Received: 25 July 2023 | Accepted: 3 April 2024

DOI: 10.1111/1467-8489.12565

The Australian Journal of Agricultural and Resource Economics AARES

ORIGINAL ARTICLE

Estimating the productivity of US agriculture: The Fisher total factor productivity index for time series data with unknown prices

Thanh Ngo[1,2] | **David Tripe**[3] | **Duc Khuong Nguyen**[4,5]

[1]School of Aviation, Massey University, Palmerston North, New Zealand

[2]VNU University of Economics and Business, Hanoi, Vietnam

[3]School of Economics and Finance, Massey University, Palmerston North, New Zealand

[4]De Vinci Research Center, Pôle Universitaire Léonard de Vinci, La Défense, Île-de-France, France

[5]International School, Vietnam National University, Hanoi, Vietnam

Correspondence
Thanh Ngo, School of Aviation, Massey University, Palmerston North, New Zealand.
Email: t.ngo@massey.ac.nz

Abstract

In this paper, we propose a straightforward way to estimate the Fisher ideal total factor productivity (TFP) index (FI) in cases where price information is unavailable, using 'shadow prices' derived from data envelopment analysis (DEA). A Monte Carlo experiment shows that the shadow price Fisher ideal TFP index (SPFI) can effectively estimate the 'true' FI with relatively small (and stable) errors. The empirical application to the US agriculture sector (1948–2017) further suggests that the SPFI is a (superior) alternative to the traditional Malmquist DEA, especially in dealing with unbalanced panel or time series data when price data are unknown.

KEYWORDS

data envelopment analysis, Fisher index, Monte Carlo simulation, total factor productivity, US agriculture

JEL CLASSIFICATION

C69, D24, O13, O47, O51, Q10

1 | INTRODUCTION

Productivity assessments have long interested researchers in analysing firms' performance, especially for accounting for growth over time. There are two main approaches to measuring growth in total factor productivity (TFP): the axiomatic and the economic approaches (Diewert, 1992; Diewert & Nakamura, 2003). The former approach, which is more traditional in the literature, requires information on the quantity of input or output and their prices in different periods (from time series data) to construct an ideal TFP index that has

a number of axiomatic properties (e.g., the Fisher ideal TFP index [FI] in Diewert, 1992). The biggest problem, however, is that economic data are often available in terms of values (Cross & Färe, 2009, 2015) or quantities (Silva Portela, 2014), that is, the prices are unobserved. By contrast, the economic approach relies on some assumptions (e.g., production or cost functions) being made, which, in turn, allow the calculation of a TFP index using only the quantities of inputs and outputs (e.g., the Malmquist TFP index [MI] in Caves et al., 1982; Färe et al., 1994). It also requires panel data and is thus a detour from the origin of the axiomatic approach as a time series analysis.

We note that the economic approach still (indirectly) uses 'shadow prices' (i.e., the estimated prices of inputs and outputs based on economic theory and the firm's behaviours) in its TFP calculation. It first estimates the common (production or cost or profit) frontier for all firms and then compares the firms from different periods (and frontiers) to measure changes in their TFP over time (Grifell-Tatjé & Lovell, 1997; Färe et al., 1998; Kuosmanen & Sipiläinen, 2009; Kerstens & Woestyne, 2014; Tziogkidis et al., 2018). Therefore, we argue that if the shadow prices of inputs and outputs can be estimated using the economic approach, one can easily use them to construct an axiomatic TFP index such as the FI.[1] The idea of using shadow prices to estimate the FI in the economic approach is not new and was examined by Kuosmanen et al. (2004), and Kuosmanen and Sipiläinen (2009), among others. Although these studies depended on balanced panel data, we show that it is possible to estimate the so-called 'shadow prices' of the Fisher ideal TFP index (SPFI) from time series data on quantities so that we can effectively measure the 'true' FI for both balanced and unbalanced data. In this sense, our paper differs from the works of Cross and Färe (2015) and Silva Portela (2014), who estimate the prices and quantities from value-based data while we estimate the (shadow) prices from quantity data. To the best of our knowledge, this is the first study to do so.

To evaluate the accuracy of our Fisher TFP estimation, a Monte Carlo simulation was used, following the framework of Giraleas et al. (2012). In particular, we generated the quantities of inputs and outputs, as well as the prices, and computed the 'true' FI measures. We then computed the SPFI only from the quantities of inputs and outputs. The comparisons between the SPFI and the FI, especially under different settings of the variance in the quantities of inputs and outputs, showed that the SPFI can effectively estimate the 'true' FI with relatively small (and stable) errors. For our empirical analysis, the US agricultural sector was chosen. This is because agriculture is a popular sector for TFP applications (Färe et al., 1985; Liu et al., 2013; Madau et al., 2017; Mao & Koo, 1997; Ngo et al., 2019; Thiam et al., 2001), in which agriculture in the United States has been extensively examined (Balk et al., 2020; O'Donnell, 2012; Plastina et al., 2021). We found that the SPFI estimates outperformed those calculated from the MI and the TFP estimations provided by the United States Department of Agriculture (USDA). The SPFI, therefore, provides a good approximation of the 'true' FI for both the cases of balanced and unbalanced panel data, and also works for time series data, even when the price data are unknown. This strengthens our argument that the SPFI is a better alternative to the MI, which has been popularly used in the field of assessing productivity and efficiency (Alexakis et al., 2019; Färe et al., 1998; Kao & Hwang, 2014; Li et al., 2022).

In what follows, Section 2 first briefly explains how the nonparametric technique of data envelopment analysis (DEA) is used to estimate the shadow prices of inputs and outputs, and how shadow prices can be directly applied to the process of calculating the SPFI. Section 3 presents a simple Monte Carlo experiment to evaluate the accuracy of our SPFI. An empirical analysis

[1]Diewert (1992) listed 20 properties that an index should have and concluded that the FI is the only function that satisfies all these requirements.

using the SPFI of the US agricultural sector (1948–2017) is presented and discussed in Section 4. Section 5 provides some concluding remarks and the implications of the results.

2 | NONPARAMETRIC SHADOW PRICES FOR THE SPFI

The original idea of DEA is to find the optimal multipliers (weights) that can maximise the ratio of weighted outputs to weighted inputs, that is, the efficiency score of the selected firm within the group of firms in an industry. Theoretically, these weights show how inputs and outputs can be combined in the production function (or cost–profit functions) of the firm, thus explaining the hidden relationships (and hence the name 'shadow prices') among their prices. For a set of n firms, each using m inputs x_i ($i=1, 2, \ldots, m$) to produce s outputs y_r ($r=1, 2, \ldots, s$), a certain j_0-th firm can maximise its constant returns to scale efficiency by solving the following nonlinear fractional programming problem[2]:

$$\begin{aligned} EF_{j_0} &= \max_{u,v} \frac{\sum_r^s u_r y_{rj_0}}{\sum_i^m v_i x_{ij_0}} \\ \text{subject to: } &\frac{\sum_r^m u_r y_{rj}}{\sum_i^k v_i x_{ij}} \le 1, j = 1, \ldots, n \\ & u_r, v_i \ge 0, r = 1, \ldots, s, i = 1, \ldots, m \end{aligned} \tag{1}$$

In this article, our approach is slightly different. We consider K period observations of a certain firm that forms a time series production frontier.[3] Hence, Equation (1) can be used to estimate the shadow prices (and the efficiency) of that firm in different periods.[4] Once these shadow prices have been estimated, they can replace the true prices in the Fisher output quantity index and the Fisher input quantity index, following Diewert (1992), Färe and Grosskopf (1992), and Diewert and Nakamura (2003), among others:

$$F_y^Q\left(p^{0,1}, y^{0,1}\right) = \left(\frac{p^0 y^1}{p^0 y^0}\frac{p^1 y^1}{p^1 y^0}\right)^{1/2} \text{ is the Fisher output quantity index} \tag{2}$$

$$F_x^Q\left(w^{0,1}, x^{0,1}\right) = \left(\frac{w^0 x^1}{w^0 x^0}\frac{w^1 x^1}{w^1 x^0}\right)^{1/2} \text{ is the Fisher input quantity index} \tag{3}$$

where p and w are the real output and input prices in the periods 0 and 1, respectively.

Consequently, the SPFI is derived as follows:

$$\text{SPFI}_{0,1} = \frac{\left(\frac{u^0 y^1}{u^0 y^0}\frac{u^1 y^1}{u^1 y^0}\right)^{1/2}}{\left(\frac{v^0 x^1}{v^0 x^0}\frac{v^1 x^1}{v^1 x^0}\right)^{1/2}} \approx \text{FI}_{0,1} = \frac{F_y^Q\left(p^{0,1}, y^{0,1}\right)}{F_x^Q\left(w^{0,1}, x^{0,1}\right)} \tag{4}$$

[2]The constant return to scale DEA model, which was introduced by Charnes et al. (1978), is used because of its simplification. Once can easily extend the model for the assumption of variable-returns-to-scale following Banker et al. (1984).

[3]This is also called a 'k-specific intertemporal production sets' (Tulkens & vanden Eeckaut, 1995). However, one can still use the proposed method for cross-sectional or panel data.

[4]The super-efficiency approach of Andersen and Petersen (1993) is also used to avoid the problem of non-unique shadow prices.

where u and v are the shadow prices of outputs and inputs estimated from Equation (1) in the periods 0 and 1, respectively.

3 | MONTE CARLO SIMULATION AND THE ACCURACY OF THE SPFI

We evaluated the accuracy of our SPFI measure using a simple Monte Carlo simulation, similar to the experiment of Giraleas et al. (2012).

For the first period, S amounts of input (x_1, x_2, …, x_S) and a single amount of output y of our unique firm were randomly drawn from a uniform distribution in the range (10, 20). Accordingly, the data in the subsequent periods were non-negative no matter what standard deviation (SD) values are chosen. Consequently, S corresponding input prices (w_1, w_2, …, w_S) were also randomly selected from a uniform distribution $U(0, 1)$. Note that this data-generating process allowed the best outcome when generating random values for the inputs and output that had equal probabilities. This process is popularly applied in DEA simulation studies (Kneip et al., 2016; Lee & Cai, 2020; Ngo & Tsui, 2021; Simar & Wilson, 2007). The output price p is then determined according to the zero economic profit theory, that is, by equating total revenues to total costs:

$$w_1x_1 + w_2x_2 + \ \ldots \ + w_Sx_S = py \tag{5}$$

$$p = \frac{\left(w_1x_1 + w_2x_2 + \ \ldots \ + w_Sx_S\right)}{y} \tag{6}$$

For the $(K-1)$ subsequent periods,[5] the quantities of inputs and outputs of that same firm were scaled by $(K-1)$ random normally distributed numbers $N(0, \text{SD})$ with a default value of $\text{SD}=0.1$. We also examined the cases where $\text{SD}=0.25$ and $\text{SD}=0.5$.[6] By increasing SD, we allow the quantity of inputs and outputs to be more volatile over time and thus increase the volatility of p, according to Equation (6). In time series analysis, this also means that the changes in productivity over time will become larger (i.e., greater fluctuation of FI over time). Consequently, the efficiency estimated using Equation (1) will be more volatile between periods, resulting in a more volatile SPFI. Meanwhile, input and output prices were randomly generated as in the first period.

In the end, we produced simulated time series data for a unique firm over K periods that used S amounts of inputs ($x_1, x_2, \ \ldots \ , x_S$), which corresponded to S input prices ($w_1, w_2, \ \ldots \ , w_S$), to produce a single amount of output y, which corresponded to the output price p. Based on that, we can easily compute the 'true' FI using x, w, y and p. We can also estimate the SPFI using the shadow prices obtained from a DEA model using only x and y. We then analyse the accuracy of the SPFI using the mean absolute deviation (MAD), the root mean square error (RMSE) and the accuracy level (AC) as follows.[7]

$$MAD = \frac{1}{B}\left(\frac{1}{K}\sum_{i=1}^{K}\left|SPFI_i - FI_i\right|\right) \tag{7}$$

[5]It is noted that if the length of the time series is too short, for example, only two to three periods, our DEA model will still suffer from the pitfall of an insufficient number of DMUs (Dyson et al., 2001).

[6]These values are chosen following Giraleas et al. (2012). We, however, also examine the case of $SD = 0.5$ since our analysis is bases on time series data in which the volatility over time may be higher than that of cross-sectional data as in Giraleas et al. (2012). Note again that we did not examine the heteroscedastic issue where SD is time-varying due to its complicatedness; we leave this task for future research.

[7]Giraleas et al. (2012) used MAD and the mean square error, a different version of the *RMSE*, for their analysis. The AC was used by Cross and Färe (2015).

TABLE 1 Summary of the accuracy with low volatility (SD = 0.10).

	S=1				*S=3*				*S=5*			
K	**10**	**20**	**30**	**50**	**10**	**20**	**30**	**50**	**10**	**20**	**30**	**50**
MAD	0.000	0.000	0.000	0.000	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
RMSE	0.000	0.000	0.000	0.000	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
AC	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
SP	1.000	1.000	1.000	1.000	0.838	0.843	0.854	0.862	0.786	0.836	0.843	0.847
nSP	300	300	300	300	283	300	300	300	270	300	300	300

Abbreviations: AC, accuracy level; *K*, the number of periods involved in the simulation; MAD, mean absolute deviation; nSP, number of cases with a 10% level of significance in Spearman's test; RMSE, root mean square error; *S*, the number of inputs involved in the simulation; SP, Spearman's coefficient.

$$RMSE = \frac{1}{B}\sqrt{\frac{1}{K}\sum_{i=1}^{K}\left(SPFI_i - FI_i\right)^2} \tag{8}$$

$$AC = \frac{1}{B}\left(\frac{1}{K}\sum_{i=1}^{K}\frac{FI_i}{SPFI_i}\right) \tag{9}$$

where B is the number of simulations.

According to Equations (7)–(9), the SPFI will be more accurate or closer to the 'true' FI when the MAD and RMSE are small and when AC is closer to unity. In addition to these tests for measurement errors, we carried out a nonparametric test for the correlation between the SPFI and the 'true' FI. We also report the average Spearman's ranking correlation coefficients (SP) between the SPFI and FI, as well as the number of cases in which this coefficient was statistically significant at the 10% level of significance (nSP).

In what follows, we report the accuracy of the SPFI in our Monte Carlo experiments where we increased the number of inputs ($S = 1, 3, 5$), the number of periods ($K = 10, 20, 30, 50$) and the data's volatility (SD = 0.1, 0.25, 0.5) across 300 simulations ($B = 300$),[8] in comparison with the 'true' FI. Since most DEA applications involve fewer than five inputs and have a period of 30 years, these settings are justified for our simulation. The MAD, RMSE, AC, SP and nSP between the two measures from different experiments are presented in Tables 1–3, with the volatility increasing as SD = 0.1 to SD = 0.25 and SD = 0.5, respectively.

First, we can see that for the case of one input and one output (i.e., $S = 1$), in all three tables, the MADs and RMSEs are zero, whereas the AC equal unity, indicating that the SPFI is identical to the 'true' FI. We argue that, for the case of a single input and a single output, the DEA requires the least estimation so that the shadow prices (u and v) can capture the relationship between the true prices (p and w); thus, the SPFI is free from estimation errors.[9]

Second, with less volatile data (i.e., $SD = 0.10$), increasing the number of periods (K) and/or the number of inputs (S) had no impact on the accuracy of the SPFI. In fact, it made the correlation between SPFI and the 'true' FI stronger, reflected in higher SP and nSP (the bottom two rows of Table 1). For moderately and highly volatile data, an increase in the

[8]We chose $B = 300$ following Cross and Färe (2015). We also tested for the case that $B = 500$ but no significant improvements were found, so we conclude that 300 simulations are sufficient.

[9]Although the 'shadow prices' estimated by DEA may not be identical to the true prices.

TABLE 2 Summary of the accuracy with moderate volatility (SD = 0.25).

	S = 1				*S* = 3				*S* = 5			
K	**10**	**20**	**30**	**50**	**10**	**20**	**30**	**50**	**10**	**20**	**30**	**50**
MAD	0.000	0.000	0.000	0.000	0.007	0.007	0.008	0.008	0.008	0.008	0.008	0.008
RMSE	0.000	0.000	0.000	0.000	0.009	0.010	0.010	0.010	0.009	0.010	0.010	0.010
AC	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
SP	1.000	1.000	1.000	1.000	0.821	0.855	0.860	0.860	0.797	0.829	0.837	0.838
nSP	300	300	300	300	278	300	300	300	270	300	300	300

Abbreviations: AC, accuracy level; *K*, the number of periods involved in the simulation; MAD, mean absolute deviation; nSP, number of cases with a 10% level of significance in Spearman's test; RMSE, root mean square error; *S*, the number of inputs involved in the simulation; SP, Spearman's coefficient.

TABLE 3 Summary of the accuracy with high volatility (SD = 0.5).

	S = 1				*S* = 3				*S* = 5			
K	**10**	**20**	**30**	**50**	**10**	**20**	**30**	**50**	**10**	**20**	**30**	**50**
MAD	0.000	0.000	0.000	0.000	0.014	0.015	0.016	0.016	0.016	0.015	0.016	0.016
RMSE	0.000	0.000	0.000	0.000	0.018	0.019	0.021	0.021	0.020	0.021	0.021	0.021
AC	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.001	1.000	1.001	1.001
SP	1.000	1.000	1.000	1.000	0.832	0.846	0.852	0.861	0.791	0.824	0.829	0.837
nSP	300	300	300	300	280	300	300	300	267	297	300	300

Abbreviations: AC, accuracy level; *K*, the number of periods involved in the simulation; MAD, mean absolute deviation; nSP, number of cases with a 10% level of significance in Spearman's test; RMSE, root mean square error; *S*, the number of inputs involved in the simulation; SP, Spearman's coefficient.

number of inputs slightly decreased the accuracy of the SPFI, but it was still acceptable as the increases were only 10–20 percentage points (in terms of MAD, RMSE and AC). In contrast, an increase in the number of periods made the SPFI more consistent (in terms of SP and nSP) (Tables 2 and 3). Consequently, we suggest that the SPFI results are robust and consistent, and it is thus a good estimate of the 'true' FI even when data on prices are unobservable.

Third, the highest errors in terms of MAD, RMSE and AC were found for the case of the highest volatility, the highest number of inputs and the highest number of periods (see Table 3 for the case where $S = 5$ and $K = 50$). This is reasonable because more estimation is now involved, and DEA becomes (relatively) less efficient in computing the 'true' relationships between the prices. However, consistent with the previous discussion, since SP and nSP were high enough, we still concluded that the SPFI could effectively estimate the 'true' FI. Figure 1 illustrates the detailed results of our simulations for this case with the highest error. Note that our simulations are multi-dimensional regarding *S*, *K*, SD and the set of measurements including MAD, RMSE, AC, SP and nSP. It is therefore not easy to illustrate the results using a two-dimensional figure. Nevertheless, it can still be observed in Figure 1 that the MAD varies within an acceptable range of 0.007 to 0.032, whereas the AC is close to unity and the SP mostly ranges from moderate (i.e., 0.550) to strong (i.e., 0.970).

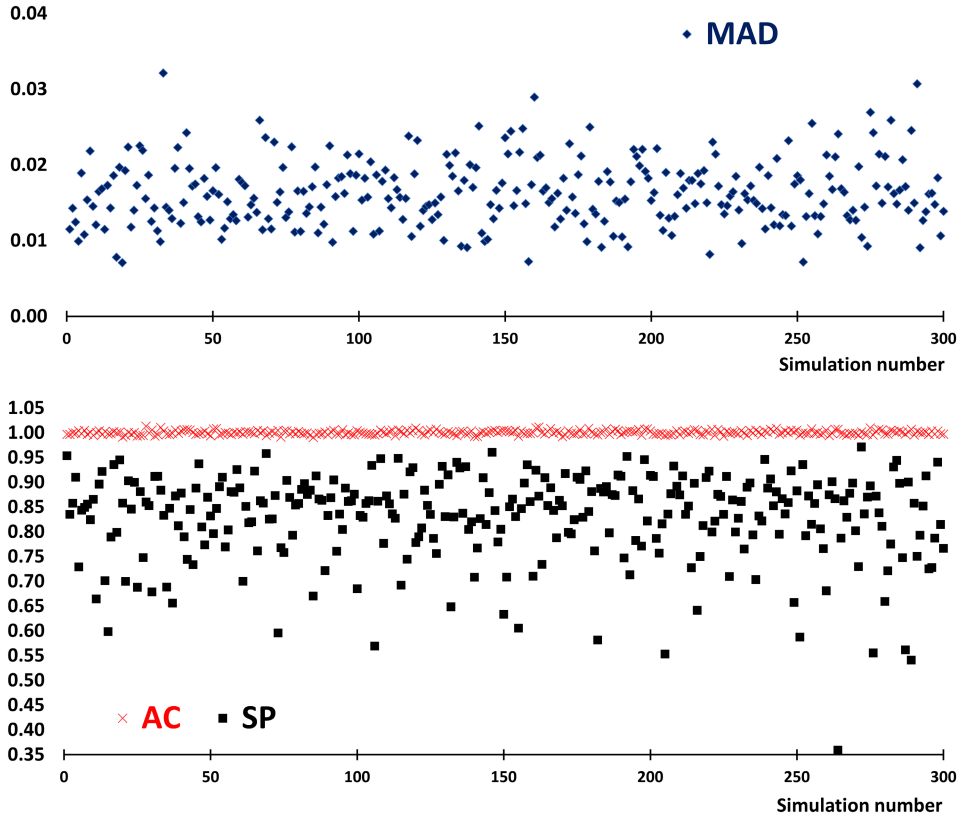

FIGURE 1 Accuracy between SPFI and the 'true' FI ($S=5$, $K=50$, SD$=0.5$).

4 | EMPIRICAL APPLICATION: PRODUCTIVITY OF THE US AGRICULTURAL SECTOR (1948–2017)

Agriculture is a popular sector for TFP applications (Balk et al., 2020; Färe et al., 1985; Liu et al., 2013; Thiam et al., 2001). For instance, Mao and Koo (1997) studied the Malmquist TFP of 29 provinces in mainland China from 1984 to 1993 and found that the increase in TFP in Chinese agricultural production mainly arose from technological innovations. In contrast, a study by Madau et al. (2017) on the European dairy sectors (22 countries, 2004–2012) found dairy farms suffered a decline in TFP. At the global level, Coelli and Rao (2005) found that Asian countries had the highest cumulative growth in agricultural productivity, followed by North America and Europe, whereas Africa and South America remained at the bottom. A short summary of such studies is that they mostly relied on the Malmquist TFP index introduced by Färe et al. (1994), which requires panel data. Our empirical application of the SPFI contributes to this strand of the literature by examining the case of both panel (balanced and unbalanced) and time series data.

To illustrate the SPFI, we, used data from the US agricultural sector. These data have been collected and tabulated by the Economic Research Section of the USDA.[10] These data were also used in previous studies such as O'Donnell (2012), Balk et al. (2020) and Plastina et al. (2021). It is noted that US agricultural data at the state level are only available for the 1960–2004

[10]The data can be downloaded from https://www.ers.usda.gov/data-products/agricultural-productivity-in-the-us/.

TABLE 4 TFP of US agriculture at the state level (1960 and 2004).

Panel A. Descriptive statistics

Variable	Mean	Standard deviation	Min	Max
FI	1.395	0.289	0.742	2.027
MI	1.549	0.440	0.522	2.688
SPFI	1.422	0.513	0.469	2.743
USDA	1.687	0.424	0.579	2.578

Panel B. Comparison results

Type of test	SPFI vs. FI	SPFI vs. MI	SPFI vs. USDA	MI vs. FI	MI vs. USDA	USDA vs. FI
t-test (H_0: *equal means*)	0.383	−1.719*	−3.755***	2.828***	−2.427**	8.713***
Mann–Whitney test (H_0: *equal ranks*)	−0.769	−2.032**	−3.246***	−1.795*	−1.957**	−3.818***
Spearman's test (H_0: *no correlation*)	0.312**	0.414***	0.425***	0.520***	0.628***	0.834***
Kendall's test (H_0: *no correlation*)	0.202**	0.303***	0.282***	0.353***	0.433***	0.644***

Note: *, **, and *** denote the 10%, 5%, and 1% levels of significance, respectively.

period, whereas the national-level data cover the 1948–2017 period. Hence, if we would like to conveniently follow the traditional (Malmquist) TFP approach using the state-level (balanced) panel data, we may need to use a narrower data set from the 1960–2004 period, that is, 45 yearly observations. In contrast, we can use the new SPFI for the whole period of 1948–2017 using the national-level time series data, that is, 70 yearly observations. Such real-life data are important for our analysis to illustrate that the proposed SPFI can work for a wide range of data.

We first compare the results of the SPFI, the Malmquist TFP and the TFP estimations provided by the USDA. For space-saving purposes, following Balk et al. (2020), we have not reported the estimation results for every year of the whole period of 1960–2004; instead, we focussed on the first and last available years of the data set to estimate the growth in TFP between 1960 and 2004. Table 4 presents the comparison among these indices, where our FI (the 'true' Fisher TFP index calculated using data on both quantities and prices) and the Malmquist TFP index (MI) estimated using only data on quantities are identical to what has been reported as the FI and the geometric mean output-orientated MI in Balk et al. (2020), respectively. Similar to Balk et al. (2020), we found that the estimates of TFP provided by the USDA had the highest values as a result of their estimation procedures (Appendix S1).

However, following the argument that the FI calculated from data on quantities and prices is the most accurate measure of TFP (Diewert, 1992), comparing the estimated TFP indices with the FI values is more important. According to Panel A of Table 4, our SPFI estimate was the closest to the 'true' FI measure, compared with the MI and USDA measures. Panel B of Table 4 also suggests that there is no significant difference between the means (i.e., in the *t*-test) and ranks (i.e., in the Mann–Whitney test) of FI and SPFI, whereas differences between FI and the two indices (MI and USDA) existed.[11]

[11]As suggested by an anonymous referee, we further employed the Diebold and Mariano (1995) forecast comparison test to compare the SPFI, USDA and MI against the 'true' FI, under the null hypothesis that the forecast accuracy of those measurements is equal. Our test results suggest that the SPFI has a similar forecasting power to the USDA but better than the MI.

TABLE 5 TFP of US agriculture at the national level (1948–2017).

Panel A. Descriptive statistics

Variable	Mean	Standard deviation	Min	Max
FI	1.016	0.050	0.862	1.175
SPFI	0.998	0.047	0.796	1.107
USDA	0.666	0.221	0.359	1.079

Panel B. Comparison results

Type of test	SPFI vs. FI	SPFI vs. USDA	USDA vs. FI
t-test (H_0: *equal means*)	−3.397***	12.055***	−12.565***
Mann–Whitney test (H_0: *equal ranks*)	−1.880*	−8.500***	−8.844***
Spearman's test (H_0: *no correlation*)	0.628***	0.067	−0.085
Kendall's test (H_0: *no correlation*)	0.459***	0.046	−0.052

Note: *, **, and *** denote the 10%, 5%, and 1% levels of significance, respectively.

In Table 5, we report the TFP estimates using the national-level time series data of US agriculture (1948–2017). Since the MI could not be estimated in this case, we only present the comparison among FI (the 'true' Fisher TFP index calculated using data on both quantities and prices), SPFI and the USDA's index (Appendix S2). We also note that the calculation of USDA is different from the other two: the USDA only accounts for the number of outputs and inputs but not their prices. For instance, according to Table 1a of the USDA,[12] the TFP for the year 1948 (i.e., $TFP_{1948} = 0.3812$) was calculated as the ratio between the total amount of output (i.e., US$146,398 million) and the total input (i.e., US$384,058 million) in this year. We thus believe that the FI estimates calculated from both quantities and prices better reflects the growth in TFP of US agriculture. Consequently, the comparison between our SPFI and FI in Panel B of Table 5 reveals that SPFI gives a good estimation of FI when price data are not available, as the two are consistent with each other (i.e., both the Spearman's and Kendall's tests were strongly significant at the 1% level, whereas the Mann–Whitney test was weakly significant at the 10% level).[13]

5 | CONCLUSION

In this article, we proposed a straightforward way to estimate the 'true' FI with the SPFI by using time series data. This is a new application of the estimated shadow prices from the nonparametric approach of DEA in cases where information on prices is not available, especially when data exist in the form of time series.

To measure the accuracy of the SPFI, we used a simple Monte Carlo experiment where the number of inputs, the length of the time series and the volatility of prices and quantities varied. We found that the SPFI effectively estimated the 'true' FI with relatively small (and stable) errors. The empirical part of our study used data from the US agricultural sector (1948–2017). For this case, following the traditional Malmquist TFP approach meant that we would end up with a shorter period of analysis (i.e., only 45 years for US agriculture at the state level from 1960 to 2004), compared with the FI and SPFI approaches using time series data (i.e., all

[12] https://www.ers.usda.gov/webdocs/DataFiles/47679/table01a.csv

[13] The Diebold and Mariano (1995) test to compare the SPFI and USDA against the 'true' FI suggests that the SPFI is a better forecast.

70 years of the 1948–2017 period). More importantly, empirical comparisons of the TFP measures (including the Malmquist TFP, the SPFI, and the TFP estimates provided by the USDA), especially against the 'true' FI, suggested that SPFI is a (superior) alternative to the traditional Malmquist DEA, especially in dealing with unbalanced panel or time series data and when data on prices are unknown.

Future research can extend our approach by examining the SPFI in other fields such as banking and finance, health care and transportation where the structure of prices is complicated and often unknown (Alexakis et al., 2019; Boubaker et al., 2024; Liu et al., 2013; Oh et al., 2017). It would be interesting to include more advanced DEA models, such as the variable returns to scale assumption (Banker et al., 1984), the slack-based measures (Tone, 2001), the common set of weights (Hammami et al., 2022), inverse DEA (Boubaker et al., 2022) or DEA-machine learning (Boubaker et al., 2023). One could also decompose the SPFI to provide better insights into different aspects of changes in productivity over time, such as technical changes, technological changes or changes in scale efficiency (Färe et al., 1994; Ngo & Nguyen, 2012).

DATA AVAILABILITY STATEMENT

The data that support the empirical findings of this study are available from the USDA at https://www.ers.usda.gov/.

ACKNOWLEDGEMENT

Open access publishing facilitated by Massey University, as part of the Wiley - Massey University agreement via the Council of Australian University Librarians.

ORCID

Thanh Ngo https://orcid.org/0000-0002-6090-8067

REFERENCES

Alexakis, C., Izzeldin, M., Johnes, J. & Pappas, V. (2019) Performance and productivity in Islamic and conventional banks: evidence from the global financial crisis. *Economic Modelling*, 79, 1–14. Available from: https://doi.org/10.1016/j.econmod.2018.09.030

Andersen, P. & Petersen, N.C. (1993) A procedure for ranking efficient units in data envelopment analysis. *Management Science*, 39(10), 1261–1264. Available from: https://doi.org/10.2307/2632964

Balk, B.M., Barbero, J. & Zofío, J.L. (2020) A toolbox for calculating and decomposing total factor productivity indices. *Computers & Operations Research*, 115, 104853. Available from: https://doi.org/10.1016/j.cor.2019.104853

Banker, R.D., Charnes, A. & Cooper, W.W. (1984) Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science*, 30(9), 1078–1092.

Boubaker, S., Le, T.D.Q., Manita, R. & Ngo, T. (2024) Balancing bank profits and nonperforming loans: a multiple objective programming approach. *Annals of Operations Research*. Available from: https://doi.org/10.1007/s10479-024-05831-x

Boubaker, S., Le, T.D.Q. & Ngo, T. (2022) Managing bank performance under COVID-19: a novel inverse DEA efficiency approach. *International Transactions in Operational Research*, 30, 2436–2452. Available from: https://doi.org/10.1111/itor.13132

Boubaker, S., Le, T.D.Q., Ngo, T. & Manita, R. (2023) Predicting the performance of MSMEs: a hybrid DEA-machine learning approach. *Annals of Operations Research*, (online first), 1–23. Available from: https://doi.org/10.1007/s10479-023-05230-8

Caves, D.W., Christensen, L.R. & Diewert, W.E. (1982) The economic theory of index numbers and the measurement of input, output and productivity. *Econometrica*, 50, 1393–1414.

Charnes, A., Cooper, W.W. & Rhodes, E. (1978) Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2(6), 429–444.

Coelli, T.J. & Rao, D.S.P. (2005) Total factor productivity growth in agriculture: a Malmquist index analysis of 93 countries, 1980–2000. *Agricultural Economics*, 32(s1), 115–134. Available from: https://doi.org/10.1111/j.0169-5150.2004.00018.x

Cross, R.M. & Färe, R. (2009) Value data and the Bennet price and quantity indicators. *Economics Letters*, 102(1), 19–21.

Cross, R.M. & Färe, R. (2015) Value data and the Fisher index. *Theoretical Economics Letters*, 5(2), 262–267.
Diebold, F.X. & Mariano, R.S. (1995) Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 13(3), 253–263. Available from: https://doi.org/10.1080/07350015.1995.10524599
Diewert, W.E. (1992) Fisher ideal output, input, and productivity indexes revisited. *Journal of Productivity Analysis*, 3(3), 211–248.
Diewert, W.E. & Nakamura, A.O. (2003) Index number concepts, measures and decompositions of productivity growth. *Journal of Productivity Analysis*, 19(2), 127–159. Available from: https://doi.org/10.1023/a:1022897231521
Dyson, R.G., Allen, R., Camanho, A.S., Podinovski, V.V., Sarrico, C.S. & Shale, E.A. (2001) Pitfalls and protocols in DEA. *European Journal of Operational Research*, 132(2), 245–259.
Färe, R., Grabowski, R. & Grosskopf, S. (1985) Technical efficiency of Philippine agriculture. *Applied Economics*, 17(2), 205–214.
Färe, R. & Grosskopf, S. (1992) Malmquist productivity indexes and Fisher ideal indexes. *Economic Journal*, 102(410), 158–160.
Färe, R., Grosskopf, S., Norris, M. & Zhang, Z. (1994) Productivity growth, technical progress, and efficiency change in industrialized countries. *American Economic Review*, 84(1), 66–83.
Färe, R., Grosskopf, S. & Roos, P. (1998) Malmquist productivity indexes: a survey of theory and practice. In: Färe, R., Grosskopf, S. & Russell, R.R. (Eds.) *Index numbers: essays in honour of Sten Malmquist*. Netherlands: Springer, pp. 127–190. Available from: https://doi.org/10.1007/978-94-011-4858-0_4
Giraleas, D., Emrouznejad, A. & Thanassoulis, E. (2012) Productivity change using growth accounting and frontier-based approaches – evidence from a Monte Carlo analysis. *European Journal of Operational Research*, 222(3), 673–683.
Grifell-Tatjé, E. & Lovell, C.A.K. (1997) A DEA-based analysis of productivity change and intertemporal managerial performance. *Annals of Operations Research*, 73, 177–189.
Hammami, H., Ngo, T., Tripe, D. & Vo, D.-T. (2022) Ranking with a Euclidean common set of weights in data envelopment analysis: with application to the Eurozone banking sector. *Annals of Operations Research*, 311, 675–694. Available from: https://doi.org/10.1007/s10479-020-03759-6
Kao, C. & Hwang, S.-N. (2014) Multi-period efficiency and Malmquist productivity index in two-stage production systems. *European Journal of Operational Research*, 232(3), 512–521.
Kerstens, K. & Woestyne, I.V. (2014) Comparing Malmquist and Hicks–Moorsteen productivity indices: exploring the impact of unbalanced vs. balanced panel data. *European Journal of Operational Research*, 233, 749–758.
Kneip, A., Simar, L. & Wilson, P.W. (2016) Testing hypotheses in nonparametric models of production. *Journal of Business & Economic Statistics*, 34(3), 435–456. Available from: https://doi.org/10.1080/07350015.2015.1049747
Kuosmanen, T., Post, T. & Sipiläinen, T. (2004) Shadow price approach to total factor productivity measurement: with an application to Finnish grass-silage production. *Journal of Productivity Analysis*, 22(1–2), 95–121.
Kuosmanen, T. & Sipiläinen, T. (2009) Exact decomposition of the Fisher ideal total factor productivity index. *Journal of Productivity Analysis*, 31(3), 137–150.
Lee, C.-Y. & Cai, J.-Y. (2020) LASSO variable selection in data envelopment analysis with small datasets. *Omega*, 91, 102019. Available from: https://doi.org/10.1016/j.omega.2018.12.008
Li, Z., Feng, C. & Tang, Y. (2022) Bank efficiency and failure prediction: a nonparametric and dynamic model based on data envelopment analysis. *Annals of Operations Research*, 315(1), 279–315. Available from: https://doi.org/10.1007/s10479-022-04597-4
Liu, J.S., Lu, L.Y.Y., Lu, W.-M. & Lin, B.J.Y. (2013) A survey of DEA applications. *OMEGA*, 41(5), 893–902.
Madau, F.A., Furesi, R. & Pulina, P. (2017) Technical efficiency and total factor productivity changes in European dairy farm sectors. *Agricultural and Food Economics*, 5(1), 17. Available from: https://doi.org/10.1186/s40100-017-0085-x
Mao, W. & Koo, W.W. (1997) Productivity growth, technological progress, and efficiency change in chinese agriculture after rural economic reforms: a DEA approach. *China Economic Review*, 8(2), 157–174.
Ngo, T. & Nguyen, L.T.P. (2012) Total factor productivity of Thai banks in 2007-2010: an application of DEA and Malmquist index. *Journal of Applied Finance and Banking*, 2(5), 27–42.
Ngo, T. & Tsui, K.W.H. (2021) Estimating the confidence intervals for DEA efficiency scores of Asia-Pacific airlines. *Operational Research*, 22, 3411–3434. Available from: https://doi.org/10.1007/s12351-021-00667-w
Ngo, T., Vu, H.V., Ho, H., Dao, T.T.T. & Nguyen, H.T.H. (2019) Performance of fish farms in Vietnam–does financial access help improve their cost efficiency? *International Journal of Financial Studies*, 7(3), 45.
O'Donnell, C.J. (2012) Nonparametric estimates of the components of productivity and profitability change in U.S. Agriculture. *American Journal of Agricultural Economics*, 94(4), 873–890. Available from: https://doi.org/10.1093/ajae/aas023
Oh, Y., Oh, D.-H. & Lee, J.-D. (2017) A sequential global Malmquist productivity index: productivity growth index for unbalanced panel data considering the progressive nature of technology. *Empirical Economics*, 52(4), 1651–1674. Available from: https://doi.org/10.1007/s00181-016-1104-6

Plastina, A., Lence, S.H. & Ortiz-Bobea, A. (2021) How weather affects the decomposition of total factor productivity in U.S. agriculture. *Agricultural Economics*, 52(2), 215–234. Available from: https://doi.org/10.1111/agec.12615
Silva Portela, M.C.A. (2014) Value and quantity data in economic and technical efficiency measurement. *Economics Letters*, 124(1), 108–112.
Simar, L. & Wilson, P.W. (2007) Estimation and inference in two-stage, semi-parametric models of production processes. *Journal of Econometrics*, 136, 31–64.
Thiam, A., Bravo-Ureta, B.E. & Rivas, T.E. (2001) Technical efficiency in developing country agriculture: a meta-analysis. *Agricultural Economics*, 25(2–3), 235–243. Available from: https://doi.org/10.1111/j.1574-0862.2001.tb00204.x
Tone, K. (2001) A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research*, 130(3), 498–509.
Tulkens, H. & vanden Eeckaut, P. (1995) Non-parametric efficiency, progress and regress measures for panel data: methodological and aspects. *European Journal of Operational Research*, 80, 474–499.
Tziogkidis, P., Matthews, K. & Philippas, D. (2018) The effects of sector reforms on the productivity of Greek banks: a step-by-step analysis of the pre-Euro era. *Annals of Operations Research*, 266(1), 531–549. Available from: https://doi.org/10.1007/s10479-016-2381-3

SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

How to cite this article: Ngo, T., Tripe, D. & Nguyen, D.K. (2024) Estimating the productivity of US agriculture: The Fisher total factor productivity index for time series data with unknown prices. *Australian Journal of Agricultural and Resource Economics*, 00, 1–12. Available from: https://doi.org/10.1111/1467-8489.12565