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A CLINICAL STUDY OF THE MATHEMATICAL
INCOMPETENCE OF SOME UNIVERSITY
STUDENTS

A THESIS PRESENTED IN PARTIAL
FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY
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ABSTRACT

The objectives of the study are to contribute to the clear identification and understanding of the factors which lie behind the severe mathematical difficulties experienced by some otherwise able university students.

A careful description of the phenomenon, which might lead to an explanation, is dependant on an understanding of the cognitive processes of the individuals concerned. Consequently a research method and theoretical perspective were chosen which would enable a study of these processes to be made as they were used in solving mathematical problems. The method was based on the Piagetian clinical interview and the theoretical background was essentially that of Skemp's (1979) model of intelligent behaviour. The principal advantages of this model were its structural rather than global features and the close relationship implied between the cognitive and affective determinants of behaviour.

Twenty six subjects were interviewed having a wide range of mathematical abilities and interest. Each subject was presented with the same sequence of tasks taken from the primary-secondary school arithmetic-algebra syllabus. The responses were probed in an unstructured manner.

The analysis of the interview data had two stages. Firstly, in order to provide an overview, a formal coding was undertaken in which the response to each item was classified according to the level of understanding indicated. The resulting data was analysed initially in an entirely descriptive manner and then was subjected to Latent Response Analysis. Following this statistical analysis a closer clinical analysis was made using a multiple-coding approach to build up a mosaic of evidence concerning the conceptual structures used by the subjects.

The principal conclusions of the study relate firstly to the vital importance of the availability of appropriate initial frameworks for the successful handling of mathematics.

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It is argued that the absence of such frameworks, or schema, interpreted in the light of Skemp's theory, explains both the affective reaction of subjects having difficulty with mathematics, and the development of intelligent behaviour within one form of knowledge but not in another.

Secondly, the evidence of the study indicated that it was unlikely that the difficulties which the students were having with mathematics were due either to the abstract nature of the concepts involved or to the logical nature of the subject matter.

Thirdly, the topic of fractions emerged clearly as the most likely source of real difficulty. It is suggested that generations of curriculum designers have seriously underestimated the difficulties associated with learning in this area.

Finally attention is drawn to the necessity for teachers to constantly monitor the development of the cognitive structures of their students and to be sensitive to signals in the affective domain which might indicate developing problems in the cognitive area. In this way the vicious interaction of cognitive and affective reactions to mathematics, which is the most distressing feature of the problem, might be avoided.

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