due to the age and productive efficiency of the stock is included in the error term used. This need not necessarily be so as one or other or both these variables may have been held constant in the situation from which the equation to be used, was derived. Since this was in fact the case for the information used in this study, the first method of accounting for these variables has been used.

6.5.3 Liveweight

The influence of liveweight on production is not well established. The liveweight changes typical in dairy cattle under New Zealand dairy farming conditions are well known, ^{4/}, but it is not clear whether these are cause or effect in production. It is commonly thought that the early lactation loss of bodyweight contributes to the energy supplies for body maintenance and milk production. This however only occurs over a period of 6 - 8 weeks. Later in the lactation it would appear that this function is reversed. As well there is usually a period in later lactation where liveweight tends to remain relatively constant. In this situation it is

^{4/} The typical pattern of liveweight change for dairy cattle under New Zealand dairy farming conditions is a loss of bodyweight after the first 6 - 8 weeks, a regain of this over 8 - 10 subsequent weeks to reach and hold a fairly static level from the middle to the end of lactation.

difficult to know whether liveweight is a cause or an effect in production.

Liveweight of dairy cattle is difficult to measure accurately. Measurements can be taken but these vary considerably according to gut fill. The assessment of true liveweight changes over short periods is therefore difficult and it is doubtful whether measured changes indicate true changes in bodyweight. Frequent weighings and assessments on an annual or 3 monthly basis give satisfactory estimates of bodyweight changes, but for shorter periods, such as a week or a fortnight, it is doubtful whether liveweight observations are sufficiently accurate to warrant inclusion in a simulation function.

In view of these considerations liveweight has been omitted from the animal production function. This omission however, is only formal. In reality, liveweight is included in the production equation as part of the error term.

6.5.4 Production

The output of dairy cattle takes three forms: milk, true liveweight gain and body maintenance. Milk production is readily measured and can be expressed in terms of a standard fat corrected (4%) milk for the purposes of calculation and comparison. This is the main form of production of interest in a simulation of grazing management.

Bodyweight gain and body maintenance also absorb energy and must also be considered in formulating a simulation. However, in as much as liveweight changes tend to balance out over the season and body maintenance can be assumed to be a continuous and fundamental function of a grazing animal, both these forms of output have been omitted from the simulation production function. Any variation which may occur in these factors can be considered to be represented by the error term in the milk production equation in the

6.6 Summary

In this chapter the grazing management problem has been defined in terms of a dairy farming situation. An overall model for this has been introduced and the main functions involved have been discussed. Problems associated with the data and the explicit formulation of the functions involved have been considered. Of these problems, the measurement of pasture production and animal intake under grazing conditions, are outstanding. Consideration is now given to the formulation of a special model of grazing management.

CHAPTER 7

NUMERICAL ESTIMATION OF A SPECIAL MODEL OF GRAZING MANAGEMENT

7.1 A Special Model

The discussion of the factors involved in a monte carlo simulation of grazing management given in Chapter 6 allows consideration to be given to the formulation of a special monte carlo model for grazing management. This has been developed within the confines of existing information for grazing management.

The special model consists of a description of the farming situation envisaged in the simulation. This includes two equations for simulating pasture production on this farm, a set of decision rules for grazing the pasture grown, an intake - output equation for simulating the milk production of cows grazed, and sundry decision rules for the particular grazing management situation envisaged. For this model three equations are required. Details of the numerical estimation of these are now considered.

7.2 Numerical Estimation of the Pasture Production Function

The pasture production function for the special model was estimated from weather and pasture production information recorded on the No. 2 Dairy Project at Ruakura Animal Research Station.

The mean or the maximum and minimum temperature recorded daily in the standard meteorological Stevenson screen at 4' above the ground, was used as the weather index for this function.

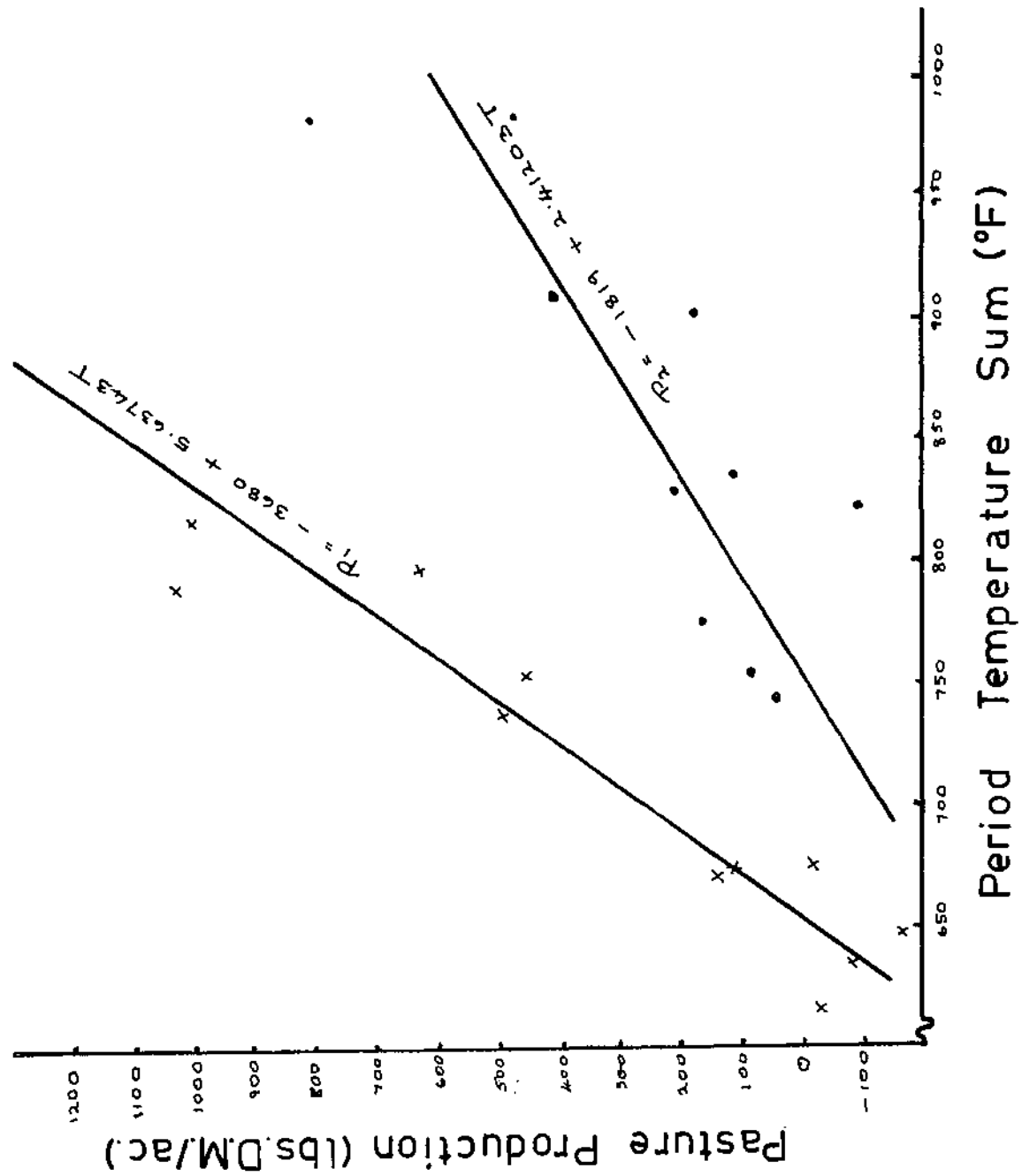


Figure 7.1 Pasture Production 1960/61.

The Uncontrolled Low (UL) treatment measurements of pasture production for the 1960/61 season were selected for the initial investigation of the weather pasture growth relationship. The Uncontrolled Low data was selected because it was the treatment least complicated by hay, silage, and autumn saved pasture practices and the 1960/61 season was selected on the grounds that it was the most recent information available. Pasture measurements on this treatment and in this season were at 14 ± 2 day intervals.

Uneven time intervals between measurements of pasture production present a minor problem initially but a solution to this was found in using cumulative totals of temperature data. These totals were found by summing the maximum and minimum daily temperatures for the number of days in the growing period (including the first and excluding the last, as cutting occurred on the morning of the last day) and averaging these two totals. The data accumulated in this way together with the observations of pasture production are shown in Table 7.1 ^{1/}. This data was graphed and the result of this is shown in Figure 7.1. Initially no pattern was obvious in this data, and a variety of equations to fit the data were tested before a satisfactory conclusion was reached. This conclusion involved the division of the data into two time periods. Once this had been done it was possible to fit two linear equations to the data. These are those shown in Figure 7.1.

One linear equation was fitted to the observations made before the 8th November. Another was fitted to the data for the remainder of the season. ^{2/}

^{1/} Appendix I

^{2/} In doing this one point from the second period (16/1) was omitted as it appeared entirely inconsistent with the pattern of observations for this time of the year.

The resulting equations for the two periods were respectively

$$P_1 = -3680 + 5.63743T ; R^2 = .88657 \quad (7.1) \\ (0.67411)$$

$$P_2 = -1819 + 2.41203T ; R^2 = .65118 \quad (7.2) \\ (0.62595)$$

Where P_1 is the pasture production (in pounds of dry matter per acre) in any 14 ± 2 day period between 7th June and 8 November, P_2 is the pasture production (in pounds of dry matter per acre) in any 14 ± 2 day period between 8th November and 22nd May. T is the mean of the 14 ± 2 day sums of maximum and minimum daily temperatures.

Equation 7.1 explains a significant proportion of the variance at the 99% level of probability. Equation 7.2 explains a significant proportion of the variance at the 95% level. ^{3/}

Having obtained these results two courses of action were open. The first of these was to increase the accuracy of the two equations fitted. This could have been done by including and testing terms of T^2 and higher powers, or by reconsidering a previous decision about the inclusion of other weather parameters with a view to including some of these.

The alternative course of action was to examine data from one or more other seasons to see if this pattern of growth was maintained in different seasons. It appeared that if this were found to be so, a new simulation equation, based on several years data could be formed. Such an equation, it was thought, would form a much stronger basis for simulation. This second line of study was chosen as it was felt that representativeness was more important in a monte carlo simulation (where variance outside the equation can be dealt with by sampling) than accuracy.

Records of pasture production on the No. 2 Dairy project had only been

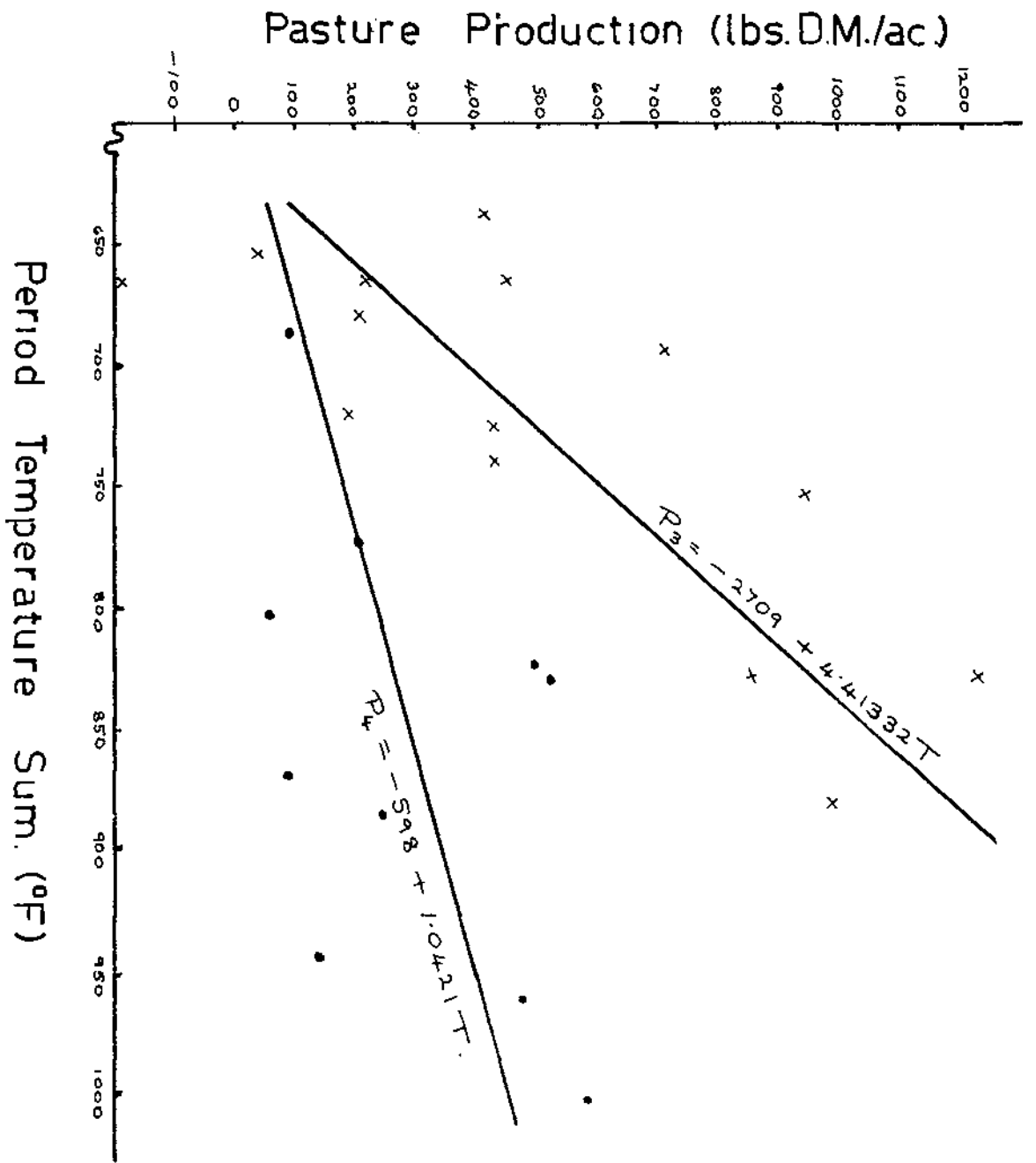


Figure 7.2 Pasture Production 1959/60

made over three seasons and of these only two were strictly comparable for the purpose outlined above. A change of stocking rate between the first and second seasons recorded meant that only the latter two could be studied with a view to forming a joint between years equation. The two comparable seasons were 1959/60 and 1960/61. The 1960/61 data had already been used to derive the equations quoted above. An examination of the 1959/60 data was therefore made.

Raw data for this season's production is recorded in Table 7.2 ^{4/} and was graphed, Figure 7.2. The pattern of growth in this season was not as distinct as in the 1960/61 season but it was nevertheless possible to fit two functions of the type used for the 1960/61 season. In this second season, growth to which the first period function could be fitted continued over an extra cutting period. This meant that, the initial equation fitted in this season, covered a slightly longer period than in the first year. In view of this, it appeared more reasonable to divide the data into two growth periods rather than two arbitrarily fixed calendar dated periods. This explains the occurrence of more points in the first period of the second year data. The first period in this season was taken to be the interval 8th June to 21st December and the second period 21st December to 9th May. Regression equations for these two periods were derived and are respectively

$$P_3 = -2709 + 4.41332T \quad ; \quad R^2 = .68563 \quad (7.3) \\ (0.86120)$$

$$P_4 = -598 + 1.04210T \quad ; \quad R^2 = .23463 \quad (7.4) \\ (0.66435)$$

Where P_3 is the pasture production (in pounds of dry matter per acre)
in any 14 ± 2 day period between 8th June and 21st December
 P_4 is the pasture production (in pounds of dry matter per acre)
in any 14 ± 2 day period between 21st December and 9th May.
 T is the mean of the 14 ± 2 day sums of the maximum and
minimum daily temperatures.

Equation 7.3 explains a significant amount of variance at the 99% level
of probability. Equation 7.4 explains a significant amount of variance at
the 85% level of probability.

Having derived these two equations, the possibility of amalgamating the
two seasons data was examined using an analysis of covariance. The basis
proposed for the amalgamation was that of combining the data of each growth
period of the first season with equivalent growth period of the second season.
That is, the combination of equations 7.1 and 7.3, and 7.2 and 7.4. The
analysis of covariance showed that, in both cases, the two seasons' data could
be combined. ^{5/} This was done and equations 7.5 and 7.6 were fitted.

$$P_s = -3171 + 4.98832T ; R^2 = .76690 \quad (7.5) \\ (0.57260)$$

$$P_r = -1159 + 1.66714T ; R^2 = .41659 \quad (7.6) \\ (0.47410)$$

Where P_s is the pasture production (lbs. DM./ac.) over the months June
to November inclusive

P_r is the pasture production (lbs. DM./ac.) over the months December
to May inclusive.

T is the mean of the 14 ± 2 day sums of the maximum and minimum
daily temperatures in these periods.

Both these equations, 7.5 and 7.6 explain significant amounts of variance in the dependant variable at the 99% level of probability.

These two equations are representative of two years' experience and provide a basis on which to simulate pasture production.

7.3 Numerical Estimation of the Animal Production Function

The animal production function for the special model was estimated from intake and milk production data recorded on Hutton's full feeding nutrition trial at Ruakura Animal Research Station. The following information was available from this trial.

TABLE 7.5
DATA AVAILABLE FROM HUTTON'S FULL - FEEDING
NUTRITION TRIAL

	1960/61	1961/62
No. of Cows Recorded:	6	7
Average Calving Date:	2nd July	1st August
Average Age Cows:	5.0 yrs.	4.3 yrs.
Parameters recorded:	Pounds fat corrected (4%) milk per cow per week. Intake, K cals energy. Liveweight in pounds by weeks. Stage of lactation by weeks.	
Period of recording:	36 weeks from calving for each cow.	

It was decided that the formulation of an animal production simulation function should be made on an individual cow basis. This allows the full

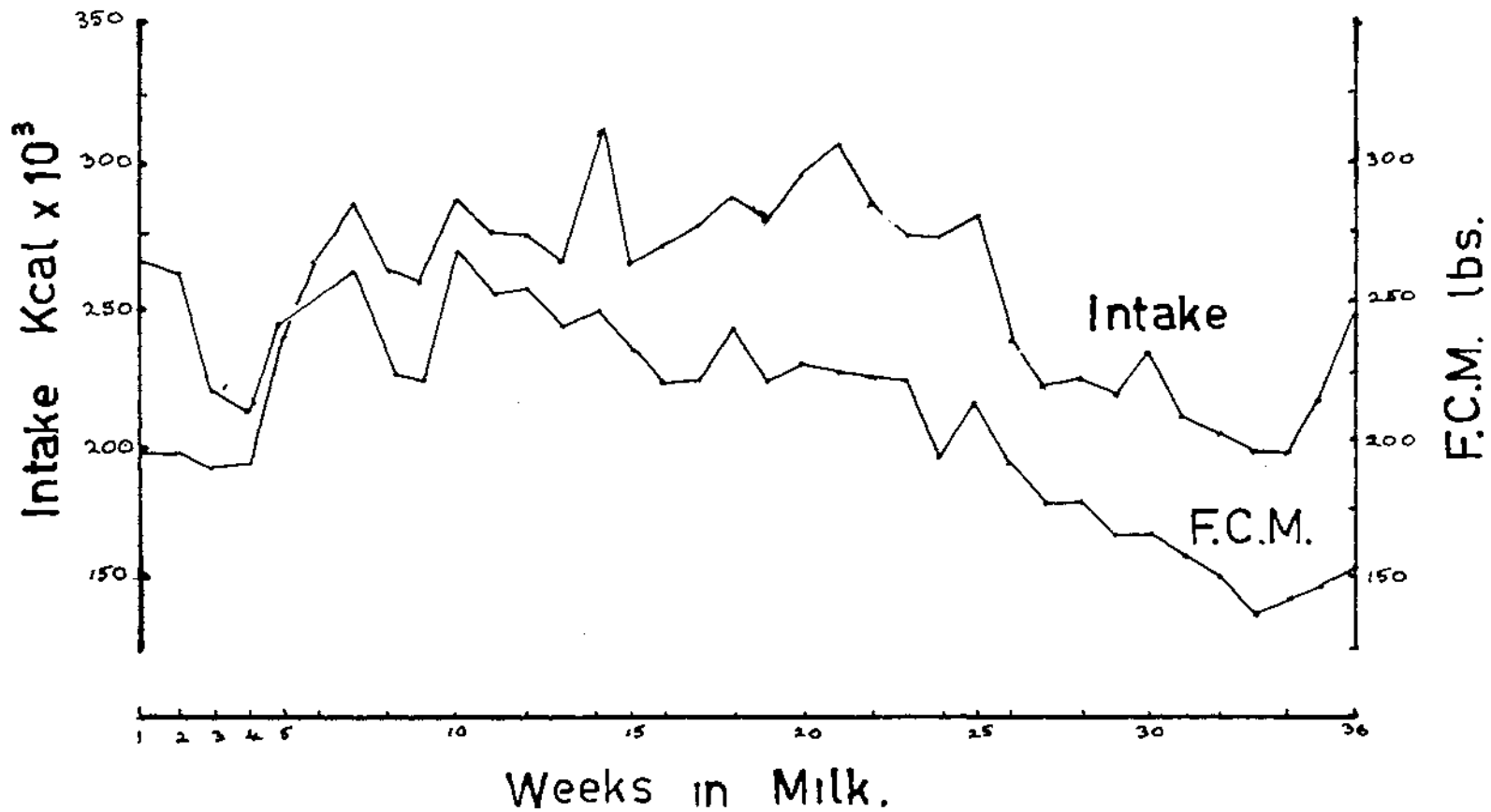


Figure 7.3 Production and Intake by Weeks.

degree of animal variability encountered in practice to be represented in the simulation.

One typical cow, a 5yr old, was selected and the records of this animal intake and production in the 1960/61 season were studied in detail. Intake and production data for this cow are given in table 7.6. ^{6/} A graphic study of this information was made before formulating the simulation function. The graphs plotted for this purpose are shown in Figures 7.3 and 7.4. From these it appeared that intake was probably exerting a linear effect on milk production, and stage of lactation a quadratic effect. In view of this evidence, the following function was fitted to the data

$$Y_t = a + b_1t + b_2t^2 + b_3I_t$$

Where Y_t = Fat corrected pounds of milk produced in period t

I_t = Intake in kilo calories $\times 10^3$ in the period t

t = The time interval from calving in weeks.

The equation estimated was

$$Y_t = + 155 \underset{(0.23167)}{-3.28271t} + \underset{(0.06377)}{0.00708t^2} + 0.4511I_t ; R^2 = 0.92624 \quad (7.7)$$

Each of the independant variables in this equation explains a significant (at the 99% level of probability) amount of the variance occuring in the dependent variable, milk production. ^{7/}

As with the pasture production simulation function it was hoped that the intake output equation could have been better substantiated by data from a further 1 or more years. This however was not possible as records of the type used to derive the equation above were only kept on each animal involved,

^{6/} Appendix II

^{7/} Table 7.6 Appendix II

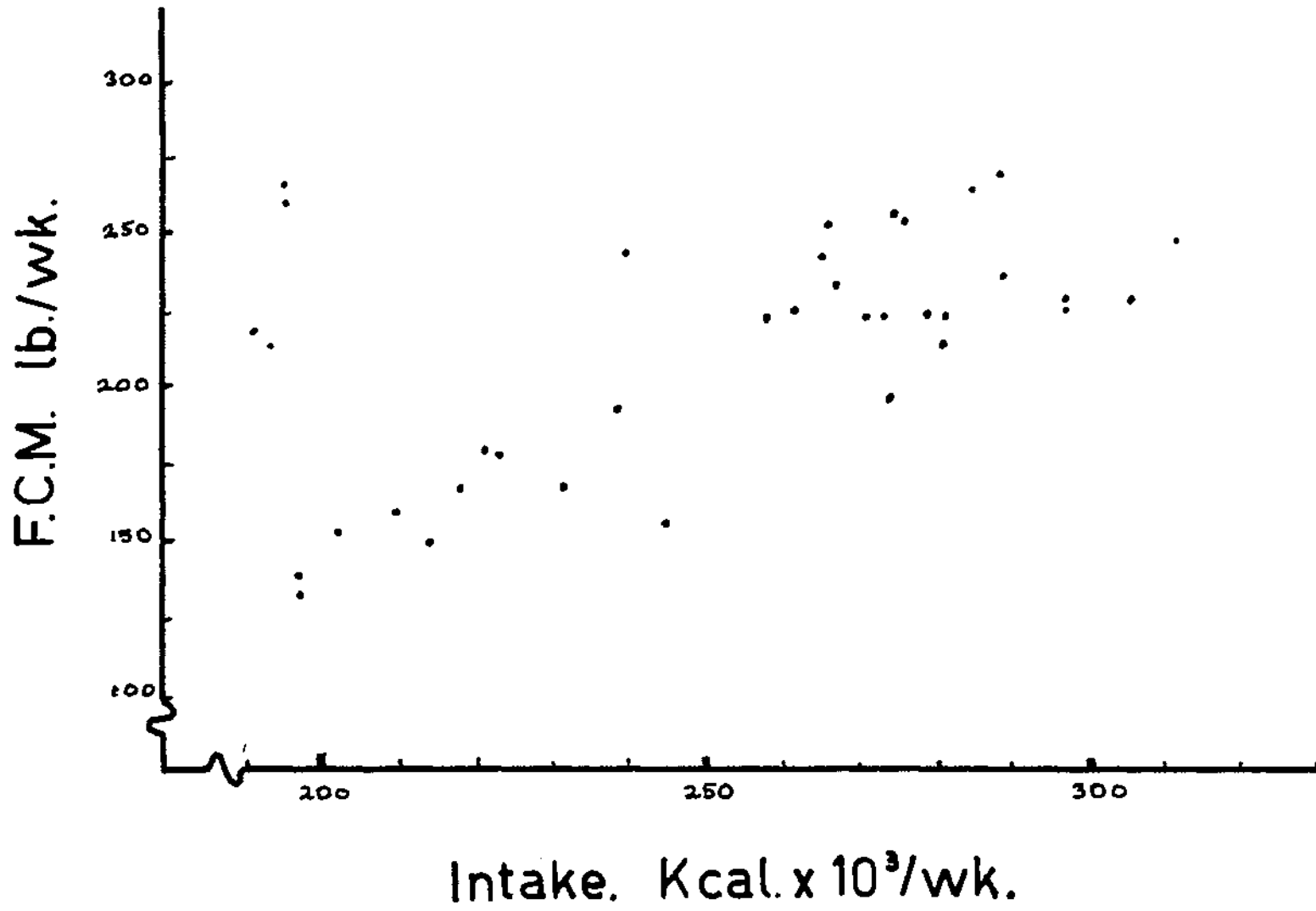


Figure 7.4 Intake / Output Relationship

for one year. Equation 7.7 is used in the special model.

7.4 The Special Model

Using the data available from Ruakura and the equations derived above, it has been possible to formulate and operate a basic monte carlo simulation of grazing management. Because of the several limitations of the data and equations used, this simulation is restricted in its application. It is perhaps best viewed as an illustrative example of the technique of simulation from which certain useful conclusions about simulation and grazing management may be drawn.

The following details give an account of the conditions, both substantiated and assumed, which describe the situation being simulated. These jointly form the special model.

7.4.1 Pastures and Soils

An area of pasture and soil of the types encountered on the No. 2 Dairy Project at Ruakura Animal Research Station Hamilton, New Zealand; which under an annual fertiliser dressing of 2 cwt. of serpentine super might be expected to grow pasture according to the two equations given below is envisaged

$$P_s = -3171 + 4.98832T \quad (7.5) \\ (0.57260)$$

For which the error term (e_1) is taken to be normally distributed with mean zero and variance equal to 43,685.

$$P_r = -1159 + 1.66714T \quad (7.6) \\ (0.47410)$$

For which the error term, (e_2) is taken to be normally distributed with mean zero and variance equal to 32,946. ^{2/}

^{2/} The numerical estimate of variance in each of these cases, is an evaluation of the error variance taken at the mean, for the equations involved. Normality of these variances is assumed.

7.4.2 Livestock Complement, Grazing Method and Stocking Rate

The grazing management plan being simulated provides for dairy cattle to be grazed continuously on this area at the rate of one cow per acre. The animals on the area are taken to be of the type and productive capacity indicated by the input output relationship.

$$Y_t = + 155 - 3.28271t + 0.00708t^2 + 0.45511It \quad (7.7)$$

(0.23167) (0.00581) (0.06377)

For which the error term, e is taken to be normally distributed with a mean of zero and a variance of 8. ^{10/}

7.4.3 Time Specifications

- (a) The simulation starts at calving on 6th July and proceeds for 36 weeks. The 6th July has been chosen as it is this date the original animal input - output data is applicable. Simulation is most likely to be accurate if this date is used.
- (b) The time unit for pasture growth simulations is 14 days. That is, equations are used to predict grass growth over 14 day intervals.
- (c) The time unit for the livestock production simulation is 1 week. The difference in time base for pasture production and livestock production is equalized by assuming the pasture production to be evenly distributed over the 14 days. Equal measures of growth are allotted to the two 7 day animal production periods involved.
- (d) Pasture production 7.5 is taken to apply from calving until the 30th November and thereafter for the remainder of the simulation equation 7.6 is used. This is purely an arbitrary choice of a change over date. It lies approximately halfway between the two dates encountered in deriving the equations.

^{10/} The numerical estimate of variance is an evaluation of the error variance taken at the mean, for the equation specified. Normality of the variance is assumed.

7.4.4 Unit Base

All simulation calculations are made on the basis of 1 acre of grazed pasture. No economies of scale are either implied or explicit.

7.4.5 Decision Rules for Livestock Feeding

In determining the values of intake, I_t , for use in equation 7.7, a grazing management system defined by the following decision rules is used.

TABLE 7.7

LIVESTOCK FEEDING DECISIONS RULES

$$P_{at} \geq 0.93I_{at} ; I_t = I_{at} \quad (1)$$

Feed pasture only.

$$0.66I_{at} \leq P_{at} < 0.93I_{at} ; I_t = P_{at} \quad (2)$$

Feed pasture available

$$P_{at} < 0.66I_{at} ; I_t = P_{at} + S_t \quad (3)$$

Feed pasture available

Supplement with hay to make $I_t = 0.66I_{at}$

Where I_{at} is intake to appetite, in week t as given in

Table 7.6 11/

I_t is the actual intake made in week t.

P_{at} is the pasture available in week t.

S_t is the supplementary feed in week t.

These three decision rules define the way the livestock are fed in this simulation. Rule (1) indicate that so long as the amount of pasture available exceeds 93% of the intake required for full feeding, an intake to

appetite is fed. The allowance for a 7% depression below full feeding before a change in intake is made, in accordance with Hutton's depression trial results (14).

The second rule (2) indicates that so long as the pasture available at any stage of the lactation is greater than two thirds the full feeding requirement, but less 93 percent of it, the pasture available will constitute the intake made. The actual amount available is the amount fed in these circumstances.

When, as in the third situation (3) the pasture available falls below two thirds of that required for full feeding, pasture supplies are considered inadequate. Hay is fed to the extent required to bring the intake level up to two thirds of that required for full feeding.

In defining a grazing management feeding plan of this type, it must be appreciated that some liberty has been taken in the use of equation 7.7. The regression coefficients of this equation are, strictly speaking only applicable to values of $I_t = I_{at}$ for $t = 1, 2, \dots, 36$. Therefore when $I_t \neq I_{at}$ values are inserted in this equation, the assumption that regression coefficient for intake when I_t does not equal I_{at} is the same as I_t equal to I_{at} , is implicit. This may or may not be substantiated in practice. In that such an assumption involves only an extrapolation from a known situation, it may in fact not be unreasonable.

7.4.6 Weather

The population of weather parameters to be sampled in the simulation of pasture production was taken from the records of mean temperatures of the 15 years 1947 - 1962 (including the first and the last years but omitting 1951, for which no records were available.). The long term mean monthly temperatures

and variances were calculated from these records. It was not possible to estimate the 14 day period means required for the pasture production simulation functions satisfactorily from the data available. In consequence monthly means have been used to define the populations to be sampled for temperature observations. Variances of the means were however adjusted for the shorter period. This was achieved by doubling the monthly variance of the mean ^{12/}.

Normality of these temperature distributions is assumed. This assumption is based on the fact that means are used to describe the monthly temperature populations. The distribution of these means tends to normality with increasing numbers of observations (⁸). For 15 observations this approximation is sufficiently accurate to be commensurate with the accuracy of the remainder of the simulation data.

7.4.7 Pasture Composition

The figures for the energy value of pasture and its digestibility at different times of the growing season are taken from unpublished records of the H3;60:61 nutrition trial at Ruakura. ^{13/} This data represents a sample of the conditions which might be expected to occur in the grazing situation being simulated here.

7.4.8 Feed Available

The feed available to the cows in this simulation is calculated as a weekly balance. From pasture production records it appeared reasonable that 500 lb. of dry matter per acre of nutritive value 2020KCal/lb should be "on hand" when the cows calve. This was taken as the starting point for all

^{12/} The temperature data for the different months of the year is given in Table 7.8 of Appendix I.

^{13/} See table 7.9 Appendix II.

availability calculations which then proceeded according to the following equation.

$$A_t = \frac{A_{(t-1)} - C_{(t-1)}}{2} + G_t$$

Where A_t is the amount of feed available in period t

A_{t-1} is the amount of feed available in period $(t-1)$

C_{t-1} is the amount of feed consumed in period $(t-1)$

G_t is the amount of feed grown in period t .

The remainder

$$A_{(t-1)} - C_{(t-1)}$$

at the end of each period is divided by two because some allowance for waste of the unused pasture must be made. Without specific factual information on this aspect of grazing it is difficult to know the exact degree of the wastage. Fifty percent is taken here to cover losses from treading, lying and loss under excreta. Decay, as previously mentioned, is covered in the estimation of pasture production.

These details then define the special simulation model for grazing management. Its operation can now be considered.

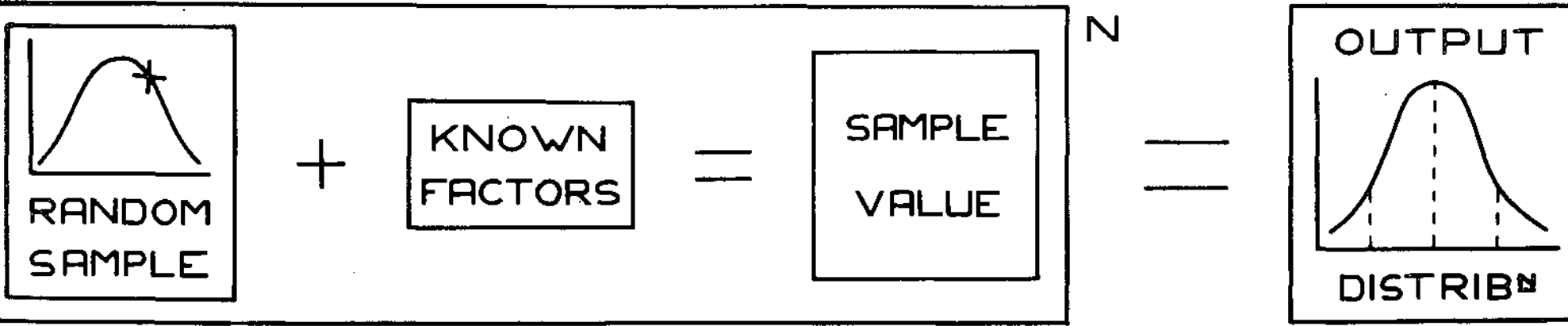


Figure 8.1 Simulation Process

CHAPTER 8

SIMULATION OF A SINGLE SEASON

In this chapter, the simulation of a single "season" using the special model defined in Chapter 7, is described. This is followed in the next chapter by a summary and discussion of results from the simulation of five such "seasons".

The process of operating or running a monte carlo simulation is shown diagrammatically in Figure 8.1. Random sample values, in this case temperature and equation deviates, are combined with the known factors, pasture growth and animal response functions, to provide a sample estimate of the outcome, pounds of butterfat, achieved under the conditions of the simulation. Because the sample outcome is only one of many which might have resulted from random sampling, these calculations of a sample value for output must be repeated. This explains the large N outside the bracket round the left hand side component of Figure 8.1. A series of N such sample estimates allow distribution function of outcome to be plotted. From this, conclusions about the most likely outcomes and the overall variance of the system can be made.

The actual steps involved in this process for a grazing management simulation under the conditions of the special model are as follows.

Within each time period of the simulation, pasture production is first predicted. To do this, a random sample value of the temperature is drawn from the population defined for the period. This value is inserted in the appropriate pasture production equation, and an estimate of pasture production is made by solving the equation. The error variance of the equation is then random sampled. The sample of variance obtained is added to the predicted (from the equation) pasture production. The result is an estimate of pasture production for the

period.

Pasture available (pasture grown in the period under consideration plus that remaining at the end of the preceeding time period) is then calculated. This estimate is then compared with the animal requirement figure for the period. An estimate of the "feed" balance is thus obtained. The feeding decision rule appropriate to this situation is invoked. This results in an estimate of intake being made. The intake estimate is inserted in the animal production equation which is then solved. The solution of this equation together with a random sample value of its error variance give an estimate of milk production for the time period under consideration.

These steps are repeated for each and every time period in the simulation. When production estimates for all the time periods have been made, these are summed. The estimate so provided is an estimate of one "season's" production. Iteration of this process for several "seasons" allows conclusions to be drawn about the overall grazing management system.

These processes will now be elaborated in greater detail.

8.1 Derivation of Random Numbers and Random Variates

The simulation of grazing management using the special monte carlo model involves random sampling on three occasions. These occasions are the determination of the temperature index to represent weather in the pasture production function, the determination of a sample of error variance for the pasture production function, and the determination of a sample of error variance for the animal production function. In each case, the variance to be sampled has been defined as being normally distributed. Random normal numbers are therefore used to sample these variances. ^{1/}

^{1/} This is the method described in Chapter 4 section 2.1. Random normal numbers for this phase of simulation have been drawn from the tables of random normal numbers given by Massey and Dixon (28).

An example of a table of these numbers is given in Table 8.1.^{2/} The method involves the following steps.

In each case, the variance to be sampled is first identified. A random normal number is then drawn from a table of these numbers. The number drawn is used to calculate a random sample value of the variance under consideration. This is done by multiplying the random normal number drawn, and the standard deviation of the variance under consideration, together. The result is a random standard deviate (deviation from the mean). This is then added or subtracted, according to sign, to or from the mean of the population being sampled.

For example, consider the sampling of the temperature index to obtain a value for inclusion in a pasture production equation in the simulation. Suppose the population to be sampled is defined as being normally distributed with mean 46.7 and variance 4. To random sample this population, a random normal number is first drawn from, for instance, Table 8.1. Suppose the number drawn is the eleventh number of Column 17, -0.527 of this table, the standard deviation of the variance for the temperature index is then multiplied by this value

$$2 \times -0.527 = -1.054$$

This figure, (-1.054) is a random deviate of the temperature population. This is then added to the mean value 46.7, to give 46.6, which is the random variate for temperature in this case.

The random sampling of the error variance of a simulation equation is achieved in the same way.

For example, consider the case where milk production has been estimated using equation 7.7. An estimate of milk production given by this equation may, for instance be 241 pounds of fat corrected milk. The error variance

for Equation 7.7 is 64. To sample this, a random normal number is first drawn from, Table 8.1. This may, for example, be the fifteenth number in Column 14, + 1.063 of that table. The sample of error variance for equation 7.7 then is, the standard deviation of the error variance multiplied by this number.

$$8 \times 1.063 = 8.504$$

This new value, (8.504), is a random deviate of the error variance. This is rounded to a whole number and added onto the value for milk production estimated from the equation.

$$241 + 9 = 250 \text{ lbs. F.C.M.}$$

This estimate, 250lbs. F.C.M., is then the monte carlo simulation estimate of milk production for the period under consideration.

These two examples indicate the procedures for random sampling used in operating the special model. The remaining details of the simulation process are now considered. Where necessary the discussion is illustrated with examples taken from the simulation of "Season" One, which is given in its entirety in Table 8.2. ^{3/}

8.2 Simulation of Pasture Production

The simulation of pasture production is conducted on the basis of a 14 day growing period. Within any one of these intervals, the process involves the following steps

- (a) Selection of a random variate of temperature by random sampling.
- (b) The calculation of the amount of pasture grown in the 14 day interval by solving the appropriate pasture production equation.
- (c) Addition or subtraction from the predicted value of a random sample of error variance applicable to the pasture production equation used.

- (d) Division of the pasture production predicted at the end of stage (c) into two equal parts, and the allotment of these two amounts of pasture growth to the two weeks comprising the 14 day period.
- (e) The expression of the seven day pasture grown figures in terms of energy.
- (f) The calculation of the pasture available to livestock.

These steps are illustrated numerically by the calculations involved in estimating the pasture grown in the first 14 day period (30th June to 13th July) of the simulation of "Season" One.

For this period the mean daily temperature population to be sampled is normal with a mean of 46.7 and variance 4.

- (a) The random normal number, -0.527, was drawn and yields, when used in the manner described in the section 8.1, a random variate for temperature equal to 46.6. This value is assumed to occur on all 14 days in the period, thus giving a total temperature observation for the period of $14 \times 46.6 = 638.4$.
- (b) The pasture production equation for this period is equation 7.5.

$$P_g = -3171 + \frac{4.98832T}{(0.57260)} ; R^2 = .76690 \quad (7.5)$$

The value (638.4) for T is inserted in this equation which is then solved. The pasture production predicted is:

$$\begin{aligned} P_g &= -3171 + (4.98832 \times 638.4) \\ &= 131\text{lb of dry matter per acre.} \end{aligned}$$

- (c) The random normal number, -0.129, was drawn. This is used to calculate the proportion of error variance to be added or subtracted to the solution of equation 7.5. For equation 7.5, the error variance is 43,685. The random standard deviate for P_g is therefore

$$-0.129 \times \sqrt{43,685} = -27 \text{ lbs of dry matter}$$

This is subtracted from P_s , the pasture growth predicted using equation 7.5.

$$13 - 27 = -14 \text{ lbs of dry matter.}$$

The pasture grown in this period is thus simulated as being -14lbs of dry matter per acre. ^{4/}

- (d) The 14 day growth estimate above is divided equally into two seven day amounts of -7lbs of dry matter each. These amounts are allotted to the separate weeks ending 6th July and 13th July respectively.
- (e) Considering only one of these two weeks, the second, the factor for converting dry matter to energy is 2041 Kilo calories per pound. ^{5/} Minus seven pounds of dry matter is thus equivalent to minus 14287 Kilo calories or $-14 \text{ Kcal.} \times 10^3$. This figure ($-14 \text{ Kcal.} \times 10^3$) is the simulation estimate of the pasture produced in the 7 day period 6th - 13th July.
- (f) The pasture remaining at the end of the previous week, ^{6/} was 1040 Kcal. $\times 10^3$. The pasture grown in the week 6th - 13th July is added to this, giving 1026 Kcals. $\times 10^3$ of available pasture. This last figure is the pasture available to livestock in the week 6th - 13th July of the simulation. The utilization of this can now be considered.

^{4/} This is interpreted as meaning that during this particular 14 day period pasture decayed faster than it grew. This is an allowable interpretation as the situation from which equation 7.5 was derived included this circumstance.

^{5/} Table 7.9 Appendix I

^{6/} In this case, at the start of the simulation

8.3 Simulation of Intake

The simulation of intake involves evaluating the balance between pasture available and animal requirements. When this has been done the appropriate decision rule for livestock feeding, is invoked. The intake estimated in this way is expressed in terms of gross energy. This must be converted to digestible energy terms for use in the animal production equation. This conversion is made by multiplying the gross energy figure by the digestibility percent. The estimate of digestible energy so derived is the intake estimate used in the simulation of animal production.

The example used in the previous section is continued. Pasture available in week one of the simulation has been estimated at 1026×10^3 Kcal. The livestock present require 259×10^3 Kcal. in this week. ^{7/} The balance is therefore in favour of the cows being fully fed from pasture. An amount of pasture equivalent to 259×10^3 Kcal. is therefore deducted from the 1026×10^3 Kcal. of pasture available, as the intake of the cows in the period. The digestibility of this material ^{8/} is 76.01% which means that the digestible intake of the cows is 197×10^3 Kcal. This is the figure to be used in the animal production equation.

The remaining pasture, $(1026 - 259 = 767) \times 10^3$ Kcal., is subject to pugging, treading losses, and general wastage. This is estimated at 50% in accordance with the wastage rule for the special model. In consequence only 383×10^3 Kcal. of pasture go forward to the next time period of the simulation.

^{7/} This is a gross energy estimate. It is derived from the digestible energy figure given for the week in Table 7.6. Appendix II.
Gross Energy = Digestible Energy $\times \frac{1}{\text{Digestibility}}$

^{8/} Table 7.9 Appendix I

8.4 Simulation of Milk Production

The simulation of milk production requires the solving of the milk production equation using the intake information derived from the simulation of intake above and a random sample of the error variance of this equation.

In the previous section an intake of 197×10^3 Kcals., of digestible energy was calculated. This is inserted in the milk production equation

$$Y_t = +155 - 3.28271t + 0.00708t^2 + 0.45511I_t ; R^2 = .92624 \quad (7.7)$$

$(0.23167) \quad (0.00581) \quad (0.06377)$

for week one of the simulation. The equation to be solved is then

$$Y_1 = 155 + (0.45511 \times 197) - (3.28271 \times 1) + (0.00708 \times 1)$$

which on solution yields

$$Y_1 = 241 \text{ lbs of fat corrected milk.}$$

The error variance at this predicted value is 64. The random normal number drawn was -1.560 which means that $(8 \times 1.560) = 13$ lbs of fat corrected milk are to be subtracted from the 241 predicted, to allow for error variance in production. The simulated milk output for week one of the simulation is therefore 228 lbs of fat corrected milk.

8.5 Updating The Simulation to Record a "Season's" Results

The simulation process for week one has been described in detail. This however is only the beginning of the simulation operation. Once the milk production for the first week has been estimated, production in the second week must be studied. The simulation parameters are therefore brought up to date ready for a repetition of the calculations just described. Further random samples are taken and production for the second week is estimated. The records are then again updated and the process continued until production in each of the 36 weeks in the simulation has been estimated. This entire

process for one such simulated season is shown in Table 8.2. ^{9/}

Once the calculations are complete, milk production over the whole year can be summed and expressed as pounds of butter fat. For "Season" One, this estimate is 7398 lbs of fat corrected milk. This is equivalent to 296 lbs of butterfat. Since the simulation is conducted on the basis of one acre of pasture, this is a production of 296 lbs of butter fat per acre.

8.6 Simulation of Five "Seasons"

The estimate of 296 pounds of butterfat achieved in "Season" One constitutes one sample value equivalent to that shown in Box 3 of Figure 8.1. This is only one of many possible outcomes which might have occurred. Before results can be examined in detail iteration of the simulation is necessary. This is to allow for the occurrence and effects of alternative random variates. Iteration, in this study, has been limited to five "seasons". The operation of the simulation model for the four seasons subsequent to that described in detail in this chapter follows exactly the same pattern as that described here, except that different random numbers are used. The derivation of the random numbers used in these remaining four seasons is summarised in Table 8.3. ^{10/}

A simulation of five seasons' grazing management using the special model has been conducted. Consideration is now given to the results of this simulation.

^{9/} Appendix III

^{10/} Appendix III

CHAPTER 9

SUMMARY OF RESULTS FOR FIVE "SEASONS"

In this chapter the results of the five iterations of the special model are presented and discussed.

9.1 Milk Production

The outcomes of the five iterations are summarised in Table 9.1. This presents the butterfat per acre production figures for the five "seasons" simulated.

TABLE 9.1

MILK PRODUCTION

"Season"	Production, Bf/ac.
1	296
2	298
3	291
4	287
5	292
Mean	292.8
Variance	18.8

These results indicate that "on average" a production of 292.8 lbs of butterfat per acre can be achieved under the management system of the special model. This production is not to be the same every year but varies. In considering this variation, normality of the distribution of outcomes can reasonably be assumed since each outcome is the result of a series of

calculations. Knowing this, it can then be said that, when this management system is employed, it can be expected to yield outputs of between 288.5 lb Bf./acre and 297.1 lb. Bf./acre in two thirds of the seasons in which it is employed.

These estimates indicate that the grazing management system simulated yields a comparatively stable output. This occurs in spite of seasonal variability in pasture production (see below).

To a degree, this outcome was to be expected. The simulated feeding plan was such that only a moderate amount of underfeeding was allowed, even when pasture production was low. In view of this, a reasonably stable set of outcomes is to be expected.

9.2 Pasture Production

A review of column E_g in the simulation of "Season" One ^{1/} indicates a seasonal pattern of pasture growth in the simulation. From a period of slow growth in July, production of pasture increased gradually through August and September to high growth rates in the months of October and November. Growth rates then decreased in December and are low for the first half of January. After this there is an increase in production to moderate levels which continue throughout the remainder of the simulation. Similar patterns were observed in the simulation of the other four seasons.

This type of seasonal variability in pasture production is typical of real grazing management situations. Its appearance in the simulation indicates that a reasonable parallel of reality has been achieved.

^{1/} Table 8.2 Appendix III

9.2.1 "Negative" Pasture Production

One anomaly was encountered in the simulation of pasture production. This was the problem of interpreting negative growth estimates in periods when all the pasture available in the previous period has been consumed. Negative growth figures can be taken to represent decay in the pasture provided there is some there to decay. In event of there being no pasture initially, "negative" growth is difficult to interpret. This results from using the normal distribution (with a range from minus to plus infinity) for sampling errors which, in fact, can only be non negative.

As this event occurred infrequently in the "seasons" studied (8 occasions from a possible 180, and these continued to 3 of the 5 seasons) it was decided to treat them as zero pasture production weeks. In the long term, however, the occurrence of these events calls for a reconsideration of the pasture production simulation functions.

9.2.2 Feed Shortages

In each week of each "season" an evaluation of the feed available from pasture is made. In certain weeks of the "seasons" studied, the pasture available fell short of that required for full feeding. The occurrence of these shortages is shown in Table 9.2

In this table, three types of shortage are distinguished: slight, significant, complete. A slight shortage represents the situation where the pasture available is between 66 and 100 percent of the intake required for full feeding. A significant shortage identifies a situation where the pasture available is below 66 percent of full feeding intake requirements and supplementary feeding is required. ^{2/} A complete shortage is a situation where no feed is available in the form of pasture and livestock have to be fed entirely on supplements. The timing and extent of these shortages varies

^{2/} Occasions when a significant feed shortage are encountered in "Season" One are indicated by asterisks in Table 8.2. (Appendix III).

between "seasons".

TABLE 9.2
DISTRIBUTION OF PASTURE FEED SHORTAGES

Season Number	Type of Shortage	Weeks of Occurrence
1	Slight	7, 11, 23, 24, 25.
	Significant	3, 5, 26, 27.
	Complete	
2	Slight	24, 25, 29.
	Significant	3, 6, 7, 30, 31.
	Complete	4, 5, 32, 33.
3	Slight	6.
	Significant	7, 24, 25, 30, 31, 32, 33
	Complete	
4	Slight	23, 29, 30, 36.
	Significant	3, 6, 7, 24, 25, 34, 35.
	Complete	4, 5.
5	Slight	5, 11, 23.
	Significant	
	Complete	32, 33.

9.3 Hay Requirement

When feed shortages occurred in the simulation, the feeding plan used provided for the use of supplementary feeds to be fed where necessary.^{3/}

^{3/} That is when the amount of pasture available is below 66% of the full feeding intake requirement.

The amount of energy required in the form of supplementary feeds for the various "seasons" is shown in the first part of Table 9.3

TABLE 9.3
SUPPLEMENTARY FEED REQUIRED

"Season"	1	2	3	4	5
	72	117	28	62	204
Energy	33	129	106	129	198
Required	61	160	126	160	
	52	12	9	93	
Digestible		31	23	106	
K.Cals.		71	85	46	
x 10 ³		54	83	53	
				22	
				89	
Total	218	574	460	760	402
Gross Energy					
K.Cal x 10 ³	363	956	766	1266	670
Hay (lb)	363	956	766	1266	670
Hay (Bales)	6	14	11	19	10

Each entry in the energy required section of this table represents one occasion on which supplementary feeding was required. ^{4/} These energy

^{4/} The energy values given have been derived using the following formula.

$$E_s = (E_r - E_a)$$

Where E_s is digestible supplementary energy required

E_r is 66% of the total gross energy required for full feeding.

E_a is the gross energy available from pasture
is the pasture digestibility percentage.

requirements have been summed up for each season and expressed as a hay requirement. To do this, a digestibility coefficient of 60% and an energy content of 1000 K.cal x 10³ per lb. has been assumed for the hay. The number of bales of hay required in each "season" is shown in the last line of Table 9.3.

The mean requirement of hay over the five seasons is 12 bales. This estimate has a variance of 23.5. This is "new knowledge" and provides information for calculating the inventory of hay which is required, for this type of grazing system.

No allowance for the making of this hay was made in the running of the simulation. It was assumed that hay was made the previous season or bought and stored till required. The haymaking activity could however have been introduced into the simulation. The method is comparatively simple. When feed starts to build up a decision to set some aside for hay would be taken. The remainder would continue to grow and be grazed in the manner already defined. The closed area would grow according to some different but appropriate function until harvest. Nutritive value, waste, and dry matter assessments could be made and knowledge of the amount of effective feeding value stored, thereby obtained. A preliminary appraisal of the surplus pasture available towards the end of the spring in each of the "seasons" actually studied indicates that there should be little difficulty in using this method to make the grazing management system used in this study, self sufficient in hay.

9.4 Comparison with Small Farm Results

The results of the simulation of the five "seasons" given in table 9.1 are results which might reasonably be anticipated in practice. The "seasons"

simulated are short owing to a restriction on the data available for the animal production equation. When due allowance is made for this, (an extra 5 weeks at 140 lbs of fat corrected milk per week; 28 lbs of butterfat) production figures of about 320 lbs of butterfat per acre are obtained for the seasons simulated. These can be compared with the results of a small farm experiment involving a similar grazing management system.

The Uncontrolled Low treatment of the No. 2 Dairy Grazing Management Stocking Rate Experiment at Ruakura provides the data for this comparison. On this, a similar, but not the same, grazing management system as that of the special model, has been used. (25) Results from this experiment include estimates of production of 370 lbs of butterfat per acre. The variance of these estimates is 568. Comparison of the simulation results with these estimates indicates that the simulation is capable of producing results which are comparable with reality.

Each of the results discussed above illustrates the way a monte carlo simulation produced new information. At the start, there is no information other than the equations describing the pasture and animal production systems, and a sample of data for several populations. Nothing is known of the integration of these factors.

Once the simulation has been completed detailed information is provided on how pasture might be expected to grow; when excess may be anticipated; when shortages are likely to be encountered; what supplementary feed is required and, what output can reasonably be anticipated. This knowledge is, of course implicit in the data but it is not until the model is actually "run", that it is exposed in detail.

CONCLUSIONS

From this study the following conclusions can be drawn.

1. Simulation is a useful and feasible technique for use in the study of grazing management problems.
2. Monte Carlo methods provide a useful way of incorporating the variability typical of agricultural production functions, into a simulation.
3. A simulation provides the opportunity to evaluate existing knowledge on all phases of grazing management and to review the integration of this critically.
4. The possibility of simulating grazing management does not lessen the number of problems involved in this complex function. It does however identify limitations in existing knowledge.
5. The extent to which the simulation of grazing management is possible is limited by the amount of data currently available for grazing management functions.
6. This study has been concerned only with the simulation of grazing management on the basis of existing knowledge. The possibility of simulating grazing management on the basis of hypothetical data has not been examined. This may possibly merit attention.

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APPENDICES

A P P E N D I X

I

PASTURE PRODUCTION

DATA

AND EQUATIONS

TABLE 7.1

PASTURE PRODUCTION/TEMPERATURE DATA
 UNCONTROLLED LOW TREATMENT
 1960/61 SEASON

Date Cut	Period (Days)	Pasture Grown lb.D.M.Net	Sum Max T Pd.	Sum Min T Pd.	Mean Sum Min -Max	
1.6.60						
7.6	6	1	360.8	256.5	308.6	
20.6	13	-80	758.3	515.3	636.8	
4.7	14	-162	774.1	521.0	647.5	
18.7	14	-24	781.8	448.7	615.2	
1.8	14	-14	805.5	547.5	676.4	
15.8	14	142	799.4	542.5	670.9	Period 1
29.8	14	131	829.5	519.3	673.4	
12.9	14	488	863.8	611.4	737.6	
26.9	14	456	859.6	646.0	752.8	
10.10	14	1034	907.4	669.1	788.2	
25.10	15	1008	976.9	658.3	817.6	
8.11	14	629	928.4	667.9	798.1	
21.11	13	175	889.3	660.2	774.7	
5.12*	14	2152	947.7	600.1	783.9	
19.12	14	108	976.7	698.1	837.4	
4.1.61	16	474	1180.6	783.6	982.1	
16.1	12	622	886.5	651.5	769.0	
31.1	15	814	1109.7	852.7	981.2	
13.2	13	202	974.1	685.4	829.7	Period 2
27.2	14	178	1044.4	763.4	903.9	
13.3	14	411	1039.7	777.6	908.6	
27.3**	14	-947	991.2	662.3	826.7	
10.4**	14	722	971.8	613.1	792.4	
24.4	14	-91	957.8	687.0	822.5	
8.5	14	43	895.4	589.6	742.5	
22.5	14	83	893.1	614.7	753.9	
30.5	8	-63	475.6	302.8	389.2	

Where T is temperature

* Inconsistent observation omitted from regression analysis.

**Confounded observations omitted from regression analysis.

Source: Ruakura Pasture Production Measurement Records for the No. 2 Dairy Project, and the official daily weather records for Ruakura Animal Research Station.

TABLE 7.2

PASTURE PRODUCTION/TEMPERATURE DATA
 UNCONTROLLED LOW TREATMENT
 1959/60 SEASON

Date Cut	Period (Days)	Pasture Grown lb.D.M.Net	Sum Max T Pd.	Sum Min T Pd.	Mean Sum Min Max	
1.6.59 8.6	7	9	381.5	219.9	300.7	
22.6	14	-185	788.9	522.9	665.9	
6.7	14	43	780.6	531.7	656.1	
20.7	14	203	785.2	490.9	638.1	
3.8	14	221	779.5	552.0	665.8	
17.8	14	189	830.8	608.3	719.5	
31.8	14	453	812.3	514.4	663.5	
15.9	15	435	918.3	558.4	738.3	Period 1
28.9	13	716	825.6	562.4	694.0	
12.10	14	425	848.7	599.9	724.3	
27.10	15	857	956.1	703.5	829.8	
9.11	13	942	880.1	627.8	753.9	
23.11	14	909	953.0	683.7	818.4	
7.12	14	1224	980.5	678.3	829.4	
21.12	14	995	1015.0	748.3	881.9	
5.1.60	15	480	1105.4	819.2	962.3	
18.1	13	58	942.9	661.3	802.1	
2.2	15	590	1135.9	872.1	1004.0	
15.2	13	533	973.5	689.8	831.6	
29.2	14	142	1071.2	818.8	945.0	Period 2
14.3	14	251	1016.9	754.5	885.7	
28.3	14	503	963.4	690.0	826.7	
11.4	14	211	935.6	606.5	771.0	
26.4	15	96	1016.0	724.1	870.0	
9.5	13	101	840.8	525.6	683.2	
23.5	14	-260	877.7	607.0	742.3	
31.5	8	1	486.3	336.0	411.1	

Where T is temperature

Source: Ruakura Pasture Production Measurement records for the No. 2 Dairy Project and the official weather records for Ruakura Animal Research Station.

THE PASTURE PRODUCTION EQUATIONS

$$P_1 = -3680 + 5.63743T \quad ; \quad R^2 = .88657 \quad (7.1)$$

(0.67411)

$$P_2 = -1819 + 2.41203T \quad ; \quad R^2 = .65118 \quad (7.2)$$

(0.62595)

Where P_1 is the pasture production (in pounds of dry matter per acre)
in any 14 ± 2 day period between 7th June and 8th November,
 P_2 is the pasture production (in pounds of dry matter per acre)
in any 14 ± 2 day period between 8th November and 22nd May.
T is the mean of the 14 ± 2 day sums of maximum and minimum
daily temperatures.

$$P_3 = -2709 + 4.41332T \quad ; \quad R^2 = .68563 \quad (7.3)$$

(0.86120)

$$P_4 = -598 + 1.04210T \quad (7.4)$$

(0.66435)

Where P_3 is the pasture production (in pounds of dry matter per acre)
in any 14 ± 2 day period between 8th June and 21st December,
 P_4 is the pasture production (in pounds of dry matter per acre)
in any 14 ± 2 day period between 21st December and 9th May.
T is the mean of the 14 ± 2 day sums of the maximum and minimum
daily temperatures.

$$P_s = -3171 + 4.98832T \quad ; \quad R^2 = .76690 \quad (7.5)$$

(0.57260)

$$P_r = -1159 + 1.66714T \quad ; \quad R^2 = .41659 \quad (7.6)$$

(0.47410)

Where P_s is the pasture production (lbs. DM/ac.) over the months June
to November inclusive,
 P_r is the pasture production (lbs. DM/ac.) over the months December
to May inclusive
T is the mean of the 14 ± 2 day sums of the maximum and minimum
daily temperatures in these periods.

Note: In deriving these equations the data was coded as

$$Y = P + 200$$

$$X = T - 600$$

The equations given are however applicable to the decoded data.

TABLE 7.3

ANALYSES OF VARIANCE FOR THE PASTURE
PRODUCTION REGRESSIONS

Season	Equation	Source	SS.	df.	MS	F	Signif.
1960/61	7.1	Total	1,802,913	10			
		Regression	1,598,416	1	1,598,416	70	**
		Error.	204,497	9	22,721		
	7.2	Total	613,428	9			
		Regression	339,448	1	339,448	9.9	*
		Error	273,980	8	34,247		
1959/60	7.3	Total	2,245,832	13			
		Regression	1,539,816	1	1,539,816	27	**
		Error	706,016	12	58,834		
	7.4	Total	386,942	9			
		Regression	87,119	1	87,119	3.0	#
		Error	229,823	8	28,727		
Both Seasons	7.5	Total	4,310,496	24			
		Regression	3,305,724	1	3,305,724	76	**
		Error	1,004,772	23	43,685		
	7.6	Total	1,016,502	19			
		Regression	408,559	1	408,559	12	**
		Error	607,943	18	33,774		

** 99% level of probability

* 95% level of probability

85% level of probability

TABLE 7.4.A

ANALYSIS OF COVARIANCE FOR EQUATIONS
7.1 AND 7.2

Source	SS	df.	MS.	F	Signif.
Joint	3,092,164	1	3,092,164	71	**
Diff.	46,071	1	46,071	1,062	NS.
Error.	910,510	21	43,358		
Total	4,310,496	24			
Overall	3,305,729	1	3,305,729	76	**
Regn Error	956,581	22	43,481		
Means	48,086	1	48,086	1.105	NS.

TABLE 7.4.B

ANALYSIS OF COVARIANCE FOR EQUATIONS
7.2 AND 7.4

Source	SS.	df.	MS.	F	Signif.
Joint	419,484	1	419,484	13.2	**
Diff.	70,751	1	70,751	2.22	NS.
Error	510,135	16	31,883		
Total	1,016,502	19			
Overall	423,465	1	423,465	12.4	**
Regn Error	580,886	17	34,170		
Means	12,151	1	12,151	21	NS

TABLE 7.8

TEMPERATURE DATA SUMMARY FOR RUAKURA ANIMAL RESEARCH
STATION

Month	Monthly Mean	Variance of the Monthly Mean	14 Day Variance
January	63.4	5.6	11.3
February	64.8	4.6	9.3
March	61.6	4.3	8.6
April	57.0	7.1	14.4
May	52.9	4.1	8.3
June	48.7	1.8	3.6
July	46.7	1.9	3.9
August	48.7	1.3	2.6
September	51.6	0.9	1.9
October	55.3	2.8	5.6
November	58.2	1.2	2.3
December	61.6	3.7	7.4

Derived from the Official Weather Record Summaries published by
the Meteorological Service of the New Zealand Air Department.

TABLE 7.9

PASTURE COMPOSITION BY WEEKS

Week No.	Calories per Gram	Kcals per pound equivalent	Digestibility %
1	4500	2041	76.01
2	4480	2032	75.34
3	4555	2066	75.43
4	4545	2061	75.04
5	4490	2036	75.30
6	4505	2043	76.25
7	4500	2041	76.11
8	4525	2052	76.22
9	4495	2038	70.67
10	4575	2075	76.62
11	4605	2088	75.36
12	4570	2072	75.45
13	4450	2018	74.78
14	4455	2020	77.95
15	4505	2043	73.23
16	4400	1995	73.27
17	4350	1973	72.30
18	4335	1975	71.84
19	4430	2009	71.12
20	4375	1984	70.49
21	4415	2002	69.67
22	4390	1991	68.40
23	4370	1982	67.82
24	4450	2018	66.79
25	4400	1995	65.89
26	4395	1993	65.26
27	4315	1957	63.99
28	4360	1977	62.84
29	4410	2000	62.73
30	4425	2007	62.21
31	4420	2004	64.04
32	4460	2023	65.36
33	4360	1977	65.53
34	4325	1961	64.80
35	4375	1984	66.33
36	4405	1995	67.93

Week 1 is 6th to 13th July.

Source: Ruakura Animal Research Station Experiment H.3.60.61 records.

A P P E N D I X

II

ANIMAL PRODUCTION

DATA

AND EQUATION

TABLE 7.6

INTAKE AND MILK PRODUCTION DATA FOR RUAKURA COW
NUMBER 972. FOR 36 WEEKS FROM CALVING (6th July 1960)

Weeks in milk.	Intake Dig. Kcal. x 10 ³	Fat Corrected Milk. lbs.
1	197	265
2	195	260
3	191	219
4	194	212
5	240	244
6	266	253
7	285	262
8	262	225
9	259	222
10	289	269
11	277	253
12	276	255
13	266	241
14	312	246
15	268	232
16	271	221
17	280	222
18	289	235
19	282	221
20	297	227
21	307	226
22	287	224
23	274	222
24	274	195
25	281	213
26	240	191
27	222	178
28	223	178
29	218	165
30	233	166
31	211	158
32	203	151
33	197	133
34	197	138
35	215	149
36	246	154

Source: Expt.
H.3.60.61
Ruakura Animal
Research
Station.

THE ANIMAL PRODUCTION EQUATION

$$Y_t = + 155 - 3.28271t + 0.00708t^2 + 0.45511I_t ; R^2 = 0.92624 \quad (7.7)$$

Where Y_t = Fat corrected pounds of milk produced in period t

I_t = Intake in Kilo-calories $\times 10^3$ in the period t.

t = The time interval from calving in weeks.

TABLE 7.6.A

CORRELATION MATRIX. FOR THE ESTIMATION OF EQUATION
7.7

	t	t ²	I _t	Y _t
t	+1.00000	+ .96995	- .14865	- .87134
t ²		+1.00000	- .33885	- .92410
I _t			+1.00000	+ .53352
Y _t				+1.00000

TABLE 7.6.B

ANALYSIS OF VARIANCE FOR REGRESSION EQUATION 7.7
(STANDARDIZED TERMS)

Source	SS.	df.	MS.	F	Signif
Total	1.00000	35			
t	.75921	1	.75921	373	**
t ²	.10529	1	.10529	51	**
Intake	.07252	1	.07252	35	**
Error	.06298	31	.00203		

** Significant at the 99% level of probability.

A P P E N D I X

III

SIMULATION DATA

TABLE 8.1

RANDOM NORMAL NUMBERS

$(\mu = 0 \quad \sigma = 1)$

14	15	16	17	18	19
-0.988	-0.445	0.964	-0.266	-0.322	-1.726
0.090	0.050	0.523	0.016	0.277	1.639
0.973	-0.058	0.150	-0.635	-0.917	0.313
-0.994	-0.807	-1.203	1.163	1.244	1.306
0.731	0.420	0.116	-0.496	-0.037	-2.466
0.233	0.791	-1.017	-0.182	0.926	-1.096
0.746	0.890	0.824	-1.249	-0.806	-0.240
0.990	0.900	-0.837	-1.097	-1.238	0.030
3.282	0.295	-0.416	0.313	0.720	0.007
0.366	-2.654	-1.400	0.212	0.307	-1.145
-0.257	1.120	1.188	-0.527	0.709	0.479
0.178	0.524	-0.672	-0.831	0.758	0.131
-1.033	1.497	-0.661	0.906	1.169	-1.582
1.220	0.471	0.784	-0.719	0.465	1.559
1.063	0.320	1.406	0.701	-0.128	0.518
-0.481	1.521	-1.367	0.609	0.292	0.048
-1.789	-1.211	-0.871	-0.740	0.513	-0.558
0.510	-0.150	0.034	0.054	-0.055	0.639
0.102	-0.939	-1.457	1.766	1.087	-1.275
0.891	1.158	1.041	1.048	-0.324	-0.404

Source: Massey and Dixon. An Introduction to Statistical Analysis.
Appendix 1 p. 372

GLOSSARY OF TERMS USED IN TABLE 8.2

Temp.	Temperature, °F
T.	Random variate of Mean Temperature, °F.
S.D.	Standard Deviation
R.S. 17/3	Random Sample. A random normal number derived from column 17 starting at block 3 of the table of these numbers given by Massey and Dixon (28)
$\sum_{14} T$	14 day sum of the temperature variate
P (14)	Pasture production in pounds of dry matter, over 14 days.
P (7)	Pasture production in pounds of dry matter, over 7 days.
E per lb. D.M.	Energy value in Kilo-calories of one pound of dry matter of pasture.
E_g	Energy value in Kilo-calories $\times 10^3$ of pasture grown.
E_a	Energy value in Kilo-calories $\times 10^3$ of pasture available to the grazing animals.
E_r	Energy in Kilo-calories $\times 10^3$ required to fully feed the animals.
Dig.%	Digestibility percentage of the pasture.
I	Intake, in digestible Kilo-calories $\times 10^3$, made by the animals.
t	time, week of lactation.
\hat{Y}	Milk Production, pounds of fat corrected (4%) milk.
$\hat{\hat{Y}}$	Simulated milk output, pounds of fat corrected (4%) milk.

TABLE 8.2

SIMULATION

Days	Temp. Mean (S.D.)	R.S. 17/3	T	$\sum_{14} T$	P (14)	R.S. 70/4	P (14)	P (7)	E per lb. D.M.	
7										
7	46.7 (2.0)	-0.527	45.6	638.4	13	-0.129	-14	-7	2041	
7								3	2032	
7	14	-0.831	45.0	630.0	-29	0.167	6	3	2066	
3								83	2061	
3	14	0.906	50.1	701.4	327	-0.770	166	83	2036	
3								173	2043	
3	14	-0.719	47.5	665.0	146	0.957	346	173	2041	
3								217	2052	
3	14	-0.701	50.6	708.4	362	0.345	434	217	2038	
3								164	2075	
3	14	0.609	52.4	733.6	488	-0.759	329	165	2081	
3								265	2072	
10	14	-0.740	50.5	707.0	355	0.834	530	265	2018	
10								231	2020	
10	14	0.054	55.4	775.6	697	-1.121	463	232	2043	
10								646	1995	
11	14	1.766	59.5	833.0	984	1.481	1293	647	1973	
11								417	1975	
11	14	1.048	59.7	835.8	998	-0.783	834	417	2009	
11								508	1985	
11	14	-1.190	56.4	789.6	767	1.190	1016	508	2002	
12								65	1991	
12	14	0.758	63.6	890.4	325	-1.077	130	65	1982	
12								183	2018	
12	14	-0.484	60.2	842.8	246	0.663	366	183	1995	
1								76	1993	
1	14	-0.410	62.0	868.0	288	-0.743	153	77	1957	
1								372	1977	
1	14	-1.339	58.8	823.2	213	2.941	745	373	2000	
2								292	2007	
2	14	(64.8) (3.0)	-0.913	62.0	568.0	288	1.643	585	293	2004
2								233	2023	
2	14		-0.848	62.2	870.8	292	0.965	467	234	1977
2								193	1961	
	14		1.475	65.8	921.2	376	0.054	386	193	1984
								190	1995	
	14	61.6 (2.9)	-1.253	57.9	810.6	192	1.041	380	190	1998

TABLE 8.3

SUMMARY OF SOURCES OF RANDOM NORMAL NUMBERS

FOR THE SIMULATION OF "SEASONS"

2 - 5

"Season" Number	Starting point in Massey and Dixon Tables ^{1/} Column, and Block Number		
	Temperature Sample	Pasture Equation Variance Sample	Milk Equation Variance Sample
2	28/1	80/1	03/1
3	73/2	14/1	41/4
4	92/4	74/6	75/3
5	91/5	60/6	37/6

^{1/} Massey and Dixon (28) pp. 371 - 380

Note: Each time sampling is required in the simulation, random numbers are drawn from the tables indicated, starting at the point indicated and working down the column.