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**VARIABLE RATE APPLICATION TECHNOLOGY IN  
THE NEW ZEALAND AERIAL TOPDRESSING  
INDUSTRY**

A thesis presented in partial fulfilment of the requirements of the degree of Doctor of  
Philosophy in Agricultural Engineering  
at Massey University, New Zealand



**Robert Ian Murray**

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## ABSTRACT

Greater use of technology to assist aerial application of fertiliser will be of benefit to the topdressing industry and farmers. Benefits arise through automating the fertiliser flow control system; reducing off target fertiliser application, and managing fertiliser inputs based on the potential outputs of the farmland; thus increasing the profitability of hill country farming systems. A case for technology assisted application is developed by investigating the field performance of conventional and enhanced flow control systems and the effect of variable rate application on hill country pasture production.

A single particle model that predicts flight trajectory from the particle force balance based on the aircraft groundspeed, axial and tangential propeller wash, wind characteristics and particle properties including sphericity was developed. Model predictions were compared to predictions from AGDISP 8.15. Results and trends were similar.

The single particle ballistics model described above was extended to predict the lateral distribution of fertiliser after release from an aircraft. To achieve this, two parameters are important, the transverse flow profile of material leaving the hopper gatebox and the sphericity of the particles. Techniques for measuring these parameters are described and experimental results are presented for superphosphate. These data were used in the model to predict the lateral distribution pattern from a Gippsland Aeronautics 200C for a known discharge mass, which was compared to a measured pattern from the same aircraft for the same discharge mass. Good agreement between the shapes of the two distributions was found. The transverse distribution model provides a practical tool for optimising the design of spreaders, or optimum particle characteristics for a given spreader. It has the ability to predict the distribution profile of any particle size distribution from each, or all, of the spreader ducts.

Culmination of the single particle and transverse distribution models led to the development of a deposition footprint model that was capable of predicting field application within a 25 ha trial site. The deposition footprint model was embedded inside a geographical information system and comparisons were made between the actual and predicted deposition across a series of transect lines. Good agreement was found.

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Following this, a comparison of the predicted field performance between an automated and manual control system were made. Economic benefits for a single application of superphosphate were identified through using automated control, where 10% less fertiliser was applied outside of the application zone when compared to the manually operated system. This equated to a net benefit of NZD \$2800 for a 1500 ha hill country farming system. The value of improving the performance of a topdressing aircraft, on an industry level, was also examined. Cost/benefit analysis between a manual and automated system revealed a benefit of NZD \$111,700 yr<sup>-1</sup> for a single topdressing aircraft using the automated system.

The economic impact of Variable Rate Application Technology (VRAT) is examined, using Limestone Downs as an example. The spatially explicit decision tree modelling technique was used to predict the annual pasture production over the entire Limestone Downs property. The resulting decision tree classes tended to follow the farm's digital elevation model. A series of six different fertiliser application scenarios were developed for comparison to a base line scenario using conventional aerial application techniques. VRAT outperformed the fixed rate applications in terms of pasture production and fertiliser utilisation. Full variable rate application and a model optimised prescription map, produced the highest annual pasture yield. Variable rate techniques were predicted to increase annual production and the spatial variability of that production. An economic analysis of the six production scenarios was undertaken. Farm cash surplus was calculated for each scenario and clearly revealed the benefits of using variable rate application technology. VRAT was found to be the most efficient and highest returning application method per hectare. Additional costs and increased charge-out rates were likely to occur under VRAT; nevertheless, the analysis indicated that significant financial incentives were available to the farmer. A sensitivity analysis revealed that even with a 20% increase in charge-out rate associated with VRAT, the farm's annual cash position varied by only \$4500 (0.4%), suggesting the cost of implementing such a system is not prohibitive and would allow aircraft operators to add value to their services.

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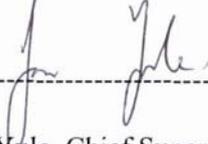
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## STATEMENT REGARDING RESEARCH CONTRIBUTION

The thesis is founded on six papers which have either been published or submitted for publication. The order of authorship reflects the contribution made to the individual pieces of work. In five of the six papers, Robert Murray is the main author. One paper, Chapter 2, has Dr Jim Jones as main author, this reflects his contribution in developing the mathematical model described. Robert Murray understands the model and worked closely with Dr Jones in verifying and testing the core mathematics used in the modelling process.

Signed  Date 18/03/2008.

Associate Professor Ian Yule, Chief Supervisor.

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## NOMENCLATURE

$A$  = area [ $\text{m}^2$ ]

$Arsh$  = inverse hyperbolic sine

$A_{sp}$  = specific area

$C_D$  = drag coefficient

$CV$  = coefficient of variation

$ch$  = hyperbolic cosine

$D_p$  = diameter of particle [m]

$du, dd$  = directly upstream, directly downstream of propeller

$g$  = gravitational acceleration [ $\text{m s}^{-2}$ ]

$H$  = height above ground [m]

$I_a$  = axial flow induction factor

$I_r$  = tangential flow induction factor

$m, n$  = wind constants

$R, r$  = radius of propeller [m]

$Re$  = Reynolds number

$S$  = aircraft planform area [ $\text{m}^2$ ]

$S_p$  = projected area of particle [ $\text{m}^2$ ]

$S_{x,y,z}$  = distance from release point in  $x, y, z$  axis [m]

$sh$  = hyperbolic sine

$SU$  = stock unit

$th$  = hyperbolic tangent

$t$  = time [s]

$U_{x,y,z}$  = velocity components [ $\text{m s}^{-1}$ ]

$|U|$  = slip velocity between particle and air [ $\text{m s}^{-1}$ ]

$V_p$  = volume of particle [ $\text{m}^3$ ]

$x, y, z$  = components along  $x, y$  and  $z$  axis respectively

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$y_{prop}$  = distance from centre of propeller in  $y$  direction

$z_{prop}$  = distance from centre of propeller in  $z$  direction

$\rho_p$  = density of particle [ $\text{kg m}^{-3}$ ]

$\rho_{air}$  = density of air [ $\text{kg m}^{-3}$ ]

$\mu_{air}$  = air viscosity [Pa s]

$\phi$  = particle sphericity [-]

$\Delta$  = incremental change [-]

$k$  = constant

$\alpha$  = integration constant

$\omega$  = angular velocity of the rotating wake at radius  $r$  [ $\text{rad s}^{-1}$ ]

$\Omega$  = the angular velocity of the propeller [ $\text{rad s}^{-1}$ ]

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## CHAPTER 1 - GENERAL INFORMATION

### *1.1 INTRODUCTION*

Precision agriculture (PA) or precision farming technologies have been defined in various ways; however, there remains little deviation from the original concept, presented over two decades ago. Johnson et al. (1983) showed considerable insight in suggesting that future machinery used for production agriculture will be automatically controlled to prescribe cultural practices, based on variation in soil, crop and climate conditions. Adding that, some soil and crop information may be sensed on-the-go and stored in a computer on board the field machine. The computer could be programmed to make real-time decisions, based on this information, and control fertiliser, herbicide and pesticide application, while pinpointing the position of the machine in the field. PA, as we know it today, encompasses an ever widening range of techniques. A major feature of PA is that it allows producers to manage previously unmanaged soil and landscape variability. In this process, several land units, traditionally managed as a single entity, can be addressed individually. Subsequently, managers can respond to the distinctive agronomic characteristics that exist within the sub-units (National Research Council, 1997).

Variable Rate Application Technology (VRAT) is one of the many subdivisions within precision agriculture. VRAT can be loosely defined as the automated adjustment of material flow rate to deliver prescribed quantities of product during field operations. The core components of a variable rate system are: the Geographical Information System (GIS), which is used to determine the prescribed application rate or, in the case of sensors, to gather continuous streams of field or location specific information; the Global Navigation Satellite Systems (GNSS) to provide the exact position within the field; and small, powerful industrial computers to record and correlate position and prescription data, as well as controlling the devices that alter application rate as the machine travels across the field.

Delineation of the required nutrient application for non-uniform land patches is dependent on the crop response to specific patch characteristics and is often difficult to determine from

historic agronomic experiments conducted in uniform conditions. A number of spatially explicit models have been developed for cropping. This has been achieved by linking crop production models to geographical information systems. Examples of this include STICS (Schnebelen et al., 2004), GIS based EPIC (Tan and Shibasaki, 2003), CROPGRO (Paz et al., 2001), and DSSAT (Lal et al., 1993). Spatial yield variability is a complex interaction of many factors, including soil properties, weather, pests, fertility, and management. Crop models are excellent tools to evaluate these complex interactions and provide insight into causes of spatial yield variability (Paz et al., 2001) and variable nutrient demand and provide a vehicle for the development of site-specific nutrient application rates, a feature that has been overlooked in aerial topdressing. Although it is technically possible to vary application rates through manual hopper door control on broad, well defined areas, there may be advantages in automating this process. In order to achieve the appropriate application rate on smaller land management units, an automated delivery system for the aircraft is required. It is well known that VRAT has been used successfully for spray materials for a number of years. Commercial spray variable-rate-control systems for agricultural aircraft are available as add-ons from several GNSS suppliers. However at the time of writing this thesis, a commercial variable-rate-control system for granular fertiliser application from aircraft was not available.

## ***1.2 POTENTIAL USE OF VRAT***

VRAT is important from a number of perspectives; consider Nitrogen (N), which is a highly mobile nutrient in the soil system. Traditional uniform N applications may in many cases result in over- and under supply of crop N demand in various parts of the field due to spatial variability of drainage and crop density. Uniform rates of application may not achieve maximum net returns (Koch et al., 2004). VRAT may provide a solution to this problem; allowing farmers to micro-manage certain areas of their land in order to increase net return and reduce environmental impact. For example, in high yielding areas where high fertility responses are likely, managers may choose to increase nutrient application rates to maximise yield and net return. Additionally, reducing fertiliser input in low yielding areas is advantageous when plant

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production is limited by factors such as, solar radiation or soil physical properties, which can not be controlled by land managers. The economic returns from uniform fertiliser application might be compromised if soil-plant relations are not understood and inputs managed accordingly (Lambert et al., 2006).

Reducing the environmental impact of fertiliser application is a further objective of VRAT. Using these techniques, it may allow us to get closer to the idealised situation, where the majority of nutrients applied are used in crop production, leaving minimal nutrients free to contaminate the environment (Pierce and Nowak, 1999; Ferreyra et al., 2006). Link et al. (2006) suggests that in Europe, environmental laws are being implemented to limit N fertilization on arable land. As a result, the State of Baden-Württemberg (southwest Germany) has passed legislation to further reduce N contamination of groundwater from agricultural sources. In certain regions, farmers are to be paid compensation for reducing N levels below a target threshold value in the soil after harvest.

Koch et al. (2004) investigated the effect of variable- and uniform rate N application on maize production. They found that less total N fertiliser was used with the variable yield goal strategy when compared with the uniform management strategy. Net returns also increased under variable rate and were found to be NZD\$26.39 to NZD\$42.85 ha<sup>-1</sup> higher than uniform N management. When the field was divided into site-specific management zones, they concluded that variable-rate N applications were more profitable than conventional uniform N applications. There are many examples of successful implementation of VRAT in cropping systems, (Fiez et al., 1994; Thrikawala et al., 1998; Thrikawala et al., 1999; Yang et al., 1999; Bronson et al., 2003). Whether these agronomic and financial improvements transfer to pastoral agricultural systems requires further investigation. It is well documented that hill country pasture production is influenced by changes in topography such as field slope or aspect (Lambert et al., 1983; Gillingham et al., 1998; Lambert et al., 2000; Lopez et al., 2003; Dodd et al., 2004). Despite this variation, maintenance phosphorous (P) and sulphur (S) fertiliser inputs to New Zealand hill country remain uniformly applied by “blanket” topdressing techniques. Although much of the previously reported variable rate work is focused on N application, the potential gains that can

be made by variable rate application (VRA), of maintenance fertiliser to New Zealand hill country farming systems, requires investigation in order to quantitatively analyse the effect of VRA, and whether VRAT is feasible and can deliver greater efficiencies, for aerial applied fertiliser.

### **1.3 AREA OF STUDY**

Nutrient inputs to highly variable New Zealand hill country currently involves uniform fertiliser recommendations and blanket application rates via aerial topdressing aircraft. There is potential to use VRAT in topdressing aircraft to fertilise hill country land units with their site specific nutrient requirements. Technological advances such as, GNSS, GIS and high-speed, micro industrial computers and wireless communication can provide a data rich environment within the aircraft supplying information such as: position, heading, ground speed, height above ground, wind speed and direction (via wireless link), rate information and representative particle characteristics. This data could be used to drive real-time deposition software that would control hopper door opening and application rate.

The trajectory of a fertiliser particle, once it has been ejected from an aircraft, can be influenced by a number of factors including wind, particle ejection velocity, particle sphericity, spreader design, mass flow rate and aircraft characteristics. Analysis of these factors and their effect on particle dynamics can be determined by modelling using the single particle approach. This has significant value for defining the minimum specifications required in a variable rate control system; firstly, for calculating shut off distances to sensitive areas for various particles in different operating and environmental conditions, and secondly it serves as an integral step towards deposition footprint modelling. To be successful, a variable rate control system must be able to modify the deposition footprint in real-time depending on the operational and environmental conditions. This is difficult to achieve, as the environment (wind speed and direction, aircraft heading and ground speed, height above ground) in which the agricultural aircraft operates varies continuously over time, or, with changes in topography.

Environmental risk of fertiliser application to permanent and ephemeral streams can be

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reduced by automated control of the deposition footprint. This footprint could be compared to any close proximity waterways and the flight path offset, or application ceased, to avoid that area.

In order to achieve these improvements, we must first determine the current performance of a typical topdressing aircraft and how operational and environmental conditions affect individual fertiliser particles, transverse distribution patterns, the application footprint, and field-scale application variability.

Being able to predict the spatial fertiliser application pattern over a field, or farm, serves two important roles. Firstly, it provides a nutrient input layer for pasture production software being used by the grazing manager and secondly, it provides quality assurance of the application that can be assessed at the completion of the application job. Currently, decision support systems, capable of predicting the variability in aerially applied fertiliser, and the spatial variability in pasture production, are not available. This study aims to provide information to support these developments.

#### ***1.4 SPECIFIC OBJECTIVES***

- a) Review the current techniques for modelling fertiliser particle dynamics from aircraft and develop a fertiliser deposition model to account for varying environmental conditions, aircraft parameters and fertiliser characteristics.
- b) Measure the performance of an agricultural aircraft applying fertiliser to a hill country trial site and collect field data for validating the fertiliser deposition model.
- c) Compare the modelled and measured fertiliser application on the hill-country trial site and develop a method to determine the value of improving application performance.
- d) Review software simulations of annual pasture production that could be used to evaluate the benefits of VRAT.
- e) Develop several application scenarios that compare current application methods to variable rate application techniques and how these influence the agronomics and economics of the farming system.

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## 1.5 STRUCTURE OF THESIS

The organisation of the Chapters follows a logical order in which the research objectives were undertaken. Six of the Chapters have been submitted to both New Zealand and international scientific journals, in order to disseminate the knowledge that was acquired while fulfilling the aims of the study. Each chapter is introduced and discussed briefly under its respective heading below. Each of the papers presented in this thesis is co-authored and represents my contribution and that of my supervisors where appropriate, as is conventional in scientific publications. The intellectual content is considered entirely my own, in all of the papers, except for the derivation of the mathematical calculations used in the single particle ballistic model, presented in Chapter 2. Although I had considerable input in the construction and formulation of the model, the advanced mathematics was completed by Dr Jim Jones and recognition for this was given in the authorship of the paper.

*Chapter 1 – General Information.* Serves as an overview of the topic and provides a broad introduction to the study area. The literature supporting PA and VRAT are briefly reviewed and the aims and direction of the study are presented, resulting in a series of specific objectives that provide a focus for this work.

*Chapter 2 – Modelling Solid Fertiliser Deposition from a Fixed Wing Aircraft – a Ballistics Model* covers the development of a single particle ballistics model from first principles. It also presents a series of cases comparing the model output to previously reported experimental data and the dry material component included within the aircraft spray dispersion model AGDISP (Teske et al., 2003b) .

*Chapter 3 – Modelling Solid Fertiliser Deposition from a Fixed Wing Aircraft – Predicting Lateral Distribution* reviews a number of factors that previous researchers have identified as strongly influencing lateral fertiliser distribution pattern of an agricultural aircraft. This Chapter also presents methods for, and results from, measuring the sphericity of superphosphate and its horizontal flow profile as it leaves the hopper gate, as well as development of a lateral

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distribution model for a Gippsland Aeronautics 200C aircraft.

*Chapter 4 – Modelling Solid Fertiliser Deposition from a Fixed Wing Aircraft – Field Scale Predictions Within a GIS Environment* describes the development and validation of a GIS based deposition model that uses measured aircraft, and environmental data, to compute the spatial distribution of fertiliser over a field or farm. In addition, cost benefit analysis was used to assess the economic impact of improving application techniques.

*Chapter 5 – Developing Variable Rate Application Technology: Modelling Annual Pasture Production on Hill Country* discusses some of the important factors governing annual pasture production, as well as providing commentary on the modelling techniques used by others in attempting to model annual pasture production. This Chapter also describes the application of the decision tree modelling technique used in Chapter 6 for predicting pasture production on a sheep and beef hill-country farm.

*Chapter 6 – Developing Variable Rate Application Technology: Scenario Development and Agronomic Evaluation.* Develops six fertiliser application scenarios for a hill-country case study farm. These scenarios were developed specifically for input into a decision tree model and were compared to a base line scenario which uses the farm's current uniform "blanket" application techniques. The aim is to identify land units that would respond positively to targeted nutrient application, in addition to areas that are likely to have a poor response to fertiliser.

*Chapter 7 – Developing Variable Rate Application Technology: Economic Impact for Farm Owners and Topdressing Operators* presents the economic impact of the six fertiliser spreading scenarios discussed in Chapter 6. Farm operating costs are considered under each of the scenarios and the economic consequences of the approach are calculated. The economic viability of VRAT, compared to blanket application techniques, is evaluated along with the likely cost implications for aircraft operators, if they chose to use a VRAT system.

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*Chapter 8 – Summary* presents the results from this work, together with their contribution to the body of knowledge. Suggested future directions for studies in the field of VRAT and ballistic modelling of granular fertiliser are presented.

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## CHAPTER 2 - MODELLING SOLID FERTILISER DEPOSITION FROM A FIXED WING AIRCRAFT - A BALLISTICS MODEL

*Publication arising from this chapter*

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Jones, J. R., Murray, R. I., Yule, I. J. 2007. Modelling solid fertilizer deposition from a fixed wing aircraft – I. A ballistics model. *Trans. ASAE (under review)*.

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### 2.1 INTRODUCTION

The ability to accurately predict the landing position of granular fertiliser particles is important not only for legislative and environmental reasons but also for achieving the desired application rate over a particular land unit. With the increased use of geographic information systems (GIS) combined with yield mapping techniques it is possible to determine the required fertiliser application rate with reasonable accuracy across a land unit. Therefore, to optimize agricultural production, the spreading equipment should be able to apply the fertiliser at a variable rate. However, current aerial spreading application rates are highly variable due to limited control of fertiliser discharge from the aircraft hopper and lack of predictive tools to estimate the resulting ground distribution, which have considerable economic consequences (Murray and Yule, 2006). This paper addresses one aspect of the greater project to develop a variable rate technology for aircraft application of fertiliser; that is, a ballistics model to predict the trajectory of a single particle leaving the non ram-air spreader device mounted under the aircraft. With this model the value of improving the spreading performance of an agricultural aircraft can be ascertained.

Ballistics models are not new. Much of the previous work is for spray applications, developed for the USDA Forest Service to predict the drift behaviour of pesticides sprayed from aircraft above forests. Teske et al. (2003a) reviews the developments in modelling these aerial sprays. Out of this body of work two experimentally validated models have arisen, the Lagrangian trajectory model called AGDISP (Bilanin et al., 1989) and a Gaussian slanted-plume model, which is combined with AGDISP into a model called FSCBG (Teske et al.,

1993). A number of researchers have attempted to adapt AGDISP for use with granular materials (Walker and Gardisser, 1989; Bansal et al., 1998a, b, c); the main drawback being the lack of information about the velocity and volume of air flow through the spreader device and the resulting particle velocities and ejection angles (Walker and Gardisser, 1989). Bansal et al. (1998b) attempted to overcome this by using computational fluid dynamics (CFD) to model velocity and particle ejection angles for a Transland eleven-duct aerial spreader (Model 23501, Transland Inc., Harbor City, California) and validated this against experimental measurements arriving at a correction factor to explain the difference. The resulting information was used as input to the AGDISP program, however under analysis, the predicted and actual deposition patterns were found to differ and no conclusions were drawn as to the accuracy of the CFD predictions. Optical methods were used by Grift (2001) and Grift et al. (2001) to measure the mass flow of granular fertiliser material in aerial spreader ducts. They found that in a low-density flow regime, the diameter of each particle could be measured individually. For granular fertilisers, the flow was measured with an accuracy of  $\pm 2\%$  for high-density flows and  $\pm 4\%$  for low-density flows. It is important that these measurements be made because the design of spreaders varies considerably. By characterizing the ejection particle size and velocity trajectories, the landing positions of individual fertiliser particles can be better predicted (Grift and Hofstee, 1997; 2002).

Other researchers have also developed single particle ballistics models for fertiliser, some for ground based spreading (Mennel and Reece, 1963; Pitt et al., 1982; Griffis et al., 1983; Olieslagers et al., 1996; Aphale et al., 2003) and others for aerial spreading. The most fundamental aerial spreading work is that of Grift et al. (1997) who developed an analytical solution across a small step integration which, with successive numerical substitution, accurately predicts the vertical fall of individual fertiliser particles in quiescent air. Yates et al. (1973) were the first to study particles falling from an aircraft. They used the drag force balance to predict trajectories as a function of ejection angle and velocity, aircraft groundspeed, and particle size for spherical particles. These predictions rely on accurate estimates of drag coefficient which Law and Collie (1973) determined for a range of spherical and non spherical

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fertiliser particles, obtaining reasonable agreement between experimental and standard drag correlations. Later Bansal et al. (1998b) used the drag force relationships to predict flight trajectories for spherical and non spherical particles assuming turbulent flow and hence a constant drag coefficient, as originally proposed by Mennel and Reece (1963). None of these models included propeller wash, or consider particle-particle interaction in the region of the spreader device. For fixed wing aircraft, the effect of propeller wash on solid particle trajectories has not been experimentally validated, although Teske et al. (2003a) do provide some simple momentum theory calculations which are included in the freely available AGDISP 8.15 software package used for comparison in this paper. It must be noted that for spray drift, the effect of the far wake on dispersion and drift has been fully validated (see Teske et al., 2003a). In this work, propeller wash is arguably less important because particles are large, 1-7 mm, and the position of the spreader device is often near the slipstream boundary of the wash. For smaller particles, the fluctuation velocities in the near wake region of the propeller wash begin to exceed the terminal settling velocities. Other factors are also important: drag depends on the square of slip velocity which, at particle release, is very high; and secondly, as side wind velocities are often small, any radial rotation of the wake is likely to be of a similar magnitude. Therefore this paper, in addition to solving the force balance acting on the particles, provides a simple method for including propeller wash that awaits validation.

## ***2.2 THE MODEL***

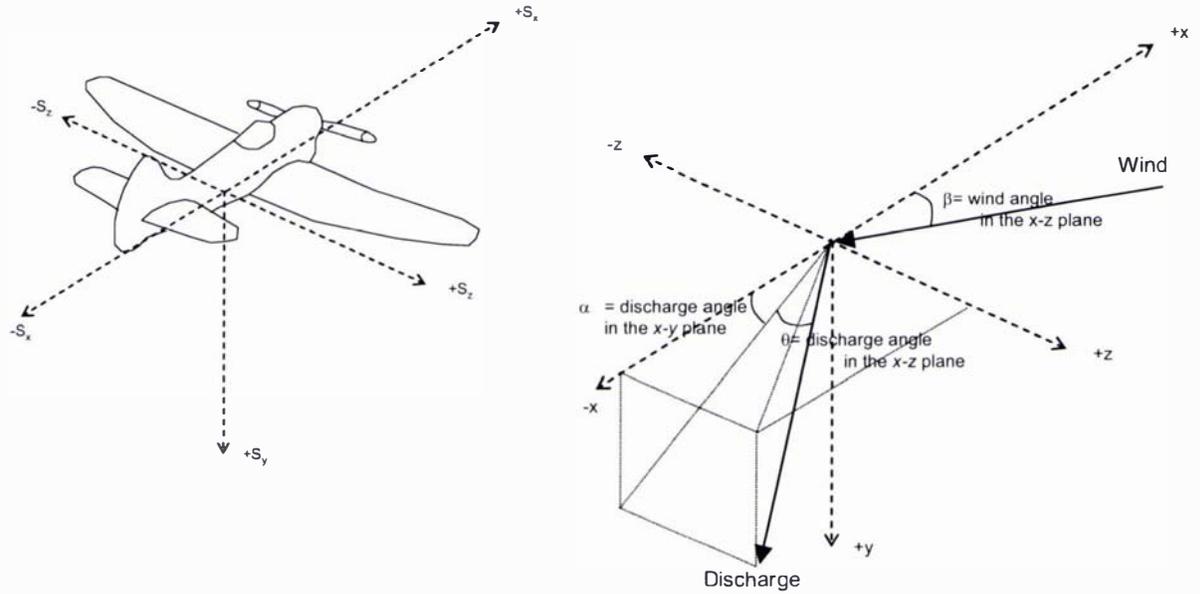
The model presented here uses the same force balances as Bansal et al. (1998b) (although different frames of reference are used) with the addition of a variable drag coefficient, and a propeller wash model. Grift et al. (1997) provide a successive substitution pseudo-analytical solution for the same balance equation (in the vertical direction only), for particle position as a function of fall time using the variable drag coefficient relationships established by Zabeltitz (1967). For each time increment in their numerical solution, they assume the drag coefficient is constant, but then update the drag coefficient as a function of the updated velocity. The same successive substitution pseudo-analytical approach is used here for predicting directional

velocities and positions except the vertical fall position, because integration is only possible by a numerical method. The variable drag coefficient used here is that of Haider and Levenspiel (1989), which is the same as described by Teske et al. (2003a) for use in AGDISP. This has two advantages; first, that the drag coefficient is a continuous function of Reynolds number thus avoiding discontinuity errors that result from the Zabeltitz relationship; and second, that particle sphericity is included. Propeller wash is treated differently here than in Teske et al. (2003). Fertiliser particles are released close to the slipstream boundary and their large size means that they are likely to fall quickly out of the influence of the wash zone. Therefore, the near wake conditions are considered more important than the far wake conditions, which have greater effect on spray drift. In developing this model, we show that the vertical fall equations reduce to those of Grift et al. (1997), then compare our results to those of Bansal et al. (1998b) using their measured discharge velocities. After this, we include aircraft flight speed, wind characteristics and propeller wash to compare to the AGDISP 8.15 model. The following describes our model and methods of solution.

### **2.2.1 FRAMES OF REFERENCE**

Ground referenced Cartesian coordinates are used to describe the location of the particle where release is defined to occur at  $(S_x, S_y, S_z) = (0, 0, 0)$  at time  $t = 0$ . Directions are defined as  $x$ ,  $y$  and  $z$ -directions. The  $x$ -direction is along the flight path of the aircraft and is positive in the direction of flight, the  $y$ -direction is positive vertically towards the ground and the  $z$ -direction is perpendicular to the flight direction and positive on the true right side (starboard side). The discharge velocity of the particle leaving the aircraft is defined relative to the aircraft and has  $x$ ,  $y$  and  $z$  direction components. These components of velocity are a function of the spreader design and the duct exit angles, but are not calculated here. The motion of the particle is determined from a force balance between the particle and the air, which allows calculation of the slip distances between the air and particle every time step. In this respect, the model has a Lagrangian frame of reference, which is defined using the same  $x$ ,  $y$  and  $z$ -directions. This is illustrated in Figure 2.1. The Cartesian position of the particle is then obtained by relating these

slip distances to the particle direction of travel and the other local variables of aircraft speed, wind and propeller wash. The Lagrangian frame slip calculations are developed below for a single time step. Later sections deal with the effect of the local Cartesian variables of flight speed, wind characteristics and propeller wash and their effect on the slip calculations.



- (a) Cartesian frame. Particle release occurs at  $(0, 0, 0)$ . (b) Lagrangian frame with wind and discharge angles (where  $180^\circ$  in both planes is directly behind the aircraft).

Figure 2.1. Coordinate frames of reference.

### 2.2.1.1 Lagrangian frame calculations: component slip velocities and slip distances

At discharge, the slip velocity is calculated from the known aircraft groundspeed, the wind velocity, the position of the particle relative to the propeller, and the calculated propeller axial and tangential wash velocities and the exit velocity from the spreader. Further motion of the particle then depends on the force balance between the particle and air, given by Equations [2.1]-[2.3] for each component direction. The inertial added mass term is neglected.

$$V_p \rho_p \frac{dU_x}{dt} = -\frac{1}{2} \rho_{air} S_p C_{D,x} |U| U_x \quad [2.1]$$

$$V_p \rho_p \frac{dU_z}{dt} = -\frac{1}{2} \rho_{air} S_p C_{D,z} |U| U_z \quad [2.2]$$

$$V_p \rho_p \frac{dU_y}{dt} = V_p g (\rho_p - \rho_{air}) - \frac{1}{2} \rho_{air} S_p C_{D,y} |U| U_y \quad [2.3]$$

The drag coefficient is defined as a function of Reynolds number (Haider and Levenspiel, 1989)

$$C_D = \frac{24}{Re} [1 + [8.1716 \exp(-4.0655\phi)] Re^{(0.0964+0.5565\phi)}] + \frac{73.69 Re \exp(-5.0748\phi)}{Re + 5.378 \exp(6.2122\phi)} \quad [2.4]$$

where

$$|U| = \sqrt{U_x^2 + U_y^2 + U_z^2} = \text{magnitude of slip velocity between particle and air [m s}^{-1}]$$

$$U_x, U_y, U_z = \text{directional components of slip velocity [m s}^{-1}]$$

$$V_p = \text{volume of particle defined as } \frac{\pi}{6} D_p^3 \text{ [m}^3]$$

$$D_p = \text{diameter of particle [m]}$$

$$g = \text{gravitational acceleration [m s}^{-2}]$$

$$t = \text{time [s]}$$

$$\rho_p = \text{density of particle [kg m}^{-3}]$$

$$\rho_{air} = \text{density of air [kg m}^{-3}]$$

$$S_p = \text{projected area of particle defined as } \frac{\pi}{4} D_p^2 \text{ [m}^2]$$

$$Re = \text{Reynolds number} = \frac{D_p U_x \rho_{air}}{\mu_{air}} \text{ or } \frac{D_p U_y \rho_{air}}{\mu_{air}} \text{ or } \frac{D_p U_z \rho_{air}}{\mu_{air}} \text{ [-]}$$

$$\phi = \text{particle sphericity [-]}$$

$$\mu_{air} = \text{air viscosity [Pa s]}$$

These equations assume that particles are of uniform density and do not spin. The above equations cannot be solved analytically because the drag coefficient is a complex function of velocity. However, by examining a plot of  $C_D$  versus  $Re$ , it is clear that the drag coefficient

changes only slowly; at most  $C_D \propto U^{-1}$  and at least  $C_D = \text{constant}$ . Therefore, it can be assumed that  $C_D$  is approximately constant across a small time period,  $\Delta t$ . This allows Equations [2.1] and [2.2] to be integrated over  $\Delta t$  for constant  $C_D$  using initial velocities  $U_{x,t}$  and  $U_{z,t}$  determined from the previous time step calculation. In each integration, it is also necessary to assume that the other velocities, which are lumped into the substitution variables  $c_{x,t}$  and  $c_{z,t}$  as shown below, are also constant over  $\Delta t$ . This gives the new projected  $x$  and  $z$  component slip velocities between the particle and air (Gradshteyn and Ryzhik, 1994; definite integral §2.266)

$$\frac{1}{U_{x,t+\Delta t}} = \frac{1}{c_{x,t}} \operatorname{sh} \left( \operatorname{Arsh} \left( \frac{c_{x,t}}{U_{x,t}} \right) + \Delta t \cdot b_{x,t} c_{x,t} \right) \quad [2.5]$$

$$\frac{1}{U_{z,t+\Delta t}} = \frac{1}{c_{z,t}} \operatorname{sh} \left( \operatorname{Arsh} \left( \frac{c_{z,t}}{U_{z,t}} \right) + \Delta t \cdot b_{z,t} c_{z,t} \right) \quad [2.6]$$

$$\text{where } b_{x,t} = \left( \frac{3\rho_{\text{air}} C_{D,x,t}}{4\rho_p D_p} \right), \quad b_{z,t} = \left( \frac{3\rho_{\text{air}} C_{D,z,t}}{4\rho_p D_p} \right), \quad c_{x,t} = \sqrt{U_{y,t}^2 + U_{z,t}^2} \text{ and}$$

$$c_{z,t} = \sqrt{U_{x,t}^2 + U_{y,t}^2}.$$

As a check, when  $\Delta t \rightarrow 0$ ,  $U_{x,t+\Delta t} \rightarrow U_{x,t}$  as expected. Integrating a second time gives the slip distance travelled by the particle relative to the air,  $\Delta x$ , over time  $\Delta t$  (Gradshteyn and Ryzhik, 1994, definite integral §2.423, eqn 1)

$$\Delta x = \frac{1}{b_{x,t}} \ln \left[ \frac{\operatorname{th} \left( \frac{1}{2} \left( \Delta t \cdot b_{x,t} c_{x,t} + \operatorname{Arsh} \left( \frac{c_{x,t}}{U_{x,t}} \right) \right) \right)}{\operatorname{th} \left( \frac{1}{2} \operatorname{Arsh} \left( \frac{c_{x,t}}{U_{x,t}} \right) \right)} \right] \quad [2.7]$$

$$\Delta z = \frac{1}{b_{z,t}} \ln \left[ \frac{\operatorname{th} \left( \frac{1}{2} \left( \Delta t \cdot b_{z,t} c_{z,t} + \operatorname{Arsh} \left( \frac{c_{z,t}}{U_{z,t}} \right) \right) \right)}{\operatorname{th} \left( \frac{1}{2} \operatorname{Arsh} \left( \frac{c_{z,t}}{U_{z,t}} \right) \right)} \right] \quad [2.8]$$

As a check, when  $\Delta t \rightarrow 0$ ,  $\Delta x \rightarrow 0$  and  $\Delta z \rightarrow 0$  as expected. Each successive time step uses the previously calculated values of  $U_{x,t}$ ,  $U_{z,t}$ ,  $h_{x,t}$ ,  $h_{z,t}$ ,  $c_{x,t}$  and  $c_{z,t}$  although local Cartesian velocity gradients do affect these and must be included, as discussed later. Care must be taken in the model when predicting the slip velocities and slip distance in the event that  $c_x = 0$  or  $U_{x,0} = 0$ , or  $c_z = 0$  or  $U_{z,0} = 0$ . In these cases the equations have a singularity point in the form written, but this can be avoided by rewriting the hyperbolic *tanh* and *sh* terms in exponential form and rearranging. However, these cases also represent elementary analytical solutions which can replace Equations [2.5-2.8] when they occur.

In the  $y$  direction, gravity influences the force balance. Equation [2.3] can be rearranged into the form

$$\frac{dU_y}{dt} - a + b_y |U| U_y = 0 \quad [2.9]$$

where  $a = \frac{g(\rho_p - \rho_{air})}{\rho_p}$  is a constant and  $b_y = \frac{\rho_{air} S_p C_{D,y}}{2V_p \rho_p}$  can be assumed constant over

a small time period,  $\Delta t$ . By using the substitution variable  $c_y^2 = U_x^2 + U_z^2$  this equation can be separated to

$$dt = \frac{dU_y}{a - b_y U_y \sqrt{U_y^2 + c_y^2}} \quad [2.10]$$

This equation can be solved for the special case when  $c_y^2 = 0$  to obtain the analytical solution published by Grift et al. (1997) for  $y$ -direction motion only. For the non-zero case, an implicit expression for slip velocity is obtained when Equation [2.10] is integrated over a small time step,  $\Delta t$ . This cannot be integrated a second time to obtain slip distance travelled between the particle and the surrounding air. Therefore Equation [2.3] is solved for slip velocity  $U_y(t) = dy(t)/dt$  and slip distance travelled  $y(t)$  and by linearization, then using the higher order Runge Kutta fourth order approximation for the system of equations

$$U_y = \frac{dy}{dt} \quad [2.11]$$

$$\frac{d^2 y}{dt^2} = a - b \frac{dy}{dt} \sqrt{\left(\frac{dy}{dt}\right)^2 + c_y^2} \quad [2.12]$$

The term  $c_y$  changes with time as the  $x$  and  $z$  velocities change. This means that when calculating the  $k_{1,1}$ ,  $k_{1,2}$ ,  $k_{2,1}$ ,  $k_{2,2}$ , ... and  $k_{4,2}$  values of the Runge Kutta approximation at each time step,  $c_y$  is also recalculated at the appropriate partial increments of the time step. For this reason, the analytical solutions for the  $x$  and  $z$  velocities across a small time step, given by Equations [2.5] and [6], are particularly useful.

Solution yields the following predictive equations for slip velocity and distance travelled between the air and the particle in a time step

$$U_{y,t+\Delta t} = U_{y,t} + \frac{1}{6} [k_{1,2} + 2k_{2,2} + 2k_{3,2} + k_{4,2}] \quad [2.13]$$

$$\Delta y_{t+\Delta t} = \frac{1}{6} [k_{1,1} + 2k_{2,1} + 2k_{3,1} + k_{4,1}] \quad [2.14]$$

where the initial Cartesian position is  $S_{y,0} = 0$  and the initial velocity  $U_{y,0}$  is determined by considering the aircraft speed, wind conditions, propeller wash and discharge velocities as discussed above. The  $k$ -values are recalculated every time step:

$$k_{1,1} = \Delta t \cdot U_{y,t}$$

$$k_{1,2} = \Delta t \cdot \left( a - b U_{y,t} \sqrt{U_{y,t}^2 + c_{y,t}^2} \right)$$

$$k_{2,1} = \Delta t \cdot \left( U_{y,t} + \frac{1}{2} k_{1,2} \right)$$

$$k_{2,2} = \Delta t \cdot \left( a - b \left( U_{y,t} + \frac{1}{2} k_{1,2} \right) \sqrt{\left( U_{y,t} + \frac{1}{2} k_{1,2} \right)^2 + c_{y,t+\Delta t/2}^2} \right)$$

$$k_{3,1} = \Delta t \cdot \left( U_{y,t} + \frac{1}{2} k_{2,2} \right)$$

$$k_{3,2} = \Delta t \cdot \left( a - b \left( U_{y,t} + \frac{1}{2} k_{2,2} \right) \sqrt{\left( U_{y,t} + \frac{1}{2} k_{2,2} \right)^2 + c_{y,t+\Delta t/2}^2} \right)$$

$$k_{4,1} = \Delta t \left( U_{y,t} + \frac{1}{2} k_{3,2} \right)$$

$$k_{4,2} = \Delta t \left( a - b \left( U_{y,t} + \frac{1}{2} k_{3,2} \right) \sqrt{\left( U_{y,t} + \frac{1}{2} k_{3,2} \right)^2 + c_{y,t+\Delta t}^2} \right)$$

In these calculations, the term  $c_v^2$  uses the previous time step values of velocity, the term  $c_{y,t+\Delta t/2}^2$  uses the half increment predictions of velocity obtained from Equations [2.5] and [2.6] by substituting  $\Delta t/2$  for  $\Delta t$ , and  $c_{y,t+\Delta t}^2$  uses Equations [2.5] and [2.6] directly. When modelled for the special case of  $y$ -direction motion only, the Runge Kutta method gives the same result as the analytical solution obtained from Equation [2.10], as given by Grift et al. (1997).

### 2.2.1.2 Cartesian frame calculations: effect of local velocity gradients on slip velocities

Equations [2.5], [2.6] and [2.13] predict the component slip velocities at  $t + \Delta t$  based on the known particle position ( $S_x, S_y, S_z$ ) and slip velocity ( $U_x, U_y, U_z$ ) at time  $t$ , which are calculated at the previous time step. However, changes in velocity gradients in the local Cartesian coordinate frame also affect the slip velocities and must be included. As an example, imagine a particle discharged into a concurrent airstream moving at  $10 \text{ m s}^{-1}$  with a slip velocity of  $30 \text{ m s}^{-1}$ . Now, imagine that particle suddenly crossing a hypothetical boundary into quiescent air. The slip velocity is now  $40 \text{ m s}^{-1}$  which means the drag forces on the particle are quite different. For a particle ejected from an aircraft, three Cartesian frame velocities cause local velocity gradients. These are the flight speed of the aircraft, the wind characteristics and the propeller wash. Each is discussed separately below.

The effect of flight speed on the slip velocity gradients is included in the initial calculation of slip at the point of discharge. As stated above discharge is a function of spreader design. The slip velocity is the difference between the aircraft groundspeed and Bansal et al.'s discharge velocities with the inclusion of wind and propeller wash, as discussed later.

Wind is a function of the height from the ground and therefore exhibits a height dependent velocity gradient. This means that a decrease in wind occurs across a time step as the particle

falls, which slightly increases the slip between the particle and air. Wind is limited to act only in the  $x$ - $z$  plane and a simple power law is used. The magnitude of the wind velocity is given by

$$\bar{U}_{wind} = m(H-S_y)^n \quad [2.15]$$

where  $H$  = release height from the ground [m],  $S_y$  = distance fallen towards the ground from the release point [m], and  $m, n$  = constants. In this work  $n = 1/7$  although it is known to vary considerably from this value (Manwell et al., 2002, p44). Wind velocity is referenced to a measured value, typically recorded 2 m above ground. Therefore, as the particle falls through an increment in height,  $\Delta S_y = \Delta(H-S_y)$ , the wind velocity decreases which increases the slip velocity between the particle and air by  $mn(H-S_y)^{n-1} \Delta(H-S_y)$ .

Propeller wash has both axial and radial velocities that differ from the surrounding air. General momentum theory is used to define the propeller wash velocity profile. The rotation of the propeller provides the thrust that flies the aircraft, but also produces changes in the axial and radial velocities of the air at the propeller on its downstream side. Thrust produces an additional axial velocity that is assumed constant within the slipstream boundary but disappears outside it. This additional axial velocity therefore increases the slip velocity on the particle both as it leaves the aircraft (which must be included when calculating the initial slip) and when the particle exits the slipstream boundary of the propeller wake. The additional radial velocity is caused by the torque of the propeller acting on the air: it has  $y$  and  $z$ -direction components and rotation of the air is assumed to occur only within the slipstream boundary of the wake. Velocity gradients exist both in the slipstream and at the boundary as the particle exits into the free-stream flow.

Axial and radial velocities are defined within general momentum theory by the axial flow induction factor,  $I_a$ , and the tangential flow induction factor,  $I_r$ , where  $I_a$  is defined as the ratio of the increase in axial velocity due to thrust and the free stream axial flow velocity. The actual axial velocity is then the free-stream velocity multiplied by  $(1+ I_a)$ . The tangential flow induction factor,  $I_r$ , is defined as the ratio of the angular velocity of the rotating air immediately downstream of the rotor and double the propeller angular velocity,  $\omega/2\Omega$ , where  $\omega$  is the angular

velocity of the rotating wake at radius  $r$  and  $\Omega$  is the angular velocity of the propeller. The factor of 2 comes from the definition of momentum flux. These two induction flow factors rely on another important dimensionless group,  $\lambda = \Omega R/U$ , the tip speed ratio defined as the speed ratio between the propeller tip and the free-stream velocity, where  $r=R$  is the propeller tip radius.

General momentum theory divides thrust and power calculations into the near and far wake, where the near wake is close to the propeller blade and the far wake is some distance downstream by which time the wake is fully developed. Calculations between these limits are difficult and it is not possible to simply determine how the axial and radial velocities decay with distance away from the propeller, other than to assume they decay similarly to axisymmetric jets (e.g., Wygnanski and Fielder, 1969). Here, we calculate the axial and radial velocity near the propeller and do not assume decay. Our reasons are that the particles are large, 1-7 mm, and are released close to the propeller, within 2-3 diameters, and often near the base of the slipstream boundary. They fall outside the boundary relatively quickly, so including a decay term will not add to the practical accuracy of the calculations. The slipstream boundary used here is the tube that encloses the swept air volume of the propeller. In addition, we recognize that the aircraft wings and fuselage are bluff bodies that act to constrain the rotation of the wake and, without using complicated fluid flow computational methods, it is not known how they affect the rotational flow. Here, their effects are ignored.

General momentum theory applies to an idealized actuator disc rather than a multi-bladed propeller and it is discussed here in this context. First, we assume the lift forces on the blades forming the disc are normal to the resultant velocity of the air relative to the blades (i.e. the observer is standing on a blade as it rotates (see Sharpe (2004) for this construct)). This means no work is done on or by the fluid and therefore Bernoulli's theorem can be applied to the flow across the disc between points immediately upstream and immediately downstream of the disc. At a given radial position the energy-per-unit-volume balance is

$$P_{du} + \frac{1}{2} \rho U_{\infty}^2 (1 + I_a)^2 + \frac{1}{2} \rho \Omega^2 r^2 = P_{dd} + \frac{1}{2} \rho U_{\infty}^2 (1 + I_a)^2 + \frac{1}{2} \rho \Omega^2 (1 - 2I_r)^2 r^2 \quad [2.16]$$

where the inertial terms have been omitted because they are redundant. Subscripts  $du$  and  $dd$  refer to directly upstream and directly downstream respectively. The axial velocity directly upstream has already increased from the free stream value  $U_\infty$  to  $U_\infty(1 + I_a)$  prior to reaching the disc. This means it is unaffected across the rotating disc and so cancels. The quantity  $\Omega(1 - 2I_r)$  refers to  $\Omega - \omega$ , the relative angular velocity between the blade and the air on the immediate downstream side of the rotor. Symbol  $r$  is the radius from the axis of disc rotation. In addition, the air is assumed incompressible and the torque of the propeller rotates the air in the direction of blade rotation. Equation [2.16] simplifies to give the pressure difference across the disc

$$\Delta P_d = P_{du} - P_{dd} = 2\rho\Omega^2 r^2 I_r (1 - I_r) \quad [2.17]$$

This pressure drop is slightly greater than that which produces thrust because some pressure losses occur between the condition far upstream and the fully developed wake downstream. However, in this simple analysis we will neglect these.

Second, we assume that the above calculated pressure drop across the rotor disc, when integrated over the disc area, equals a pressure force which is equal to the thrust force which, in turn, equals the aircraft drag in steady flight. The thrust force is defined by the change in axial momentum of the air between the conditions far upstream and at the rotor disc. Because momentum is conserved, this also represents the change in momentum between the conditions far upstream and the fully developed wake. Equating these three quantities yields

$$\int_0^R \Delta P_d 2\pi r dr = \int_0^R \rho U_\infty^2 a (1 + I_a) 2\pi r dr = \frac{1}{2} \rho C_{D,aircraft} S U_\infty^2 \quad [2.18]$$

where  $C_{D,aircraft}$  is the drag coefficient and  $S$  is the planform area of the aircraft. The terms  $I_a$  and  $I_r$  in Equations [2.17] and [2.18] are both regarded as functions of  $r$ , so integration by parts is required for the first and second terms in Equation [2.18]. To simplify this, we introduce the assumption that circulation is uniform along the blade span, which means that angular momentum is radially uniform ((Sharpe (2007) justifies this assumption for the rotating flow from wind turbines),

$$2\Omega I_r r^2 = \text{constant} = 2\Omega I_{r,\text{tip}} R^2 \quad [2.19]$$

where  $r = R$  defines the slipstream boundary at the rotor (i.e., the propeller radius) and  $I_{r,\text{tip}}$  is the tangential flow induction factor evaluated at this boundary. Using this definition  $I_r$  can be substituted in terms 1 and 2 in Equation [2.18]. Each term is integrated separately below

*Term 1*

$$\begin{aligned} 2\pi \int_0^R \Delta P_d r dr &= 4\pi\rho\Omega^2 I_{r,\text{tip}} R^2 \int_0^R (1 - I_r) r dr = 4\pi\rho\Omega^2 I_{r,\text{tip}} R^2 \left[ \frac{R^2}{2} - \int_0^R I_r r dr \right] \\ &= 4\pi\rho\Omega^2 I_{r,\text{tip}} R^2 \left[ \frac{R^2}{2} - \left\{ \frac{R^2}{2} I_{r,\text{tip}} - \int_0^R \frac{r^2}{2} \frac{dI_r}{dr} dr \right\} \right] \end{aligned} \quad [2.20]$$

The second term of the integration by parts (after substitution of  $I_r$ ) reduces to  $\int_0^R \frac{I_{r,\text{tip}} R^2}{r} dr$

, which does not give a realistic answer unless an inner hub radius is used, i.e. the integration limits are between  $R_{\text{hub}}$  and  $R$ . Term 1 then becomes

$$\begin{aligned} 2\pi \int_0^R \Delta P_d r dr &= 4\pi\rho\Omega^2 I_{r,\text{tip}} R^4 \left[ \frac{1}{2} - I_{r,\text{tip}} \left( \frac{1}{2} + \ln \left( \frac{R}{R_{\text{hub}}} \right) \right) \right] \\ &= 2\pi\rho\Omega^2 I_{r,\text{tip}} R^4 \left[ 1 - I_{r,\text{tip}} \left( 1 + 2 \ln \left( \frac{R}{R_{\text{hub}}} \right) \right) \right] \end{aligned} \quad [2.21]$$

*Term 2*

Using the common assumption that the axial flow induction factor,  $I_a$ , is radially uniform, term 2 becomes

$$\int_0^R \rho U_\infty^2 I_a (1 + I_a) 2\pi r dr = 2\pi\rho U_\infty^2 \int_0^R I_a (1 + I_a) r dr = \pi\rho U_\infty^2 I_a (1 + I_a) R^2 \quad [2.22]$$

Substituting Equations [2.21] and [2.22] into [2.18] links the axial and tangential flow induction factors,  $I_a$  and  $I_r$ , to the tip speed ratio  $\lambda$ , the known aircraft drag coefficient  $C_D$ , the

aircraft planform area  $S$  and the propeller radius  $R$ . Terms 1 and 3 of Equation [2.18] can be used to evaluate  $I_r$  and terms 2 and 3 to evaluate  $I_a$ . Both relationships produce quadratics in each term which can be solved using the root formula. However, it is reasonable to assume that the second term inside the square brackets of Equation [2.21] is small compared to term 1, i.e.,

$1 \gg 2I_{r,tip} \ln\left(\frac{R}{R_{hub}}\right)$ , so simplifying the expression for  $I_{r,tip}$  to

$$I_{r,tip} = \frac{1}{4\lambda^2} \frac{C_{D,aircraft} S}{\pi R^2} \quad [2.23]$$

From terms 2 and 3 evaluating the positive root of  $I_a$  gives

$$I_a = -\frac{1}{2} + \frac{1}{2} \sqrt{1 + 2 \frac{C_{D,aircraft} S}{\pi R^2}} \quad [2.24]$$

A typical topdressing aircraft is the Gippsland Aeronautics GA-200C which has  $C_{D,aircraft} = 0.1$ ,  $S = 10.42 \text{ m}^2$ ,  $R = 1.07 \text{ m}$ , and flies at 110 knots ( $203 \text{ km h}^{-1}$ ) with the propeller rotating at 2500 rpm, which gives  $I_{r,tip} = 0.0029$  and  $I_a = 0.13$ . These equate to an additional axial air velocity of  $26 \text{ km h}^{-1}$  and a rotation speed of the air at the tip of  $2\Omega I_{r,tip} R = 2.1 \text{ m s}^{-1}$ . Both results are significant compared to the airspeed and typical wind velocities.

As explained above, this simplified propeller wash model assumes all propeller induced air flow occurs within a slipstream boundary defined by the swept volume of the blades. Therefore, apart from the initial slip velocity calculation as the particle exits from the spreader device mounted underneath the aircraft, there is no axial flow velocity gradient until the particle reaches the slipstream boundary where a step change decrease in stream velocity occurs. This decrease is equal to  $I_a U_\infty$  which increases the slip velocity between the particle and air by the same amount. In contrast the particle will experience a radial velocity gradient, defined from the conservation of radial momentum in Equation [2.19] until it reaches the slipstream boundary where a step change occurs from  $2\Omega I_{r,tip} R$  to zero, which increases the slip velocity between the particle and the air by the same amount. It is clear these step changes in slip velocity are not physically realistic because the velocity gradient at the boundary will entrain air from outside

the stream tube, which will result in a more gradual and fluctuating velocity gradient. However, the model is sufficient to explain that a radial component to flow exists and the observation that (for example) clockwise rotation of the propeller on a still day results in particle drift to the true left of the aircraft. This model is therefore viewed as satisfactory for the modelling of granular fertiliser release from aircraft. This simplification of constraining propeller wash into the stream tube does cause one anomaly: because step changes in slip causes similar step changes in the forces acting on the particle, it is possible that the particle can be projected back into the slipstream. To avoid this, the model is constrained so that the slipstream flow ceases to influence the particle when it (first) reaches the slipstream boundary. After this, no subsequent slipstream velocity gradient calculations are made even if the particle re-enters the stream tube. Of course, in a more complex physical model this could be accommodated.

### 2.2.1.3 Cartesian frame calculations: drift

As well as the local Cartesian velocity gradients affecting the slip velocities, they also cause drift. Wind is the easiest to imagine. For example, a wind blowing from the true right of the aircraft will cause a particle dropped vertically from the aircraft to drift to the left. This needs to be viewed from both the Lagrangian and Cartesian frames of reference. In the Lagrangian frame, wind blows over the particle from the (true) right and the slip decreases with time as drag acts on the particle. In the Cartesian frame, the particle will be drifting with an increasing velocity which, in the limit, will equal the wind speed. In a similar way, the additional axial and radial air flows induced by the propeller cause drift within the slipstream boundary. Therefore, the Cartesian position of the particle is calculated from the slip distance travelled plus drift at each time step. For each coordinate direction, these are:

$$S_{x,t+\Delta t} = S_{x,t} \pm \Delta x - U_{x,wind} \Delta t - I_a (U_{grdspd} - U_{x,wind}) \Delta t \quad [2.25]$$

$$S_{y,t+\Delta t} = S_{y,t} \pm \Delta y + 2\Omega I_r \frac{R^2}{r^2} z_{prop,t} \Delta t \quad [2.26]$$

$$S_{z,t+\Delta t} = S_{z,t} \pm \Delta z - U_{z,wind} \Delta t - 2\Omega I_r \frac{R^2}{r^2} y_{prop,t} \Delta t \quad [2.27]$$

where  $U_{grdspd}$  is the groundspeed of the aircraft, and  $y_{prop,t}$  and  $z_{prop,t}$  are the  $y$  and  $z$  coordinates of the particle at time  $t$  with respect to the axis of propeller rotation when the particle lies inside the propeller stream tube. After the particle first reaches the slipstream boundary, none of the axial or radial induction flow terms apply, i.e., after  $y_{prop}^2 + z_{prop}^2$  equals or exceeds  $R^2$ .

Equations [2.25-2.27] require the sense of relative particle airflow to be specified correctly. Earlier calculations of slip velocity and slip distance between the particle and air determined only their magnitudes but not their sense. This was not possible because drag always opposes motion and is proportional to the square of velocity. Sense depends on the relative motion between the particle and air. For the  $x$ -direction, the updated position at time  $t+\Delta t$  relies on whether the particle is moving forward relative to the air ( $+\Delta x$ ) or backward ( $-\Delta x$ ). Wind is assumed positive if blowing from one of the forward quadrants (1<sup>st</sup> and 4<sup>th</sup>) in the  $x$ - $z$  plane. Also, it is reasonable to expect that the aircraft groundspeed minus the  $x$ -direction component of wind,  $U_{grdspd} - U_{x,wind}$ , is always faster than the discharge velocity of the particle from the spreader (although the model will account for the unusual case when this is not true). For the  $y$ -direction, the updated position relies on whether the particle is moving downward relative to the air ( $+\Delta y$ ) or upward ( $-\Delta y$ ), but also depends on the rotating air flow of the near wake given by the third term, which becomes negative in the 3<sup>rd</sup> and 4<sup>th</sup> quadrants (in the  $y$ - $z$  plane) with respect to clockwise rotation of the propeller where  $z_{prop} < 0$ . For the  $z$ -direction, the slip distance travelled between the particle and the air is positive ( $+\Delta z$ ) if the particle is moving to the true right relative to the air and negative ( $-\Delta z$ ) if it is moving to the true left relative to the air. Wind is regarded as positive when it blows from the true right (1<sup>st</sup> and 2<sup>nd</sup> quadrants) in the  $x$ - $z$  plane. The third term becomes positive when the particle is located in the 1<sup>st</sup> and 4<sup>th</sup> quadrants (in the  $y$ - $z$  plane) relative to clockwise rotation of the propeller where  $y_{prop} < 0$ .

The following section compares this model to the simpler model of Bansal et al. (1998b) for the  $x$  and  $z$  direction landing position when there is no wind,  $U_{x,wind} = U_{z,wind} = 0$ , then to AGDISP when wind and propeller wash are included and particles are no longer spherical.

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### 2.3 RESULTS AND DISCUSSION

Direct comparison to experimental data is not available, so comparisons are made to the ballistic model of Bansal et al. (1998b) and to output from AGDISP 8.15. Table 2.1 compares predicted landing positions from this model to those predicted by Bansal et al. (1998b). Their model assumed turbulent conditions which meant they used a constant drag coefficient, although did not state its value. To make the comparison as close as possible, we fixed  $C_D = 0.44$  (Mennel and Reece, 1963) and used the ejection angles and velocities provided by Bansal et al. from their FLUENT simulations. Fall height to the ground is set at 15 m, quiescent air is assumed and the material is defined as urea with density,  $1220 \text{ kg m}^{-3}$ , and sphericity  $\phi = 1.0$ . Comparison shows similar landing positions and provides confidence in the model results obtained here. The shortcomings of Bansal et al.'s (1998b) work lie in their explanations. They did not discuss the comparison of the predicted mean landing positions from AGDISP to those predicted using the single particle model. This may just be an oversight of the authors, because by comparing the values in their Table 2.3, with the DUCT1 results plotted in Figure 2.2, the mean landing positions do appear to concur. In addition, the authors make no conclusion with respect to comparing the models, which seems to indicate some uncertainty about validation. Also, it is difficult to determine which features of the AGDISP model were turned on and off. Nevertheless, a fair comparison is obtained.

**Table 2.1. Comparison of predicted transverse ( $z$ -direction) landing position to the model of Bansal et al., (1998b) for the special case of constant drag coefficient at  $C_D = 0.44$ . The offset is the position of the duct relative to the centreline of the aircraft. Exit velocities are relative to the aircraft using our coordinate reference system (+ $x$ , flight direction; + $y$ , towards ground; + $z$ , true right of aircraft). The ducts are described in Bansal et al. (1998b)**

DUCT2						
GMD of particles [mm]	exit point offset [m]	$U_{x,exit}$ [ $ms^{-1}$ ]	$U_{y,exit}$ [ $ms^{-1}$ ]	$U_{z,exit}$ [ $ms^{-1}$ ]	Bansal et al z-position [m]	This model z-position [m]
1.3	-1.09	-45.3	+4.7	-51.3	-13.0	-12.9
1.54	-1.08	-41.9	+2.2	-47.6	-14.8	-14.2
1.85	-1.09	-37.6	+2.3	-43.8	-16.5	-15.5
2.2	-1.11	-35.5	+0.5	-40.7	-18.2	-17.1
2.82	-1.07	-32.3	+2.6	-36.9	-20.0	-18.4
3.66	-1.08	-32.3	+4.4	-37.3	-23.0	-21.3
DUCT4						
GMD of particles [mm]	exit point offset [m]	$U_{x,exit}$ [ $ms^{-1}$ ]	$U_{y,exit}$ [ $ms^{-1}$ ]	$U_{z,exit}$ [ $ms^{-1}$ ]	Bansal et al z-position [m]	This model z-position [m]
1.3	-0.52	-51.5	+4.9	-37.6	-10.7	-11.2
1.54	-0.52	-47.7	+5.5	-36.2	-12.3	-12.3
1.85	-0.52	-43.8	+4.9	-33.1	-13.6	-13.4
2.2	-0.51	-40.6	+4.6	-31.1	-14.8	-14.4
2.82	-0.52	-36.5	+4.4	-29.3	-16.4	-15.9
3.66	-0.51	-34.8	+4.0	-26.9	-17.7	-17.3
DUCT6						
GMD of particles [mm]	exit point offset [m]	$U_{x,exit}$ [ $ms^{-1}$ ]	$U_{y,exit}$ [ $ms^{-1}$ ]	$U_{z,exit}$ [ $ms^{-1}$ ]	Bansal et al z-position [m]	This model z-position [m]
1.3	-0.03	-63.3	+7.2	0.5	0.2	-0.03
1.54	-0.04	-53.9	+4.7	0.5	0.3	-0.04
1.85	-0.04	-47	+4.8	0.7	0.5	-0.04
2.2	-0.05	-41.1	+3.8	3.3	2.1	-0.05
2.82	-0.04	-41.5	+4.0	0.7	0.5	-0.04
3.66	-0.04	-43.7	+4.2	0.4	0.3	-0.04

The next level of comparison includes height dependent wind profiling, propeller wash and sphericity. Here we compare our model to AGDISP 8.15 for discharge from a Gippsland Aeronautics GA-200C aircraft (see Figure 2.2). This type of aircraft is commonly employed for aerial spreading of fertiliser in New Zealand. The discharge angle of particles leaving the spreader (set 0.75m below the axis of the propeller ( $r = 1.07$  m)) is set at  $160^\circ$  in the  $x$ - $z$  plane and  $0^\circ$  in the  $x$ - $y$  plane (our model defines  $180^\circ$  as directly behind the aircraft in both planes). The equivalent maximum spread angle is set at  $20^\circ$  in AGDISP (which defines  $0^\circ$  as directly behind the aircraft in the  $x$ - $z$  plane). The user specified discharge velocity is set in both models at  $10 \text{ m s}^{-1}$  relative to the aircraft along this trajectory, for wind speeds of 2 and  $4 \text{ m s}^{-1}$

(measured 2 m above ground) for particle sphericities 0.6, 0.8 and 1.0. Discharge occurs from the spreader device mounted underneath the aircraft, which is positioned 0.75 m below the propeller axis and particle release, for this comparison, is offset 0.10 m in  $+z$  direction (true right side). It must be noted that the discharge velocity used here is lower than that used by Bansal et al. (1998b), because the Gippsland GA200C aircraft is not powerful enough to use the air ram spreader device reported by them. Without the high air flowrates, lower exit velocities occur. Wind obeys the power law given in Equation [2.15].



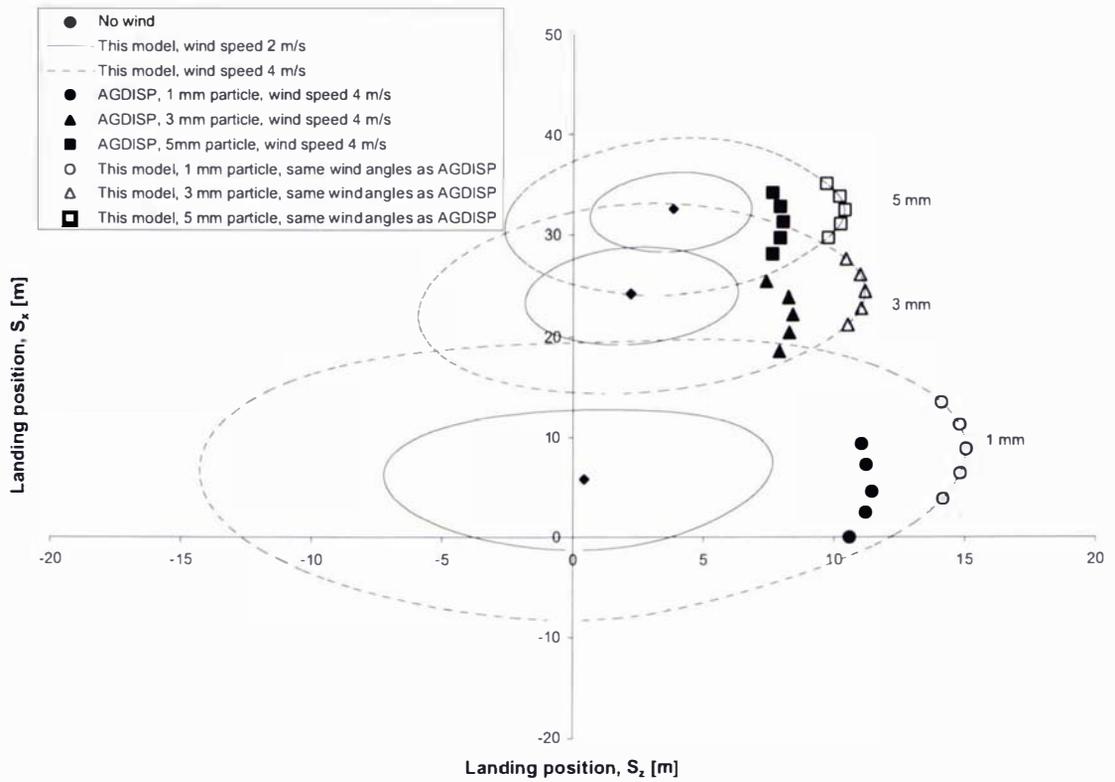
**Figure 2.2. Illustration of the GA200C aircraft including location of the six duct spreader device (located 0.75 m below the axis of the propeller)**

Contour plots clearly illustrate the flexibility of the model proposed here and enable easy comparison to AGDISP predictions. Figure 2.3 shows the  $x$ - $z$  landing position for spherical 1, 3 and 5 mm particles of superphosphate, density  $1760 \text{ kg m}^{-3}$ , for wind speeds of 0, 2 and  $4 \text{ m s}^{-1}$  blowing from all points of the compass and Figure 2.4 shows the effect of sphericity on landing position for a 3 mm particle. As expected larger particles travel further and stronger winds cause more drift. The AGDISP predictions trend the same way as our model, but with less transverse drift. In addition, the behaviour trace on Figure 2.4 shows that AGDISP has a much increased effect of drag on non-spherical particles. It also shows a discontinuity in AGDISP

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behaviour below  $\phi = 0.7$ . It is difficult to pinpoint the exact reasons why these differences occur but, because both models use the same drag relationship, it is likely to be due to the slightly different treatment of propeller wash. The discontinuity may well be a sign error.

Figures 2.3 and 2.4 show that, under certain wind conditions, the particle can land behind its release point. This is examined in Figure 2.5, where trajectories shown for the case of a head wind of  $2 \text{ m s}^{-1}$  for a range of sphericities. The particles in this example are released at an angle of  $160^\circ$  in the  $x$ - $z$  plane and  $180^\circ$  in the  $y$ - $z$  plane at a velocity of  $10 \text{ m s}^{-1}$ . As particles become less spherical drag increases, which means particles slow down more rapidly and do not travel as far forward. For the lowest sphericities of 0.6 and 0.7, the particle actually lands behind its release point. This occurs because the particle has slowed to below the additional propeller stream velocity,  $I_d U_{\text{prop}}$ , so that, when it crosses the slipstream boundary, it is now travelling backwards in the Cartesian frame. Figure 2.6 shows for the same conditions that the radial flow from the propeller drifts the least spherical particles towards the true left of the aircraft.



**Figure 2.3.** Phase diagram showing effect of particle size on landing position for spherical particles of 1, 3, and 5 mm diameters at wind speed of 0, 2 and 4 m s<sup>-1</sup> where the wind blows from all points of the compass. Comparisons from AGDISP correspond to five wind angles from 250 to 290° for wind blowing at 4 m s<sup>-1</sup>. Our model parameters: particle ejection angles are 160° (*x-z* plane) and 180° (*y-z* plane), particle ejection velocity with respect to the aircraft = 10 m s<sup>-1</sup>, transverse discharge offset  $z_{offset} = 0.1$  m, discharge point relative to propeller axis of rotation  $r_{discharge} = 0.75$  m, release height = 15 m, particles are superphosphate of density 1760 kg m<sup>-3</sup>, Gippslands GA200C aircraft ( $C_{D,aircraft} = 0.1$ , planform area  $S = 10.42$  m<sup>2</sup>, propeller radius  $R = 1.07$  m, groundspeed 203 km h<sup>-1</sup>, propeller rotating at 2500 rpm).

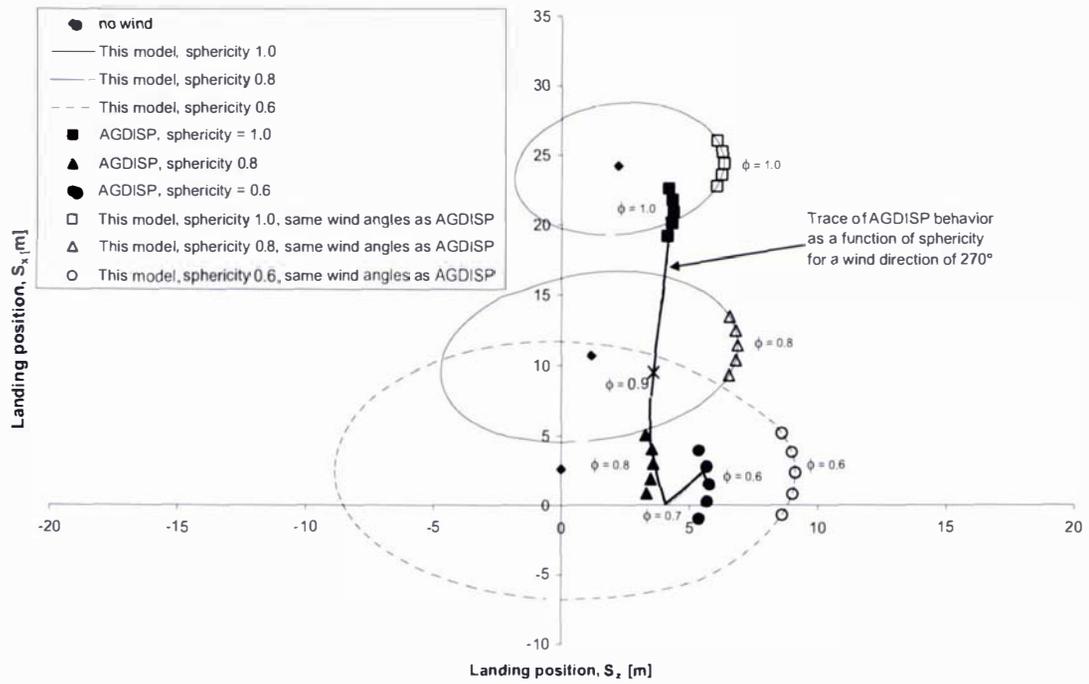


Figure 2.4. Phase diagram showing the effect of sphericity on landing position for particles of 3 mm diameter with sphericities 0.6, 0.8 and 1.0, at a wind speed of  $2 \text{ m s}^{-1}$  where the wind blows from all points of the compass. Comparisons from AGDISP correspond to five wind angles from  $250^\circ$  to  $290^\circ$  for wind blowing at  $2 \text{ m s}^{-1}$ . Our model parameters: particle ejection angles are  $160^\circ$  ( $x$ - $z$  plane) and  $180^\circ$  ( $y$ - $z$  plane), particle ejection velocity with respect to the aircraft =  $10 \text{ m s}^{-1}$ , transverse discharge offset  $z_{\text{offset}} = 0.1 \text{ m}$ , discharge point relative to propeller axis of rotation  $r_{\text{discharge}} = 0.75 \text{ m}$ , release height =  $15 \text{ m}$ , particles are superphosphate of density  $1760 \text{ kg m}^{-3}$ , Gippslands GA200C aircraft ( $C_{D,\text{aircraft}} = 0.1$ , planform area  $S = 10.42 \text{ m}^2$ , propeller radius  $R = 1.07 \text{ m}$ , groundspeed  $203 \text{ km h}^{-1}$ , propeller rotating at  $2500 \text{ rpm}$ ).

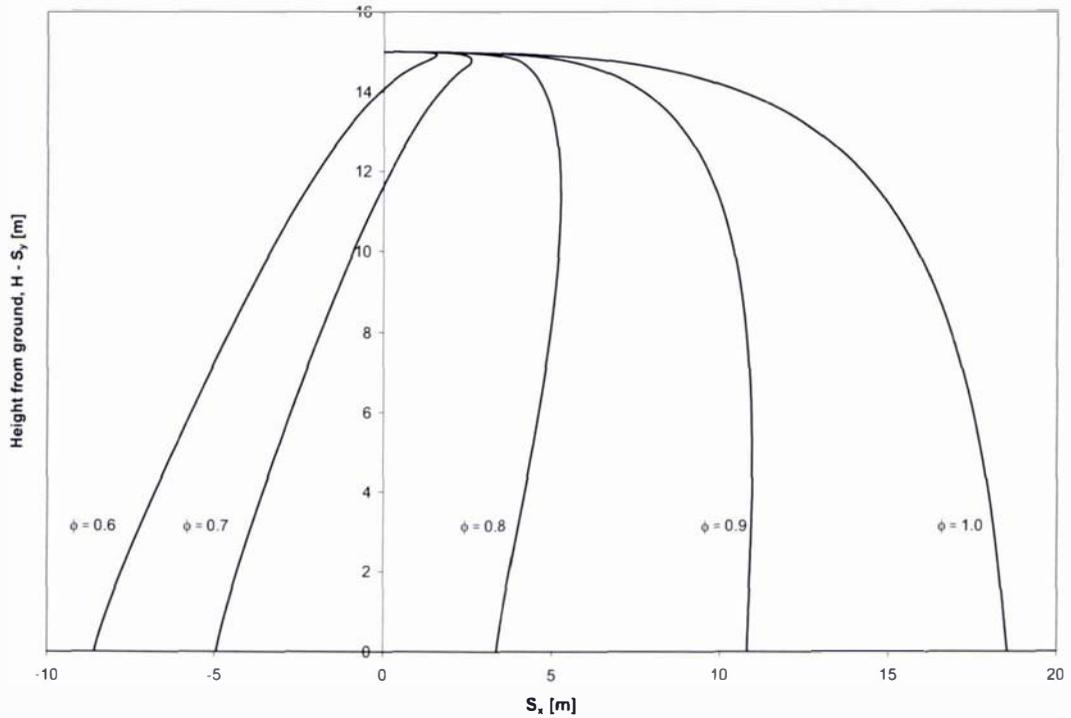
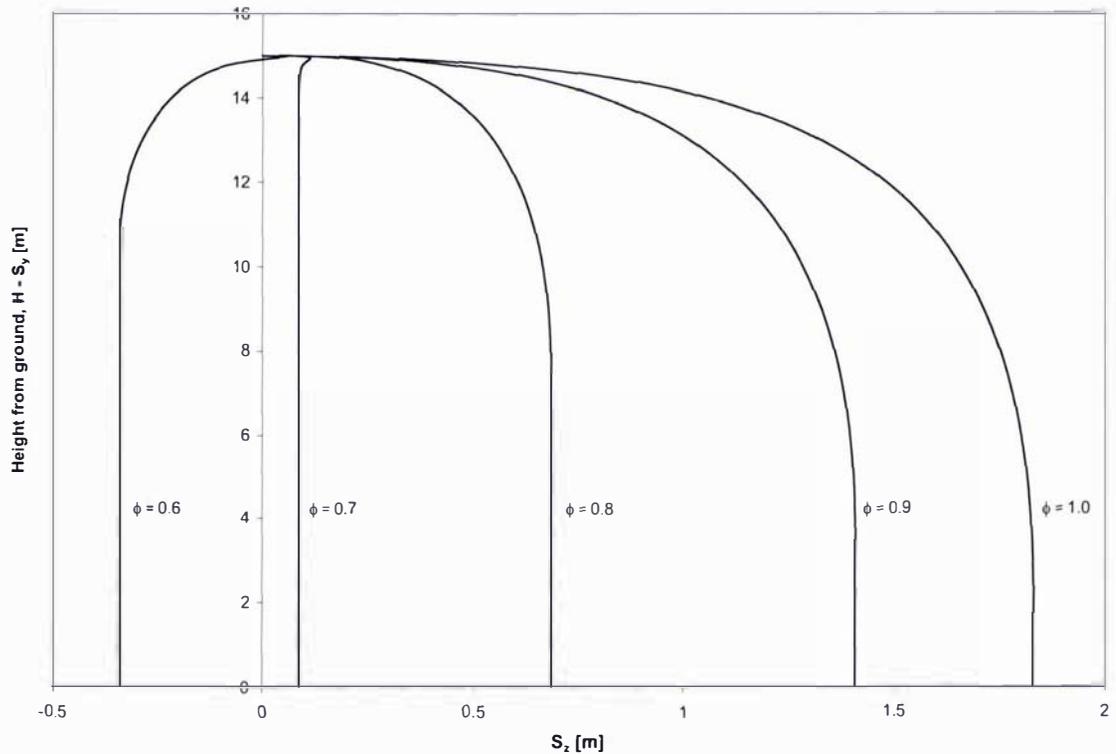


Figure 2.5. Effect of sphericity on flight trajectory in the  $x$ - $y$  plane.  $S_x$  is the axial position relative to the release point of the particle. Particle size = 3 mm, wind speed  $2 \text{ m s}^{-1}$ , a head wind with wind direction  $0^\circ$  ( $x$ - $z$  plane), particle ejection angles are  $160^\circ$  ( $x$ - $z$  plane) and  $180^\circ$  ( $y$ - $z$  plane), particle ejection velocity with respect to the aircraft =  $10 \text{ m s}^{-1}$ , no transverse discharge offset  $z_{\text{offset}} = 0 \text{ m}$ , discharge point relative to propeller axis of rotation  $r_{\text{discharge}} = 0.75 \text{ m}$ , particles are superphosphate of density  $1760 \text{ kg m}^{-3}$ , release height 15 m, Gippslands GA200C aircraft ( $C_{D,\text{aircraft}} = 0.1$ , planform area  $S = 10.42 \text{ m}^2$ , propeller radius  $R = 1.07 \text{ m}$ , groundspeed  $203 \text{ km h}^{-1}$ , propeller rotating at 2500 rpm).



**Figure 2.6.** Effect of sphericity on flight trajectory in the  $y$ - $z$  plane.  $S_z$  is the transverse position relative to the release point of the particle. Particle size = 3 mm, wind speed  $2 \text{ m s}^{-1}$ , a head wind with wind direction  $0^\circ$  ( $x$ - $z$  plane), particle ejection angles are  $160^\circ$  ( $x$ - $z$  plane) and  $180^\circ$  ( $y$ - $z$  plane), particle ejection velocity with respect to the aircraft =  $10 \text{ m s}^{-1}$ , no transverse discharge offset  $\tilde{z}_{offset} = 0 \text{ m}$ , discharge point relative to propeller axis of rotation  $r_{discharge} = 0.75 \text{ m}$ , particles are superphosphate of density  $1760 \text{ kg m}^{-3}$ , release height 15 m, Gippslands GA200C aircraft ( $C_{D,aircraft} = 0.1$ , planform area  $S = 10.42 \text{ m}^2$ , propeller radius  $R = 1.07 \text{ m}$ , groundspeed  $203 \text{ km h}^{-1}$ , propeller rotating at 2500 rpm).

## 2.4 CONCLUSION

This paper proposes a single particle ballistics model for a particle falling out of an aircraft which forms part of a program of work to determine the value of improving the spreading performance of an agricultural aircraft with the longer term view to develop a variable rate application system. While ballistics models are not new, this model has a number of new features. First, in addition to the force balance equations, it includes the constitutive relationships of vertical wind profile, a continuous function for the drag coefficient, the axial and radial components of propeller wash, and the effect of wind and propeller wash velocity gradients on slip velocity. Second, it is flexible to include all wind directions, and can handle

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any discharge velocity from any offset position relative to the axis of propeller rotation and position relative to the centre-plane of the aircraft. And third, it is fully documented; not only are equations listed but the solution methods and calculations steps are described, and all assumptions are listed.

Simulation results of the model compare favourably to the simpler model of Bansal et al. (1998b) and provide similar trends to the output of AGDISP 8.15. Research will now be directed at conducting field trials to compare the spread patterns obtained to those predicted by running the model multiple times.

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## CHAPTER 3 - MODELLING SOLID FERTILISER DEPOSITION FROM A FIXED WING AIRCRAFT – PREDICTING LATERAL DISTRIBUTION

*Publication arising from this chapter*

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Murray, R. I., Yule, I. J., Jones, J. R. 2007. Modelling solid fertilizer deposition from a fixed wing aircraft - II. Predicting lateral distribution. *Trans. ASAE (under review)*.

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### 3.1 INTRODUCTION

The single particle trajectory model described in the first paper of this series (Jones et al. 2007) predicts the landing position of a particle discharged from an aircraft as a function of the material properties, release point and release velocity relative to the aircraft, the aircraft airspeed, wind profile and propeller wash characteristics. With further information relating to material properties and discharge parameters, this model can be used to predict a transverse spread pattern.

The required material properties are particle density and shape which together determine their drag coefficients when falling in air. For non-spherical particles, the calculation of drag coefficients is well documented and relatively simple (Haider and Levenspiel, 1989; Ganser, 1993; Chhabra et al., 1999; Xie and Zhang, 2001; Yow et al., 2004). Earlier researchers modelling particle trajectories assumed turbulent conditions and used a constant drag coefficient (Mennel and Reece, 1963; Law and Collier, 1973) but this can cause errors because conditions are not always turbulent, particularly for small and low density particles (Walker et al., 1997). Also, when the turbulent assumption is made, the value of the constant drag coefficient is dependent on the shape of the particles. Shape can be defined a number of ways. The simplest is a single value, the sphericity, defined as the ratio of the surface area of the particle to that of a sphere with the same volume. Others are the shape factor, defined by Walker et al. (1997) as the sum of the absolute differences between the image radii and the best-fit circle radius, added

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from two perpendicular views, and the Q factor (Grift et al., 1997) which is defined as the ratio of the 'equivalent' diameter to the 'corresponding' diameter, producing a dimensionless number that relates an irregular fertiliser particle to a perfectly spherical particle, based on equalized fall times. Using sphericity, Haider and Levenspiel (1989) suggest two forms of the drag coefficient correlation for isometric particles, to which Teske et al. (2006) obtained a good fit for spherical particles with the experimental data of Langmuir and Blodgett (1945). Using shape factor, Walker et al. (1997) explained particle fall time based on turbulent airflow theory and a regression equation involving particle shape factor and mass. Alternatively, Grift et al. (1997) concluded that using Q factors are a practical way to express non-sphericity of fertiliser particles, presenting Q factors for calcium ammonium nitrate (CAN), nitrogen-phosphorous-potassium (NPK) and potassium fertilisers. Although each of these approaches explains some of the variation in measured versus predicted fall times, Walker et al. (1997) indicated that much of the variation which remained unexplained by particle mass and shape was seemingly random in nature. In this work, superphosphate was selected as the material to be spread and a method for measuring the sphericity of these particles was developed. With the assumption that superphosphate granules are approximately isometric, the Haider and Levenspiel correlation can be used for the drag coefficient.

The material discharge parameters required for modelling the spread distribution are the flow rate out of the aircraft hopper, the distribution of material to the various ducts of the spreader device (mounted underneath the aircraft) and the velocity vectors and distribution of these vectors from each of the ducts. Previous researchers have assumed a uniform discharge flowrate from each spreader duct (Walker and Gardisser, 1989; Bansal et al., 1998b). Bansal et al. (1998a) conducted FLUENT v4.32 (Fluent inc., Lebanon, N.H.) simulations to predict spreader duct ejection velocities from a Transland Model 23501 ram-air spreader (Transland Inc., Harbor City, California). Bansal et al. (1998b) used these velocities as input into a constant drag coefficient force-balance model to predict particle trajectories and hence a ground spread pattern. They found that the distribution of mass flow through the ducts had a profound effect on ground spread pattern. Their simulated distribution pattern produced an "M" shape

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transverse distribution but the experimentally measured pattern was the classic normal distribution shape. This suggests a breakdown in the assumptions of the ballistics model, or, more likely, that there is an uneven flow of fertiliser across the spreader ducts, in any case the authors did not simulate a distribution pattern with different flowrates within the ducts.

Gardisser (1992) suggests that the amount of material going through each duct is a fraction, based on two factors; the percentage of the total spreader intake width from which each duct receives material; and, the airspeed in each duct compared to the average across all ducts. The author hypothesized that, if the airspeed in a particular duct is above average, it receives an additional amount of material, as the material is being swept away more quickly from the gate opening. This would result in an increased flow-rate at that duct. By accepting this hypothesis, it provides an incentive to measure the airflow through each duct. To further complicate matters, the material flow into the spreader device will be affected by both the design of the aircraft hopper and the type of fertiliser flowing from it. Aircraft hoppers are often small, irregularly shaped cavities, to which it is difficult to apply standard Jenike design methods. Therefore, it is necessary to calibrate for the mass flowrate of fertiliser as a function of aircraft type, spreader type, the type of fertiliser being spread (and the size grade if it varies) and airspeed. The calibrated mass flowrate, together with particle velocity and sphericity, can be used with the single particle ballistics model (Jones et al., 2007), to generate spread patterns of fertiliser on the ground.

This paper satisfied four objectives: first, the sphericity range is determined for a sample of superphosphate typically spread from aircraft in New Zealand; second, the velocity profiles of fertiliser are measured as it exits the hopper gate beneath the aircraft; third, these material properties and flow parameters are combined with the single particle ballistics model to predict a lateral swath pattern; and fourth, the predicted swath pattern is compared to field data. Achieving these objectives will provide some understanding about how the deposition pattern is influenced by varying environmental conditions, material properties and discharge parameters. This will then enable the economic and environmental effects of fertiliser application to be investigated with a minimal time and financial investment. The third paper in this series

addresses these issues (Murray and Yule, 2007c). Beyond this, the end goal in this program of work is to develop a predictive spread pattern model that can be used as part of a variable application rate service provided by spreading operators to farmers.

### **3.2 MATERIALS AND METHODS**

In New Zealand, superphosphate is known for inconsistent particle size and shape. To investigate this further, fertiliser from a single on-farm storage heap was selected for analysis. This fertiliser represents a spot sample and is not necessarily representative of all superphosphate in New Zealand. A representative sample of the heap was taken by progressive sampling each time the loader removed loads to fill an aircraft over a series of sequential flights. The samples were combined together into one batch of about 10 kg. The batch was rolled in a bucket to mix it and 3 samples of 170 grams were removed for sieving using the hand-held nine compartment Spreadmark (FQCNZ, 2003) sieve box. Table 3.1 shows the collated size distribution, from which are calculated the industry measures of Size Guide Number of 313 ( $SGN = d_{50}$  on a cumulative fraction smaller than plot  $\times 100$ ) and a Uniformity Index of 15 ( $UI = d_{10}/d_{90}$  on a cumulative fraction smaller than plot). Under the Aerial Spreadmark Code of Practice (FQCNZ, 2003) a superphosphate fertiliser should normally be within SGN 245-300 and have a UI of greater than 11. These values indicate that this fertiliser contains a higher proportion of larger particles but has an acceptable uniformity index. Also calculable from Table 3.1 is the Sauter mean particle size which is 2.42 mm.

Particle density is an important variable because it affects the gravity body force and the inertia of particles. It was measured using the displacement technique where a measured mass of fertiliser was crushed, and a known volume of oil added to determine the mass per unit of volume. In this case the particle density was found to be  $1760 \text{ kg m}^{-3}$ .

**Table 3.1. Superphosphate particle size distribution, averaged from 3 samples of 170 grams within a bulk silo from field trials on a hill country farm, 15 km northeast of Kimbolton, New Zealand. A nine compartment, Spreadmark (FQCNZ, 2003) hand-held sieve box was used, < 0.5 mm material was ignored.**

Sieve size (mm)	Log mean of particle size (mm)	Amount of material (%)
7.0	7.6	3.6
5.8	6.4	5.2
4.7	5.2	6.9
3.7	4.2	19.5
2.9	3.3	20.6
1.9	2.3	27.8
1.0	1.4	10.6
0.5	0.7	5.8

To determine the sphericity of the fertiliser particles, a small sample of 38 g was removed from the bucket and hand sieved using sieves of apertures 8, 5, 4, 3.35, 2.8, 2, 1, and 0.4 mm. These were selected to approximately match the FQCNZ sieve box series even though they do not represent the standard practice of selecting sieves according to a power series. The mass of particles collected in each sieve range was measured, and the number of particles counted. The particles from each sieve mesh were then spread across a black non-reflective surface and photographed using a Sony Cybershot 3.3 mega pixel camera with a focal length of 0.52 m under florescent lighting. The images were imported into Labview Vision Assistant 7.0 (National Instruments, 2004) where they were calibrated, enhanced and filtered to reveal the major and minor axis length, and projected area of each particle in the image. Calibration involved converting the image pixels to length, in this case millimetres. Enhancing and filtering involved differentiating the colour of the superphosphate from the background through colour thresholding, while IMAQ Vision particle filtering (National Instruments, 2004) was used to extract individual particle parameters. The two-dimensional images were converted to a volume by rotating each particle about its major axis. Doing so generates a population of prolate spheroids for each sieve fraction. This was then checked against the volume determined by dividing the measured mass on each sieve by the known particle density. It is difficult to comment directly on these comparisons, because the log-mean of the sieve apertures assumes the population of particles is log-normally distributed on each sieve. Further comment would be

possible if individual particle mass measurements were made to correlate against individual major and minor axis values, but this was not done. Despite this, it is useful to compare the difference between volume calculated from image analysis and that from the measured mass divided by particle density. The difference in volume for each sieve size was always positive, as the method for measuring particle density above does not take into account internal porosity. The comparisons also show that the difference in volume increases as sieve size decreases this suggests that the image processing technique presented here may not be suitable for small particles less than 1.5 mm. The sphericity is then obtained as the ratio of the sphere equivalent surface area to that of the prolate spheroids. This is listed in Table 3.2 for each sieve fraction.

**Table 3.2. Particle characteristics determined by photographic analysis corresponding to sieve size.**

Sieve size [mm]	Log mean size on sieve [mm]	Measured mass [g]	Total volume [mm <sup>3</sup> ]	Mass Fraction [%]	Total volume of prolate spheroid [mm <sup>3</sup> ]	Total surface area of prolate spheroid [mm <sup>2</sup> ]	Mean major axis [mm]	Mean minor axis [mm]	Sphericity [-]	Std. Dev. Sphericity [-]
0.4	0.63	0.5	278	1	700	4280	1.1	0.8	0.84	0.16
1.0	1.41	1.6	926	4	1480	4320	2.7	1.7	0.76	0.13
2.0	2.37	4.2	2400	11	2490	5940	3.6	2.2	0.74	0.09
2.8	3.06	7.8	4430	20	4460	9540	3.9	2.2	0.72	0.15
3.4	3.66	5.1	2870	13	2950	5540	4.6	2.7	0.72	0.11
4.0	4.47	9.8	5540	27	6180	8350	6.6	3.8	0.71	0.11
5.0	6.32	8.7	4920	23	5340	5070	10.0	5.3	0.67	0.12
8.0	-	-	-	-	-	-	-	-	-	-
	Total	37.6	21400		23600					
		Sauter mean	3.4							

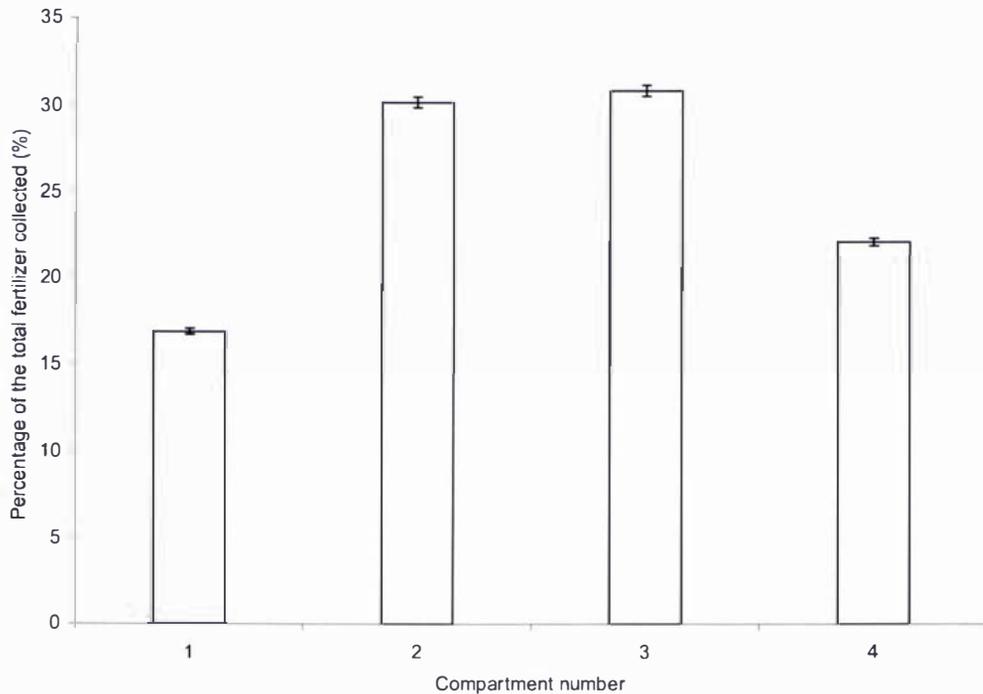
Both the net flowrate and the lateral flow distribution of fertiliser leaving the aircraft are important. The net flowrate is controlled by opening gates beneath the hopper, in this paper the device containing these gates is referred to as the gatebox. The application rate across the swath on the ground depends on the lateral flowrate distribution across the width of the gatebox and ground speed. Each of these calibrations are discussed below. They were conducted on a Gippsland Aeronautics GA200C.

Flowrate was calibrated to gatebox opening by measuring the total mass leaving the hopper in a given time at 27 mm and 35 mm gatebox openings. The net flowrate ( $\text{kg s}^{-1}$ ) from the gatebox as a function of the gatebox opening (mm) yields the relationship  $y = 0.6623x - 12.247$ ,

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( $R^2 = 0.92$ ,  $n = 10$ ), where  $x$  is the opening in millimetres.

The lateral flow distribution across the width of the gatebox was determined by a static experiment that involved constructing a small collector to fit under the fuselage gatebox to distribute the discharged material into four buckets. The gatebox was 0.80 m wide, so four compartments of 0.2 m  $\times$  0.2 m were placed under the width of the gatebox. The hopper capacity is normally 1200 kg but, for this test, 220 kg of superphosphate was placed in the hopper. This is an amount that, in the opinion of the authors, was enough to ensure normal flow conditions without requiring an unwieldy amount of fertiliser. Eight sequential samples were taken after the initial release of the material until the hopper was emptied. The gatebox opening was set at 27 mm for this single trial. Each sample run was stopped when one of the compartments approached full by closing the gates on the gatebox. In addition, a stopwatch was used to record time in order to calculate flowrate. The test data was averaged for each collector to generate a horizontal flow profile from the hopper (Figure 3.1). The resulting percentages and 95% confidence limits of 17% ( $\pm 0.26\%$ ), 30% ( $\pm 0.26\%$ ), 31% ( $\pm 0.48\%$ ) and 22% ( $\pm 0.29\%$ ) for the four compartments respectively, clearly showing that the flow is greater into the central compartments, and that the right hand side flow is greater than the left hand side. At the 27 mm opening, the mean flowrate across the entire gatebox was 338 kg min<sup>-1</sup>. Proportional flowrate distributions were not measured for any other gatebox openings; therefore, we assume the proportional flowrates are the same for all openings.



**Figure 3.1. Static flow results showing the proportion of flow from a Gippsland Aeronautics GA200C gate outlet. Error bars indicate the range of measurements over the eight tests.**

Table 3.3 lists the sequentially measured flowrates for each compartment, showing that this non randomized test had differences in flowrate. A lower flowrate was observed at the start of the test as opposed to the flow at the end when the hopper was nearly empty; nevertheless, the proportions of fertiliser landing in each bin remained constant throughout. This was not unexpected; the lower initial flowrate was likely caused by particle consolidation occurring when the hopper was filled; in this case, the fertiliser was dumped from a loading truck some distance above the hopper opening. Also, on inspection of the internal hopper walls, it was noted that the moulding of the hopper around the fuselage was different on the right and left hand sides and could be responsible for uneven distribution of material. No attempt was made to measure the particle size distribution issuing into each compartment. Without this data, we assume for later modelling that no segregation of particles occurred in the hopper during filling or discharge and that the particle size distribution remained uniform across all compartments.

**Table 3.3. Compartment flowrates for superphosphate from a Gippsland Aeronautics 200C gatebox.**

Test no.	Compartment			
	1	2	3	4
	Flowrate [ $\text{kg s}^{-1}$ ]			
1	0.7	1.3	1.4	1.0
2	0.8	1.4	1.4	1.0
3	0.9	1.5	1.6	1.1
4	0.9	1.7	1.7	1.2
5	1.0	1.7	1.8	1.3
6	1.0	1.8	1.9	1.3
7	1.2	2.1	2.1	1.5
8	1.2	2.0	2.0	1.5

When aerial spreading, the aircraft sometimes has a spreader device fitted underneath the gatebox. The device contains six ducts divided by vanes (see Figure 2.2). It is not possible to do a static proportional flowrate measurement from the spreader device because air must flow through it. Therefore, we assume that the net flowrate measured in the static tests, as a function of gatebox opening, is not inhibited by the spreader device, nor is it enhanced by air flow or vibration of the aircraft during flight. The proportion of the flow leaving the six ducts was predicted by interpolating the above four compartment measurements using polynomial regression. The predicted percentage of flow for each duct and is shown in Table 3.4.

Further, the vane angles were measured to define the discharge angle of particles assuming that the air rushing through the device during flight throws them against the vanes as they accelerate out of the spreader device. Vane angle measurement was achieved by digital imaging ArcMap (ESRI, 2004) and determined at the tip of the outer edge of each duct. The same principles were applied for calculating the distance from the centre of the hopper gate to the centre of each duct. Both the distance from centre and discharge angle for each duct are shown in Table 3.4. This information is necessary as input to the single particle ballistics model when generating a predicted ground spread pattern.

**Table 3.4. Particle exit parameters for a six duct spreader mounted on a Gippsland Aeronautics GA200C. A discharge angle of 180° corresponds to directly behind the aircraft, i.e., opposite to the direction of travel.**

	Spreader Duct					
	1	2	3	4	5	6
Distance from centre (m)	-0.26	-0.11	0.00	0.9	0.17	0.28
*Angle of discharge (°)	225	220	210	180	150	135
Flow (%)	14	16	18	19	17	16

\* This angle refers to the  $x$ - $z$  plane. The  $x$ - $y$  plane angle is 180° for all ducts; this is horizontal and directly behind the aircraft.

Discharge velocities were not able to be experimentally measured from this spreader device and thus they become an adjustable parameter within the single particle ballistics model. Published velocities by Bansal et al. (1998b) from an air ram type Transland spreader are significantly higher than is achievable from the spreader used here. This is because the Gippsland Aeronautics GA200C is not powerful enough to carry a similar ram-air type spreader. Bansal et al. (1998b) also noted that small particles acquire greater ejection velocities than larger particles, which have more inertia. In the spreader used here, the velocities will be lower because less air is present and more particle-particle interactions are expected to occur, therefore reducing the velocity range. Also, the functionality between size and velocity remains unknown. Thus a single mean discharge velocity is used for all particles as an adjustable parameter within the model. Later it is demonstrated that a discharge velocity of 25% of the aircraft velocity (relative to the aircraft) gave a predicted swath pattern that matched that of a field test.

Stochastic variation always occurs in both discharge trajectory velocities and in-flight trajectories due to many interacting factors that are not covered in the physics of the single particle ballistics model (Jones et al., 2007) or the initial conditions discussed above. Therefore, some variation in the initial conditions was introduced to avoid particles landing in the exact same position. A Gaussian random number generator was used for discharge speed, discharge angle and sphericity for each of the modelled particle trajectories. The standard deviations about discharge speed and discharge angle were set at  $5 \text{ m s}^{-1}$  and  $5^\circ$  respectively. These are unknown quantities so represent addition fitting factors; they were determined based on fitting the predicted spread pattern to that obtained in the field test. The standard deviation of

sphericity is determined by the method described above and in this case the overall mean of 0.79 and standard deviation of 0.16 were used.

After defining the above factors, the model can be used to generate landing position distribution profiles for each of the eight particle size fractions (log-mean values in Table 3.1) across each of the six ducts of the spreader device. It is essential that the model be run a sufficient number of times to generate an accurate probability distribution, so 1000 runs were completed at each of the 48 size/duct combinations. The landing positions from these runs were collected into longitudinal bins of 0.5 m width and spaced at 1 m intervals; these correspond to the tray width used in the field trial and therefore represent the industry standard. Probabilities were obtained by dividing the number collected in each bin by 1000. Each generated probability distribution was then multiplied by its specified number flowrate of particles. Ideally, these number flowrates are calculated from the particle size distribution (Table 3.1), the mass flowrate given by the gatebox opening and the proportional mass flowrates from each duct (Table 3.4). The resulting 48 distributions can then be added in a variety of ways to extract the total mass per bin, or the spread profile of given particle size or from a specified duct.

Spreader auditing data (R. Horrell, unpublished data, 16<sup>th</sup> October 2002, Feilding, New Zealand) was then compared in an attempt to validate the model prediction of spread pattern. In this field test, they used thirty trays (tray dimension, 0.5 × 0.5 m) spaced at 1 m intervals, each containing plastic inserts acting as baffles to prevent particles bouncing out. These trays were laid out in a transverse row beneath the flight path of a Gippsland Aeronautics GA200C aircraft, spreading superphosphate in still air over flat farmland from an altitude of 25 m, flying at 51 m s<sup>-1</sup> (100 knots). The aircraft made a single pass. The fertiliser spread was superphosphate, but not from the same on-farm heap as reported above. Therefore, with no alternative, we assume here that the distributions are the same and that the sphericity range is also the same. The flowrate of material leaving the gatebox was calculated to be 1175 kg min<sup>-1</sup>, determined retrospectively from the amount of material collected in the trays and the reported groundspeed (51 m s<sup>-1</sup>). All model parameters are listed in Table 3.5. Using the above mentioned initial condition data and including stochastic variation, the predicted distribution for the trays is compared to the

measured tray results. This is discussed below.

**Table 3.5. Model parameters used to simulate the field test spread pattern**

Parameter	Value	Units
<b>Constants</b>		
Aircraft speed	51.0	m s <sup>-1</sup>
Aircraft drag coefficient	0.1	-
Aircraft planform area	10.42	m <sup>2</sup>
Propeller radius	1.07	m
Propeller rotation speed	2500	RPM
Radius of discharge relative to propeller axis of rotation	0.75	m
Particle density of superphosphate	1760	kg m <sup>-3</sup>
Air density	1.229	kg m <sup>-3</sup>
Air temperature	15	°C
Air viscosity	0.0000173	Pa s
Wind speed	0	m s <sup>-1</sup>
Wind direction	N/A	-
Wind height profile constants, m = 0 n = 1/7	N/A	-
Fertiliser release height	25	m
<b>Variables*</b>		
Particle size	[per Table 3.1]	m
Particle sphericity	[per Table 3.1]	-
Duct offset positions	[per Table 3.4]	m
Ejection angles from ducts	[per Table 3.4]	°
Mass (and number) proportional flowrates from ducts	[per Table 3.4]	%
<b>Adjustable parameters</b>		
Material application rate (fixed in these simulations)	1175	kg min <sup>-1</sup>
Ejection velocity	15, 25 and 35% of aircraft speed	m s <sup>-1</sup>
Standard deviation of ejection velocity	2.5, 5.0, 7.5	m s <sup>-1</sup>
Standard deviation of ejection angle	2.5, 5.0, 7.5	°

\*Variables were changed between runs of the single particle ballistics model.

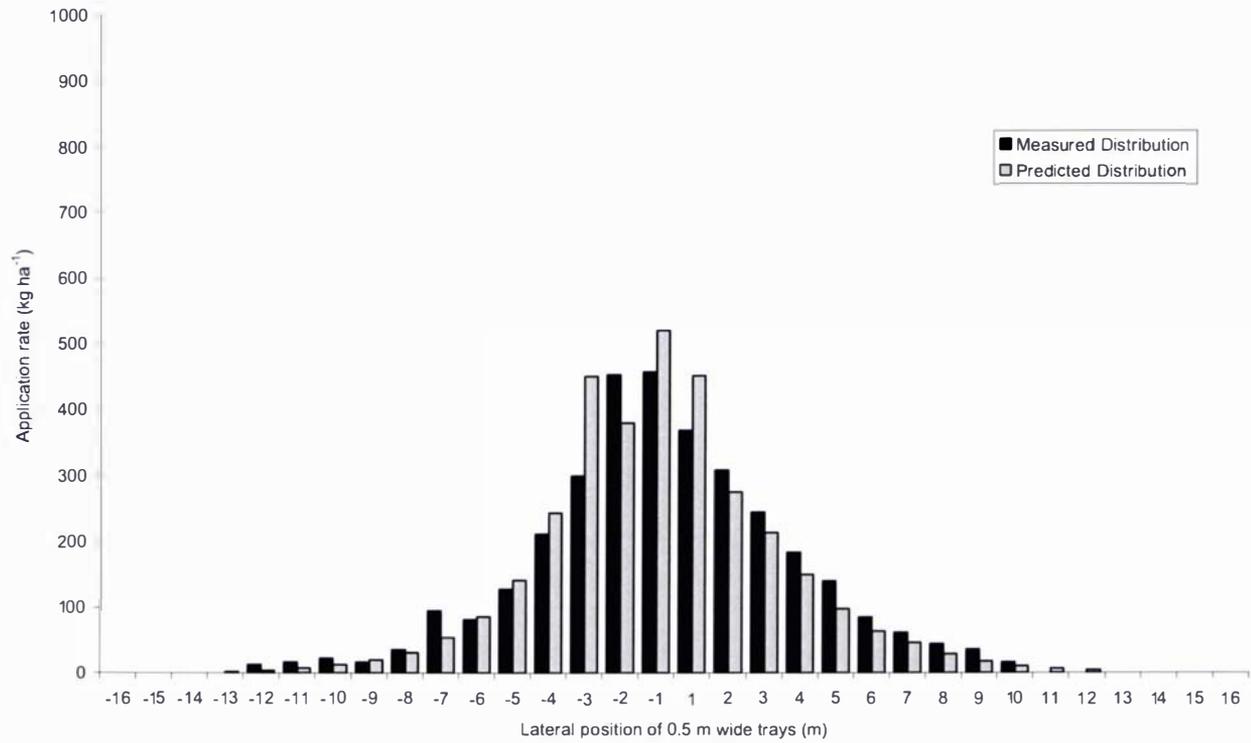
### 3.3 RESULTS AND DISCUSSION

Figure 3.2 shows that the predicted and measured spread patterns are very similar when expressed as an application rate in each tray. Some predictive capabilities were not used in the model. Because the hopper gatebox opening was not recorded, the overall flowrate used in the model was set equal to that measured in the field test. This means that the empirical prediction method described above, where the discharge flowrate is correlated to gatebox opening, was not

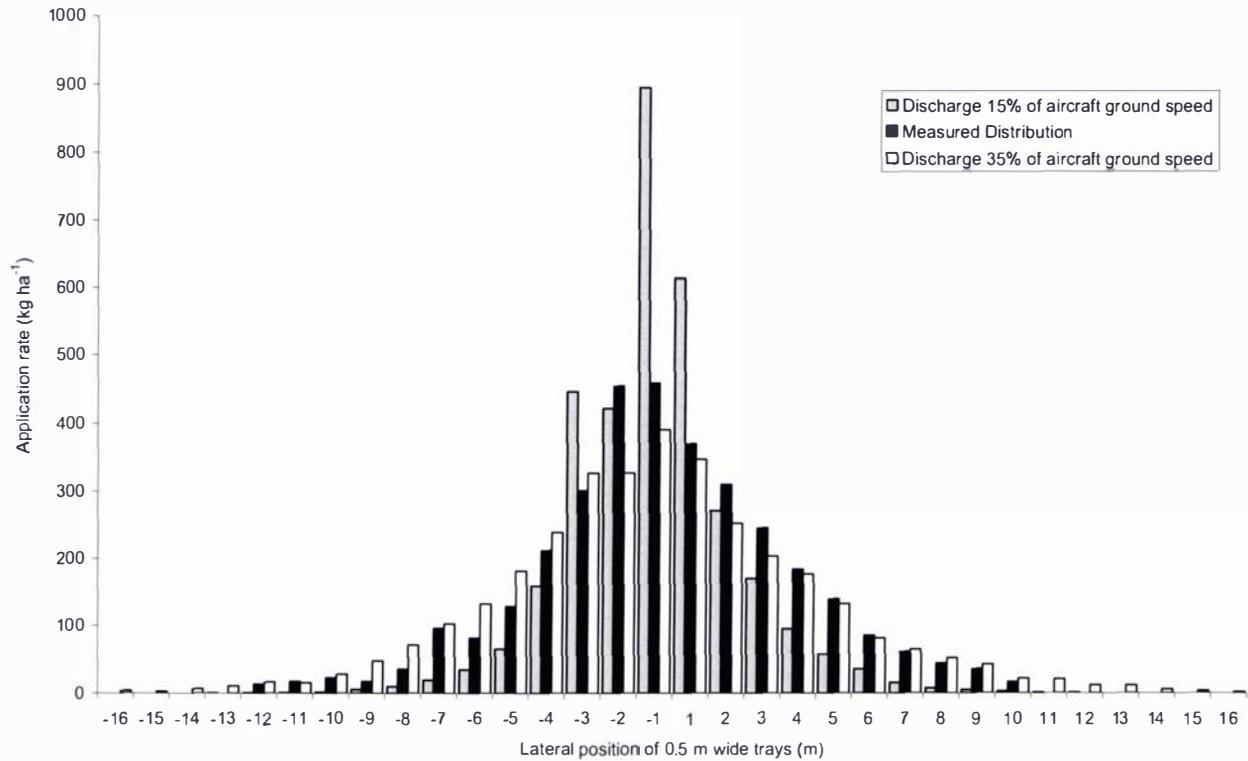
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tested in this study. Figure 3.2, therefore, just compares spread patterns. Secondly the spreader auditing data (R. Horrell, unpublished data, 16<sup>th</sup> October 2002, Feilding, New Zealand) did not contain a measure of the particle size distribution on any of the trays, so no comparison can be made about the predictive ability of the model for different particle sizes. Nevertheless, the spread patterns are similar which gives some confidence in the model.

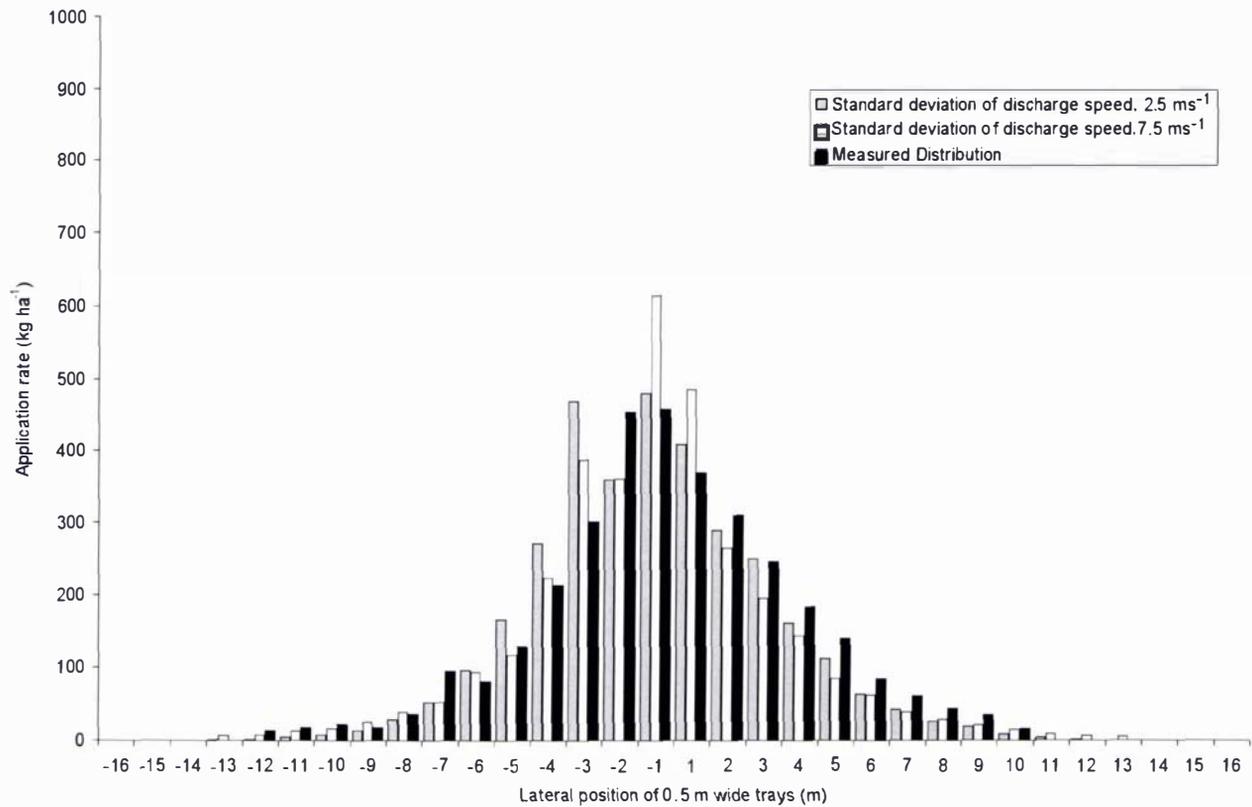
The predictive accuracy of the model depends on three adjustable parameters and the effect of these is examined in Figures 3.3, 3.4 and 3.5. Figure 3.3 examines the effect of changes in mean discharge velocity, reporting the spread pattern at 35%, 25% and 15% of aircraft ground speed. Figure 3.4 examines the effect of the variability in the discharge velocity by altering the best model fit standard deviation of  $5.0 \text{ m s}^{-1}$ , to  $2.5 \text{ m s}^{-1}$  and  $7.5 \text{ m s}^{-1}$ . And lastly, Figure 3.5 examines the effect of variability in the discharge angle by altering the best model fit standard deviation of  $5.0^\circ$ , to  $2.5^\circ$  and  $7.5^\circ$ . The other adjustable parameters are kept constant for each examination.



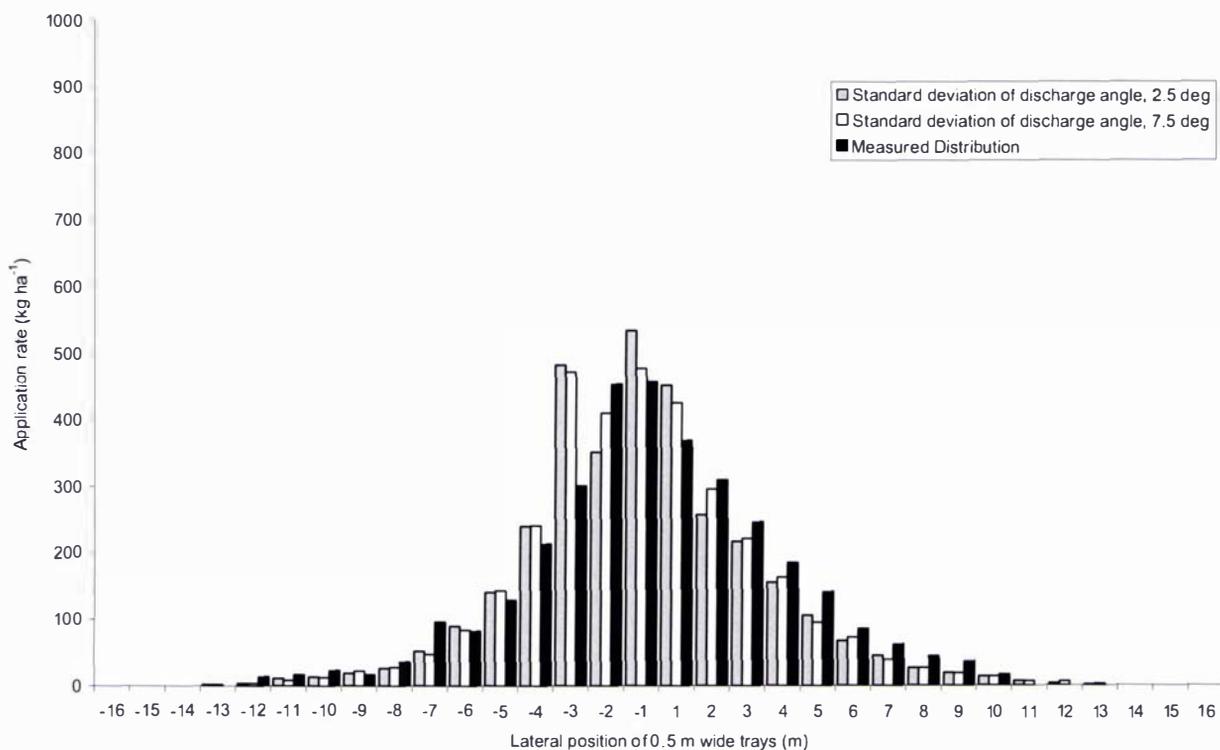
**Figure 3.2. Comparison of the measured and predicted lateral distribution pattern for a thirty-tray transverse test for a Gippsland Aeronautics GA200C fixed wing aircraft with a six duct spreader. Spreader auditing data (R. Horrell, unpublished data, 16th October 2002, Feilding, New Zealand), trays were spaced at 1 m intervals, aircraft flying conditions – altitude 25 m, superphosphate, ground speed 51 m s<sup>-1</sup>, wind velocity 0 m s<sup>-1</sup>, wind angle 0° from flight direction. Discharge velocity set to 25% of flight speed. Negative lateral positions are on the true left of the aircraft.**



**Figure 3.3. Comparison of the predicted lateral distribution pattern of a Gipsland Aeronautics GA200C fixed wing aircraft with a six duct spreader using mean discharge velocities of 15%, 25% and 35% of aircraft ground speed. Trays were spaced at 1 m intervals. Aircraft flying conditions – altitude 25 m, superphosphate, ground speed 51 m s<sup>-1</sup>, wind velocity 0 m s<sup>-1</sup>, wind angle 0° from flight direction.**



**Figure 3.4. Comparison of the predicted lateral distribution pattern of a Gippisland Aeronautics GA200C fixed wing aircraft with a six duct spreader by altering the best model fit standard deviation for discharge velocity of 5.0 m s<sup>-1</sup>, to 2.5 m s<sup>-1</sup> and 7.5 m s<sup>-1</sup>. Trays were spaced at 1 m intervals. Aircraft flying conditions – altitude 25 m, superphosphate, ground speed 51 m s<sup>-1</sup>, wind velocity 0 m s<sup>-1</sup>, wind angle 0° from flight direction. Discharge velocity set to 25% of flight speed. Negative lateral positions are on the true left of the aircraft.**

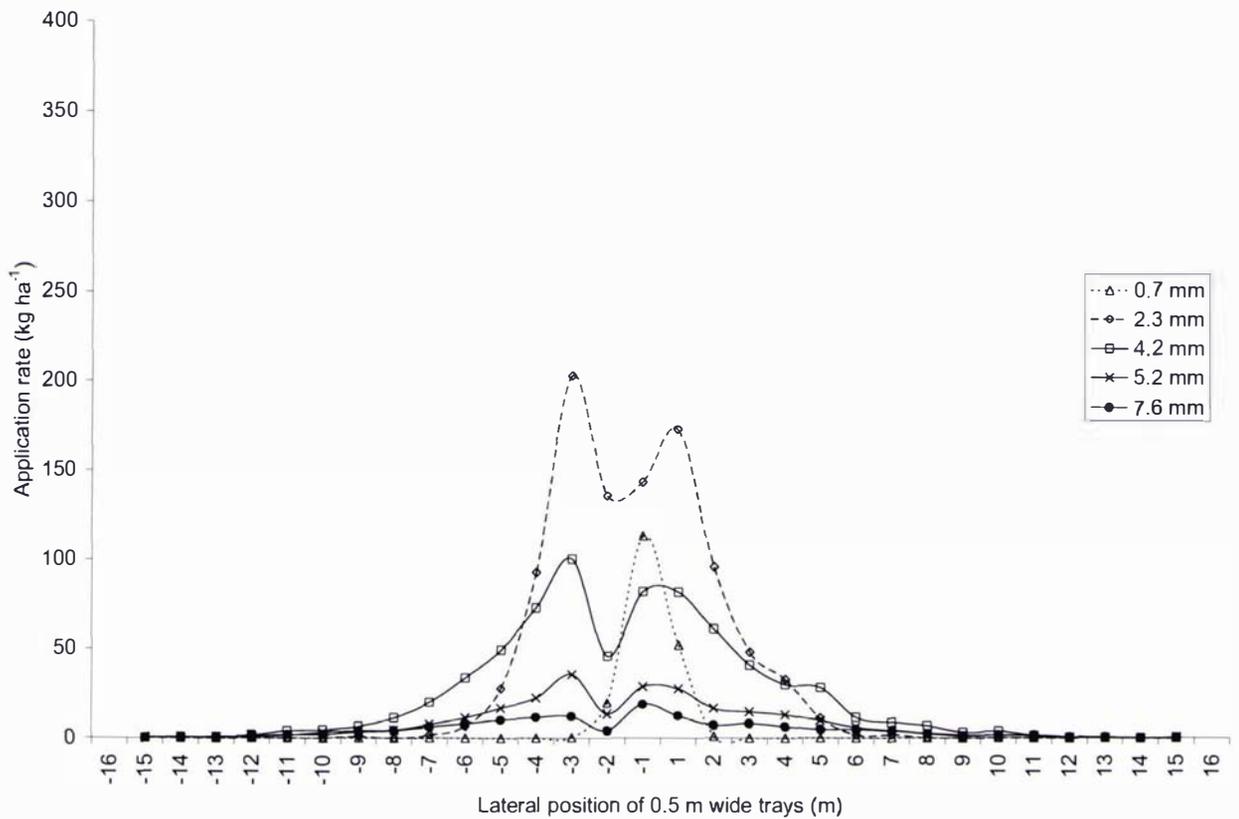


**Figure 3.5. Comparison of the predicted lateral distribution pattern of a Gipsland Aeronautics GA200C fixed wing aircraft with a six duct spreader by altering the best model fit standard deviation for discharge angle of 5.0°, to 2.5° and 7.5°. Trays were spaced at 1 m intervals. Aircraft flying conditions – altitude 25 m, superphosphate, ground speed 51 m s<sup>-1</sup>, wind velocity 0 m s<sup>-1</sup>, wind angle 0° from flight direction. Discharge velocity set to 25% of flight speed. Negative lateral positions are on the true left of the aircraft.**

The effect of altering the two adjustable parameters creates a small amount of variability in the particles landing position. By altering these parameters, the physical process of a particle moving out of the hopper, through the spreader duct and to the ground, is changed only from the point of view of the particles initial discharge into the oncoming airstream. From there, the eventual landing position is a function of drag forces acting on the particle and drift caused by wind. Therefore, we can conclude that the standard deviations of the discharge velocity and ejection angle make little difference to the spread pattern when particle landing positions are gathered into the 0.5 m longitudinal bins that simulate field test results. This leaves the discharge velocity as the major adjustable parameter governing the distribution pattern.

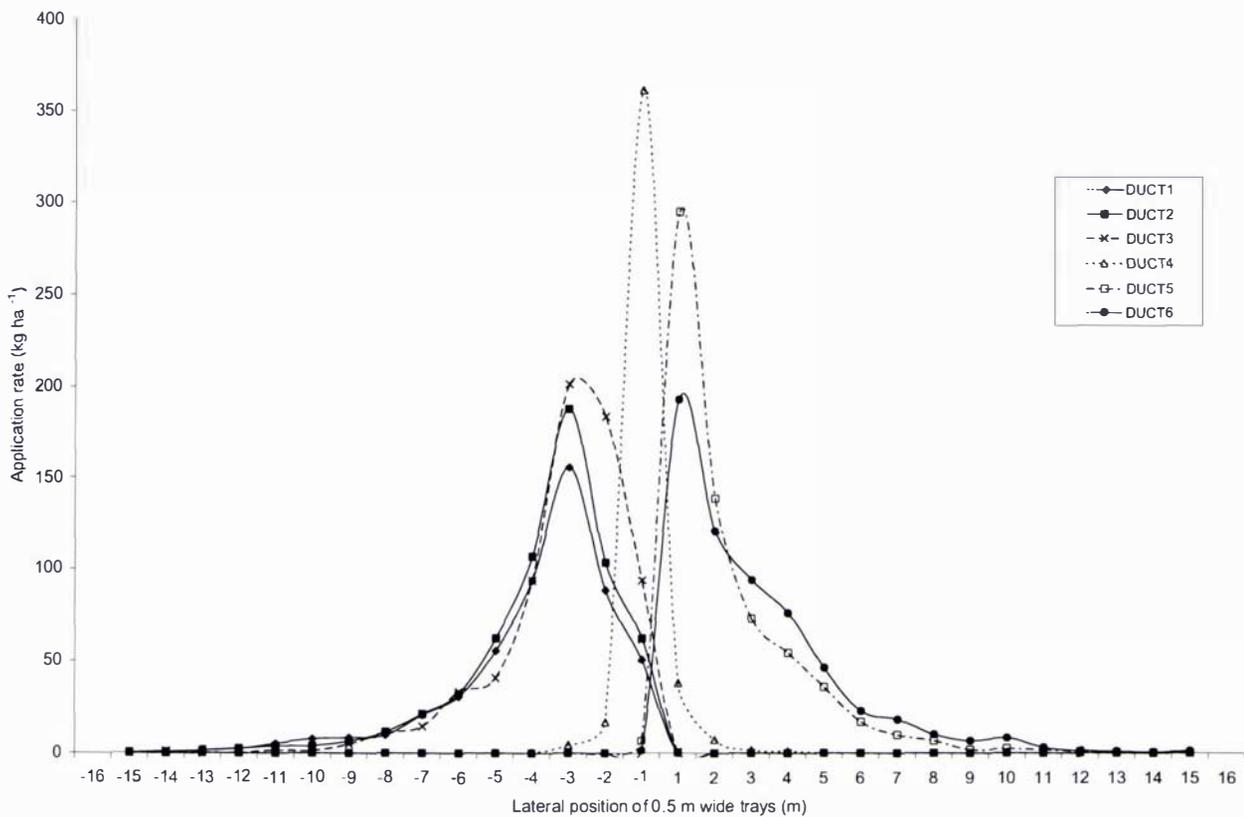
It is then interesting to use the model to predict the spread pattern of particles of different sizes. Figure 3.6 shows the predicted spread patterns for several of the particles used in the

model of Figure 3.2. It shows that larger particles are distributed more evenly across the swath than smaller particles, which are slowed more quickly by drag.



**Figure 3.6. Comparison of the predicted lateral distribution pattern of 0.7 mm, 2.3 mm, 4.2 mm, 5.2 mm, and 7.6 mm diameter particles from a Gipsland Aeronautics GA200C fixed wing aircraft with a six duct spreader. Trays were spaced at 1 m intervals. Aircraft flying conditions – altitude 25 m, superphosphate, ground speed 51 m s<sup>-1</sup>, wind velocity 0 m s<sup>-1</sup>, wind angle 0° from flight direction. Discharge velocity set to 25% of flight speed. Negative lateral positions are on the true left of the aircraft.**

Also interesting is the predicted relative distributions of particles issuing from the six ducts. These are shown in Figure 3.7, in which the effect of the rotating air flow from the propeller wash can be seen when viewing the distribution of duct 4. The predicted deposition profile reflects the clockwise rotation (from the pilot's point of view) of propeller causing a slight drift to the true left of the aircraft. In the longer term, such duct profiles can be used to better design the spreader device mounted underneath the aircraft.



**Figure 3.7. Comparison of the predicted lateral distribution pattern of ducts 1-6 from a Gipsland Aeronautics GA200C fixed wing aircraft with a six duct spreader. Trays were spaced at 1 m intervals. Aircraft flying conditions – altitude 25 m, superphosphate, ground speed 51 m s<sup>-1</sup>, wind velocity 0 m s<sup>-1</sup>, wind angle 0° from flight direction. Discharge velocity set to 25% of flight speed. Negative lateral positions are on the true left of the aircraft.**

Further validation work is required. This paper reports a promising result where the single particle ballistics model accurately predicts the spread pattern for one field trial using three adjustable parameters, two of which (the standard deviation of the discharge velocity and the standard deviation of the ejection angle) make little difference to the spread pattern. Therefore, the discharge velocity becomes the major adjustable parameter. It is expected that discharge velocity should be easily adjusted for each combination of aircraft, spreader device and fertiliser type, in which case this model can be used to accurately determine spread patterns for agricultural spreading of granular products. Nevertheless, further validation field trials should be conducted to include the prediction of discharge flowrate, at a range of application rates and different fertiliser types, on a range of aircraft and spreader devices typically used for spreading,

in a variety of wind conditions. The particle size distributions should also be collected. If validated, and the adjustable parameters are determined for each combination of aircraft, spreader and fertiliser type, the model becomes a powerful tool. It can be used to assess the benefit of variable rate application technologies, both to the extent that variable rate can be applied to the ground and the economic return possible when combined with GPS technology. As the end goal, it can form part of a variable application rate service provided by aerial spreader operators to farmers for pre-flight planning of fertiliser application rates over land units, as well as providing real-time predictions of the distribution pattern for the pilot, while in flight.

### **3.4 CONCLUSION**

The single particle ballistics model reported in Chapter 2 has been extended to predict lateral spread patterns by accounting for the material flow from the hopper, its profile across the width of the hopper gatebox and its dispersal into the ducts of the spreader device mounted underneath the aircraft. Techniques for measuring these parameters were described and experiments were conducted in order to define the values for input to the model.

The model has three adjustable parameters, the discharge speed of the particles from the spreader device, the standard deviation of this discharge speed, and the standard deviation of the discharge angle about the various vanes that divide the ducts. Data from a single field trial was used to test the predictive ability of the model. Material flow was fixed and so the comparison is effectively between the shapes of the predicted and measured lateral distribution patterns. The modelled pattern showed good agreement with the measured pattern using the optimized adjustable parameters which were, discharge speed set equal to 25% of the aircraft groundspeed, standard deviation of the discharge speed set to  $5 \text{ m s}^{-1}$  and standard deviation of the discharge angle set to  $5^\circ$ . A cursory sensitivity study indicates that the stochastic parameters make little difference to the spread pattern leaving the major adjustable parameter as the mean particle discharge speed. Further validated work is required at a range of flight conditions, for a range of fertilizers discharged at a range of rates.

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This model could serve as a useful tool in establishing and monitoring the performance of an aerial topdressing aircraft whilst in flight, it would provide a tool to compare the fertiliser deposition using improved application techniques, such as variable rate, to those currently employed. It could provide the pilot with real-time feedback about the distribution pattern whilst in flight. In addition, the transverse distribution model could be used as a tool for optimising the design of spreaders or optimum particle characteristics for a given spreader. It has the ability to predict the distribution profile of any particle size distribution from each, or all, of the spreader ducts. Furthermore, the optimum particle size distribution for a fertiliser product could be obtained by varying the particle diameter and size fraction and examining the resulting distribution pattern. Operators could sieve a representative sample of a fertiliser heap and determine the likely distribution pattern of the material given the likely environmental conditions expected on the day of field application. They may also request specific particle characteristics and size fractions from the fertiliser manufacturers in an attempt to optimise spreading with their particular aircraft/spreader configuration.

The usefulness of this work may be further enhanced when embedded within a GIS, this will enable the prediction of fertiliser application variability and deposition on a farm-scale to be investigated and would provide a means to assess the agronomic, economic and environmental impacts of fertiliser application by agricultural aircraft. The third paper in this series (Murray and Yule, 2007c) addresses these issues.

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## **CHAPTER 4 - MODELLING SOLID FERTILISER DEPOSITION FROM A FIXED WING AIRCRAFT - FIELD SCALE PREDICTIONS WITHIN A GIS ENVIRONMENT**

*Publication arising from this chapter*

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Murray, R. I., Yule, I.J. 2007. Modelling solid fertilizer deposition from a fixed wing aircraft – III. Field scale predictions within a GIS environment. *Trans. ASAE (under review)*.

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### **4.1 INTRODUCTION**

Aerial topdressing of hill country pastures is used to achieve a productivity response. Highly manoeuvrable aircraft are the only practical means of access, and large areas can be covered quickly with the expectation of higher yield and improved economic return from the land. The actual spreading performance of aircraft is difficult to measure, therefore it is not well documented, and the effects of the operating environment are not sufficiently understood. Transverse distribution testing remains the standard measure for determining an agricultural aircraft's spreading performance for aerially applied fertiliser products in New Zealand. Presently, operators have the option to take part in the Aerial Spreadmark Code of Practice (FQCNZ, 2003), where a single pass over a series of collectors, perpendicular to the flight line, determines the effective swath width for that particular aircraft at a designated coefficient of variation (CV). In this process the application of fertiliser is captured in a snapshot of time, thus the environmental and operating conditions at that point can be considered constant. This does little to account for the variables that actually occur in reality. Wind speed, particle exit velocity, particle size distribution and density are known to affect fertiliser deposition, both laterally and longitudinally, and these variables are rarely constant for any length of time, often varying from one minute or load to the next. Furthermore, there is considerable uncertainty when attempting to obtain a representative sample of the particle size distribution; a load taken from one part of the silo can differ markedly in particle size distribution from a load taken from another part. Each of these factors have the ability to alter the distribution pattern on their own,

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however their combined effect can cause considerable variability.

Presently, once an aircraft is certified for a particular product under the Aerial Spreadmark Code of Practice (FQCNZ, 2003), that aircraft can apply the product at rates  $\pm 30\%$  of the tested rate, in any conditions and with any pilot. Although this form of certification is desirable for the operator, it has limitations, and can only predict the spreading performance in the environmental conditions prevailing at the time of the test. It does not provide an indication of how the spread pattern can be affected in variable environmental conditions and what this could mean for application at ground level. A further limitation of this certification process is that it does not reveal the longitudinal displacement of different particles, so accurately judging buffer distances, or run start and stop times, would be difficult and some level of guess work may be required. Clearly there is a need for aircraft certification, as it gives an indication of the likely performance. Unfortunately a more comprehensive testing regime is likely to be costly and the ability to complete multiple certifications in different conditions may not always be achievable. As well as transverse distribution, a measure of longitudinal displacement from the flow cut-off position to the landing position would be a useful addition to the testing procedure. An alternative approach is to develop a model capable of determining the deposition footprint in a range of operating and environmental conditions. The advantages of this method are that the operator could enter the current environmental conditions and representative particle characteristics, and the model would return the appropriate track width and offset distance from buffered or boundary edges. This may be extended further to include: route planning, where the pilot would have the ability to set A –B lines and offset distances for flight paths depending on forecasted conditions; risk assessment, where material may drift onto neighbouring properties or sensitive areas and variable flow and rate control, for real-time monitoring and adjustment of shut off and buffer distances. Verification of the application job for presentation to regional bodies, or, proof of placement for farmers, is also likely to be a future requirement of aircraft spreading operations in New Zealand.

This paper follows on from the earlier single particle (Jones et al., 2007) and transverse distribution modelling (Murray et al., 2007b) work and describes the methods used to capture

and model field scale fertiliser application through the use of Geographical Information Systems (GIS), data acquisition, and software development. The objectives of this work were to firstly, measure field scale application using a series of collector transects, secondly, collect flight information and environmental conditions throughout the flight (this data was used as input for the model), and finally, to compare the actual and predicted fertiliser application within the collectors and across the transects under field application conditions.

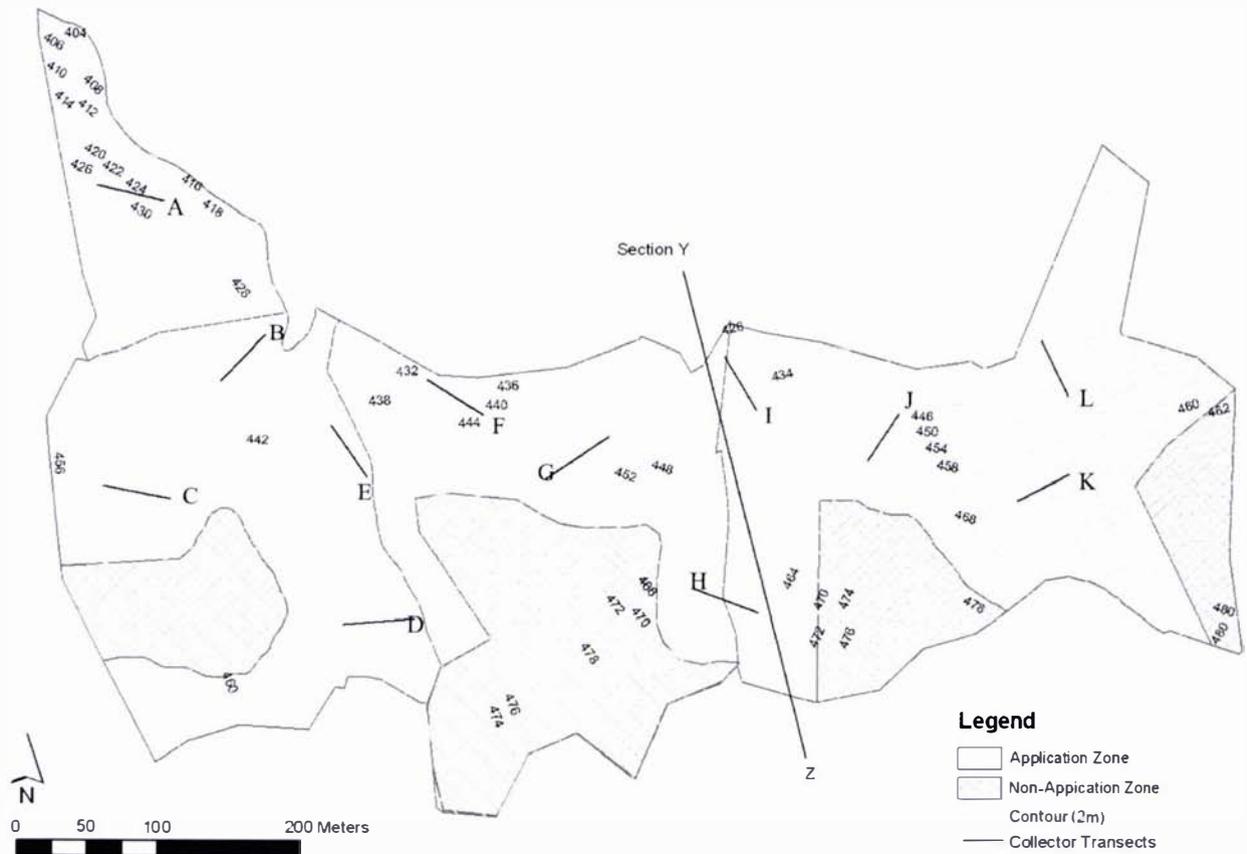
#### **4.2 MATERIALS AND METHODS**

The trial site, a 25 ha segment of a hill country farm, was located 6.5 km north of Kimbolton, in the Manawatu region, New Zealand. The site was separated into two distinct management units, either application or non-application zones. Application zones were targeted for fertiliser application, while non-application zone consisted of four stock camps located on the tops of the main ridges, these constituted approximately 6 ha in total (Figure 4.1), these areas were not to receive fertiliser.

ArcMap 9.0 (ESRI, 2006) was used to divide the 19 ha application into twelve  $100 \times 100$  m zones spread across the application area. Within each of these zones, a 45 m long transect line was created from the centroid of the zone at a randomly selected angle between  $1^\circ$  and  $360^\circ$  (north being  $360^\circ$ ). A total of 120 conical collectors from the New Zealand Agricultural Aviation Association were obtained. Each collector consisted of a circular frame with a flexible plastic funnel which narrowed and was fixed to a section of PVC pipe where the collector bag could be connected. The diameter of the collector opening was 0.6 m, with a cross sectional area of  $0.28 \text{ m}^2$ . Each collector was attached, approximately one meter above the ground, to a metal pole driven into the ground. The transects, A through L, contained ten of the conical collectors spaced at five meter intervals; these were pegged out in straight lines across the topography using an RTK GPS pre-loaded with the latitude and longitude (datum WGS1984) for each collector position.

A small electronic potentiometer, type F200 (linearity 0.1%, resistance  $5\text{k } \Omega$ ), Japan Servo Co. was fitted to the pivot point of the hopper gatebox; the output was connected to a USB-

1208FS, Measurement Computing Corp., data acquisition device. The digital signal produced by the potentiometer was calibrated with the actual gate opening of the hopper prior to the flight and the relationship  $y = 0.6623x - 12.247$  (Murray et al., 2007b) was used to determine flowrate from the hopper opening. Aircraft speed, height above mean sea level and position were obtained from a NovAtel Allstar GPS receiver (12 channel, DPGS, 5Hz output). A simple data collection program was written in Microsoft Visual Basic .Net (Microsoft Corporation, 2002) that time stamped, collated and recorded the data to a portable laptop computer (Compaq Presario, 450MHz). Further information about wind direction and magnitude was obtained via a store-on-board anemometer (Maximum Anemometers, type #41) and wind vane (NRG, #200P) combination, setup at a 1 Hz recording interval.

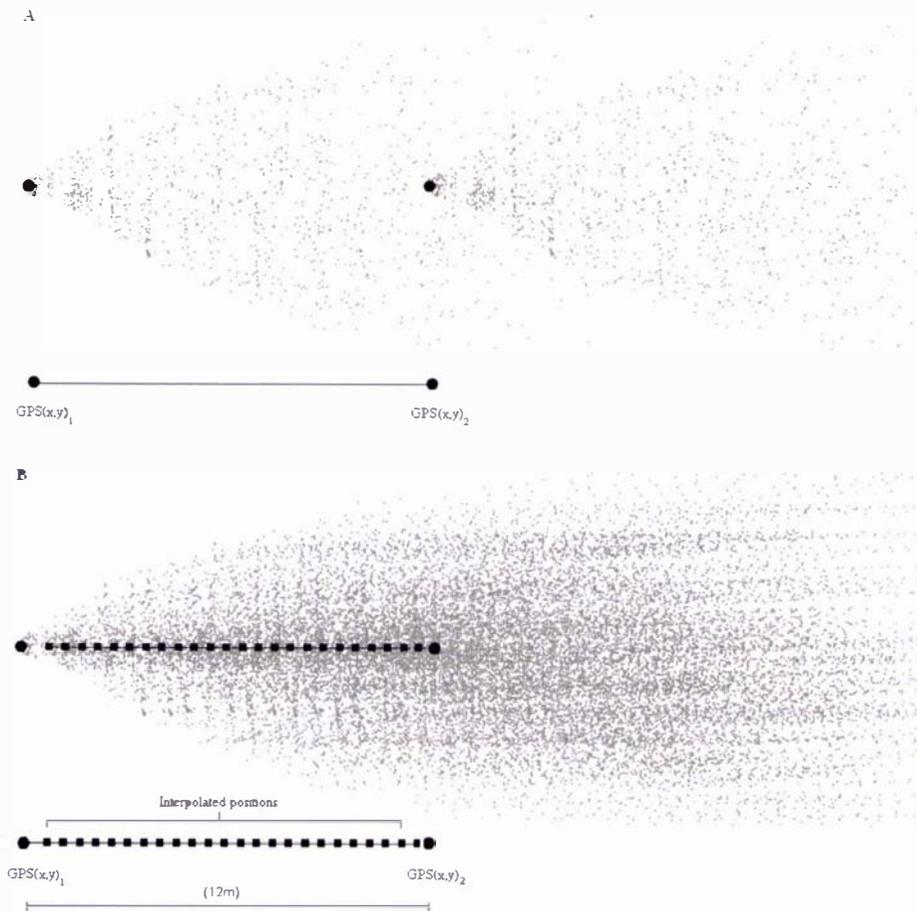


**Figure 4.1.** Field trial collector layout on 25 ha of hill country sheep and beef farmland, includes four non-application zones. Transects A - L consisted of ten collectors spaced at five meter intervals. Individual collector area 0.28 m<sup>2</sup>, target application rate was 150 kg ha<sup>-1</sup> of superphosphate fertiliser, cross section Y - Z is referred to below.

A Gippsland Aeronautics GA200C aircraft was used to spread the superphosphate fertiliser at a target application rate of 150 kg ha<sup>-1</sup>. The aircraft was fitted with a Satloc M3 navigation system (SLXg3 DGPS receiver, 12 channel, 5Hz output) which was used for parallel track guidance. A six duct, non ram-air type spreader was attached to the hopper gatebox. The aircraft's hopper capacity was 1050 litres and the intended track width was 12 m flying in an east-west direction. Three loads of fertiliser were required in order to cover the 19 ha application area. Once complete, each of the 120 plastic bags containing fertiliser were collected and weighed. In total, 1087 geo-spatial data points were recorded on the portable laptop, for each of these; wind speed and direction, taken from the on site wind meter, was matched to its respective GPS and hopper door opening data via the time stamps.

The characteristics of the superphosphate fertiliser, duct flow rates, discharge angle, release height, prop wash and stochastic variability used in this work were identical to that presented in Murray et al. (2007b). What differed were the run parameters and the initial conditions for the model, these were tagged to each geo-spatial data point obtained during field application and because of this the output of the GPS receiver was considered a limiting factor. Therefore, the distance between two subsequent points was divided into sub-locations to increase the number of times the model could be run and hence the number of particles impacting the ground; the effect of this is illustrated in Figure 4.2. This was considered appropriate given the small time period involved and the likelihood that conditions would remain relatively constant over that small time period. Without interpolation (Figure 4.2A), the particles deposition footprint lacks sufficient density and does not represent the reality of continuous flow of fertiliser material from the hopper. In comparison, Figure 4.2B increases the density of the deposition footprint and better reflects the continuous flow condition of the fertiliser being discharged. Thus the distance between two successive data points was divided into 0.5 m increments and the concentration of each particle size reduced proportionally as a function of the particle size distribution (refer to Chapter 3, Table 3.1) and flowrate (as determined above). A total of 48 particle landing positions were calculated every time the model was run, these were from the eight different particle sizes travelling through each of the six ducts described in Murray et al. (2007b). At each

sub-location, the initial conditions for wind speed and direction, aircraft speed, height above ground and flow rate were identical to the preceding GPS point attributes, this was necessary as real-time information was not available at the sub-locations. To aid simplicity, both of these procedures were automated and built into the model.



**Figure 4.2. Schematic illustration of the effect of interpolating GPS positions to create sub-positions and increase point density A) deposition footprint without interpolation B) deposition footprint with interpolation.**

**Note: Particle concentrations were reduced as a function of the interpolation distance.**

The dataset that was collected on the day of the field trial contained 1087 geo-referenced data points, each with heading, speed, wind direction and magnitude, aircraft height and flow rate attributes. This was imported into ArcMap 9.1 (ESRI, 2006) where it was used as the initial conditions for the remainder of the modelling process.

The same set of GPS input data was used to determine if any improvement in application performance could be made using variable-rate and flow-control technology. In this case, the flight path data described above was imported into ArcMap 9.1 (ESRI, 2006). Data were

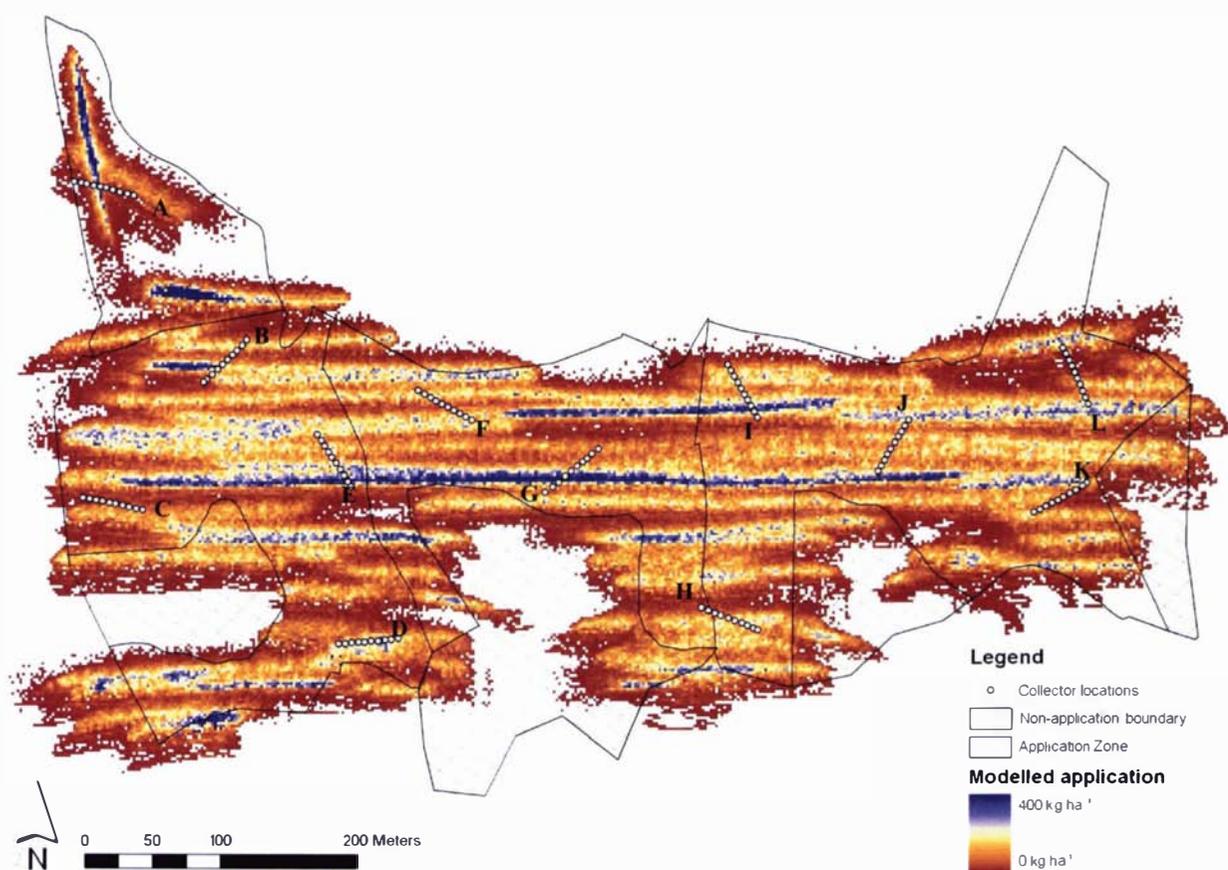
removed when the aircraft fell within a 45 m buffer zone around a non-application or field boundary when heading towards, or into, these areas. In contrast, data were removed when the aircraft was located within an internal 45 m buffer zone when flying out of, or away from a non-application area or field boundary. All data was removed when aircraft was positioned inside a non-application area. This ensured a reduction in the amount of fertiliser landing within non application zones and imitated automated hopper door control. One further modification to the input data was required for realistic emulation of automated flow control. In order to maintain a designated application rate, adjustments in the flow rate were made depending on the aircraft ground speed and swath width (Eq. 4.1). The case was included to determine the effect of flow and automated hopper door control on the application performance; therefore, swath offsets to compensate for cross winds were not included.

$$\text{Flow rate [kg s}^{-1}\text{]} = \text{application rate [kg m}^{-2}\text{]} \times \text{ground speed [m s}^{-1}\text{]} \times \text{swath width [m]} \quad [\text{Eq. 4.1}]$$

With a target rate of 150 kg ha<sup>-1</sup>, swath width of 12 m and ground speed of 54 m s<sup>-1</sup>, the flow rate from the hopper was calculated to be between 9 - 10 kg s<sup>-1</sup> (≈ 580 kg min<sup>-1</sup>), varying proportionally with changes in ground speed.

### **4.3 RESULTS AND DISCUSSION**

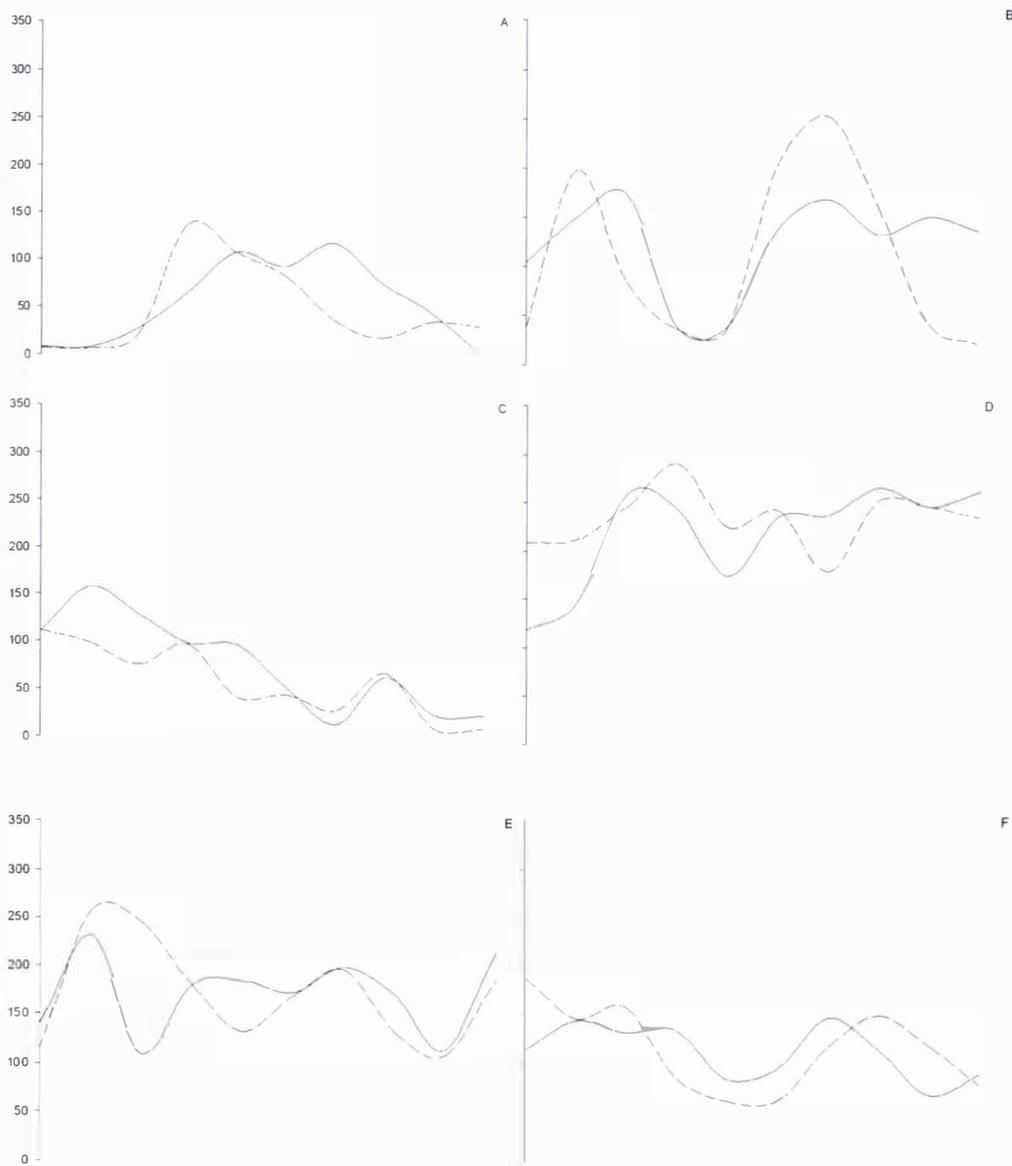
The dataset produced from the field scale application model contained well over one million data points, each with the particles physical characteristics, landing position, relative concentration, as well as, the aircraft and environmental initial conditions. The individual particle concentrations were converted to a raster surface by summing the concentration of each particle that fell within a 2 × 2 m cell, the resulting layer was converted to application rate in kg ha<sup>-1</sup> by dividing the summed concentration by the cell area then converting through to hectares. The resulting application map (Figure 4.3) showed the spatial variation in application rate over the entire trial site. A high predicted application rate is shown in blue, whereas a low application rate is represented by brown colouring. This trial was conducted with a manual hopper door control system that was operated by the pilot.

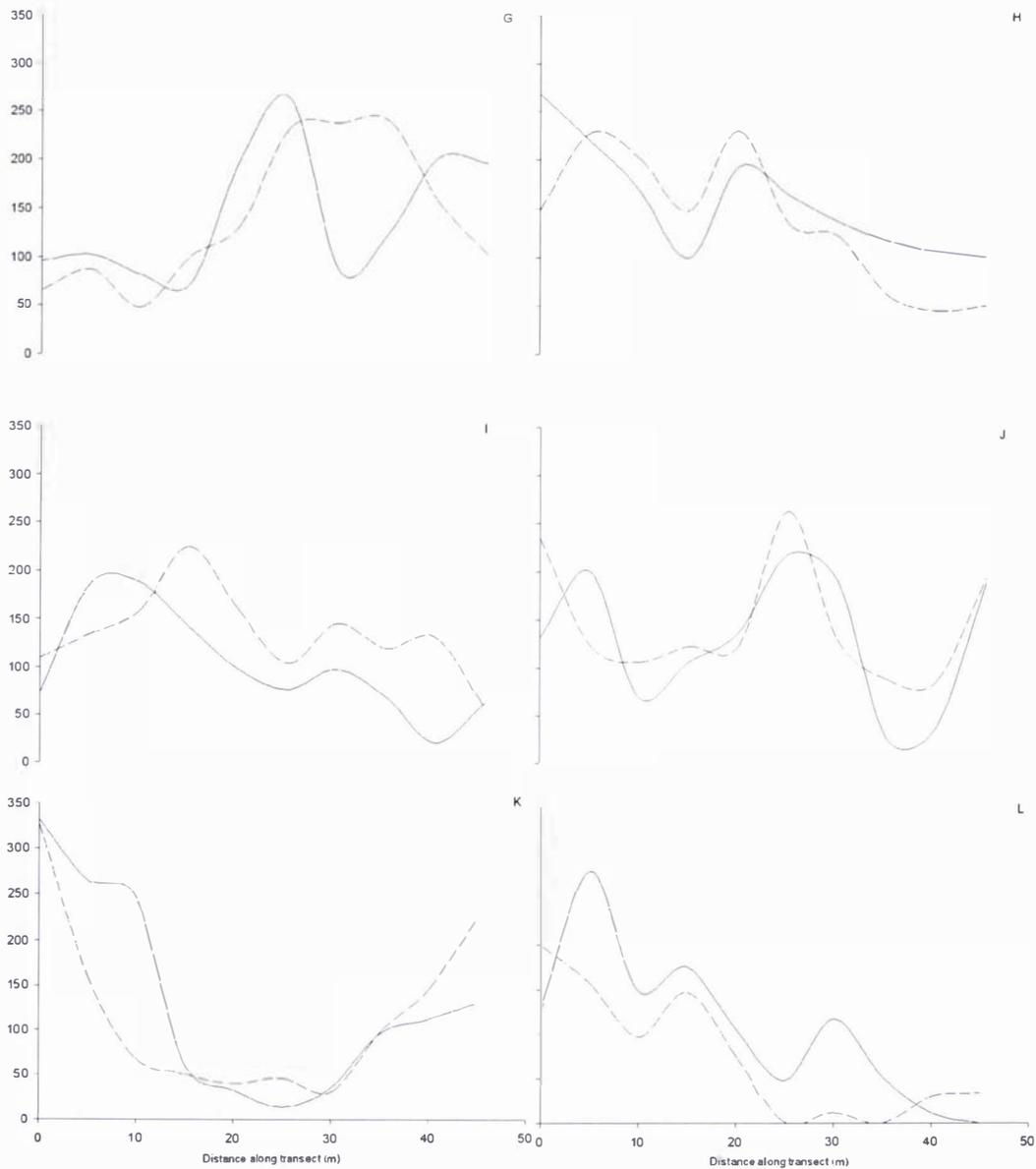


**Figure 4.3. Predicted field scale application ( $\text{kg ha}^{-1}$ ) on a 25 ha trial site, 15 km North of Kimbolton, Manawatu, New Zealand.**

Data from the collectors revealed a mean application rate of  $123 \text{ kg ha}^{-1}$  (std. dev.  $74 \text{ kg ha}^{-1}$ ) and a coefficient of variation (CV) of 0.6 (It should be stated that this was a very challenging trial situation for the pilot due to the small application area; hence this should not be considered a typical application scenario). In comparison, over the area shown in Figure 4.3, the predicted mean application rate was  $93 \text{ kg ha}^{-1}$  (std. dev.  $92 \text{ kg ha}^{-1}$ ) and the total quantity of fertiliser applied was predicted to be 1.9 tons at a CV of 0.97. Separating this into application and non-application zones allowed further comparisons to be made. When considering only material landing within the application zone, the mean application increased to  $107 \text{ kg ha}^{-1}$  (std. dev.  $95 \text{ kg ha}^{-1}$ ) constituting 1597 kg of fertiliser over 14.87 ha and the total area receiving lower than  $10 \text{ kg ha}^{-1}$  was found to be 1.52 ha. In the non-application zone, approximately 193.3 kg of superphosphate was predicted to be applied over 3.0 ha. The remaining 103.9 kg was predicted to be applied outside the trial boundary.

Further comparisons can be made between the predicted and measured values across the collector transects. Figure 4.4 presents the predictions and indicates the level of variability within the transect. In Figure 4.4 many of the prediction trend similar to the actual pattern, however trends were less evident when several passes were made over the one transect, such as in G, H and L. There are a number of possible reasons for the pattern discrepancies; however, it is most likely a result of fluctuations in the particle size distribution or errors in wind measurement for that location. A small change in either variable can significantly alter the distribution pattern; this is covered in more detail below.





**Figure 4.4.** An illustration of the predictions of fertiliser distribution along the collector transects A-L. Each transect consisted of ten collectors spaced at five meter intervals. — Solid line is measured application rate, --- dashed line is the predicted application rate

Regression analysis of the mean measured and predicted application rate for each transect (A-L) in Table 4.1 reveals a coefficient of determination of  $R^2 = 0.88$ . Table 4.1 shows that although variation between measured and predicted mean application rates in individual collectors exists (Fig. 4.4), similar means are obtained if averaged over the entire transect. Also shown is the residual analysis of the actual and predicted mean application rates for each transect. This revealed errors ranging between 0.5% and 32.4%. Interestingly, it was not always

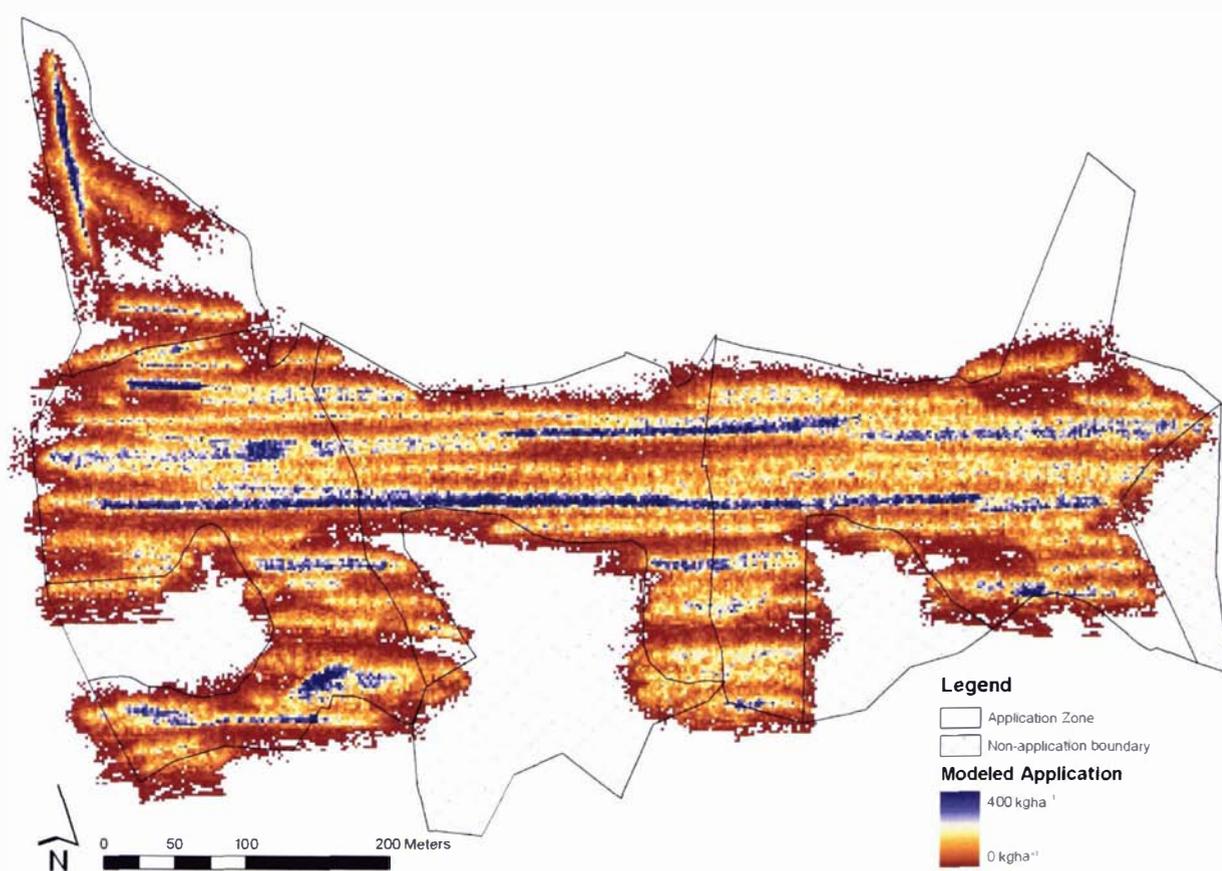
transects with the lowest residual error that trended similar distributions patterns in Figure 4.4. Transect G is an example of this, it had a residual error of only 1.4% but the modelled and predicted patterns shown in Figure 4.4 deviated markedly between 25 - 35 m along the transect. Although the residual errors were in some cases quite high, overall, the predicted mean application rates showed similar trends to the measured mean application rates.

**Table 4.1. Comparison of measured and predicted mean application rate ( $\text{kg ha}^{-1}$ ) from transects A – L, each consisting of 10 collectors spaced as 5 m intervals.**

Transect	Mean application ( $\text{kg ha}^{-1}$ )		Residual	Error (%)
	Measured	Predicted		
A	54.0	47.7	6.3	11.6
B	121.9	106.2	15.7	12.9
C	75.3	57.2	18.1	24.0
D	217.4	232.4	-15.0	-6.9
E	170.9	171.7	-0.9	-0.5
F	111.6	115.8	-4.2	-3.8
G	142.5	140.5	2.0	1.4
H	158.0	137.2	20.7	13.1
I	101.8	134.7	-33.0	-32.4
J	130.5	147.0	-16.5	-12.6
K	132.4	119.1	13.2	10.0
L	106.5	75.3	31.2	29.3

Comparisons of the performance between the modelled application (Figure 4.3) and modelled application using an automated flow and hopper door control system (Figure 4.5) were made. Over the area shown in Figure 4.5, the predicted mean application rate was  $107 \text{ kg ha}^{-1}$  (std. dev.  $99 \text{ kg ha}^{-1}$ ) and the total quantity of fertiliser applied was predicted to be 1.8 tons at a coefficient of variation (CV) of 0.93. When considering only material landing within the application zone, the mean application increased to  $118 \text{ kg ha}^{-1}$  (std. dev.  $100 \text{ kg ha}^{-1}$ , CV 0.85) constituting 1719 kg of fertiliser over 15.59 ha, the total area receiving lower than  $10 \text{ kg ha}^{-1}$  in the application zone was found to be 1.47 ha. In the non-application zone, approximately 71.1 kg of superphosphate over 1.49 ha was applied. The remaining 45.7 kg was applied outside the trial boundary. In the automated control system case, only 6% of the total fertiliser spread was outside the application area, compared to 16% for the case shown in Figure 4.3. Clearly an automated system would deliver better spreading performance and, not only did it achieve a marginally better CV, it increased the mean application rate closer to the target application and

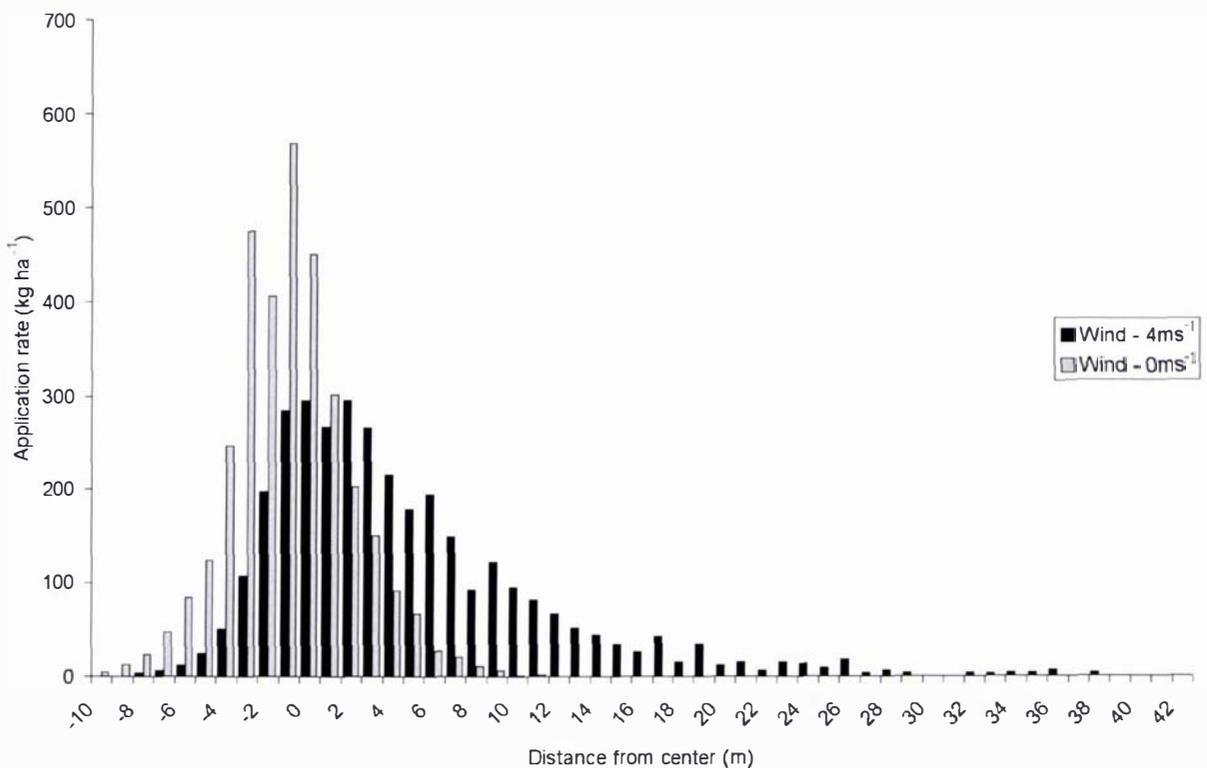
could potentially free up more time for the pilot to navigate the aircraft safely along the correct flight path. The data from the trial also indicated that the pilot maintained a fairly constant ground speed with a mean of  $56 \text{ m s}^{-1}$  (std. dev  $2.5 \text{ m s}^{-1}$ ). These minor speed fluctuations had little effect on the flow rate, thus, in hindsight it was probably not an ideal situation for comparing a manually controlled flow system to an automated constant flow system. Studies by Murray and Yule (2006) on a large hill country property, indicated greater aircraft groundspeed variation than that reported here, this would indicate that using automated hopper door control may actually be of greater benefit than this small field study suggests.



**Figure 4.5. Predicted field scale application ( $\text{kg ha}^{-1}$ ) using automated hopper door control on a 25 ha trial site, 15 km North of Kimbolton, Manawatu, New Zealand.**

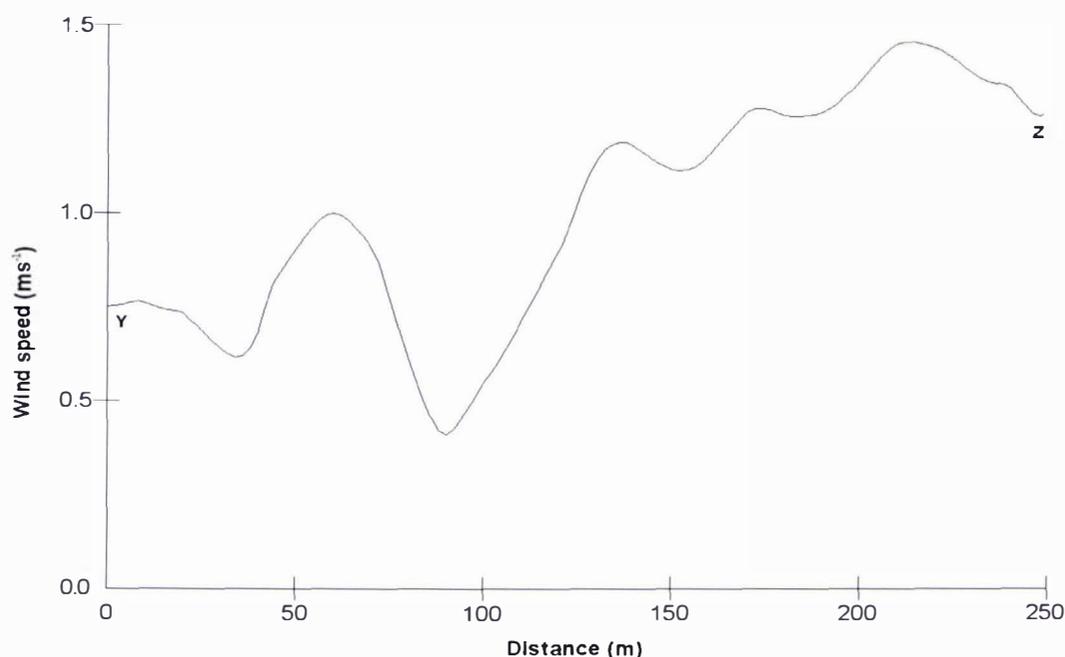
The effect of cross wind was evident in Figures 4.3 and 4.5. In both cases the average wind speed was  $0.8 \text{ m s}^{-1}$  (std. dev.  $0.4 \text{ m s}^{-1}$ ) from  $353^\circ$  (std. dev.  $19.7^\circ$ ) with the aircraft flying predominantly in the east west direction. It can be shown that cross winds alter the transverse

distribution pattern and potentially the swath width that the pilot could fly whilst maintaining an acceptable CV. This is illustrated in Figure 4.6 using the two extremes of wind speed, either zero wind or  $4 \text{ m s}^{-1}$  blowing from  $315^\circ$ . It indicates how improvements in the CV could be made when overlapping a distribution pattern that has been smoothed and extended by cross winds. Clearly the peak application rate was reduced, and the distribution skewed to the right of the flight line, the combined effect has implications for application uniformity. At an arbitrary 10 m track spacing, the transverse CV is 0.37 in zero wind conditions, this is greatly reduced to 0.18 in the case of a  $4 \text{ m s}^{-1}$  wind from  $315^\circ$ .



**Figure 4.6. Modelled transverse fertiliser distribution of particles ejected from a Gippsland Aeronautics GA200C fixed wing aircraft with a six duct spreader – altitude 25 m, aircraft heading  $360^\circ$ , superphosphate, ground speed  $54 \text{ m s}^{-1}$ , no wind and  $4 \text{ m s}^{-1}$  wind blowing from  $315^\circ$**

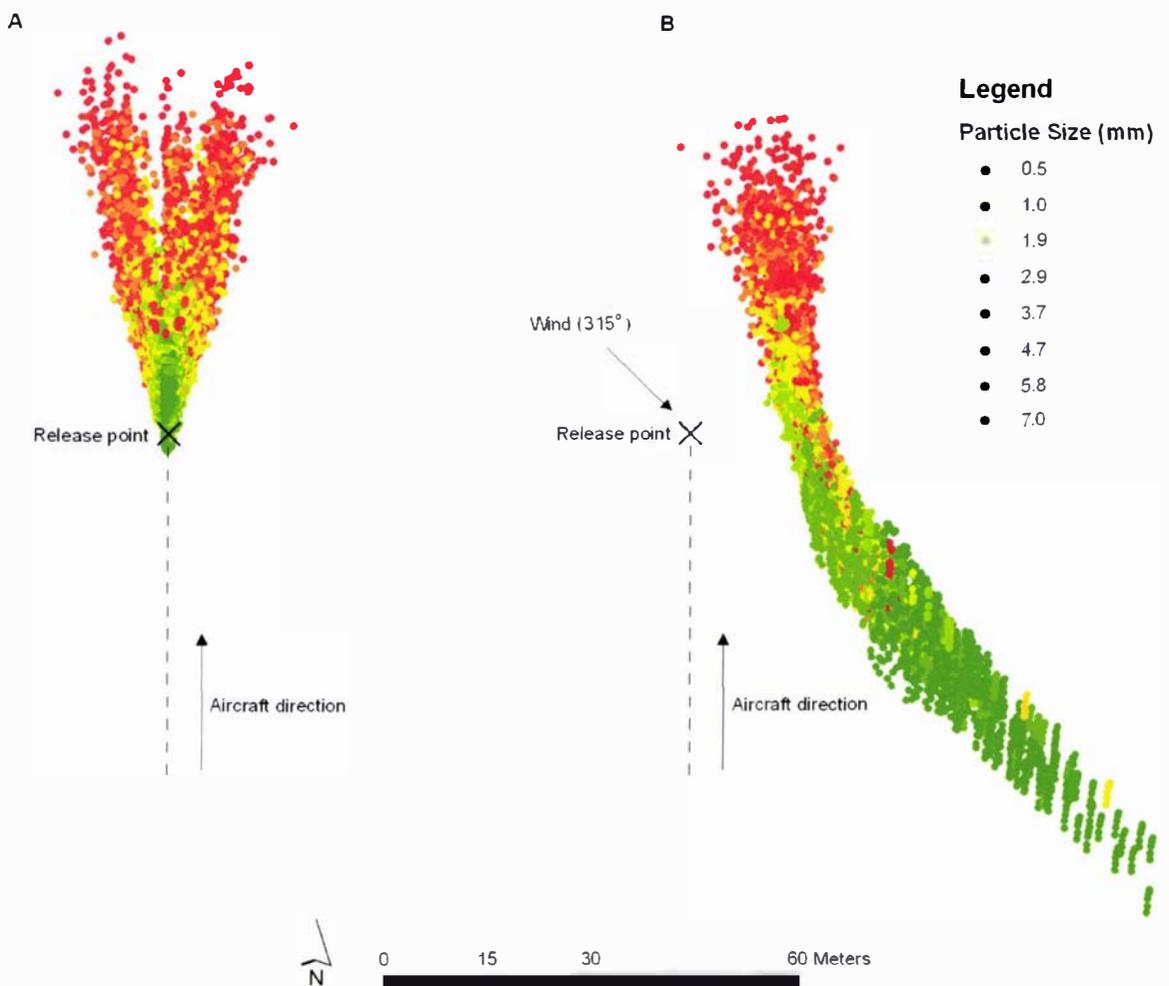
The uniformity of application from pass to pass is undoubtedly improved in the case of stable cross winds of low magnitude ( $<1.5 \text{ m s}^{-1}$ ) (Figure 4.6); however this is unlikely to occur in reality. Consider a cross section of wind speed (Figure 4.7) taken across the trial site and across time (Figure 4.1, section Y – Z) in the north/south direction. Assuming a swath width of 12 m, ignoring wind direction, and considering the transect is 250 m long, the pilot would need to cross this line approximately twenty times to complete the application job. Each time the pilot crosses this line, the winds magnitude and thus the transverse distribution pattern would differ, causing variations in the overlapped deposition at ground level.



**Figure 4.7.** Cross section (Y-Z) of wind speed in the north/south direction taken from an interpolated ArcGIS raster surface. Wind speed data was obtained during fertiliser application on a 25 ha trial site, 15 km North of Kimbolton, Manawatu, New Zealand (refer to Figure 4.1).

Linking this back to the field examples (Figures 4.3 and 4.5), the transverse distribution pattern from the aircraft was both displaced and expanded to the south as a result of the cross winds. Although this doesn't greatly affect the application uniformity under moderate and stable wind conditions (such as those recorded in the field trial), there are issues when applying fertiliser near application-zone boundaries. The smaller diameter particles that were exposed to

a cross wind and were ejected close to a southern boundary were blown further south and often landed outside the trial area. This highlights the need for cross wind compensation in setting each flight line, and is an issue that requires further research. Clearly the transverse distribution patterns will be far less affected by wind blowing along the flight path, in this case cross wind compensation will not be required, however the displacement of particles along the flight path will be affected and buffer distances would need to be adjusted depending on the wind direction and magnitude. Although, the effect of wind on individual particles and the transverse distribution pattern has been covered in previous work, far less is known about its effect on the spatial distribution.



**Figure 4.8.** Modelled spatial fertiliser deposition of particles ejected from a Gippsland Aeronautics GA200C fixed wing aircraft with a six duct spreader – altitude 25 m, superphosphate, ground speed 54 m s<sup>-1</sup>, wind in case A) no wind, B) 4 m s<sup>-1</sup> wind blowing from 315°

Figure 4.8 (A) and (B) show the contrast in spatial distribution using the exact wind conditions as that illustrated in Figure 4.6. The deposition footprint in (A) follows the classic “V” shaped pattern where smaller diameter particles were not displaced very far forward, or to the sides, of the release point when compared to larger particles. The total deposition width and length was found to be 35 m and 60 m respectively in zero wind conditions. In case (B), the data reflected significantly more spatial variability. Almost all of the particles were blown to the right of the release point, and although the larger particles still formed a “V” forward of the release point, their forward displacement was marginally reduced. Smaller diameter particles (< 2.0 mm) were highly influenced by the cross wind. The total deposition width and length was found to be 70 m and 115 m respectively when a  $4 \text{ m s}^{-1}$  wind was present.

This has serious consequences for agricultural aviation and the application of granular material to the land, particularly for applying near sensitive areas and correctly judging buffer distances in order to avoid contamination. In the case of Figure 4.8 (B), when applying into and out of application zones, opening the door too early may mean small fertiliser particles are blown back from the release point and outside the application zone. The release height in Figures 4.6 and 4.8 was 25 m, however, in the field trial the mean release height was 58 m above ground level (std. dev. 7 m). At this increased height, and in the same conditions, the smaller particles would drift much further than what was presented in Figures 4.6 and 4.8. Unfortunately, controlling the fine particles is very difficult; their eventual landing position is determined predominantly from the wind. One alternative may be to remove sub 3.0 mm diameter particles; this would make the deposition footprint more manageable in windy conditions, in contrast it would likely increase the products manufacturing cost and significantly alter the lateral distribution pattern. It has been shown that larger particles have greater momentum and have the potential to travel greater distances from their release point, this may cause gaps in the distribution pattern.

#### **4.4 ECONOMIC ANALYSIS**

Calculating the economic implication of uneven P application is difficult; however, the cost

of off-target superphosphate application can be approximated from the model output. The economic analysis presented here is limited to the fertiliser applied outside the application zone. An assumption has been made where fertiliser applied outside the trial site is of no economic benefit. Other studies relating to the effectiveness of applied superphosphate over time indicate that superphosphate previously applied, has an effect beyond that in its immediate year of application (Barrow, 1974; Suckling, 1975; Lambert et al., 1983). Therefore, although fertiliser has been under- or over-applied in many areas within this trial, the natural buffering of the soil compensates, to some degree, for variability in P supply and it is not considered in the economic analysis (Table 4.2).

**Table 4.2. Economic analysis for the application of superphosphate fertiliser on a 25 ha trial site, 15 km North of Kimbolton, Manawatu, New Zealand. Results are also extrapolated to a hypothetical 1500 ha (effective) farm scale. Superphosphate cost NZ\$ 191 ton<sup>-1</sup> (Ravensdown, 2005), target application rate 150 kg ha<sup>-1</sup>.**

Parameter	Trial Modelled	Automated modelled	Extrapolated Trial	Extrapolated Automated	Units
Area of application zone	19	19	1500	1500	[ha]
Area of non-application zones	6	6	474	474	[ha]
Total fertiliser applied	1.9	1.8	150	145	[t]
Fertiliser applied outside field boundary (a)	104	46	8211	3632	[kg]
Fertiliser applied in non-application zone (b)	193	71	15237	5605	[kg]
Total quantity of fertiliser applied off target (a + b)	297	117	23447	9237	[kg]
Cost of off target application	57	22	4500	1737	[NZD \$]
Cost per hectare	3.0	1.1	3.0	1.1	[NZD \$ ha <sup>-1</sup> ]

The economic data from the 25 ha trial was extrapolated to a larger scale. New Zealand hill country farms can often comprise over 1000 ha of rolling grassland pastures, where it is not feasible to use land based fertiliser spreading vehicles. In these situations agricultural aircraft are used to apply the fertiliser product. The example in Table 4.2 uses a 1500 ha (effective) farm area with around 470 ha of non application areas and helps put the field trial figures into a New Zealand farming context. Of greatest interest is the comparison between the extrapolated trial and automated results, in particular, the total amount of fertiliser applied off-target, and the total cost of that off-target application. The results suggest that a significant quantity of fertiliser could be applied off target using conventional techniques compared to automated hopper door control. In the extrapolated case, around 14 tons of superphosphate could be saved

by improving the performance of the machine. The cost of off-target application using a manually controlled system was found to be NZD \$4500 ( $\$3 \text{ ha}^{-1}$ ) compared to NZD \$1730 ( $\$1 \text{ ha}^{-1}$ ) with automated control (using the techniques described above), this constitutes a net benefit of NZD \$2770 or 10% between the two methods.

In Murray and Yule (2007b) a simple aircraft work-rate model was presented for superphosphate application on hill country pastures (refer to Chapter 7, Table 7.3). In that example, the application area was 2518 ha and the job was predicted to take 18.7 hours to complete, this equates to an approximate work rate of  $134 \text{ ha hr}^{-1}$ . By assuming a typical topdressing aircraft has an annual work load of 720 hours, then the total application area for that period will be around 96,500 ha. By applying the cost per hectare (refer to Table 4.2) to the annual area covered by the aircraft, the difference in value for the off-target fertiliser application, between a manual and automated system, can be calculated. In this example it equates to NZD \$111,700 annually.

Clearly, the automated approach offers significant reductions in off-target application which will be of interest to the farmer, in addition to the wider aerial topdressing industry. It has been shown that environmental, safety, and performance advantages may be available using improved application techniques. Although these improvements will come at a cost to the operator, the cost of not adopting these techniques may outweigh the initial capital required to enhance aerial application performance. How operators justify installing the required equipment, may depend on government legislation, regional regulations, industry pressure, or from farmer requests. However, before this happens, a larger field trial and more detailed cost benefit analysis should be performed.

#### **4.5 CONCLUSION**

Earlier work by Jones et al. (2007) and Murray et al. (2007b) was used to develop a field scale application model for a Gippsland Aeronautics 200C aircraft, spreading superphosphate fertiliser. The model was embedded inside ArcMap 9.0 (ESRI, 2006) and data from a 25 ha field trial was used as input to model the performance of an agricultural aircraft operating on a

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hill-country farming system. Comparison of the model output against the amount of fertiliser captured within a series of collectors showed the ability of the model to predict the deposition of an actual fertiliser application event.

Cross winds were shown to reduce the peaked nature of the transverse spread pattern while improving or maintaining an acceptable CV. Unfortunately, this was also shown to make application harder to manage when we consider the spatial footprint of the material on the ground. An example of the effect of a  $4 \text{ m s}^{-1}$  cross wind on the spatial footprint highlighted the difficulties one would expect to encounter when applying fertiliser in windy conditions. The model described here could be very useful when combined with an automated hopper door control system. Combined with real-time wind speed and direction data it would be possible to determine the deposition footprint whilst in flight, this could be compared to the aircraft current position and its proximity to any non application zones, and action taken to avoid contaminating those areas. It would also have been very interesting to compare the flight lines that the pilot could have achieved on the 25 ha trial site, had he not had to manually adjust the hopper opening for flow rate and avoidance of non application zones, thus leaving all of his concentration to fly the aircraft safely along the parallel swath lines.

The data from the field trial was extrapolated to reflect a 1500 ha hill country sheep and beef farming system and showed considerable cost savings and better utilisation of available resources using the automated approach. The cost of off-target application (either applied outside the paddock boundaries or into non-application zones) using a manually controlled system was estimated to be NZD \$4500, compared to NZD \$1730, with automated control. In this simple cost/benefit analysis, the net benefit of improving the performance of agriculture aircraft was clearly evident. Further benefits between a manual and automated system are anticipated on a larger scale farm trial. In this situation, the amount of longitudinal variation in application rate is expected to be greater due to the influence of topography on the ground speed of the aircraft.

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## **CHAPTER 5 - DEVELOPING VARIABLE RATE APPLICATION TECHNOLOGY: MODELLING ANNUAL PASTURE PRODUCTION ON HILL COUNTRY**

*Publication arising from this chapter*

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Murray RI, Yule IJ, Gillingham AG 2007. Developing variable rate application technology: modelling annual pasture production on hill country. *New Zealand Journal of Agricultural Research* 50: 41-52.

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### **5.1 INTRODUCTION**

Sheep and beef farming accounts for more than seventy five percent of pastoral land use in New Zealand (Statistics New Zealand 2002), with the vast majority of this land classified as hill country. Hill country farmland generally consists of a combination of flat, rolling/easy and steep topography. Each land class has a differing input requirement, growth potential, grazing efficiency and influences annual production more, than in dairy farm land.

Due to the open market conditions and volatility of the New Zealand dollar, combined with the uncertainty of climatic conditions, producers are forced to become more and more efficient in order to succeed. To increase their efficiency they are turning to technological advances that can help them streamline managerial decisions and improve efficiency of input usage with the expectation of increased production and net return.

It is widely accepted that climate, particularly rainfall, combined with soil characteristics, fertiliser history and topography are factors that provide a wide range in production potential and herbage composition. Fertiliser, which is the largest cash expenditure item (Thompson and Matthews, 2005), continues to be applied inefficiently. With the dramatic variation in production across a typical hill country farming system and the need for improved efficiency, this could be targeted at areas that can provide increased returns.

Current aerial topdressing practice is limited to broad application rates with little or no differentiation across a range of soil types, micro-topography and potential growth conditions.

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Traditional application techniques involving phosphatic, and more recently nitrogenous fertiliser, generally consist of a uniform rate over large areas, and in some cases the entire farm receives the same blanket application rate. This method does not take into consideration the variability in pasture nutrient requirements.

The overriding reasons for the current use of aerial blanket application are twofold. Firstly, there is a lack of relevant information for making alternative decisions. Up until now there has been no way to accurately predict pasture production (a precursor to making fertiliser recommendations) within the highly variable micro-topography. At present in New Zealand, fertiliser recommendations are based on broad interpretation of farm topography, management and nutrient requirements such as in Overseer Nutrient Budgets 2 (AgResearch, 2003). However, this does little to address the more variable nutrient requirements of hill country grazing land. Because of the spatially variable characteristics of pasture production, a model which takes account of this is required. Secondly, there is low awareness and understanding of alternative techniques such as Variable Rate Application Technology (VRAT).

This paper is the first in a series of three describing the development of VRAT for hill country, from the principles of modelling annual pasture production to the agronomic and economic benefits that can result. This paper reviews a number of important factors that contribute to variations in annual pasture production, as well as providing a brief commentary on previous pasture production models. An attempt has been made to model annual production on a 2700 ha hill country farm using both the decision tree approach and by using animal grazing records and predicted monthly animal intake values. The second paper in the series (Murray and Yule, 2007a) describes the scenario development and provides an agronomic evaluation of the case study farm, and the third (Murray and Yule, 2007b) presents the economic impact of VRAT for farm owners and topdressing operators.

## **5.2 REVIEW OF LITERATURE**

New Zealand hill country is an amalgamation of contrasting topography. This naturally occurring variation in terrain, combined with the diverse environmental conditions and differing

management techniques, makes it difficult to model pasture and animal production with a high degree of certainty, especially when considering the effects of temporal variations, such as seasonal climatic events and grazing management.

Contrasting land topography leads to significant variability in both the macro and micro-climates. Topographic variations in slope and aspect influence the micro-climate, soils, pasture species and animal grazing behaviour. Many studies have been undertaken to better understand the relationship between some, or all, of these key variables, both in New Zealand (Gillingham, 1982; Ledgard et al., 1982; Radcliffe, 1982; Lambert et al., 1983; Sheath, 1983; Rowarth et al., 1988; Moir et al., 2000; Lopez et al., 2003; Dodd et al., 2004) and overseas (Lieffers and Larkin-Lieffers, 1987; Sala et al., 1988; Zimmermann and Kienast, 1999; Goldin, 2001; Belesky et al., 2002; Clark et al., 2003; Garden et al., 2003).

A number of long-term, multiple criteria field trials were established in New Zealand hill country, such as those at Whatawhata Hill Country Research Station, located 25 km west of Hamilton in the North Island, at an altitude of 220 m above mean sea level (MSL) (Gillingham, 1980; Bircham and Sheath, 1986; Dodd et al., 2004), at Ballantrae Hill Country Research Station a moist, low-fertility hill country farm, located 20 km north-east of Palmerston North, in the foothills of the Ruahine Range, southern Hawke's Bay with an altitude ranging from 125 to 350 meters MSL and annual rainfall of 1270mm (Lambert et al., 1983; Lambert et al., 2000; Lopez et al., 2003), and at Waipawa Research Farm with a low annual average rainfall site, situated about 4 km west of Waipawa in Central Hawke's Bay (Gillingham et al., 1998; Morton et al., 2005) and Whareama, Gladstone and Mauriceville representing low medium and high rainfall regimes in the Wairarapa hill country (Moir et al., 2000).

## **5.2.1 FACTORS OF SIGNIFICANCE**

### **5.2.1.1 Meteorology**

Analysis of data collected at 9500 sites throughout the central United States confirmed the overwhelming importance of water availability as a control on production (Sala et al., 1988). Annual precipitation and its influence on pasture production, has been measured in experiments

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under New Zealand growing conditions and seems to support the findings of Sala et al. (1988). Clark et al.(2003) found that on a regional and farm scale the total annual rainfall, and in particular the distribution of rainfall during the year, were more important than other weather variables in determining the amount of pasture grown in a year. The effect of moisture on pasture yield is further highlighted in Moir et al. (2000) where the authors created a model identifying soil moisture, climate and soil fertility as the three major factors governing annual pasture production.

Gillingham et al. (1998) conducted within-field moisture analysis, finding that soil moisture levels varied greatly during each year but were always higher on south, rather than north, facing slopes. Within north-facing slopes, soil moisture levels were higher on easy, than on steep slopes, but on south-facing aspects the opposite was often the case. The finding that moisture was higher on southerly slopes coincides with the findings of Lambert and Roberts (1976), and later Bircham and Sheath (1986), who found that moisture loss through evapotranspiration was greater on the north rather than for the south aspect, possibly due to exposure to increased solar radiation or temperature on northerly slopes.

Although soil moisture is one of the most important factors in determining annual production, its relationship with other factors also plays an important role. Moir et al. (1997, cited from Moir et al. 2000) used data from sites differing in fertility and rainfall to demonstrate that as rainfall increased, the size of the pasture response to improved fertility also increased. However, in order to predict yield for a given combination of soil, moisture and fertility, other factors must be considered.

Lambert and Roberts (1976) measured a number of climatic variables on north, south, east, and west aspects of a hill country farm over a period of 12 months. They found that during January to August, the north aspect was warmer than the south aspect, and for the remainder of the year, the east aspect was warmest. The authors suggest that the significance of wind in the hill pasture environment is often underestimated and that its warming and cooling effect can either enhance or impede pasture production.

Pasture production is influenced by temporal variations associated with the climatic

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conditions. Both Gillingham (1980) and Suckling (1975) found that the proportion of annual production occurring within each season was similar on all slope categories, and a higher proportion of the annual pasture growth was grown on north, rather than south aspects during spring with the opposite effect in summer. This highlights the difficulties that can be expected when attempting to model pasture production on a seasonal basis.

### ***5.2.1.2 Topography and Grazing Management***

Gillingham et al. (1998) reported that slope category and management history significantly influenced soil condition and pasture production, with management being of secondary importance to slope. In a number of studies, pasture production increased with both fertiliser application and decreasing slope (Lambert et al., 1983; Gillingham et al., 1998; Lambert et al., 2000; Lopez et al., 2003; Dodd et al., 2004), with Lambert et al. (1983) reporting 100% more herbage being grown on camp sites than on hillsides, also finding that small slope differences on hillsides had major effects on herbage accumulation rate.

Another topographical feature that influences species and pasture production is slope aspect (Radcliffe, 1982). Lieffers and Larkin-Lieffers (1987) found that slope position and aspect were the most important factors influencing the distribution of prairie species and consequently herbage accumulation in complex terrain in central Canada. In Gillingham et al. (1998), production levels were found to be highest on north-facing easy slopes. As in central Canada, both slope and aspect significantly affected total annual pasture production within the New Zealand trial sites. Gillingham (1980) and Lambert et al. (1983) concluded that the effect of aspect on pasture production is less than that of slope. In general, shady aspects have higher production than sunny aspects during periods of moisture stress, whereas sunny aspects produce more herbage at other times. Although these variations are frequently observed in pasture production systems, successful management of the variation is difficult. Sheath (1983) believes that the difficulties in achieving adequate pasture utilisation in hill pastures are further complicated by animal grazing requirements and preferences. The combination of varying feed, complex terrain and animal behaviour make modelling pasture removal by animals difficult.

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In the northern hemisphere, Belesky et al. (2002) found that topography strongly influenced grazing duration, where duration decreased as slope steepness increased, and was least on steep sunny slopes. In southern hemisphere sites, Sheath (1983) found a similar relationship to slope and grazing duration, as the slope decreased, pasture disappearance rates increased, indicating an obvious preference for grazing flat to easy sloping terrain. The observation has also been made that sheep graze low slopes in preference to high slopes in spring and summer indicating a seasonal pattern probably due to the more abundant pasture in spring, and the lack of pasture on steep slopes in summer. Gillingham (1980) found further relationships between seasonal grazing patterns, stating that greater pasture growth occurred on north slopes in spring and a higher legume content on south slopes in summer and autumn. The author also suggested that if the variation in pasture growth could be fully utilised, it may help to smooth out fluctuations in stock numbers across high or low growth periods. Sheath (1983) goes on to say that subdivision in hill country, in particular separating the preferred north from south-facing aspects, has long been an accepted method of improving the efficiency of pasture development and utilisation. However, the literature suggests that because of the dominating effect of topography on pasture growth and utilisation, sub-division into homogeneous land strata, including both slope and aspect, will be required for full grazing control and increased utilisation.

### ***5.2.1.3 Nutrient transfer***

Environmental concern about nutrient discharge to and around waterways and sensitive areas is gaining considerable momentum with environmental groups and regional authorities. Recent studies (Nash and Halliwell, 1999; Gillingham and Thorrold, 2000; McDowell et al., 2003; ECAN, 2004; Hart et al., 2004) highlight efforts by legislative agencies and research institutions to discern the effects of phosphorus (P) and or nitrogen (N) application on localised water quality, as well as educating farmers and consultants on efficient fertiliser usage and practices that can reduce nutrient flow into ground and surface water.

*Phosphorus* - A considerable proportion of the phosphorus taken up by the pasture and ingested by the animal is in fact returned to the soil as dung or as plant residues such as roots or uneaten litter. Phosphate returned to the pasture as dung is not redistributed evenly (McLaren and Cameron, 1996). The net transfer of nutrients in the faeces and urine of grazing animals is a major contributing factor to the effect of micro-topographical slope on soil fertility (Gillingham and During, 1973). For instance, steep slopes may receive very little phosphate through faeces and urine transfer, whereas stock camps receive large amounts.

The importance of dung, as a pathway for P return to the soil, was found to be greatly affected by topography (Gillingham and During 1973; Gillingham 1980, 1982; Gillingham et al. 1980; Saunders et al. 1987; Lambert et al. 2000). Soil fertility was found to decrease with increasing slope of measurement sites (Lambert et al., 2000). Further studies by Gillingham et al. (1980) found that on campsites faecal P supply was two or three times annual pasture uptake, and P returned to the slopes, which occupied the majority of each paddock, was equivalent to only a small fraction of the total P uptake by pasture. In that study, the total return of P across all slope classifications was less than that required to compensate for pasture uptake (refer to Table 5.1), hence the need to supply varying rates of additional nutrients through aerial application and the development of a suitable control system to fully compensate for nutrient transfer effects.

**Table 5.1 Net total P balance ( $\text{kg ha}^{-1}$ ) including standard errors of estimate for each net value. Adapted from Gillingham et al. (1980)**

	North	South
Campsite	+ 60.7 ( $\pm$ 22.1)	+119.8 ( $\pm$ 21.0)
25 degree slope	- 19.5 ( $\pm$ 2.4)	- 10.0 ( $\pm$ 2.4)
45 degree slope	- 15.3 ( $\pm$ 1.3)	- 13.8 ( $\pm$ 1.4)

As suggested previously, one way of partially controlling dung distribution is through subdividing paddocks into homogenous land strata. This may help to reduce the transfer of nutrients by animals from high slope to low slope areas, however in some hill country properties this could be entirely unpractical, and a valid alternative in balancing supply and return of

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nutrients may be VRAT.

In the study by Gillingham (1980), total P return in dung and litter on moderately steep slopes (25°), was less than that taken up annually by pasture, and therefore fertiliser is necessary to avoid some depletion of soil P. On the steepest slopes (45°), only a small proportion of total P uptake was returned. Consequently it is suggested that on this stratum the role of topdressing in maintaining soil P levels and therefore also plant P uptake, would be more important than in any other areas of hill pasture (Gillingham et al., 1980). This differs from the precision agriculture concept, which suggests that lower producing and potentially less economic terrain should not be given preferential fertiliser application (Ferguson et al., 1999), instead low fertility tolerant grass species should be encouraged in those areas and fertiliser applied on land that provides the greatest economic return.

Fertiliser application to New Zealand hill country is dominated by the use of phosphate fertiliser. In most New Zealand farming regions white clover forms the basis of pasture production by fixing nitrogen (N) and supplying it to associated grasses which contribute the majority of dry matter production. In order to stimulate satisfactory clover growth, phosphate (P) fertilisers are applied at levels which are generally regarded as more than adequate for grass. A requirement of this clover-grass partnership is that there is adequate moisture, especially during spring and summer, for vigorous clover growth and persistence (Gillingham et al., 1998).

There are a variety of commercial fertilisers available for application of phosphorus to hill country. Once in the soil, the phosphate moves relatively slowly as it reacts with the inorganic soil fraction (particularly in soils containing high amounts of iron and aluminium compounds). This process is often referred to as phosphate retention and greatly influences phosphate fertiliser application recommendations. Generally phosphate retention in New Zealand soils is divided into three classes: low (0-30 %), medium (31-85 %), and high (86-100 %) (McLaren and Cameron, 1996)

Morton and Roberts (1999) indicate that there are very few leaching losses of P (less than 1 kg ha<sup>-1</sup> y<sup>-1</sup> in all but very sandy soils (McLaren and Cameron, 1996)) and it does not matter when P fertilisers is applied, but if the soil test levels are low and an immediate increase in

production is required, the sooner it is applied the sooner there will be benefits. Other studies relating to the effectiveness of applied superphosphate over time indicate that superphosphate previously applied has an effect beyond that in its immediate year of application (Barrow, 1974; Suckling, 1975; Lambert et al., 1983). Suckling (1975) suggests that where superphosphate treatments were changed, effects of fertiliser history could outweigh those of current treatment for many years. This is in line with comments by Lambert et al. (1983) that as fertiliser history extends, phosphate applied in a particular year becomes relatively less important and the annual pasture response to P will likely have a time lag, due to the requirement for legume growth and N fixation to respond before there is additional associated grass growth. Gillingham et al. (1998) did, however, find a seasonal P response occurring from spring to autumn on southern aspects, and on easy north facing slopes which showed a large increase in clover growth.

This literature suggests that any modelling of annual pasture production on hill country, particularly for the P fertiliser component, should emphasise the influence of long-term P responses, as well as the less significant annual P response and how changes in topography stimulate both clover and grass growth.

*Nitrogen* - Lately, there has been an increase in the use of nitrogen (N) fertilisers to quickly overcome and avoid feed shortages in middle to late winter and early spring. Morton and Roberts (1999) suggest that N fertiliser can be used as a management tool, by producing extra pasture dry matter at times when animal feed requirements exceed pasture growth, the authors proposing that N could be thought of as a form of "supplementary feed". The aim when applying N, as with phosphorus, is to achieve the most economic response per unit of nutrient applied; however, studies indicate that applying large amounts of N to ryegrass/clover pastures can significantly suppress clover growth and hence N supplied through N fixation (Steele and Dawson, 1980; Lambert and Clark, 1986; Gillingham et al., 1998). Inadequate N supply can occur in summer-dry climates where white clover does not persist, primarily because of moisture shortage in other seasons. Therefore, in some situations, P supply will be more than adequate as a result of routine fertiliser application and N supply may be the major factor limiting pasture growth (Gillingham et al., 1998).

Morton and Roberts (1999) indicated that pasture responses to N applied in autumn in low summer rainfall areas are variable, but are more consistent in areas with higher rainfall. In all regions, reliable N responses can be obtained between early June (northern North Island) and September (southern South Island) to provide extra pasture in early spring. Gillingham et al. (1998) investigated N responses further, in particular the relationship between N response, topography and P fertiliser history. In large scale field trials all areas showed significant responses to N fertiliser, with the best results obtained on steep, north aspects with a low soil P status. The authors suggest that application of P to south aspects and moist north slopes, and for steep, north facing slopes to receive N, plus limited P, will be the most efficient fertiliser policy in terms of net pasture production.

### **5.2.2 PASTURE PRODUCTION MODELS**

There are three major approaches to pasture production modelling: these being regression based statistical models (Lambert et al., 1983; Sala et al., 1988; Scott, 2002), theoretically based models (Gilmanov et al., 1997; Riedo et al., 1998; Moir et al., 2000) and more recently decision tree or data mining (Zhang et al., 2005b).

The statistical or empirical model approach uses regression techniques and mathematical response functions to derive predictions of pasture production (Lambert et al., 1983; Sala et al., 1988; Scott, 2002). In these models the simulated pasture production data was analysed to determine if there were any relationships with the management and environmental factors investigated.

Lambert et al. (1983) focused on the influence of fertiliser and grazing management on the herbage accumulation on a low fertility hill country site near Woodville, New Zealand. It was found that spring plus summer rainfall accounted for 23% of the variation in the mean annual herbage accumulation, with high vs. low fertiliser treatments also providing significant variation in herbage growth. Slope had a strong negative relationship in the 15-27° slope range, and this seems to concur with the findings of other field trials mentioned earlier. The author concluded by stating that herbage accumulation rate on steep areas was consistently lower than that on

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flatter areas. Again this is a common finding, suggesting that any spatial pasture production model would have to take this into account. Analysis of aspect revealed that under moisture stress, shady aspects produce more while the opposite occurs when moisture is not limiting, this is due to the increased photosynthetically active radiation (PAR).

Scott (2002) developed several empirical mathematical response functions between sheep carrying capacity (CC), fertiliser and grazing management, soil moisture and other climatic factors. The trials were conducted at the AgResearch Mt John site, Lake Tekapo, South Island (New Zealand) and in this case soil fertility, particularly annual phosphatic and sulphur applications, were deduced as the main factors determining potential CC. As was found by Lambert et al. (1983), climate accounted for part of the variation, with CC correlating positively with the average September to December total rainfall, and negatively to mean March temperature. In both cases, rainfall rated highly as a governing factor in annual pasture production and subsequently CC.

Scott (2002) remarked that caution was required when selecting response functions. This is to ensure that no unlikely biological functions are introduced and that consideration is given to whether the models will behave as intended on data outside that which was used in its generation and validation. This seems to be a generic problem with the regression based empirical method, as both Lambert et al. (1983) and Scott (2002) expressed trepidation in extrapolating models outside similar sites. Empirical models, in the regression form, would therefore seem an unlikely choice as a model that could be extrapolated to quite different environmental and managerial conditions.

The theoretical model design, as used by Moir et al. (2000) used an alternative approach. They described a pasture production model that takes into account the effect of climate as it influences soil moisture and evapotranspiration, and how these factors relate to soil fertility. The validation data was obtained from trials at Whareama (low rainfall), Gladstone (medium rainfall) and Mauriceville (high rainfall) in the Wairarapa hill country. Evapotranspiration was calculated using the Priestley and Taylor (1972) method and was linked to growth by a proportionality constant ( $k$ ). Both Moir et al. (2000) and Ritchie (1983) argue that the

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proportionality constant depends primarily on soil fertility. This is in direct contrast to the comments of Sinclair et al. (1997) who suggested that unless pasture is malnourished,  $k$  is hardly affected by nutrient status. Experimental data was compared to substantiate the link between ( $k$ ) and soil fertility. It was found that at a number of different soil fertility levels the effect on ( $k$ ) was clear, regardless of rainfall regime at the particular sites (Moir et al. 2000). Although being a more suitable choice for predictions outside the validation area, this model does not account for variations in topography, although the authors suggest it would be a relatively simple addition and they are currently working on incorporating the effects of slope and aspect (M. J. Hedley 20 Oct 2005, pers. comm).

The third modelling method used in predicting annual yield is the decision tree or data mining approach as used by Zhang et al. (2005b) to model annual and seasonal hill country productivity over a number of sites spanning most of the North Island of New Zealand. Although relatively new, this method is increasing in popularity because of its ability to predict new cases based on previously known information (Scheffer, 2002; Zhang et al., 2005b). The decision tree approach is a non-parametric machine-learning modelling method, which recursively splits the multidimensional space defined by independent variables into zones that are as homogenous as possible in terms of the response of the dependent variable (Vayssières et al., 2000). Zhang et al. (2005b) discuss its increasing use in environmental modelling using the example of tree species range changes over time (Iverson and Prasad, 1998) and also in analysis of remotely sensed data of alternative cultivation techniques (Yang et al., 2003). In both cases the authors commented on the technique's accurate prediction of the variable in question.

Zhang et al. (2005b) found that the decision tree models for annual and seasonal pasture productivity resulted in a smaller average squared error and a higher percentage of adequately predicted cases than the corresponding regression models. The decision tree model for annual pasture productivity was reported to adequately predict 90.1% of the cases in the model validation, some 10.8% higher than that of the regression model. In addition to the higher predictive accuracy of the decision tree, the models revealed the relative importance and interaction of environmental and management variables in influencing pasture productivity,

therefore presenting a valid decision support tool that lends itself to assisting in strategic farm decisions.

Again, spring rainfall was identified as the most significant factor influencing annual pasture productivity. This was followed by slope and then fertiliser application history. The authors mentioned the limitation that the model did not generate a continuous prediction, and thus could not detect the influence of small changes in environmental and management variables on pasture productivity. Despite this constraint, it may still provide a level of spatial detail that has, in the past, been unattainable by other pasture production modelling techniques. Therefore, this method of data mining and decision tree analysis warrants further investigation.

### 5.3 DECISION TREE MODEL EVALUATION

Zhang et al. (2004) created a generic decision tree model that could be used throughout the North Island in order to predict annual pasture production. This model was subsequently embedded into ArcGIS 9.0 (ESRI, 2004) a powerful Geographical Information System (GIS) package and used to predict annual pasture production at Limestone Downs, a 2500 hectare hill country farm located on the west coast of New Zealand in Franklin County, and situated 15 kilometres south of Port Waikato. Limestone Downs was chosen because it has a favourable climate with a good blend of rainfall, temperature and solar radiation and a detailed management history. All of the variables used and their respective units are shown in Table 5.2.

**Table 5.2 Data inputs, common to all scenarios used in the decision tree pasture production model.**

Input Description	Input Units
Aspect	degree
Slope	degree
Spring Rainfall	mm
Summer Rainfall	mm
Spring and summer Rainfall	mm
Autumn Temperature	°C
Winter Solar radiation	MJ m <sup>-2</sup> d <sup>-1</sup>
Annual Nitrogen input	kg N ha <sup>-1</sup>
5 Year Cumulative Phosphate input	kg P ha <sup>-1</sup>
Annual Phosphorous input	kg P ha <sup>-1</sup>

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The model itself and the input variables associated with it are shown in Figure 5.1. Several of the variables were adjusted by the cosine of the slope angle as described in Zhang et al. (2005b). Each condition and its relating split value is shown within the square objects, the resulting annual pasture production results are shown in the circular objects. For example, an area where the spring rainfall was found to be greater than 228 mm and had a slope of less than 16° with a cumulative five year phosphorus application of greater than 160 kg P ha<sup>-1</sup> with a sunny (north) aspect is predicted to produce 16,701 kg DM ha<sup>-1</sup> yr<sup>-1</sup>.

The resulting annual pasture production, as predicted by the model, was, adjusted for grazing efficiency across differing topography. These correction factors relate to estimated annual pasture utilisation levels (%) for each land class. Steep country, classified as having a slope greater than or equal to 26°, was corrected to a value 60% of the prediction, due to the animals preference to graze less steep land. Secondly, rolling country was land that had slopes between 12° and 26° and was corrected by a factor of 70%. Finally flat to easy country was designated as having slopes less than or equal to 12° and was adjusted by 80% of predicted annual pasture production.

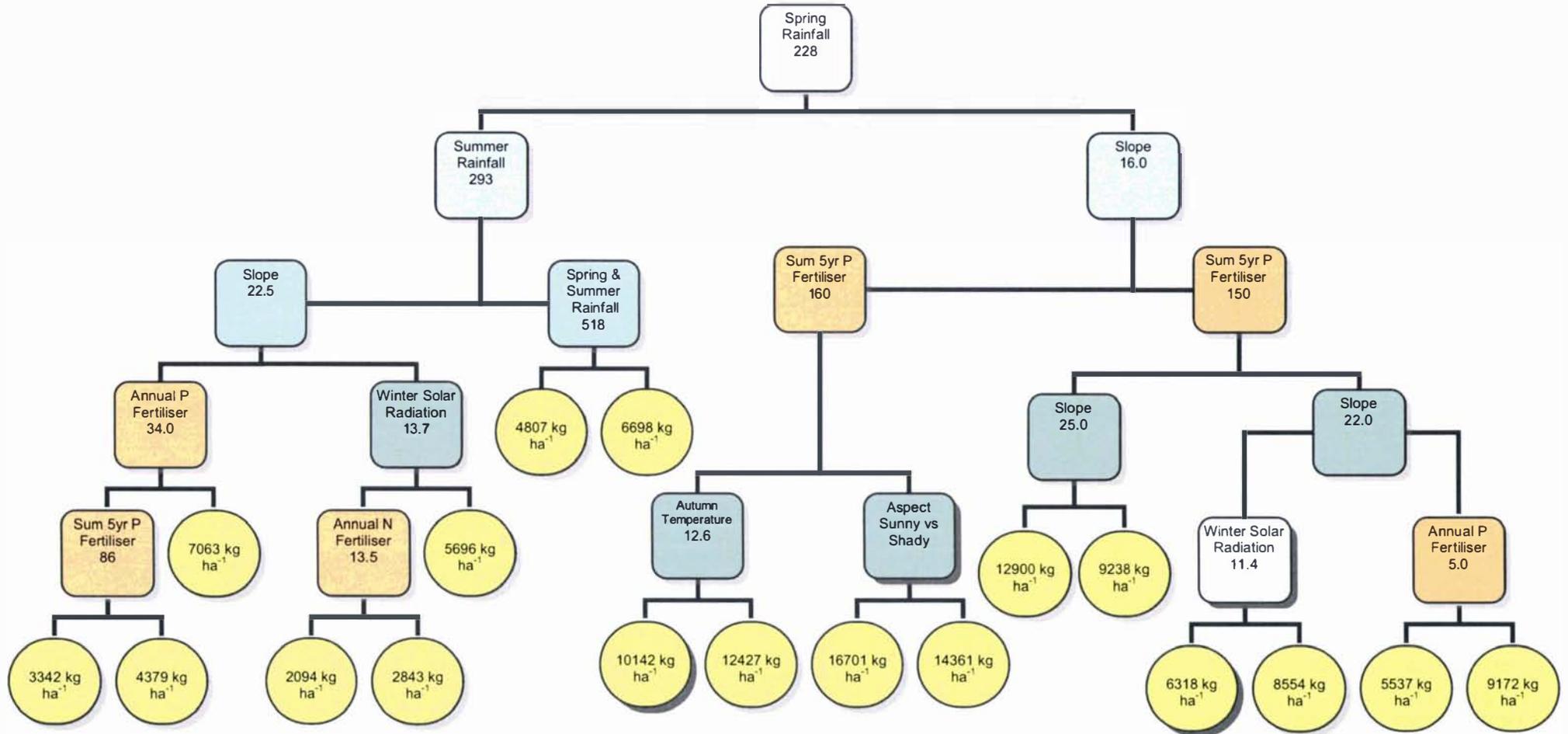
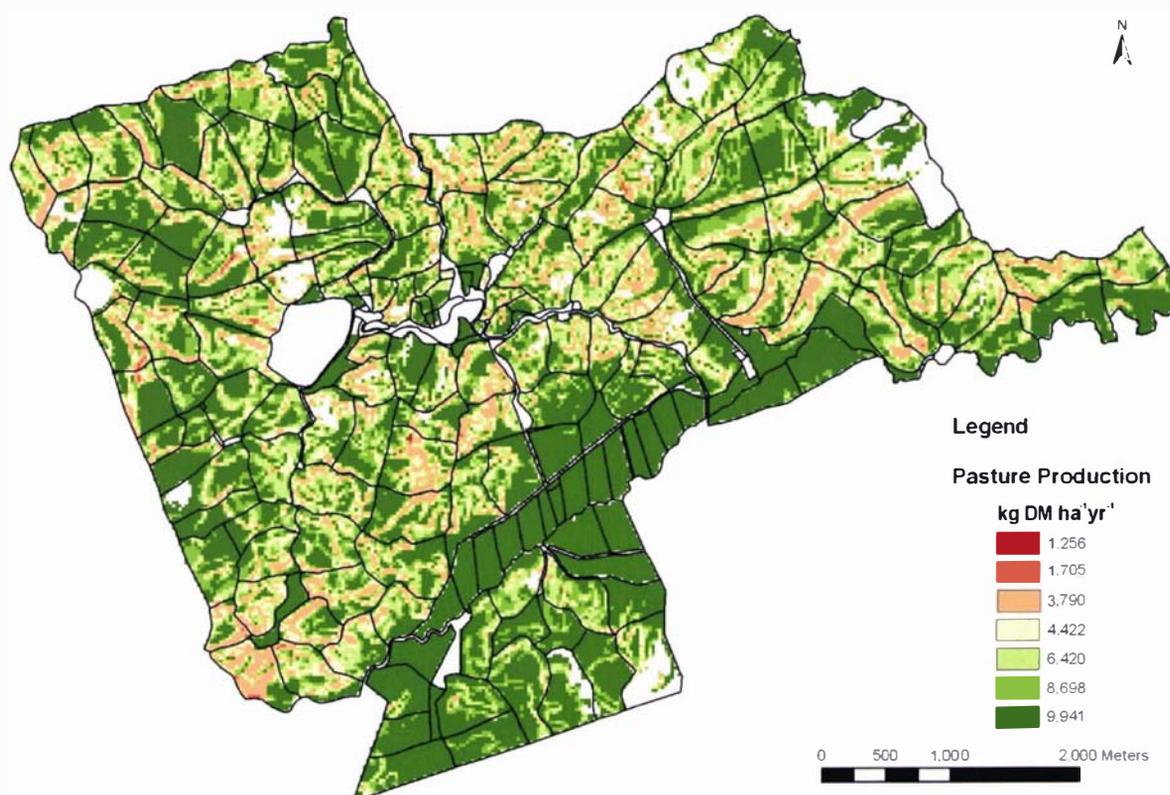


Figure 5.1 Annual pasture production model for North Island of New Zealand as developed by Zhang et al. (2004). Each rectangular object contains an input variable and a split value. Where the variable is less than the split value then follow the left branch, where greater, follow the right branch. This continues until a production value is found.

Once adjusted for grazing efficiency, the predicted utilisable annual production, shown in Figure 5.2, highlights the variable nature of hill country pasture production. Estimates range from 1256 kg DM ha<sup>-1</sup> yr<sup>-1</sup> to 9941 kg DM ha<sup>-1</sup> yr<sup>-1</sup> with a mean production of 7918 kg DM ha<sup>-1</sup> yr<sup>-1</sup> and a standard deviation of 2349 kg DM ha<sup>-1</sup> yr<sup>-1</sup>. With 47% of the farm expected to produce 9941 kg DM ha<sup>-1</sup> yr<sup>-1</sup> and 79 % to produce over 6420 kg DM<sup>-1</sup> ha<sup>-1</sup> annually (refer to Table 5.3). The total annual pasture produced from the 2518 ha farm is predicted to be 19,936 t DM.



**Figure 5.2 Predicted annual pasture production on Limestone Downs, North Island, New Zealand resulting from the decision tree model.**

An attempt was made to validate the annual pasture production model using animal grazing records and predicted monthly animal intake values. The information was provided by Limestone Downs management and extracted from Farmtracker 6.2 (Farmworks, 2005). This method relies on highly detailed and stringent record keeping of stock moving in and out of the farming system. It also relies on an accurate stock class identification and stock unit (SU) allocation for each stock class.

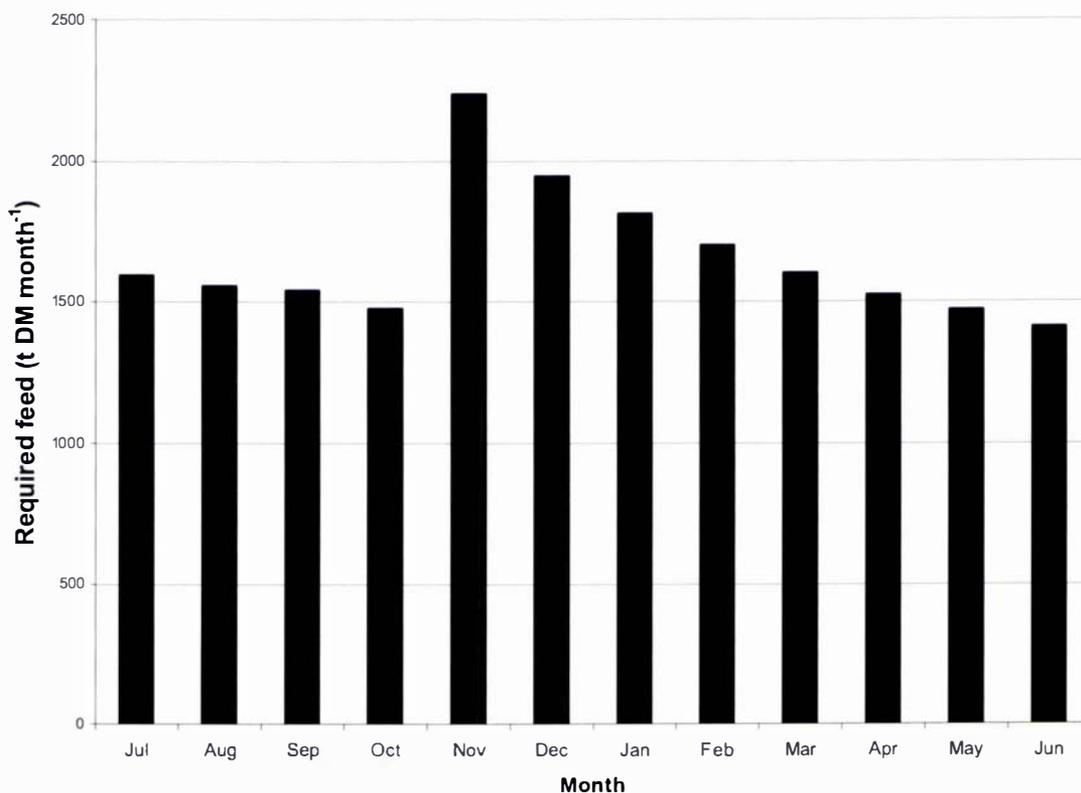
**Table 5.3 Summary of predicted annual pasture production on Limestone Downs, North Island, New Zealand, as derived from the decision tree model.**

Pasture Production (kg DM ha <sup>-1</sup> yr <sup>-1</sup> )	Area (ha)	Total Pasture Production (t DM)
1256	<1	<1
1705	4	7
3790	293	1111
4422	220	974
6420	444	2849
8698	383	3331
9941	1173	11664
<b>Total</b>	<b>2518</b>	<b>19936</b>

Total monthly grazing data, specifically animal class and number of head, were summarised each month for the period 1st July 2003 – 30th June 2004. Required animal intake, based on 550 kg DM yr<sup>-1</sup> for 1 SU (Cornforth and Sinclair, 1984), for each animal class was calculated by multiplying the total SU carried in each month by 45.8 kg DM m<sup>-1</sup>, one month's feed allocation per SU. The total feed requirement for sheep is 13,712 t DM while for cattle 6,156 t DM is required annually. This totals approximately 19,868 t DM yr<sup>-1</sup> of pasture required from the 2,518 ha of effective grazing land.

The resulting feed profile is shown in Figure 5.3 and highlights the gradual increase in feed required over October through to December. This is due mainly to the additional stock numbers in the period after lambing and the increase in feed required once lambs are weaned. This is followed by a gradual decline in numbers as weaned lambs and some mixed age ewes are sold. This is a common feature in New Zealand sheep and beef farming systems as farmers attempt to match feed demand and supply: in this case the extra feed produced in spring coincides with increased feed demand.

When comparing the results of the decision tree modelled production with intake using the technique described above, it is apparent that the two results agree with each other remarkably well, with 19,936 t DM yr<sup>-1</sup> predicted from the decision tree model and 19,868 t DM yr<sup>-1</sup> for the animal intake and feed allocation result. This equates to a difference of only 67 t DM annually; i.e. some 0.3% of the predicted annual production. Due to the close agreement of these figures, we conclude that the decision tree model is a relatively good predictor of annual pasture produced on this property.



**Figure 5.3 Monthly feed requirements for the period of July 2003 to June 2004 on Limestone Downs sheep and beef hill country farm, North Island, New Zealand. Derived from feed intake model**

Further validation of the model is underway. In these field trials, three cages were placed in each of the seven production classifications resulting from the decision tree model. Topography in each of these classifications was quite diverse, the highest production class was flat land while the lowest production class was very steep terrain. Pasture cuts from a total of 21 cages are being collected throughout 2005 -2007 in an attempt to test the model predictions of spatial variation in annual production.

Because the model has the ability to predict yearly production based on a change of one or more key variables, it provides a useful tool for strategic planning of fertiliser application and stock management. This enables the development of continuously variable GIS based prescription maps where fertiliser is applied in either the most efficient or in a less productive manner, depending on farm owner's production or lifestyle goals.

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#### 5.4 CONCLUSION

This paper presents a summary of the factors that have contributed to variations in pasture production in numerous field trials both in New Zealand and overseas. Rainfall and slope, both spatially variable factors, were identified as the major factors governing annual pasture production. The amount of rainfall influences pasture production on a regional or farm basis, while topography dictates pasture production within the field. With the advent of technology, in particular GIS and computer modelling, we are now in a position to interrogate these variables and develop decision support systems that take into account the unique variations within a farm and field environment. The decision tree model developed by Zhang et al. (2004) was capable of utilising spatially variable inputs as well as weighting the governing factors such as rainfall and slope.

Calculation of a farm's annual pasture production is useful as it helps to identify strategic opportunities for either increased production or greater efficiency of inputs, particularly if within-field variation is scrutinised. The decision tree model predicted a total annual production of 19,936 t DM yr<sup>-1</sup> from the 2518 ha of effective grazing land at Limestone Downs, and showed the considerable range of pasture productivity within a single paddock. Further validation of the spatial variation is underway by a series of cage cuts across each of the seven production classes. Comparison of the results between the decision tree and animal intake models revealed homogeneity in the respective predictions of annual production with a difference of only 67 t DM yr<sup>-1</sup>. While the techniques resulted in similar predictions, more rigorous validation is required. Given the difficulty in measuring annual pasture production and the close agreement between the techniques, extension work seems justified.

The decision tree model has several distinct advantages. Firstly in managing fertiliser quantity and placement, secondly in identifying potential increases in grazing utilisation through greater control of stock, and thirdly, in risk assessment to changes in environmental conditions. The animal intake model has the advantage of simplicity but relies heavily on accurate grazing history. In comparison, the decision tree model requires a sound knowledge of GIS and a

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substantial amount of spatial-data gathering, both historical and environmental, which is often expensive to obtain.

Because of the current interest in increasing the effectiveness or efficiency of fertiliser application and due to the ranking of fertiliser in total cash farm expenditure, it has been targeted as a practice that requires future research. The decision tree model and its use on Limestone Downs will enable further analysis of the influence of fertiliser application on annual pasture production. This may help drive the use of VRAT.

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## CHAPTER 6 - DEVELOPING VARIABLE RATE APPLICATION TECHNOLOGY: SCENARIO DEVELOPMENT AND AGRONOMIC EVALUATION

*Publication arising from this chapter*

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Murray RI, Yule IJ 2007. Developing variable rate application technology: scenario development and agronomic evaluation. *New Zealand Journal of Agricultural Research* 50: 53-63.

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### **6.1 INTRODUCTION**

New Zealand relies heavily on the exportation of agricultural products to overseas markets. Agriculture accounts for 53% of the total merchandise export value of which, sheep and beef exports totalled around \$5.29 billion in 2005 (MAF, 2005a). Production comes from nearly 9 million hectares of pastoral grazing land (Statistics New Zealand, 2002). Therefore, every hectare of grazing land in a sheep and beef farming system does, on average, generate approximately \$588 ha<sup>-1</sup> in export earnings. For a 2500ha sheep and beef farm such as Limestone Downs this could equate to \$1.47 million dollars in export earnings or 0.02% of New Zealand's total agricultural export value from this property alone. This highlights the significance of a single farm on a national level and helps to position the value of farming in the New Zealand economy. It also shows why it is important for farmers and research organisations to further developments that could lead to increased productivity or efficiency in agricultural systems.

Discussions on sustainability of agricultural production reflect that farmers are more likely to be interested in how their production figures compare against regional statistics and particularly how their situation relates to their neighbours. With this neighbourly competition and the struggle for increased production or efficiency from the available resources, farmers are searching for methods that could assist in the decision making process and increase net return. Given a discrete amount of natural resources and the need for improved efficiency, farmers have expressed an interest in reducing expenditure or targeting it at their more productive land. This

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is, of course, very difficult in reality. Clark et al. (2003) suggest that farmers have no control over the climate and it is difficult to predict weather patterns more than a few days in advance, thus success can depend on how well the farmer identifies and manages the risk of climate variation. The development of models to help identify risk and input factors has been the focus of much research in the past (Bluett et al., 1998; Riedo et al., 1998; Clark et al., 2000; Herrero et al., 2000; Belesky et al., 2002; Lopez et al., 2003).

In order to make informed managerial decisions on stocking rate, labour requirements, supplementary feed, fertiliser application and frequency, farmers desperately require a method of calculating annual pasture production across their farmland. In the past there has been a lack of accurate methods to evaluate a change in one or more of the inputs on a particular farm under its unique and often variable environmental conditions. With the development of precision agricultural technologies such as variable rate application of fertiliser, the method for evaluating annual production would need to be capable of accepting spatial variability within a field and factoring this into the result. For this reason, Zhang et al. (2004) created a decision tree model that could be used throughout the North Island to predict annual pasture production based on unique spatial input variables. The dataset used to generate the model was created from many climatic, environmental and management variables, with the aim of covering the most important factors influencing pasture productivity. The decision tree models for annual and seasonal pasture productivity were found to clearly reveal the relative importance of environmental and management variables in influencing pasture productivity (Zhang et al. 2005a, b).

This model was subsequently embedded into ArcGIS 9.0 (ESRI, 2004) and applied to a 2500 ha hill country farm as a means to explore the relationship between fertiliser input and its effect on annual pasture production. Over recent years, intensive soil classification of the farm has been undertaken by staff and students at Massey University, this information has been used in various parts of the modelling process.

Although considerable information exist about the relationship between the application of phosphorus, pasture growth and the influence of topography (Lambert et al. 1983, 2000; Gillingham et al. 1998; Lopez et al. 2003; Dodd et al. 2004), up until now we have lacked the

mechanism to apply variable rates of fertiliser to match the potential of the land. A variable rate control system designed by the New Zealand Centre for Precision Agriculture, Massey University in conjunction with Wanganui Aerowork Ltd has been developed, and commercial trials using this system have resulted in a ground resolution of 18 m by 18 m (representing the smallest management unit that the system was able to respond to with a rate adjustment). Figures such as 15-20 kg superphosphate  $\text{SU}^{-1} \text{yr}^{-1}$  wintered have been used as a broad recommendation (Morton and Roberts, 1999); however the within field variability of hill country pasture production is often overlooked. What is further limiting the use of this technology is some economic justification of the benefits variable rate application technology (VRAT) has for the farmer.

This is the second paper in a series of three describing the development of VRAT, from the principles of modelling annual pasture production to the agronomic and economic benefits that can result. This paper describes six scenarios that have been developed to test the farm's response to a change in the fertiliser application policy. The agronomic impact for each of the scenarios was presented and summarised. The first paper (Murray et al., 2007a) reviews a number of important factors that contribute to variations in annual pasture production, as well as providing a brief commentary on previous pasture production models, while the third (Murray and Yule, 2007b) presents the economic impact of VRAT for farm owners and topdressing operators

## **6.2 MATERIALS AND METHODS**

The decision tree model as described in Zhang et al. (2004) was used to model annual pasture production at Limestone Downs. Currently, Limestone Downs comprises approximately 2500 hectares of grazing land, with mostly flat to rolling (46%) and easy (42%) country with some steep faces (12%). Aspect is predominantly southwest, with a split of North (22%), East (20%), South (31%) and West (27%) respectively. The soil type varies considerably across the property, with soils from peat to volcanic. The climate is favourable for intensive livestock farming, with a good blend of rainfall, temperature and solar radiation. The farm is managed

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using a 70% sheep, 30% beef ratio, with the cattle usually confined to the flatter land. Detailed grazing records have been kept in Farmtracker 6.2 (Farmworks Precision Farming System Ltd, 2005) for the last 10-15 years so animal movements and grazing patterns can be easily identified and monitored.

Waiuku Forest, 12 km North of Limestone Downs at 52 m above sea level (a.s.l.) was selected for temperature and rainfall data. Solar radiation data was obtained from Pukekohe Environmental Weather Station, 29 km North of Limestone Downs and 82 m above sea level. These sites were considered to adequately represent the meteorological condition at Limestone Downs. The meteorological datasets were obtained from the National Institute of Water and Atmospheric Research Limited (NIWA), New Zealand.

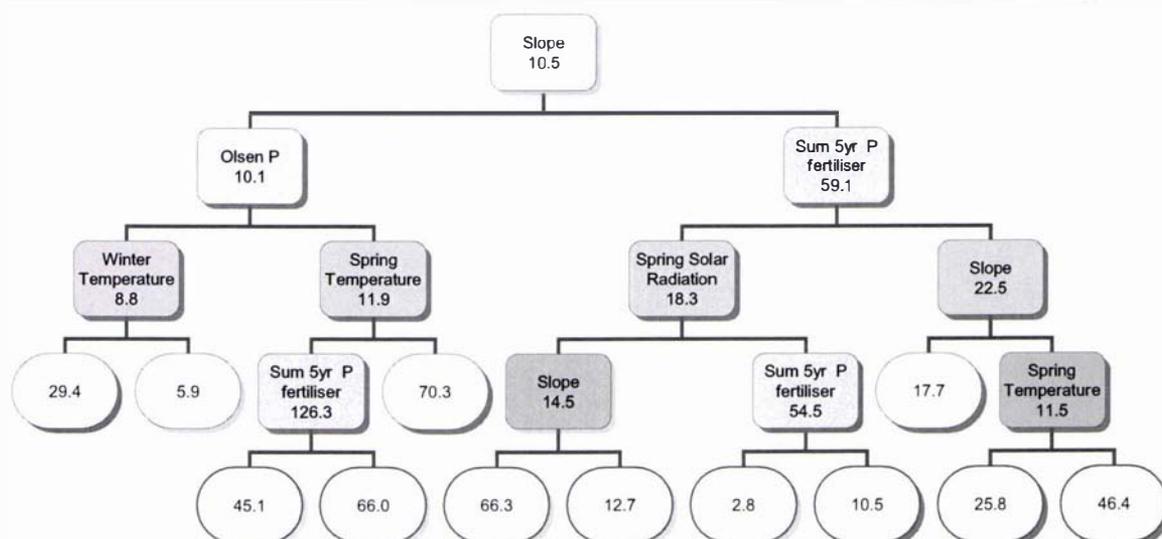
A 10 m digital elevation model (DEM) was used as the basis for topographic analysis. Each class, affected by the slope of the land, was adjusted by the cosine of the slope angle as described in Zhang et al. (2005b). This applied to the rainfall and fertiliser data and was based on the assumption that vertical land (i.e., 90°) received no fertiliser and perfectly flat land (i.e., 0°) received the entire fertiliser application rate. Each of the input datasets is shown in Table 6.1 and in some cases there was homogeneity between input dataset in both the annual production and species composition models.

**Table 6.1 Data inputs, common to all scenarios used in pasture production and species composition models. Seasons are define as summer (Dec – Feb), autumn (Mar – May), winter (Jun – Aug), spring (Sep – Nov).**

Input Description	Input Units	Pasture Production	Species composition
Aspect	degrees from north	x	
Slope	degrees from horizontal	x	x
Spring rainfall	mm	x	
Summer rainfall	mm	x	
Spring and summer rainfall	mm	x	
Mean autumn temperature	°C	x	
Mean winter solar radiation	MJ m <sup>2</sup> d <sup>-1</sup>	x	
Annual nitrogen input	kg N ha <sup>-1</sup>	x	
5 year cumulative phosphate input	kg P ha <sup>-1</sup>	x	x
Annual phosphorus input	kg P ha <sup>-1</sup>	x	
Olsen P	µg g <sup>-1</sup>		x
Mean spring temperature	°C		x
Mean spring solar radiation	MJ m <sup>2</sup> d <sup>-1</sup>		x
Mean winter temperature	°C		x

### 6.2.1 Defining unresponsive pastures

To spatially distinguish areas that would be unlikely to respond efficiently to nutrient application, another model was required. Zhang et al. (2005a) used data mining to create a decision tree model that delineates high fertility response grasses (HFRG) from other grass species (Figure 6.1). This classification system was embedded in ArcGIS 9.0 (ESRI, 2004) and was used to identify areas that would be either targeted or ignored when aerially applying fertiliser. Figure 6.1 shows each decision and its respective split point. For instance if an area has a 10° slope, Olsen P > 10.1, spring temperature < 11.9 and cumulative five year P fertiliser application > 126.3, the model predicts 66% of the pasture to contain high fertility response grasses. The remaining 34% being a mix of other pasture species, such as legumes, flat weed or low fertility tolerant grasses.



**Figure 6.1** The decision tree model that was used to define high fertility response grasses as described in (Zhang et al., 2005a). Each rectangular object contains an input variable and a split value. Where the variable is less than the split value then follow the left branch, where greater, follow the right branch. This continues until the circular object, containing a percentage of HFRG is reached.

Soil test data was acquired from the farm's major fertiliser supplier, Ravensdown Fertiliser Cooperative Ltd. Sample transect lines were digitised and overlaid across Limestone Down's slope and aspect spatial datasets. Both the slope and aspect data were classified so that the existing soil characteristics could be extrapolated. Aspect was split into two classes, the first being shady, which consisted of south-east to south-west aspect and secondly, sunny designated as all other aspects. Both aspect classes were split into three slope classifications, slope < 10° (easy), 11° - 20° (mid) and > 20° (steep) as used by Morton and Roberts (1999). Each classification was assigned the average measured soil properties as indicated from actual soil tests.

The spatial datasets were embedded into the ArcGIS environment and the pasture species composition model run on the data.

### **6.3 ANNUAL PASTURE PRODUCTION MODEL**

Raster datasets for each of the input variables shown in Table 6.1 were created and served as spatial input variables in the model. The decision tree that was used to model annual pasture production is shown in Murray et al. (2007). Each condition and its relating split point is shown

within the square objects, the resulting annual pasture production results are shown in the circular objects. For example, an area where the spring rainfall was found to be greater than 228 mm and had a slope of less than 16° with a cumulative five year phosphorus application of greater than 160 kg P ha<sup>-1</sup> residing on a sunny aspect is predicted to produce 16,701 kg DM ha<sup>-1</sup>.

The resulting annual pasture production as predicted by the model was then adjusted for grazing efficiency across differing topography. These correction factors relate to estimated annual pasture utilisation levels (%) for each land class. Steep country, classified as having a slope greater than or equal to 26°, was corrected by 60%, due to the animals preference to graze less steep land. Rolling country was land that had slopes between 12° and 26° and was corrected by a factor of 70%. Flat to easy country was designated as having slopes less than or equal to 12° and was adjusted by 80% of predicted annual pasture production (Gillingham, A. G., 12 February 2005, pers. comm).

Areas of trees and infrastructure were identified from a recently acquired high resolution (1 × 1 m) orthophotograph of the property. These areas were digitised to create an effective area surface that was used to clip the dataset to the actual farm extents, the total effective area was calculated to be 2518 ha.

## **6.4 SCENARIO DEVELOPMENT**

Six application scenarios were developed to test the hypothesis that VRAT would provide greater efficiency or agronomic returns, per unit of fertiliser applied, than that currently achieved through fixed rate application. For consistency, fertiliser input rate and the spatial location of the material applied were the only input parameters that changed between scenarios. All other inputs remained the same to ensure that there were no interaction effects between fertiliser application and other meteorological, managerial and topographic conditions.

### **6.4.1 Scenario A - Blanket Application**

A blanket fertiliser application, based on the farm's current fertiliser application policy, was the first situation run through the pasture production model. This was used as a base to compare the agronomic impact of current methods vs. VRAT and was important as it ensured

that fertiliser placement and quantity and its effect on annual pasture production were comparable. Current application of phosphorus fertiliser on the entire Limestone Downs property was based on a maintenance rate of 24 kg P ha<sup>-1</sup> yr<sup>-1</sup>. The total quantity of superphosphate fertiliser (9% P) applied in this scenario was 671 T.

#### **6.4.2 Scenario B - Simple VRAT**

In this situation the application rate used was the same as currently used on the property. The difference was that unresponsive pastures were set up as exclusion zones which would not receive fertiliser application. The unresponsive pastures indicated in Figure 6.2 totalled 484 ha, which, at the current fertiliser application rate, would result in a saving of 129 t or NZ\$20,392 of superphosphate, reducing the total applied to 542 T.

#### **6.4.3 Scenario C - Full VRAT**

In the VRAT scenario, fertiliser was applied at rates optimised for the decision tree model (Zhang et al. 2004). This was instigated on all areas except unresponsive pastures which received no phosphorus application. Fertiliser was strategically applied to areas containing between 46 – 76% high response fertility response grasses. To make use of VRAT continuous adjustment of fertiliser application rate, the levels of fertiliser input were dictated based on each area receiving the optimum nutrient input regardless of the slope, such that:

$$a_1 = \frac{a_0}{\cos(\theta)} \quad [\text{Eq. 6.1}]$$

Where  $a_1$  = actual application rate,  $a_0$  = required application rate, and  $\theta$  = slope angle expressed in degrees

The resulting annual P application dataset ranged from 0 (for pastures < 17% HFRG) to 34.8 kg P ha<sup>-1</sup>, with a mean application on responsive areas of 32.8 kg P .ha<sup>-1</sup>. Assuming that the fertiliser was spread evenly in subsequent years then the five-yearly cumulative P application would be five times the annual application. The total amount of superphosphate used annually in this situation was 738 T, approximately 67 t more than that used in scenario A.

#### **6.4.4 Scenario D – Blanket Application (increased fertiliser input)**

This is identical to scenario A, however the quantity of fertiliser spread is matched with scenario C (738 T) thus, 26 4kg ha<sup>-1</sup> is spread over the entire farm area. This was included to compare the agronomic impact of an increased phosphorus fertiliser regime without targeting higher producing land.

#### **6.4.5 Scenario E –Simple VRAT (increased fertiliser input)**

In this scenario, the total tonnage of fertiliser used in Scenario C (738t) was divided by the farm area less the area designated as unresponsive pasture. This increased the annual blanket application rate to 32.9 kg P ha<sup>-1</sup> over the application area while excluding non-responsive pastures. This was included to investigate the extent to which the transfer of fertiliser inputs from, low production potential zones, to higher production zones, can influence annual pasture yield.

#### **6.4.6 Scenario F - Full VRAT (reduced fertiliser input)**

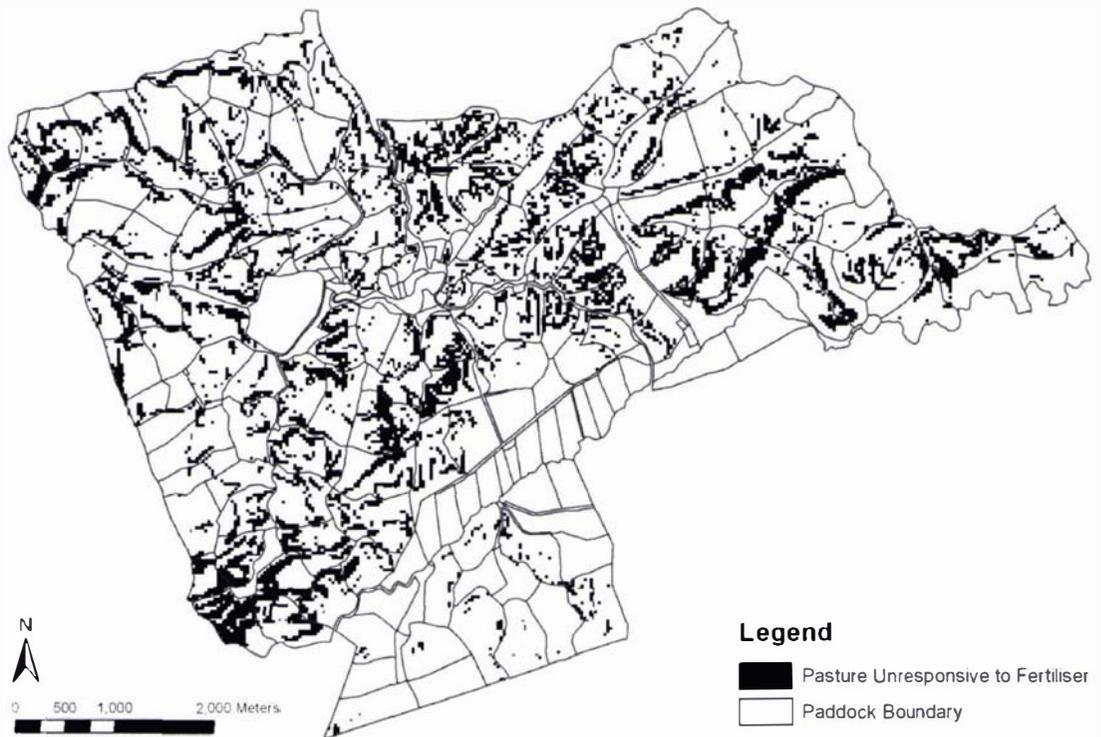
This is essentially a reallocation of the 671 t of superphosphate currently applied annually. In this case, the 738 t used in the full VRAT scenario was reduced by a correction factor of 0.91 to decrease the total amount spread to 671 t with application rates ranging from 29.06 to 38.12 kg P ha<sup>-1</sup>. As in scenario C, potentially higher producing land received preferential fertiliser application. This scenario was used to show how fertiliser use efficiency could be altered when utilising VRAT, and how this process would impact on annual pasture production.

#### **6.4.7 VRAT extension**

To assess the impact of adopting VRAT over time, a further five scenarios were run through the decision tree model. Annual pasture production for each of the five years was modelled, with changes only in the annual and 5 year cumulative phosphorus inputs. Therefore, in the first year of VRAT the annual P input would be equal to that of scenario C. This continued until year five when all subsequent annual P inputs were made using VRAT. For example after year three of VRAT the cumulative P input used in the decision tree model would be equal to three times the annual VRAT input plus two years of blanket application.

## 6.6 RESULTS

The locations of HFRG are shown in Figure 6.2. The dark shaded areas indicate predicted pastures containing only 17% HFRG and were used to determine the spatial location of areas that should not receive fertiliser application. The total area represented by unresponsive pastures is 484 ha.



**Figure 6.2 Classification of Limestone Downs, North Island, New Zealand, into pastures unresponsive to fertiliser application, due to the combination of topography, pasture composition and meteorological conditions**

Because of the combination of topography, meteorological conditions, animal grazing habits and pasture composition, there was little financial incentive in applying fertiliser to those areas. This was later reinforced by the pasture production model. Even with fertiliser input to the unresponsive areas there was no resulting change in the predicted annual pasture production.

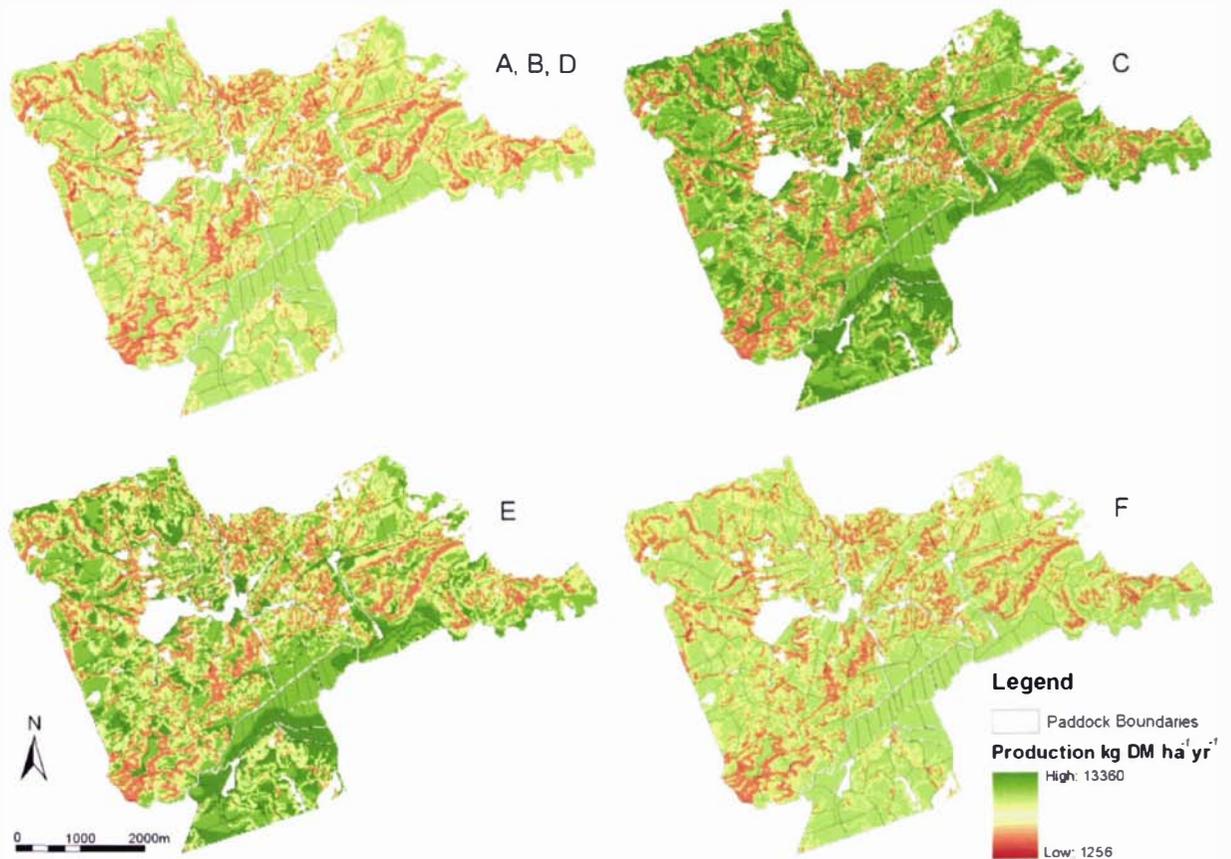
**Table 6.2 Results from an agronomic evaluation of six fertiliser application scenarios (applied for five years) and their effect on modelled annual pasture production on Limestone Downs, North Island, New Zealand.**

Parameter	Scenario					
	A	B	C	D	E	F
Mean (kg DM ha <sup>-1</sup> yr <sup>-1</sup> )	7918	7918	9846	7918	9491	8436
Pasture production range (kg DM ha <sup>-1</sup> yr <sup>-1</sup> )	1256 - 9941	1256 - 9941	1256 - 13360	1256 - 9941	1256 - 13360	1256 - 9941
Available pasture (t DM yr <sup>-1</sup> )	19935	19935	24789	19935	23894	21239
Superphosphate used (t)	671	542	738	738	738	672
Fertiliser response (t DM yr <sup>-1</sup> t superphosphate <sup>-1</sup> )	29.7	36.8	33.6	27.0	32.4	31.6

The outputs of the annual pasture production model for each scenario are shown in Table 6.2. Scenarios A, B and D returned similar pasture production predictions, both spatially (Figure 6.3), and in total annual pasture yield, with mean annual production of 7918 kg DM ha<sup>-1</sup> and a production range between 1256 - 9941 kg DM ha<sup>-1</sup>.

Scenario F produced a mean pasture yield of 8436 kg DM ha<sup>-1</sup> with a range between 1256 - 9941 kg DM ha<sup>-1</sup>. In contrast, the fertiliser application used in scenario C and E greatly increased the spatial variation in annual pasture production both within field and across the farm, with a mean production of 9846 and 9491 kg DM ha<sup>-1</sup> respectively and a range of 1256 - 13360 kg DM ha<sup>-1</sup>.

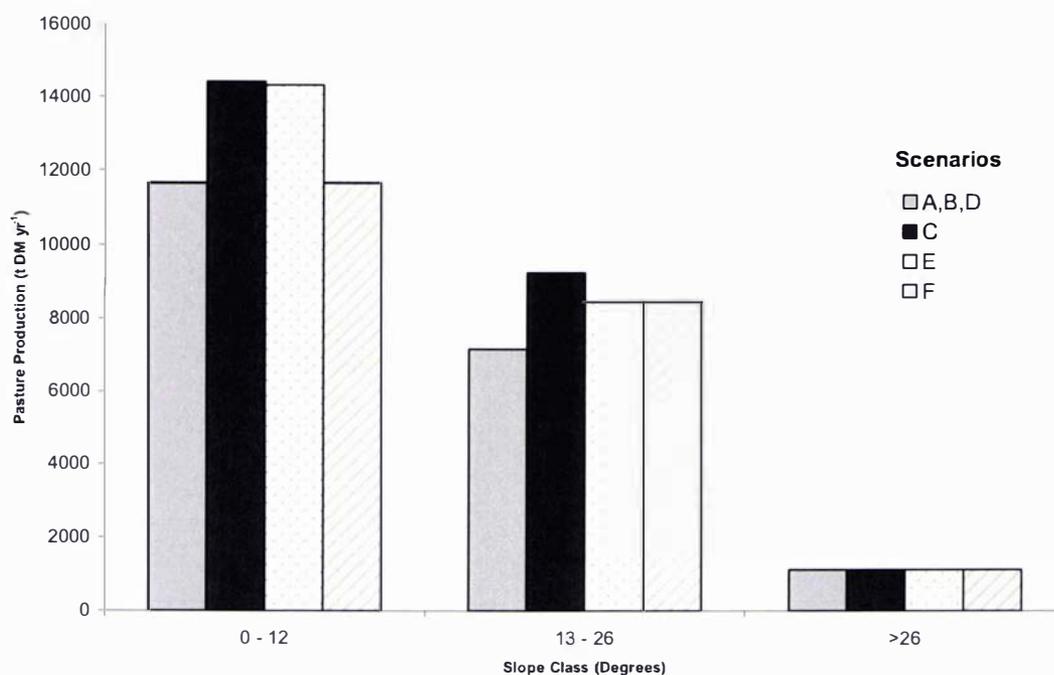
Scenario F had similar production to A, B and D in the 0-12° classification (Figure 6.4) with 11664 t DM yr<sup>-1</sup>, but was predicted to produce additional pasture in the 13-26° category with 8455 t DM yr<sup>-1</sup> compared to 7154 t DM yr<sup>-1</sup> for A, B and D. In all of the scenarios the steep (> 26°) classification returned identical production with 1118 t DM yr<sup>-1</sup>, regardless of the quantity of fertiliser applied. Scenario C performed best across the most slope categories followed by scenario E, F and finally the grouping A, B, D.



**Figure 6.3** Spatial variation in annual pasture production resulting from the six scenarios. Pasture production maps correspond to the scenarios A – F.

Scenario B used the least amount of fertiliser (542 T), some 129 t less than the amount currently applied on this property (671 T). The productive land still received the same application rate as in scenario A, even with the total quantity reduced. Scenarios C, D and E had their application limited to 738 T, the difference between each scenario being the method and placement of the fertiliser across the farm.

When considering fertiliser use efficiency (Table 6.2), scenario B produced the greatest fertiliser response with 36.8 t DM yr<sup>-1</sup> t superphosphate<sup>-1</sup>. In contrast, scenario D had a lower response at 27 t DM yr<sup>-1</sup> t superphosphate<sup>-1</sup> and represented the most inefficient method of applying fertiliser to this particular property. Scenario C made more efficient use of the 738 t of fertiliser than either scenario D or E, with 33.6 compared to 27.0 and 32.4 t DM yr<sup>-1</sup> t superphosphate<sup>-1</sup> respectively.



**Figure 6.4** Summary of annual production across the three different slope classes, flat to rolling 0-12°, easy 13-26° and steep > 26°.

Agronomic results from the application of VRAT over a five year period indicated that there was very little change in annual pasture production after three years. After four years of VRAT however, production increases were expected, particularly in the easy sloping land scattered across the property. Spatial variation is likely to increase as a result of VRAT, as evident from Figure 6.5. In year 4, annual pasture production increased to 21242 t DM with a mean of 8437 kg DM ha<sup>-1</sup> yr<sup>-1</sup>. Further production gains were expected in year 5 of VRAT due to increased soil P levels, year 5 production was predicted to be 24789 t DM yr<sup>-1</sup>.

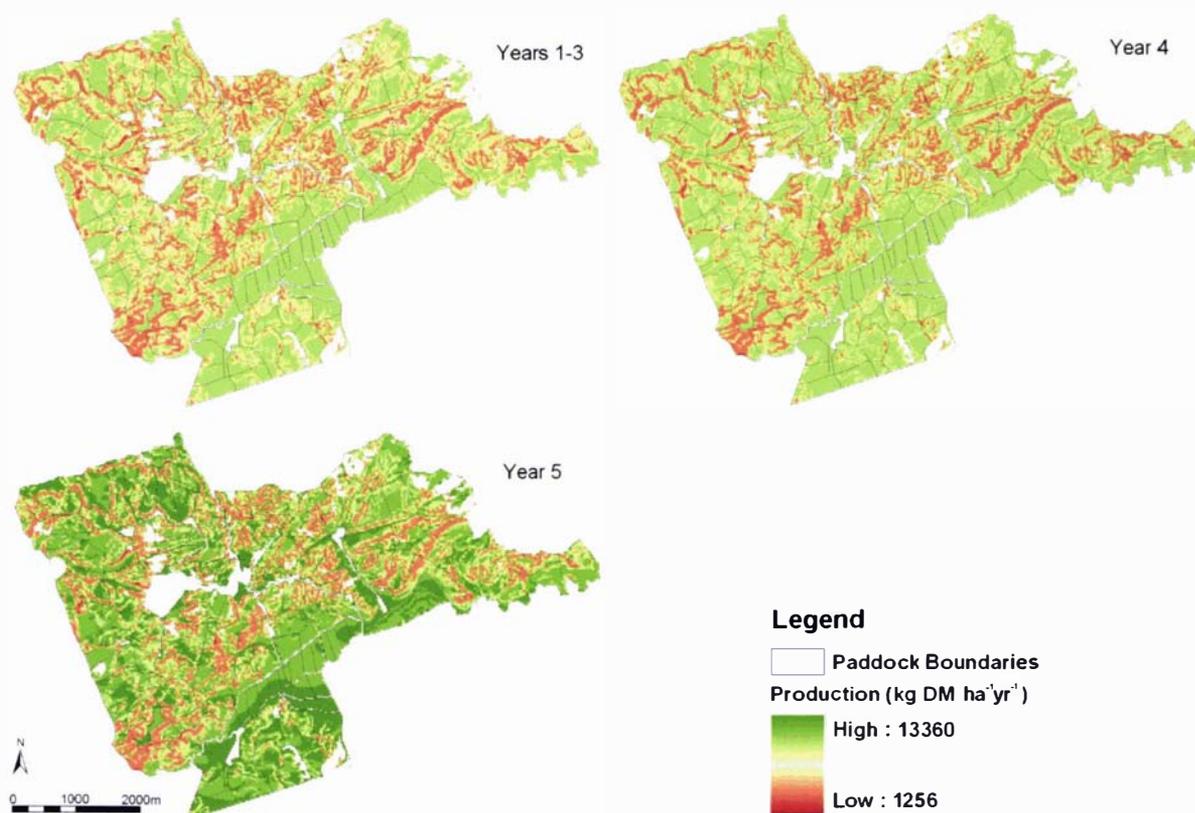


Figure 6.5. Predicted spatial variation in annual pasture production over a five year period using variable rate application technology (VRAT).

## 6.7 DISCUSSION

Zhang (2005) discusses a limitation in the decision tree approach, the author suggests that due to the binary nature of the decision tree, the responses of pasture productivity to the environmental and management variables are not continuous. This prevents the decision tree reflecting the influence of small changes to the input variables. This limitation often occurs where step changes are predicted. If the threshold is not met, no change is recorded. Scenario C is unaffected by this occurrence as the application of P is optimised for the split points described in the decision tree (Murray et al., 2007a). The predicted annual pasture production for scenarios A, B and D remained the same, even with a change in the blanket application rate. This is a logical result for scenarios A and B as the application rate remains constant, however, in scenario D the application rate is increased from 24 to 26.4 kg P ha<sup>-1</sup> across the property. Although this is a small increase, in reality an increase in annual pasture production and Olsen P would be expected. It seems equally likely that scenario F may have suffered from this also, but

perhaps not to the same extent. This could be remedied by further refining the model to include smaller step changes, although this would increase model complexity and computer processing time.

The spatial variation in pasture production increased markedly in scenarios C and E (Figure 6.3). However, Figure 6.4 indicates that most of the additional production was confined to more readily accessible flat to easy sloping topography that tends to be preferentially grazed by sheep and beef cattle (Sheath, 1983; Belesky et al., 2002). Sheath (1983) goes on to say that subdivision in hill country, in particular separating the preferred north from south-facing aspects, has long been an accepted method of improving the efficiency of pasture development and utilisation. However, the literature suggests that because of the dominating effect of topography on pasture growth and utilisation, sub-division into homogeneous land strata, including both slope and aspect, will be required for full grazing control and increased utilisation.

Fertiliser is often applied with the assumption of increased pasture production, and the larger the quantity of fertiliser applied, the higher the farms annual pasture production is expected to be, however this is not always the case, particularly with current blanket application techniques represented in scenarios A and D. In both cases the total annual pasture production from the application of both 671 and 738 t of superphosphate was the lowest out of all of the scenarios. By using either simple VRAT or full VRAT the annual pasture production yields could be substantially increased.

Scenario C is the most productive method of fertiliser application in terms of maximising yield per hectare ( $9846 \text{ kg DM ha}^{-1}$ ) thus, it would probably yield the highest returns to the farmer. In the end however, the management goals of the individual farmer will dictate what fertiliser management policy will be employed. Scenario B had the highest response per unit of superphosphate applied. This could be a feasible alternative to scenario C, if management utilised a simple VRAT and ceased the application of fertiliser to unresponsive areas, whilst retaining the current application rate on the more productive land. This analysis also highlighted the inefficiency of current fertiliser application methods (scenarios A, D). These fall far behind VRAT techniques, even though the same total tonnage of fertiliser is being spread.

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This simulation, along with numerous other studies (Barrow, 1974; Suckling, 1975; Lambert et al., 1983) suggests that the effect of P application is not instantaneous and that superphosphate previously applied has an effect beyond that in its immediate year of application. This is in line with comments by Lambert et al. (1983) which indicate that as annual fertiliser application continues, phosphate applied in a particular year becomes relatively less important and the annual response of P will likely have a time lag. Unlike nitrogen, soil phosphate levels take a number of seasons to increase and the nutrient is not totally available. These conditions help to explain, in part, why the additional yield developed under VRAT takes several years to eventuate and why the prediction of annual yield is heavily influenced by the five year cumulative P input.

With the strategic use of high analysis fertilisers and the need for accurate, efficient and environmentally safe application of nutrients to agricultural farmland, variable rate control systems are gaining significant interest. VRAT, combined with an accurate method of identifying spatially variable N responses across contrasting topography, would make an exciting and potentially lucrative area of research. It could also help to maintain the application of fertiliser by air as an environmentally acceptable practice and, provide for safer operating conditions for the pilots by replacing the manual hopper door control with an automated system thus reducing pilot fatigue and the number of tasks they must cope with.

The next major step in determining the feasibility of VRAT is to perform a full economic analysis that includes converting additional available dry matter through to additional stock units which lead to changes in stock gross margins, cash and farm expenditure and finally net returns.

## **6.8 CONCLUSION**

A decision tree model developed by Zhang et al. (2004) was used to model several pasture production scenarios under differing fertiliser application techniques and application rates. The application techniques used in this modelling approach consisted of either fixed rate blanket fertiliser application, simple VRAT (using Geographical Information Systems (GIS) and

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computer modelling to remove unproductive zones) and full VRAT which uses GIS and computer modelling, combined with a GPS and computerised control system, to continuously adjust application rates as defined by a prescription map.

Scenario C, which used full VRAT and an optimised prescription map, was predicted to produce the highest annual pasture yield across the 2518 ha Limestone Downs property at 2478 t DM, some 24.4% more than the property is currently modelled to produce. Both simple and full VRAT techniques were predicted to increase annual production by between 6.5 – 24.4%. In all accounts, the blanket application techniques which are currently employed on this property produced the lowest annual pasture production with 19935 t DM yr<sup>-1</sup>. If farm management chose to maintain fertiliser application at current levels, scenario B, which uses simple VRAT, may be a valid alternative to full VRAT. However, the analysis clearly revealed that the most productive method of application would be full VRAT using 738 t of superphosphate.

The analysis of the agronomic impact of several fertiliser application scenarios on a large hill country farm highlights the potential of spatial modelling techniques such as the prediction of annual pasture production. This coupled with VRAT and automated control systems, could prove to be a valuable proposition in the future of aerial topdressing in New Zealand.

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## CHAPTER 7 - DEVELOPING VARIABLE RATE APPLICATION TECHNOLOGY: ECONOMIC IMPACT FOR FARM OWNERS AND TOPDRESSING OPERATORS

*Publication arising from this chapter*

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Murray RI, Yule IJ 2007. Developing variable rate application technology: economic impact for farm owners and topdressing operators. *New Zealand Journal of Agricultural Research* 50: 65-72.

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### **7.1 INTRODUCTION**

Primary producers, such as those in the sheep and beef industry, are faced with many technological developments that could assist in maximising production. Unfortunately, they do not have sufficient resources to invest in all technologies and must therefore choose the most appropriate developments for their situation.

Researchers often speculate about the most important profit criteria in the sheep and beef farming sector. Animal scientists may focus on lambing rates, weaning weights and lamb deaths as important production factors, while economists may focus on gross income, total production cost and cash flows as a means of increasing production efficiency (Nudell et al., 1998). Technologists may lean more towards the adoption of emerging technologies to streamline information flow, increase input usage efficiency and enhance the management of spatial variation. Although any one of these doctrines could increase farm profitability, it is likely that a mix of all three would have the greatest potential to maximise the net return to the farmer or land owner.

Technology orientated and computer-literate farmers will probably favour the technology approach over their non-technologically minded counterparts. Information technology, in both dairy and sheep and beef farms, has undergone significant development in the last five years through the mainstream adoption of farm management software such as OVERSEER nutrient budget 2 (AgResearch, 2003), Farmtracker 6.2 (Farmworks Precision Farming System Ltd 2005) and Resolution mapping software (AgResearch, 2005). Digital records of stock

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movements, stock purchases and sales, chemical and fertiliser application history and pasture cover have often been kept and in many cases each attribute is tagged with a spatial reference such as the paddock number and saved as a layer within a GIS database. Although this information is important for traceability purposes, it has other uses, particularly when modelling farm specific performance variables, such as animal and pasture production. The industry is now in a position to make use of the wealth of information and advance with new technologies such as variable rate application of nutrients by air.

Although considerable information exists about the relationship between the application of phosphorus, pasture growth and the influence of topography (Lambert et al. 1983, 2000; Gillingham et al. 1998; Lopez et al. 2003; Dodd et al. 2004), up until now we have lacked the mechanism to apply variable rates of fertiliser to match the potential of the land. A variable rate control system designed by the New Zealand Centre for Precision Agriculture, Massey University in conjunction with Wanganui Aerowork Ltd has been developed and commercial trials using this system have resulted in a ground resolution of 18 m by 18 m. Figures such as 15-20 kg superphosphate  $\text{SU}^{-1} \text{yr}^{-1}$  wintered have been used as a broad recommendation (Morton and Roberts, 1999); however the within field variability of hill country pasture production is often overlooked. What is further limiting the use of this technology is some economic justification of the benefits Variable Rate Application Technology (VRAT) has for the farmer.

This is the last paper in a series of three describing the development of VRAT, from the principles of modelling annual pasture production to the agronomic and economic benefits that can result. This paper seeks to test the hypothesis that VRAT will provide economic advantages over what is currently achieved with the fixed rate application of phosphorus to large blocks of sheep and beef farmland. The first paper (Murray et al., 2007a) reviews a number of important factors that contribute to variations in annual pasture production, as well as providing a brief commentary on previous pasture production models. The second paper in the series (Murray and Yule, 2007a) describes the scenario development and provides an agronomic evaluation of the case study farm.

## 7.2 METHODS

Several topdressing techniques, each with multiple fertiliser input levels, were developed in order to evaluate the agronomic and economic impact of different fertiliser application techniques. Limestone Downs was chosen as the case study farm; it is located on the west coast of New Zealand in Franklin County, and situated 15 kilometres south of Port Waikato. The farm comprises approximately 2500 hectares of grazing land, with mostly flat to rolling (46%) and easy (42%) country with some steep faces (12%). Aspect is predominantly southwest, with a split of North (22%), East (20%), South (31%) and West (27%) respectively. The soil type varies considerably across the property, with soils from peat to volcanic. The climate is favourable for intensive livestock farming, with a good blend of rainfall, temperature and solar radiation. The farm is managed using a 70% sheep, 30% beef ratio, with the cattle usually confined to the flatter land.

The development of the scenarios, along with their agronomic impact was discussed in Murray and Yule (2007a). The results from that paper form the basis for the economic assessment which follows (see Table 7.1).

**Table 7.1 Results from the agronomic assessment of annual pasture production on 2518 ha of Limestone Downs as described in Murray and Yule (2007a)**

Parameter	Scenario					
	A	B	C	D	E	F
Mean (kg DM ha <sup>-1</sup> )	7918	7918	9846	7918	9491	8436
Available Pasture (TDM yr <sup>-1</sup> )	19935	19935	24789	19935	23894	21239
Superphosphate Used (T)	671	542	738	738	738	671
Fertiliser response (kg DM kgP <sup>-1</sup> )	29.7	36.8	33.6	27.0	32.4	31.6
Application method	Blanket	SVRAT	FVRAT	Blanket	SVRAT	FVRAT

Blanket: Fixed rate application of fertiliser by aircraft

SVRAT: Simple Variable Rate Application: utilising GIS modelling and GPS to define unresponsive pasture production areas which are subsequently avoided in aerial P applications.

FVRAT: Full Variable Rate Application: utilising GIS modelling to define unresponsive areas and variable application rates. Fertiliser spread using GPS and computer control variable rate application system capable of automatic changes in application rate.

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### 7.2.1 Calculating cash flow

The calculation of the total carrying capacity within each scenario was required in order to derive an economic outcome. Thus, a single Stock Unit (SU) equivalent of 550 kg DM yr<sup>-1</sup> for a 55 kg ewe with 100% lambing was used to convert available feed to potential SU (Cornforth and Sinclair, 1984) using Equation 7.1.

$$\text{Potential SU} = \frac{\text{Total Available Feed (kg DM yr}^{-1}\text{)}}{\text{SU Annual Feed Requirement (kg DM yr}^{-1}\text{)}} \quad [7.1]$$

To calculate a farm cash flow statement for comparison between scenarios, both revenue and expenditure under each scenario needed to be determined. After some minor modifications, the Ministry of Agriculture and Forestry's national sheep and beef budget model (MAF, 2005b) was used to calculate both the revenue and expenditure streams from the potential SU. Firstly, phosphate fertiliser was the variable being interrogated, therefore a spreadsheet was developed in Microsoft Excel® and was used to calculate the total fertiliser application costs for each of the blanket, Simple Variable Rate Application Technology (SVRAT) or Full Variable Rate Application Technology (FVRAT) scenarios. The aircraft and application parameters used in these calculations are shown in Table 7.3 and are discussed further in the section calculating fertiliser application cost. Secondly, in 2005 approximately 272 T of urea was applied to 500 ha of flat to rolling land on the case study farm, this along with spreading costs of \$60 T<sup>-1</sup> (Burt, 2004), was converted into SU equivalents and replaced the Fertiliser (Nitrogen) value (Table 7.2) presented in the Ministry of Agriculture and Forestry's national sheep and beef budget model .

**Table 7.2 Farm cash inflows and outflows used to calculate cash farm surplus in each of the six scenarios defined in Murray and Yule (2007a). Note: Excludes superphosphate fertiliser and application cost as this was calculated independently (refer to Table 7.3).**

Revenue	\$NZ SU <sup>-1</sup>		\$NZ SU <sup>-1</sup>
Sheep	64.35	<b>Less:</b>	
Wool	15.14	Sheep purchases	8.35
Cattle	91.46	Cattle purchases	33.86
Other farm income	1.32	<b>Gross farm revenue</b>	<b>69.86</b>
<hr/>			
<b>Farm working expenditure</b>			
Staff wages	3.93	Weed and pest control	1.09
ACC levy	0.73	Vehicle costs (includes fuel)	2.99
Animal health + breeding	3.39	Repairs and maintenance	3.86
Electricity	0.77	Administration	2.03
Feed	2.04	Rates	1.45
Fertiliser (Superphosphate)	NA	Insurance	0.91
Fertiliser (Nitrogen)	4.17	Other expenditure	1.67
Lime	0.69	<b>Cash Farm expenditure</b>	<b>36.80</b>
Regrassing costs (contractors)	1.27	Interest	6.97
Shearing costs	5.81	Rent and/or lease	1.69
<hr/>			
<b>Cash farm surplus</b>	<b>24.40</b>		

Stock unit revenue (Table 7.2) varies according to stock type, for example, sheep and wool income, sheep purchases and shearing costs are per sheep stock unit, while cattle income and purchases are per cattle stock unit. All other income and expenditure items are calculated from the combined total of sheep and beef stock units. Due to the sheep and beef model being stock specific, the initial sheep to cattle ratio as well as sheep purchases for the period from 1st July 2003 – 30th June 2004 were extracted from Farmtracker 6.2 (Farmworks Precision Farming System Ltd 2005). This was recorded as a percentage of total stock units and was used to calculate revenue and purchases for each stock category. Grazing income was not included in this example as Limestone Downs have no off-farm grazing policies. The farm's animal and grazing database showed that only a small number of rams were purchased each year, as opposed to the beef cattle, where all cattle were reported as being purchased.

### 7.2.2 Calculating fertiliser application cost

A fertiliser application cost spreadsheet was created in order to test the hypothesis that alternative fertiliser application techniques and regimes may be more economic than conventional methods and policies. This quantified, in financial terms, how changes in application rate and the total amount of fertiliser used could affect fertiliser application time and subsequently the overall cost of the fertiliser application.

The scope of this model was limited to a non-spatial analysis, therefore no account was given to the fact that, under VRAT, flow rates and time taken to empty the hopper will fluctuate over time. An example of the data requirements is shown in Table 7.3; user input was required for values in bold text.

**Table 7.3 Parameters used in calculating fertiliser application, an example of the parameters is provided for the application of 671 T of superphosphate to 2518 ha of Limestone Downs.**

Parameter	Value	Units
<b>Aircraft variables</b>		
Hopper capacity	<b>2000</b>	kg
Aircraft speed	<b>60</b>	m s <sup>-1</sup>
Bout width	<b>12</b>	m
Time per refill	<b>30</b>	s
Aircraft charge out rate	<b>1200</b>	SNZ hr <sup>-1</sup>
<b>Application information</b>		
Farm area	<b>2518</b>	ha
Rate	<b>267</b>	kg ha <sup>-1</sup>
Average distance to airstrip	<b>2000</b>	m
Cost of superphosphate fertiliser	<b>158</b>	SNZ T <sup>-1</sup>
Application time	9.7	hr
Flow rate	19.2	kg s <sup>-1</sup>
Total fertiliser used	671	T
Number of hopper refills	336	
Time taken to refill (once landed)	2.8	hr
Non-application time	6.2	hr
Total time for application	18.7	hr
<b>Cost summary</b>		
Total cost inc. fertiliser and spreading	128541	SNZ
Cost per ton	191.56	SNZ
Cost per hectare	51.05	SNZ

Several important assumptions were made; firstly, the aircraft flew the entire property regardless of application technique. In reality, it may prove to be very difficult to compute alterations to flight path of the aircraft under VRAT. Secondly, it was assumed the aircraft used

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in this example was a 550 kW Cresco topdressing aircraft developed by Pacific Aerospace Corporation, New Zealand. Thirdly, the time taken to refill, refers to loading the aircraft with fertiliser and fuel (if required). Fuel can be pumped into the aircraft in parallel with refilling the hopper. Likewise, non-application time refers to the time taken to travel from application zone to airstrip and back. Fourthly, the aircraft charge out rate includes the cost associated with the loader and driver. Lastly the average distance to the airstrip was calculated by randomly inserting 1000 points from within the Limestone Downs property boundary into ArcMap (ESRI 2004) and querying the dataset for the average distance from all points to the airstrip.

An additional cost was applied to all of the VRAT scenarios included in the farm's financial evaluation. It was approximated that for simple VRAT application, a charge of \$1000 was required to cover the costs of producing a prescription map. This was increased to \$1500 for full VRAT, as it would require further development time due to the more complex prescription maps. This would be considered a one-off payment, but if prescription maps or avoidance zones required recalculation, the cost would recur.

### **7.2.3 Sensitivity analysis**

A simple sensitivity analysis was performed on Scenario F. This scenario was chosen as it represented an appropriate method for comparing yield variations and elasticity of aircraft charge-out rate under VRAT. The price elasticity is an important component to the adoption of VRAT for hill country farms; it is assumed that agricultural aircraft operators would charge a price premium for these new application techniques, thus this was used to test how such a change would impact the financial evaluation of the scenario. Annual yield predictions were exposed to a variation of  $\pm 5\%$  and  $\pm 10\%$ , while charge out rate experienced variations of  $\pm 10\%$  and  $\pm 20\%$ .

## **7.3 RESULTS**

Using a ratio of approximately 70:30 sheep to beef cattle, both SU equivalents and total equivalent numbers of sheep and cattle were calculated for each of the scenarios. The resulting dataset is represented in Table 7.4 and was used in identifying SU numbers for the computation

of cash farm surplus. The conversion factors of 1.11 (sheep) and 4.67 (cattle) were used to convert SU equivalent values back to equivalent numbers of sheep and cattle, these were the actual conversion factors used by Limestone Downs staff and were extracted from Farmtracker 6.2 (Farmworks Precision Farming System Ltd 2005).

**Table 7.4 Summary of the total SU and equivalent numbers of sheep and cattle. Data is calculated based on annual pasture production (refer to Table 7.1), SU equivalent factors Farmtracker 6.2 (Farmworks Precision Farming System Ltd 2005), and a sheep to cattle ratio of approximately 70:30.**

	Scenario					
	A	B	C	D	E	F
Total SU	36246	36246	45070	36246	43444	38617
Sheep SU	25015	25015	31105	25015	29983	26651
Cattle SU	11231	11231	13965	11231	13462	11966

Cash farm surplus for each of the scenarios was calculated by multiplying the cash revenue and expenditure items present in Table 7.2 with the total SU and stock ratios used for that scenario.

The results from the sensitivity analysis are shown in Table 7.5 and indicate the farm's cash position is greatly influenced by annual yield. A 5% shift in annual production could lead to approximately  $\pm \$143,000 \text{ yr}^{-1}$  in cash surplus at current aircraft charge-out rates, while a 10% shift could result in  $\pm \$286,000 \text{ yr}^{-1}$ . The sensitivity of farm cash surplus to a change in aircraft charge-out rate was shown to be relatively inelastic, if operators charged  $\$1440 \text{ hr}^{-1}$  for VRAT, this would still result in a cash surplus of  $\$1,037,861 \text{ yr}^{-1}$ .

**Table 7.5 Sensitivity analysis of Scenario F showing the relationship between aircraft charge out rate and yield variation and how this effects the cash position of Limestone Downs.**

		Aircraft charge out rate (\$NZ hr <sup>-1</sup> )					
		%	-20	-10	0	10	20
Production (TDM. yr <sup>-1</sup> )	Value		960	1080	1200	1320	1440
	-10	19115	760937	758689	756442	754191	751948
	-5	20177	903865	901617	899370	897120	894876
	0	21239	1046850	1044602	1042355	1040105	1037861
	5	22301	1189722	1187474	1185227	1182977	1180733
	10	23363	1332651	1330402	1328155	1325905	1323661

There are several important figures in Table 7.6 that highlight the differences between blanket application and information technology based fertiliser application techniques, such as VRAT. Firstly, in both cases when full VRAT was used (Scenarios C, F) the highest cash surplus resulted for a given quantity of fertiliser applied. Thus Scenarios C and F yielded annual cash surpluses of \$1,226,682 and \$1,042,355 respectively, comparing this to blanket application techniques which resulted in \$971,944 (A) and \$960,256 (D). Simple VRAT also showed increased annual cash surpluses over conventional blanket application techniques returning \$993,532 (B) and \$1,177,900 (E).

**Table 7.6 Summary of the economic findings from several alternative fertiliser application methods on the 2518 ha Limestone Downs property, SSU refers to Standard Stock Unit.**

	Scenario					
	A	B	C	D	E	F
Farm Carrying Capacity (SSU)	36246	36246	45070	36246	43444	38617
P fertiliser (\$NZ)	128541	105952	140222	140228	140138	128622
Cash Surplus (\$NZ)	971944	993532	1226682	960256	1177900	1042355
P Fertiliser : expenditure ratio	0.09	0.08	0.08	0.10	0.08	0.09
Return from P fertiliser (\$NZ kg <sup>-1</sup> )	1.45	1.83	1.66	1.30	1.60	1.55
Cash Surplus per ha (\$NZ ha <sup>-1</sup> )	386	395	487	381	468	414

Obviously, the higher the stocking rate, the higher the N requirement in times of feed shortages. Nitrogen use ranged between \$151,146 and \$187,943 across the scenarios. Fertiliser (combined cost of nitrogen and phosphorus) was the highest annual cash expenditure item, ranging from \$257,098 (B) to \$328,165 (C), nitrogen constitutes over half of this cost.

Although Scenarios A, B and D had the same carrying capacity, both (A) and (D) used additional superphosphate fertiliser for no further agronomic benefits. This additional spending

reduced cash surplus, particularly in (D) which resulted in an annual cash shortfall of \$11,688 when compared to Scenario A. It also reveals several important parameters that provide a means for an economic assessment. The P fertiliser used in Scenarios B, C and E accounted for 8% of total farm expenditure, while A and F were slightly higher at 9% and finally Scenario D was raised to 10%.

The economic return from the superphosphate fertiliser ranged from \$1.30 kg<sup>-1</sup> (D) to \$1.83 kg<sup>-1</sup> (B), this clearly indicates the most efficient level and method of fertiliser application on this property.

Maximum return per hectare was achieved in Scenario C (\$487 ha<sup>-1</sup>), followed by Scenario E (\$468 ha<sup>-1</sup>). In both instances, the total amount of fertiliser was higher than that currently applied on this property. In comparison, the highest returning Scenario using the current amount of fertiliser was F, with \$414 ha<sup>-1</sup>.

#### **7.4 DISCUSSION**

The results show the financial impact of how a change in fertiliser use and alterations to application rates can lead to dramatic changes in Limestone Downs, annual cash surplus. The results presented in this paper reinforce the hypothesis that VRAT, in whatever form, would increase fertiliser use efficiency and positively impact on farm cash surplus beyond that of current blanket application techniques.

When applying 671 T (Scenarios A and F), significant increases in the farm's carrying capacity, annual cash surplus and cash surplus per hectare were apparent. This was repeated in the application of 738 T of superphosphate fertiliser. Scenario C (FVRAT) out-performed E (SVRAT) which, in turn, out-performed D (Blanket). In summarising this trend, there was a strong indication that FVRAT was the most efficient and highest returning application method per hectare, followed closely by SVRAT. In addition, the most productive quantity of superphosphate spread across this farm was 738 T utilising one of the VRAT techniques. This produced cash surpluses of \$487 ha<sup>-1</sup> for Scenario C and \$468 ha<sup>-1</sup> for Scenario E.

Previous studies by Murray and Yule (2005) indicate that SVRAT could take place with the

present technology. In order to accommodate SVRAT, operators or farmers would need to supply several spatial datasets to a trained GIS technician; this data would be run through a model to identify locations of low fertility tolerant grasses (Murray and Yule, 2007a). Once this is done, and provided the aircraft had Global Navigation Satellite System (GNSS) guidance, the non-responsive zones would be uploaded into the aircraft's job file. This job file contains all the relevant spatial information for the job, such as farm and zone boundaries, avoidance areas and rate information. The guidance system would then alert the pilot to cease application in non-responsive areas and start application when entering the responsive zone.

Although FVRAT is slightly more complex, a prototype of the system has been trialled in New Zealand over the last few years. If FVRAT costs an approximated \$50,000 to install, and the aircraft operates for 300 hours per year using FVRAT and charges 20% or \$240 hr<sup>-1</sup> more than the current charge out rate, then extra revenue gained from VRAT would be \$72,000. Therefore, assuming those earnings were taxed at 33% and an opportunity cost of 8% was applied; the payback period for the equipment would be close to one year.

The sensitivity analysis revealed that this farm is susceptible to annual pasture production variations. If a 10% change in annual yield results over the entire 2518 ha, this equates to an annual and weekly difference of 790 kg DM ha<sup>-1</sup> and 15 kg DM ha<sup>-1</sup> respectively. Considerable yield variations can occur in hill country pastures. Lambert et al. (1983) found that annual pasture accumulation ranged between 7042 kg DM ha<sup>-1</sup> and 10450 kg DM ha<sup>-1</sup> in a low fertility treatment group from 1977 to 1980. Over this period an annual maintenance P rate of 12 kg ha<sup>-1</sup> was applied to each site.

Unlike annual production, aircraft charge out rate remains relatively inelastic in terms of its effect on annual cash surplus. If operators charged \$1440 hr<sup>-1</sup> for FVRAT, this would result in an annual cash surplus of \$1,037,861, some \$65,917 more than the \$971,944 achieved using conventional methods (Scenario A). The worst case for farmers would be a poor growing season, combined with higher fertiliser application costs of \$1440 hr<sup>-1</sup> reducing cash surplus to \$751,948 yr<sup>-1</sup>. Even a 20% change in charge-out rate is unlikely to greatly affect the farm's cash position in VRAT situations. Although it seems improbable that aircraft charge-out rates

will fall below their current levels, if competition between operators did occur and the price was driven down, it would provide further incentives for the adoption of VRAT on this property.

Although superphosphate application would benefit from VRAT, the method should not be limited to this fertiliser alone. Farmers are increasing their use of N fertiliser to bolster feed supplies, particularly when feed shortages are predicted. Many more farmers are using N fertiliser as they realise the worth of this in boosting pasture through the winter and early spring period. Some farmers favour autumn N dressings rather than in winter or spring, as they find the winter feed more valuable. Some farmers are now applying only urea or sulphate of ammonia to areas of the farm that are dry and sunny (MAF, 2005a). This activity could be managed far more efficiently using VRAT and GIS, and may help drive the adoption of variable rate application of fertiliser in hill country, particularly as nitrogen application tends to provide rapid and clearly visible results.

## **7.5 CONCLUSION**

This paper presents an economic analysis of six production scenarios under differing fertiliser application techniques. Cash farm surplus was calculated for each scenario and clearly revealed the benefits of using variable rate application technology. FVRAT was found to be the most efficient and highest returning application method per hectare. This had the potential to increase annual cash surplus to \$487 ha<sup>-1</sup> some 26% higher than what the farm is currently modelled to produce using blanket application of 24 kg P ha<sup>-1</sup>.

Even though additional costs and increased charge-out rates are likely to occur under VRAT, the analysis indicates that significant financial incentives are available to the farmer. Understandably, farmers will be unlikely to adopt new application techniques until the perceived benefits are identified and the information spread to the wider farming community. This paper addresses this issue and seeks to quantify the economic implication of changing to a variable rate application system. In all cases, VRAT was found to increase farm cash surplus on the Limestone Downs property, and could provide farmers with a value-added service.

There is little evidence to suggest that farmers will adopt VRAT, instead they will likely

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select the aerial topdressing contractor that can deliver a blanket application of fertiliser at the lowest price. The sensitivity analysis revealed that even with a 20% increase in VRAT charge out rate, the farm's annual cash position varies by only 0.4%, therefore the cost of implementing such a system is not prohibitive and would allow aircraft operators to add value to their services, which up until now has been dominated by very slim margins.

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## CHAPTER 8 - SUMMARY

### *8.1 OVERVIEW*

Potentially, economic gains and environmental cost reductions may arise from being able to place aerially topdressed fertiliser where its dry matter production efficiency is high. One problem, limiting this achievement is that the field performance of an agricultural aircraft, in terms of delivering the correct quantity of fertiliser in the correct location, has in the past, been practically immeasurable and therefore, the benefits to be gained from improved placement have not been assessable. Although transverse distribution tests are commonly extrapolated to the flight path to describe a likely application pattern, it is clear from research conducted in this thesis that this method is unlikely to represent what actually occurs in the field because variability in operating conditions alters the fertiliser application footprint. Although the transverse test provides some important information, it is limited to the particular operating conditions, environmental conditions and fertiliser characteristics, on the day of the test. Thus, care should be taken when extrapolating transverse test results to represent the typical field performance of an aircraft.

The deposition footprint model allowed the relative field performance of an agricultural aircraft to be simulated. This enabled assessment of the value of improving the application performance in incremental steps, from manual to automated control. With the fundamentals of automated control clearly defined, and benefits of the technology well understood, the technical feasibility and the agronomic and economic implications of implementing VRAT on pastoral hill country farms can be assessed. This work was the first step towards a quantitative description and assessment of the value of improved application techniques through automated flow control of fertiliser applied by air. Once an automated hopper door control system is developed, it is a small step to programme a GNSS – GIS based VRAT system. In the latter case, prescription maps are required which contain rate information specific to each management area within the application zone. Several GIS data layers are required to generate

prescription map information, this data can also be used as input for a spatial explicit pasture production model.

Although the technology for VRAT is now available, what has been lacking is a suitable decision support system for developing spatially variable prescription application maps. Innovative modelling techniques such as GIS, GNSS, and improved computing power, have enabled researchers to measure, quantify and increase awareness of spatial variability in pasture production. Unfortunately, many of the commonly used decision support systems available to the farming community do not take account of the inherent in-field variability. Thus, the benefits of tailoring inputs to meet demand and break-feeding stock have been largely ignored. This has made the economic implications of VRAT difficult to quantify, therefore it has often been overlooked. The philosophy of this thesis was to take the first steps in presenting a case for VRAT adoption in New Zealand hill country farming. This work firstly examines the performance of a topdressing aircraft using manual and automated flow control. Secondly, it takes the technological and engineering leap and assuming control of fertiliser deposition is achieved, it examines how the economics and agronomics of the farming systems are influenced by delivering targeted nutrient application on variable pasture production land which is a feature of New Zealand hill country.

## **8.2 SUMMARY**

The work presented within Chapters 2 – 4 provides methods for modelling single particle trajectories, transverse distribution patterns and field-application performance. The single particle ballistics model described in Chapter 2 was used to predict the landing position of granular fertiliser particles ejected from fixed wing agricultural aircraft. The model is a single particle model that predicts particle flight trajectory from the particle force balance based on the aircraft groundspeed, axial and tangential propeller wash, wind characteristics and particle properties including sphericity. As an input, the model relies on known exit velocities from the spreader device mounted underneath the aircraft. When simplified, the mathematics for a

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particle falling vertically in quiescent air was the same as that developed by Grift et al. (1997). Without the propeller wash model, comparison was made to the lateral prediction of Bansal et al. (1998b) who used a simpler model, for which similar results and similar trends were obtained. Lastly, with propeller wash model functionality, model predictions were compared to predictions from AGDISP 8.15. Again, results and trends were similar. The model takes the first step in creating a system to evaluate the performance of agricultural aircraft and to test if they are capable of delivering variable rate application technology.

A transverse distribution model proposed in Chapter 3 built on the single particle model. Two parameters were identified as requiring further investigation; these were the flow profile of material leaving the hopper gatebox, and the sphericity of superphosphate. Techniques for measuring these parameters were described and experiments were conducted in order to define the values for input to the model. Comparison of the modelled distribution pattern from a Gippsland Aeronautics 200C was made with a measured pattern from that aircraft, good agreement was found. This model could provide the pilot with real-time feedback about the distribution pattern whilst in flight. In addition, the transverse distribution model could be used as a tool for optimising the design of spreaders or optimum particle characteristics for a given spreader. It has the ability to predict the distribution profile of any particle size distribution from each, or all, of the spreader ducts. Furthermore, the optimum particle size distribution for a fertiliser product could be obtained by varying the particle diameter and size fraction and examining the resulting distribution pattern. Operators could sieve a representative sample of a fertiliser heap and determine the likely distribution pattern of the material given the likely environmental conditions expected on the day of field application. They may also request specific particle characteristics and size fractions from the fertiliser manufacturers in an attempt to optimise spreading with their particular aircraft/spreader configuration.

Both the single particle and transverse distribution models culminated in the development of a deposition footprint model that was capable of predicting field scale application variation within a 25 ha trial site. In Chapter 4, a field scale application model for a Gippsland

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Aeronautics 200C aircraft spreading superphosphate fertiliser was developed. The model was embedded inside ArcMap 9.0 (ESRI, 2006) while data from a 25 ha field trial was used to model performance of an agricultural aircraft operating on a hill-country farming system and for validation of the model.

Such modelling techniques can also be used to aid the design of enhanced flow control mechanisms, aircraft hopper and spreader configurations. The model predicted the deposition of fertiliser at the collector locations and good agreement between the predicted and actual results was achieved ( $R^2 = 0.88$ ). Economic benefits were identified through using an automated hopper door control system. In the automated system, 10% less fertiliser was applied outside of the application zone compared to the manually operated system, this equated to net benefit of NZD \$2800 when calculated on a 1500 ha (effective) hill country farming system.

The work presented within Chapters 5 – 7 describes the methods used to model annual pasture production and assess the economic impact of VRAT on the Limestone Downs property. Chapter 5 identified some of the important governing factors that previous researchers have found when determining annual pasture production, as well as a brief review of the modelling techniques used. The spatially explicit decision tree modelling technique was used to predict the annual pasture production over the entire Limestone Downs property. The resulting production of 19,936 t DM yr<sup>-1</sup> from the 2518 ha of effective grazing land was compared with an animal intake model which revealed a difference of only 67 t DM yr<sup>-1</sup>. Embedding a pasture production model within a geographical information system is useful as it helps to identify strategic opportunities for either increased production or greater efficiency of inputs, particularly if within field variation is scrutinised. Given the difficulty in measuring annual pasture production, and the close agreement between the techniques, extension work was justified.

In Chapter 6, a series of six different fertiliser application scenarios (A-F) were developed for input into the decision tree model for comparison to a base line scenario using conventional aerial application techniques. Throughout this study, VRAT outperformed the fixed rate applications in terms of pasture production and fertiliser utilisation. Scenario C, which used full

variable rate application and a model optimised prescription map, produced the highest annual pasture yield approximately 4850 t DM more than what the property is currently modelled to produce. Variable rate techniques were predicted to increase annual production and the spatial variability of that production. If farm management chose to maintain fertiliser application at current levels, Scenario B, which uses simple VRAT, may be a logical alternative to full VRAT.

Although Chapter 6 highlighted the agronomic impact of different fertiliser application scenarios on Limestone Downs, the economic impact of these techniques needed to be determined, this was the focus of Chapter 7. Here, an economic analysis of the six production scenarios covered in Chapter 6 was undertaken. Farm cash surplus was calculated for each scenario and clearly revealed the benefits of using variable rate application technology. FVRAT was found to be the most efficient and profitable application method per hectare. Even though additional costs and increased charge-out rates were likely to occur under VRAT, the analysis indicates that significant financial incentives were available to the farmer. A sensitivity analysis revealed that even with a 20% increase in VRAT charge out rate, the farm's annual cash position varied by only 0.4%, suggesting the cost of implementing such a system is not prohibitive and would allow aircraft operators to add value to their services, which up until now has been dominated by very slim margins.

It could also be argued that a further 10% improvement in performance, as a result of reducing off-target application between the manual and automated flow systems (Chapter 4), could have been included in the economic analysis presented in Chapter 7. However, the aim of that exercise was to show how varying nutrient application, depending on specific growth related factors, could increase cash farm surplus. The assumption was made early in the conceptual stages of Chapters 5 - 7 that fertiliser could be spread at the correct rate and location, therefore making the 10% improvement in spreading performance between the systems irrelevant in that work. Perhaps the best way to highlight the value of improving the performance of a topdressing aircraft is to apply the cost per hectare (Table 4.2) to the annual work load of a typical topdressing aircraft. A simple cost/benefit analysis revealed the

difference between a manual and automated system was NZD \$111,700 yr<sup>-1</sup> for a single topdressing aircraft.

### ***8.3 CONCLUDING REMARKS AND FUTURE STUDY DIRECTIONS***

This Thesis deals with two important emerging technologies within the field of precision agriculture and, more specifically, variable rate application technology. The first of these, modelling the performance of agricultural aircraft, was required as a precursor to the studies modelling the spatial pasture production on a hill-country sheep and beef farming system. The main objectives of this research were to contribute to the body of knowledge in this field, and to introduce new ideas that overcame limitations seen in current practice. In this Thesis, these objectives have been achieved through the consolidation of previous knowledge and the development of techniques for determining the performance benefits of enhanced control systems and their impact on the agronomic and economics outlook of the farming system.

The work carried out within this Thesis on pasture production modelling has been of interest to scientific and industrial communities and has been published in a respected peer reviewed journal. It links the pasture production model to spatially variable information, such as topography, climatic data and fertiliser history within the GIS allowing quantification of the annual pasture production on hill-country farms. The lateral distribution and field deposition modelling techniques break new ground in this field; these are also under review in an international journal. Furthermore, while it does not introduce new content, the building of a particle ballistic model, in accordance with the current state of the art, serves to consolidate know-how in the public domain. The lateral distribution, and field deposition models, will prove to be a useful tool for the aerial topdressing industry. The major advancement from the distribution modelling work is that it links the fertiliser applied from the aircraft to a spatially variable GIS layer in a format suitable for input into the decision tree model described above. It allows the impact of technology to be assessed in terms of the location of fertiliser applied and how the spatial variability impacts pasture production. This is the first time this link has been

achieved.

Conclusions of the work are discussed in detail at the end of each chapter and the key findings summarised above, however, as a guide to future work, several recommendations are presented and summarised in the following paragraphs.

This ballistic modelling work was carried out for one aircraft / fertiliser combination; further validation of the ballistics models is recommended. Once fully validated, it could be extended to include the effect of fertiliser products with differing physical characteristics and how this impacts the lateral distribution pattern and deposition footprint. This would provide a guide to buffering around water bodies and swath offsets which could be included within the codes of practice for aerially applied fertiliser products. This approach could theoretically be adopted to investigate the likely spreading characteristics of the entire national aircraft fleet; however this would obviously require significant work. If achieved, best practice application techniques for each aircraft / spreader, in a variety of wind conditions, could again be included in the code of practice to guide operators of their likely deposition footprint.

An investigation of the wind velocity profiles at intervals between the aircraft flying height and ground level on hill country would be useful. If a wind measurement device fitted to the aircraft could provide a representative measure of the wind speed and direction, and the spreading performance could be improved or maintained, then as the swath width is increased (due to a cross wind) the control system could increase flow to maintain application rate at the extended swath width.

The models could also be used to assess the economic viability of producing products with optimised spreading characteristics. This would demonstrate the likely effect of eliminating very small particles and dust from fertiliser and how, from both an economic and environmental perspective, this could be compared to the cost of producing these spread-optimised products.

Although physically measuring the particle exit velocity is difficult, it is an area of great practical consequence. It is one of the key initial conditions required in particle ballistic modelling from aircraft and, in the absence of measured values, an approximation must be

made. A device that measures the velocity of the airflow through the duct would have been a useful addition to this work as it would indicate the level of particle acceleration in the air stream and provide a more robust indication of the main tuneable factor, which is the mean particle discharge speed.

Further investigation of the effect of enhanced flow control systems (e.g. automatic VRAT) on the performance of agricultural aircraft should be made. Theoretically, reducing the complexity of the pilot's cognitive tasks should provide more capacity for concentration on navigation, therefore improving performance and application uniformity. To investigate this further, equipment would need to be installed on the aircraft and a large-scale field trial completed to test the ability of the pilot to fly parallel track lines, as well as the equipment's ability to deliver uniform or variable rate flow control. This was out of the scope of this work, but is essential if the true value of the system is to be determined.

Further validation of the pasture production model is required. Extension work is underway on Limestone Downs; however, at the time of writing this thesis the field trials were ongoing. The annual pasture production data will provide a further basis for comparisons of results between measured and predicted data. The decision tree modelling approach proved to be a useful tool in identifying the benefits of spatial variable superphosphate application; however it may be equally as useful for nitrogen fertiliser application on hill country farms.

The environmental impact of VRAT is not discussed in detail in this Thesis. Fertiliser applied to dry, steep slopes, where there is little plant uptake of phosphorus, is at serious risk of runoff into rivers and streams during intense summer rainstorms. The use of VRAT to avoid application of fertiliser to these areas stands to have significant environmental benefits.

Finally, the extent of adoption of the new technology will in the final analysis depend on whether or not the farmer or aerial operator is going to achieve a greater net return. On the other hand, is the new technology going to enable policy makers to make better decisions through proof of placement and mitigation of nutrient loading in sensitive areas. What has been shown is that technology assisted application techniques will be of benefit to the industry, and there is

significant value in managing fertiliser inputs based on the potential outputs of the farmland. The thesis presents a solid case in favour of variable rate application technology; however it is up to the farmers and policy makers to make the final pivotal decisions.

#### **8.4 DISSEMINATION OF KNOWLEDGE**

Every attempt has been made to disseminate knowledge to academic and industry related parties throughout this doctoral study. The media used to convey these messages included: journal publications; conference presentations, papers and posters; technical workshop presentations; popular press articles; and personal communications. Listed below are the contributions made to the body of knowledge from this doctoral study.

##### ***Journal publications***

- Murray RI, Yule IJ, Gillingham AG 2007. Developing variable rate application technology: modelling annual pasture production on hill country. *New Zealand Journal of Agricultural Research* 50: 41-52.
- Murray RI, Yule IJ 2007. Developing variable rate application technology: scenario development and agronomic evaluation. *New Zealand Journal of Agricultural Research* 50: 53-63.
- Murray RI, Yule IJ 2007. Developing variable rate application technology: economic impact for farm owners and topdressing operators. *New Zealand Journal of Agricultural Research* 50: 65-72.
- Jones, J. R., R. I. Murray, and I. J. Yule 2007. Modelling solid fertilizer deposition from a fixed wing aircraft – I. A ballistics model. *Trans. ASAE (under review)*.
- Murray, R. I., Yule, I. J., Jones, J. R. 2007. Modelling solid fertilizer deposition from a fixed wing aircraft - II. Predicting lateral distribution. *Trans. ASAE (under review)*.
- Murray, R. I., and I. J. Yule 2007. Modelling solid fertilizer deposition from a fixed wing aircraft – III. Field scale predictions within a GIS environment. *Trans. ASAE (under review)*.

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***Conference presentations and proceedings***

- Murray, R.I., Yule, I.J. (2007). Modelling Particle Ballistics from a Fixed-Wing Aircraft – poster presentation. 20<sup>th</sup> Annual FLRC Workshop (Designing sustainable farms: Critical aspects of soil and water management). Fertiliser and Lime Research Centre, Massey University, Palmerston North, New Zealand.
- Yule, I.J., Zhang, B, Murray, R.I., Tillman, R. (2007). GIS and variable rate technologies for optimised pasture management in hill country. 20<sup>th</sup> Annual FLRC Workshop (Designing sustainable farms: Critical aspects of soil and water management). Fertiliser and Lime Research Centre, Massey University, Palmerston North, New Zealand.
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- Yule, I. J., Murray, R.I, Lawrence, H.G. (2006). Variables in paddock management. Landwise Conference, keeping our soils productive. May 16<sup>th</sup> – 17<sup>th</sup> 2006, Havelock North, New Zealand.
- Murray, R.I., Yule, I.J. (2006) Measuring the application variability of an agricultural aircraft: a case study on Limestone Downs. In: Currie, L. D, J. A Hanly ed. Proceedings 19<sup>th</sup> Annual FLRC Workshop (Implementing Sustainable Nutrient Management Strategies in Agriculture). Occasional Report No. 19. Fertiliser and Lime Research Centre, Massey University, Palmerston North, New Zealand.
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- Yule, I.J., Murray, R.I., Irwin, M., Hedley, M., Jones, J. (2005) Variable-Rate Application Technology for Aerial Topdressing Aircraft. ASAE Paper Number: 051121. American Society of Agricultural Engineers Annual International Meeting, 17th to 20<sup>th</sup> July 2005, Florida, USA
- Murray, R.I., Yule, I.J., Lawrence, H.G. (2005) Economic and Environmental Opportunities from Utilizing VRAT from Aircraft for Improved Placement of Fertilizer. ASAE Paper number 051075. American Society of Agricultural Engineers Annual International

Meeting, 17th to 20<sup>th</sup> July 2005, Florida, USA

Yule, I.J., Murray, R.I., Lawrence, H.G. (2005) Describing variability in fertiliser application – a statistical approach. In: Currie, L. D, J. A Hanly ed. Proceedings 18<sup>th</sup> Annual FLRC Workshop (Developments in fertiliser application technologies and nutrient management). Occasional Report No. 18. Fertiliser and Lime Research Centre, Massey University, Palmerston North, New Zealand.

Murray, R.I., Yule, I.J. (2005) The specification and use of GPS systems in agricultural aviation. In: Currie, L. D, J. A Hanly ed. Proceedings 18<sup>th</sup> Annual FLRC Workshop (Developments in fertiliser application technologies and nutrient management). Occasional Report No. 18. Fertiliser and Lime Research Centre, Massey University, Palmerston North, New Zealand.

Murray, R.I., Yule I.J. (2005) Variable rate application in New Zealand's agricultural aviation industry. Precision agriculture '05. Poster presented at the 5th European Conference on Precision Agriculture, Uppsala, Sweden.

Yule, I.J, Murray, R.I. GPS and GIS in Aerial Spreadmark - Survey Results (2004). New Zealand Agricultural Aviation Conference 15<sup>th</sup> October 2004. Masterton.

#### ***Technical workshop presentations***

Murray, R. I. (2006). Current and future role of GIS and GPS in aerial application of fertiliser. *Lecture to:* Sustainable nutrient management short course. Fertiliser and Lime Research Centre, Massey University, Palmerston North, New Zealand.

Murray, R. I. (2006). Variable rate application technology in the New Zealand aerial topdressing industry. *Lecture to:* New Zealand fertiliser quality council. Massey University, Palmerston North, New Zealand.

#### ***Farmer field days***

Yule, I.J, Murray, R.I (2007) Improving Aerial Topdressing Technology. Limestone Downs Field Day 20th February. Limestone Downs, Franklin County, Waikato

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