Learning to participate in a mathematical community of inquiry: Insights from cogenerative dialogues

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Abstract

Mathematics teaching and learning in New Zealand is currently in a state of change, with classroom learning environments transitioning from more traditional classrooms focused on individual performance outcomes towards more collaborative communities of mathematical inquiry. This study is set in the context of ongoing school-wide professional development initiative focused on developing mathematical inquiry communities (DMIC), with the overall aim of the study being to develop an understanding of how students experience learning mathematics as they learn to participate in a developing mathematical inquiry community.

The study uses cogenerative dialogues (cogens) to generate student voice to explore students' experiences and perceptions within this new context for mathematical learning. Providing a forum for students to discuss their experiences and thoughts about their learning environment, the cogenerative dialogues (cogens) served to cogenerate understandings which would facilitate student and teacher learning within the emergent community of mathematical inquiry, allowing students to collaborate with their teacher and co-learners to co-construct their learning environment.

The study examines students' perceptions of the collaborative learning environment and explores how students negotiate this change as they become part of a mathematical inquiry community. Findings revealed that students' social interactions and experiences within their groups have a significant influence on learning and that negotiating the collaborative learning environment is of primary concern for these students. The data offer a glimpse of how these students experience and work within groups, how they go about their learning in groups, and how they feel groups should be organised. Findings also considered to what extent students develop student agency through collaborative work and being able to share student voice.

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Chapter 1 Introduction

1.1 Background to the Study

Mathematics teaching and learning in New Zealand is currently in a state of change. In particular, there is a shift in classroom learning environments from more traditional classrooms focused on individual performance outcomes towards more collaborative communities of mathematical inquiry. These changes are driven by the need for both 21st-century skills and for the development of more equitable classrooms that better support all students' well-being and mathematical learning (OECD, 2016).

Current research advocates that student learning outcomes can be improved by modifying the learning environment and adopting equitable pedagogical practices (Hunter, 2008). The New Zealand Curriculum (Ministry of Education, 2007) states that students learn best when they feel accepted, when they enjoy positive relationships with their teachers and fellow students, and when they can be active, visible members of a learning community. Effective, cohesive learning communities develop when academic norms are strong, and language and classroom practices are inclusive of all students (Alton-Lee, 2003). The New Zealand Curriculum advances the position that learning communities that encourage learning conversations, learning partnerships, and reflective discourse afford opportunities for the challenge, support, and feedback needed to advance learning.

This study is set in the context of ongoing school-wide professional development initiative focused on developing mathematical inquiry communities (DMIC). DMIC is a New Zealand based professional development programme which aims to raise expectations of the achievement of all students and improve mathematics learning outcomes for all. To this end, DIMIC advocates replacing traditional, more procedurally based teaching and learning practices with pedagogical practices aligned with a rich mathematical discourse and collaborative approach (ERO, 2018; Hunter, 2008: Hunter & Anthony, 2011). As Hunter, Hunter, Bills, and Thompson (2016) explain DMIC is designed to

> …integrate best pedagogical mathematics practice within culturally responsive

> teaching. This transformative approach calls for significant changes in teacher practices as a way to reverse the persistent underachievement of Māori and Pāsifika and other diverse groups of students—changes which hold many challenges. (p. 59)

1.2 Research Objectives

Participating in a mathematical inquiry community represents a significant change for students, and whilst there is evidence in the research literature (Boaler,2006b) that this approach can raise student achievement, there is less research into how students experience and perceive these changes. This new environment is potentially challenging for both students and teachers. As learning becomes more student-driven, students are required to develop greater student agency, taking more ownership of their learning and the learning of others. For learning within groups to be facilitated for all students, students must participate collaboratively.

This study uses cogenerative dialogues (cogens) to generate student voice to explore students' experiences and perceptions within this new context for mathematical learning. Providing students with a voice empowers them as members of their learning community and gives all members of the learning community, including teachers, insights into how teaching and learning can be optimised (Bondi, 2013; Roth & Zimmerman, 2002). Importantly, efforts to change the way that students learn requires a significant change in how teachers teach. In moving from a transmission approach to one in which students become more active and self-directed learners, teachers need to understand more about how students experience these changes in order that they can develop strategies to support students as they learn to participate within inquiry communities.

The overall aim of the study is to explore how students negotiate change and become part of a mathematical inquiry community. Drawing on data from cogenerative dialogues (cogens) – used as part of the teacher student community building process–the research questions are:

1. How do students experience learning to engage and participate in a mathematical inquiry community and its associated mathematical practices?
2. What do students understand about their role within a mathematical inquiry community?

The use of student voice to gain insight into how students experience learning mathematics as part of a developing mathematical inquiry community was both the research object and part of the co-constructed learning (for both teacher and students) across the first 8 months of efforts to develop a mathematical inquiry community within our classroom. In the context of this 3-year professional development, these 8 months represent the initial stages of the establishment of ambitious teaching and learning, so the responses reflect this.

1.3 Overview

This chapter has provided the rationale for the study and introduced the research aim. Chapter 2 reviews the literature on the nature of engagement within a mathematical inquiry community and provides the theoretical underpinning of this study. Research design is discussed in Chapter 3. Chapter 4 reports the findings of the study and Chapter 5 discusses these findings. Chapter 6 provides a summary and provides concluding thoughts from the study.

Chapter 2 Literature Review

2.1 Introduction

This study examines how students experience and negotiate the changes they encounter as they learn mathematics within a developing mathematical inquiry community. The literature reviewed examines the nature of group work and what it means to be a learner in an inquiry community. Learners must be able to participate, collaborate, take risks, seek help and engage in productive discourse. The literature argues that participating in such a community requires participants to reconstruct ways of learning and engaging in the mathematics classroom, and that navigating these changes can be exciting, challenging, or even frustrating for learners. However, the benefits of working in such a learning community are wide-reaching.

Participation in a mathematical community of inquiry can prepare all students for participation in society (Ball, Goffney, & Bass, 2005; Hunter & Anthony, 2011) Ball et al. argued that a community of inquiry is a democratic society in which students learn that mathematics is not an area where differences are resolved by voting, but by reasoned argument; solutions are governed by mathematical reasoning rather than a desire for power. Students must listen carefully and check for understanding before disagreeing, give credit to others' ideas and seek agreement on meanings and solutions.

In order for students to be able to think critically and understand differences of opinion, the learning environment must be positive and fair (Kwasnica, 2000). Each student needs to feel comfortable and safe, empowered to state personal views. Open dialogue allows students to value a wide range of contributions and develop an understanding of diversity. Such practices within a community of learning can increase equity as a student with lower attainment can feel more comfortable pushing others to explain their thinking (Boaler, 2006a).

Collaborative communities of learning can address the widening gap between low and high achievers in mathematics (Boaler, 2006b; Hunter, 2008; Sullivan, 2011; Watkins, 2005). Such communities are inclusive and caring; inclusive because the mathematics and culture that individuals bring to the classroom is valued and respected, and caring because if students feel safe they become involved, think for themselves and take risks with their mathematics (Anthony & Walshaw, 2009a).

2.2 Developing a Community of Learners

Developing a community of learners takes time. There are many changes for both students and teachers to accept, adopt, and embed into their classroom practices. As new social and mathematical practices develop, a community of learners emerges.

Watkins (2005) described three developmental stages of class community. In the first stage, the classroom is a *community* in which students actively participate in collaborative learning and are encouraged to take an active role in classroom governance. The second stage sees the community develop into a *community of learners,* characterised by engagement in intentional learning. In the third stage, the community of learners becomes a *learning community.* Participants now support each other in their learning, taking responsibility for knowing what they and others need to learn and student agency is developing. In a fully developed learning community responsibility for and control of knowledge is shared.

2.2.1 Student Agency

According to Ferguson, Phillips, Rowley, and Friedlander (2015), "success in life requires the capacity and propensity to take purposeful action. In other words, it requires agency" (p. 13). The context for the study, a collaborative learning environment, provides students with the opportunity to develop student agency and prepare for their future.

An important feature of classrooms that support communities of mathematical inquiry is that the teacher is not positioned as an unquestioned authority (Goos, 2004); students are expected to propose and defend mathematical ideas and conjectures and engage in mathematical argumentation with their peers, in both the context of group work and whole-class discussions.

The first requirement of group work is for the teacher to delegate authority to students. Responsibility to work on tasks in the way that they choose gives students the freedom to struggle on their own and make mistakes (Cohen & Lotan, 2014). In this way, inquiry-based classrooms position students as active participants in their own learning, promoting the development of learning dispositions which will strengthen student agency. As students recognise the need to be actively involved in their own learning, they begin to see competence as related to their role in the community and the learning associated with it (Hunter & Anthony, 2011). Students begin to develop agency, which can be described thus:

> Agency is the capacity and propensity to take purposeful initiative—the opposite of helplessness. Young people with high levels of agency do not respond passively to their circumstances; they tend to seek meaning and act with purpose to achieve the conditions they desire in their own and others' lives. The development of agency may be as important an outcome of schooling as the skills we measure with standardized testing. (Ferguson et al., 2015, p. 1)

Ferguson et al. (2015) describe how agency can be expressed by self-regulatory behaviours such as punctuality, effort, good conduct, help seeking, and conscientiousness. Motivated by mastery goals, students will develop positive self-efficacy and be more satisfied with their achievements. They may have a growth mindset (Boaler, 2013), so know that by working hard they will achieve more. Students with agency may also have aspirations for the future. In contrast, a lack of agency, or disengagement, may present as pretending to make an effort, or making little effort, lacking perseverance, or avoiding help, even when it is needed.

However, the development of student agency within a community of learning cannot be presumed. Indeed, agency can be developed or repressed by various factors, including teaching practices (Ferguson et al., 2015). For example, agency may be 'dampened' when there is an imbalance between how teachers support students and how much they ask of them. Too much support may result in students not taking initiative or learning to persevere. It can also reduce opportunities for students to identify and remedy misunderstandings, which may reduce future effort. On the other hand, practices which develop student voice or require students to think rigorously, grapple with new concepts, and explain reasoning can boost agency. For a successful problem-solver, evidence of developing agency takes the form of a fully self-regulated approach with an awareness of how thinking can be used to provide problem-solving options and strategies (Darr & Fisher, 2014). However, if learners are to be able to lead others in their learning, co-plan and make decisions, student voice must be developed (Bray & McClaskey, 2016).

2.2.2 Student Voice

Cogenerative dialogues (cogens) are a learner-centred pedagogy which use student voice to help students take control of their own learning environment. Cogens enable students to be active participants in the teaching and learning process, co-generating shared understandings and outcomes each session (Emdin, 2008; Tobin, 2014). Cogens can set a context for change in the learning environment (Roth, Tobin, & Zimmermann, 2002) and can address power dynamics in the classroom (Boss & Linder, 2016). They can produce the culture that both teachers and students need to create and maintain productive learning environments (Tobin, 2014; Saunders, Averill & McCrae, 2018) and, through evaluating learning processes rather than content (Boss & Linder, 2016) provide an opportunity to improve engagement (Beltramo, 2017; Bondi, 2013),

The multiple ways and opportunities to participate in cogens include listening, expressing opinions, initiating dialogue, and clarifying or elaborating ideas. Moreover, cogens afford students the opportunity to express their struggles and give teachers the opportunity to encourage them to do so (Boss & Linder, 2016). All participants have the opportunity to speak before the collective agrees on the actions which aim to improve learning outcomes. As such, they are *shared contribution*, rather than shared participation, and can create a space for students to develop a greater understanding of events through dialogue about an experience (Murphy & Carlisle, 2008). Cogens according to Saunders, Averill, and McCrae (2018) reflect the process of ‘ako” described as a two-way process of teaching and learning: students learn from teachers while teachers, as partners in the conversation rather than expert holders of knowledge, learn from students. Saunders et al. therefore, regard cogens as a way to develop culturally responsive teaching.

Cogens can be used as a lens to describe and explain what is happening in the classroom and to interpret student’s perspectives, providing insight into the workings of learning environments and generating locally relevant theory of classroom life (Higgins & Bonne, 2014; Roth & Tobin, 2004). Boss and Linder (2016) looked at cogens through the lens of CHAT, exploring how learning opportunities are transformed by the collaborative efforts of teachers and students to improve the learning environment. The learning environment is the activity system while students and instructors are subjects. Acting as a primary tool, the introduction of cogens to this activity system mediates deeper learning with all members of the cogens group who are involved in making a personal commitment to improving future class sessions based on shared feedback. In this socio-cultural approach to learning, which positions learning as a social process, students are shaping the learning process as they participate in what is learned instead of merely accepting what should be learned (Bondi, 2013).

2.2.3 Supporting Learning Theories

A sociocultural perspective of learning regards learning as contextualised, occurring through participation in practices of a community (Askew, 2012; Hunter, 2008; Selling, 2016). Socio-constructivists regard social process and language as an important element in helping to shape knowledge (Walshaw, 2007). Therefore, when learning takes place within a mathematical community of practice, learning develops alongside increased participation, a "dynamic process of change which involves shifts in positioning of all members of the community" (Hunter, 2008, p. 32).

When learning is considered as taking part in activities that are located within communities, Askew (2012) argued that we should regard the collective, not the individual, as the unit of learning. Rather than focusing on the individual as the main cause of difference, we must look at the whole picture of why some students succeed while others do not. Askew stressed that focusing on the individual risks the group being treated as a group of individuals, missing much that could be gained by focussing on the learning of the group as a whole.

In questioning whether the discrete and decontextualized nature of traditional classroom learning is as "effective as learning within the communities in which what is practised is learnt, and vice versa" (Handley, Sturdy, Fincham, & Clark, 2006, p. 641), situated learning theories, with their focus on social participation, offer a framework to understand a shift in focus from traditional, individualised learning to learning within a community. According to Wenger (1999), traditional classrooms were designed to be free of the distractions of their participation in the outside world, with the assessment of learning involving the out-of-context demonstration of individual knowledge. Here, collaboration would be seen as cheating and much learning regarded as irrelevant. However, Wenger argued that by adopting a different perspective, one which places learning in the context of the learner's world, learning would become integral to both daily life and participation in our communities. Situated learning suggests that individual learning and the development of a learning identity emerges through engaging in opportunities to participate in practices of community (Handley et al., 2006). Wenger (1999) argued that the aim of learning is to enable learners to experience and engage meaningfully in the world. Learning for individuals is thus seen as engaging in and contributing to the practices of their communities. For communities, learning is the refining of practice and ensuring new generations of members are enculturated into mathematical ways of knowing and being.

Language is a fundamental component of participation within a mathematical inquiry community. Framed in a sociocultural discourse perspective Mercer (2004) regards language as a social mode of thinking. Mercer proposed that language is used as a collective tool for thinking and that when we interact together, we *interthink*. Used as a tool for thinking collectively and as a tool for teaching and learning, language supports constructing knowledge, creating joint understanding, and tackling problems collaboratively. This sociocultural perspective raises the possibility that "educational success and failure may be explained by the quality of educational dialogue, rather than simply in terms of the capability of individual students or the skill of their teachers" (Mercer, 2004, p. 139).

2.2.4 Supporting Pedagogy

Complex instruction (Cohen, Lotan, Scarloss, & Arellano, 1999) is an underlying pedagogical model of the mathematical inquiry classroom. This approach derives from the premise that status differences do not come from particular students but from group interactions. It addresses equity issues by focusing on inclusion through using heterogeneous grouping and monitoring participation and student status (Boaler 2006b). Additionally, the use of open-ended, interdependent tasks increases student interaction, allowing a diverse range of students to be taught to a high academic level.

As Boaler (2016b) describes, complex instruction involves the creation of multidimensional classrooms, which define success more broadly than unidimensional classrooms that measure success in terms of finding the correct answer, paying attention and following procedures. Valued practices of the multidimensional classroom include asking questions, helping others, using different representations, justifying methods and bringing a different perspective to problems. Multiple ways to achieve success can lead to students feeling more positive, working harder and developing higher levels of understanding.

Status in the classroom can privilege or marginalize a student's engagement in collaborative work, which can lead to social or intellectual dominance. The use of complex instruction can mitigate such issues by publicly positioning lower-status students with mathematical competence whenever their contributions to collaborative work are meaningful (Langer-Osuna, 2018). The use of open-ended, challenging tasks requires students to interact, drawing on each other's expertise as they support each other. The teacher's role is to deliberately intervene to produce equal status relationships within the groups in order that all students have equal access to learning (Cohen et al., 1999).

Challenge is a stimulus for mathematical thinking, and mathematical practice based around collective problem solving provides opportunities for cognitive conflict and challenge (Swedosh & Clark, 2000; Swan, 2001). In this context, social interaction is essential for learning; challenging work that cannot be attempted individually can be scaffolded when working within a group (Askew, 2012). A collaborative zone of proximal development (Vygotsky, 1976, as cited in Walshaw, 2007) can be created through interactions between peers, leading to successful problem-solving outcomes (Goos, Galbraith, & Renshaw, 2002). Mathematical learning thus becomes a process of active construction of knowledge and a process of acculturation into the mathematical practices of wider society (Yackel & Cobb, 1996), with mathematical practices constructed intrapersonally before being appropriated by the individual (Selling, 2016). Learning that is "most personally transformative turns out to be the learning that involves membership in those communities of practice" (Wegner, 2009, p. 212).

Activating cognition describes a pedagogical approach in which learning is initiated through a challenging task (Sullivan, Borcek, Walker, & Rennie, 2016). It is based on the theory that students learn mathematics best when they engage in mathematics prior to any specific instruction, building their own mathematical ideas at the beginning of the learning sequence rather than at the end. Using challenging tasks activates earlier knowledge, allowing new knowledge to be constructed and connections to be made to previous learning. This approach can be challenging for students, who may feel that they are struggling with mathematics without the support of prior teaching, but it may also present a challenge for teachers who may have to change their practice of teaching first and providing opportunities to practise learning afterwards. When planning mathematics in this way a teacher must identify the mathematical potential of the task, plan for students to create their own solutions, anticipate the need for task differentiation, and consolidate learning through similar but varied tasks (Sullivan et al., 2016).

Incorporating problem-solving and reasoning into mathematics classrooms whilst catering to the diverse needs of students can be difficult (Sullivan et al, 2016; Hunter & Hunter, 2018).

With changes in interaction norms, teachers may be pedagogically challenged to develop classroom practices where student reasoning is the focus of mathematical discussion (Hunter & Hunter, 2018). Teachers must now make the implicit explicit, making clear why some comments are accepted as valid and some are not; the collective awareness of the class must be raised (Selling, 2016). Teachers may be concerned about potential negativity towards challenging tasks (Sullivan et al., 2016) yet they must allow students to make mistakes, take risks and overcome problems through effort, using specific actions to scaffold students who have previously been limited by lack of challenge and low expectations (Hunter & Hunter, 2018).

Social norms can "provide members of the learning community with guidelines for acceptable ways to participate and communicate mathematical reasoning" (Hunter & Hunter, 2018, p. 3). Mullins (2018) identified three key components to help teachers establish and implement social norms that promote sustainable inquiry-based classrooms. Firstly, norms must be developed through the collaboration of the teacher and students; secondly, the classroom environment must be supportive of classroom discussion; and thirdly, norms should be renegotiated to help change students' ways of thinking. A challenge noted by Mullins when negotiating social norms is the teacher's control of power. Teachers must realise that there is an "appropriate amount of power that students need to have in the classroom to create a community of discourse" (p. 14). It is the negotiation of social norms that supports the balance of students and teacher control.

Sociomathematical norms differ from classroom norms in that they are specific to mathematical aspects of an activity. They include what counts as mathematically different, sophisticated or elegant, or what constitutes an acceptable mathematical explanation and justification (Yackel & Cobb, 1996). Whilst facilitating classroom discussion, the teacher acts as a participant who can legitimise certain aspects of a student's mathematical activity and implicitly sanction others. A teacher's response to a student's solution can be interpreted as an implicit indicator of how it is valued mathematically; students must infer what aspects of mathematical activity is valued. Thus, sociomathematical norms are developed and embedded in the classroom community, a process to which both teachers and students are interactive contributors (Yackel & Cobb, 1996).

Collectively drawing on these norms, Chan and Clarke (2018) identified three distinct modes of student social interactions during collaborative problem solving: *mathematical interaction,* concerning mathematical correctness of fact or procedure; *sociomathematical interaction*, concerning didactical norms of the classroom; and *social interaction,* concerning social obligations within the group, such as group dynamics. Chan and Clarke suggested that all three modes of interaction co-exist in an entangled form, and that understanding this entanglement is important for an understanding of the dynamics of collaborative problem solving and associated learning. These negotiative patterns, they argue, are an essential precursor to the development of student collaborative group work and associated learning in mathematics.

2.3 Being a Learner in a Mathematical Inquiry Community

This section outlines key requirements of working within a mathematical inquiry community. *Participation* and *collaboration* are essential if the learning environment is to become an effective community of learners with equitable access to learning outcomes. *Productive discourse* is the medium through which learning will take place, although it cannot be expected to occur without scaffolding over time. *Help-seeking* and *risk-taking* are also important for individual and collective learning but can present barriers to participation for those who are not secure within the learning environment.

2.3.1 Participation

It can be argued that the change from learning in a more passive, traditional mathematics classroom to the more active learning environment of a mathematical inquiry classroom is one of the biggest changes that students face. However, being present within a community of learners is not enough to guarantee active participation. Barriers to participation can result in passivity, a sense of not belonging, or even a negative disposition towards mathematics (Gibbs & Hunter, 2018).Viewed as an equity issue (Cohen et al. 1999; Foote & Lambert, 2011; Gibbs & Hunter, 2018) there are numerous reasons why students may not participate. In an environment which necessitates risk-taking and the sharing of thinking, participation reduces if there is a perception that not all group members are contributing, if tasks are too challenging or not sufficiently challenging, or if an individual's contributions are not valued by the group (Howe & Mercer, 2007).

The deliberate assigning of competence is key to contributions being respected (King, 2018). Most notably, research by Cohen et al. (1999) and Boaler (2006a) has found that public recognition of a specific intellectual contribution can raise an individual's status within the group. This positive evaluation relies on the teacher's power, which must be seen as a legitimate source of evaluation for students to believe it. All concerned must know exactly what was done well, with the intellectual contribution being essential to the work of the group. In contrast, research notes that without effective teacher intervention, some students may be positioned in a way that reduces participation, which can have a detrimental effect on achievement (Hunter & Gibbs, 2018). Indeed, Foote and Lambert (2011) suggested that equity can only be considered to be achieved when patterns of participation can no longer be predicted.

Although non-participatory behaviours are not necessarily passive (Mulryan, 1992), understanding the reasons for passivity would be helpful in establishing and maintaining an effective learning community. Mulryan determined that passivity may manifest itself in six ways: a passive student may be discouraged, unrecognised, despondent, unmotivated, bored, or an intellectual snob. A discouraged student may find tasks so difficult that it is felt better to leave the tasks to those who are perceived more able to tackle them. A student may withdraw from the group after efforts to contribute go unrecognised or if he or she feels uncomfortable working with group members. A lack of motivation can occur if tasks are not perceived as important which could turn to boredom if tasks are too hard to connect with or too easy to engage. Finally, a high achiever may fail to engage in cooperative work if they feel their peers are less competent than they are.

Active off-tasks behaviours can also affect participation. For example, Mulryan (1992) describes a *social opportunist* as one who regards group work as a chance not to work but to socialise; an *intentional loafer* may watch others while remaining uninvolved, or may try to encourage others in off-task behaviours; and the *alternatively involved* will, for whatever reason (e.g., they are bored, unhappy with their group, or dealing with other work pressures) be engaged in other tasks.

Participation in an inquiry community also raises the importance of relational qualities such as trust, friendliness, and inclusion (Askew, 2012). Associated with participation in classroom activities as co-learners is communication of high expectations for every student to learn mathematics and to contribute to the mathematics learning of others (Allen & Schnell, 2016). These relational qualities contribute to students’ development of mathematical identity; in this way, learning is framed as participation in the mathematical inquiry community (Goos, 2004).

2.3.2 Collaboration

Participation is a prerequisite for effective collaboration–an essential component of a mathematical inquiry classroom. Collaborative learning can be defined as "a situation in which particular forms of interaction among people are expected to occur, which would trigger learning mechanisms, but there is no guarantee that the expected interactions will actually occur" (Dillenbourg, 2011, p. 5). Seen as both a learning process and a learning outcome, students learn from the elaboration and sharing of knowledge whilst also learning collaborative skills (Remedios, Clarke, & Hawthorne, 2008); skills which will prepare them for an increasingly collaborative workforce (Groff, 2012).

For many mathematics classrooms, working collaboratively is a significant change. For the teacher, developing a collaborative classroom environment requires a shift from being at the centre of the classroom to being a group supporter, then a system facilitator, and finally a system coordinator. For students, collaborative learning involves students not only working together but also being responsible for their own learning and the learning of others (Groff, 2012). As such, collaboration can be seen as an opportunity to share the workload; distributing the cognitive load as a way to learn from multiple perspectives and as a way to develop social skills (Remedios et al., 2008). In the mathematics classroom, the collaborative process of mathematical inquiry can support student wellbeing, enhance mathematical understanding, and help develop mathematical proficiency, a productive mathematical disposition, and confidence as a mathematician (Alton-Lee, Hunter, Sinnema, & Polegato-Diggins, 2011).

Importantly, a higher level of attainment can be achieved when students work together (Askew, 2012; Brown & Thomson, 2000). Students working collaboratively work like real mathematicians, approaching open-ended problems in the genuine sense of a mathematician contributing jointly to a body of knowledge (Edwards & Jones, 2003). Working as a team, students can analyse and synthesise material and build their critical thinking skills, so have greater potential for deeper understanding than individuals working alone (Hunter & Anthony, 2011; Remedios et al., 2008). This contrasts with a competitive environment which only allows some to succeed. For example, a group which works competitively or individualistically may rely on one or two members to achieve success with others reduced to the role of supporters, non-participants or even saboteurs (Brown & Thomson, 2000).

Collaboration can also be seen as the as the route to innovation as it helps idea generation, with small breakthroughs contributing to the end result (Sawyer, 2007). Effective collaborative teams have the ability to listen well. They understand that when generating ideas it may not be obvious which ideas are to be discarded and which ideas are going to be pivotal to finding the solution. So, there must also be time for failure and for learning from this (Sawyer, 2007).

Collaborative learning only works when students *know how* to work together, which cannot always be assumed to be the case (Askew, 2012; Howe & Mercer, 2007; Simpson, Mercer & Majors, 2010). Indeed, a big change for students is the acceptance of the benefits of working collaboratively and recognising the need for multiple skills and expertise (Hunter & Hunter, 2018). Some students may not adapt easily to group work. For example, if they have previously achieved individual success, they may be unwilling to devote time and energy to group cohesiveness or may resist their own grades being dependent on the effort of others (Lotan, 2003). Uneven distribution of effort amongst the group, with perhaps some members excluded while others opt-out is likely to lessen the success of group collaboration. (Boaler, 2006b).

To support student change, teachers need to be explicit about the strategies used to promote collaboration (Alton-Lee, 2003; Hunter, 2007; Sullivan, Zevenbergen, & Mousley, 2003). In making strategies explicit, Sullivan et al. described a process termed 'regeneration of classroom social norms', involving sessions in which a particular aspect of collaborative work, such as agreeing or disagreeing or valuing multiple answers and different strategies, is focused upon. Together, these become part of the values embedded in the teacher's pedagogy.

2.3.3 Productive Discourse

A key responsibility of school is to develop students' use of language for communicating, reasoning, and learning across contexts (Ministry of Education, 2007) Simpson, Mercer, & Majors, 2010). Howe and Mercer (2007) argued that educational success or failure is related less to the abilities of students or the skills of teachers, or particular education methods, than to the nature and quality of social and communicative processes in classrooms. However, successful enactment of classroom discourse is challenging (Bruce, 2007; Hunter, 2008). Students who have previously learned that their role is to listen and follow are now required to talk and actively question. Likewise, teachers need to "move past their own history as learners and teachers in traditional mathematics classrooms where teacher-talk dominates" (Hunter & Hunter, 2018, p. 15).

Talk provides a tool for exploring different ways of thinking and understanding (Barnes, 2010; Clarke, 2013): However, although research affirms that students learn more through discussion, evidence suggests that “not just any interaction will do” (Wood & Kalinec, 2012, p. 109). Indeed, studies confirm that language is not always used in a way that will help students learn (Black, Harrison, Lee, Marshall & Wiliam, 2004; Chapin & Connor, 2007). For example, disputational talk (Mercer, 2004) is competitive and argumentative; individuals make their own decisions and there is little or no constructive criticism. Such talk has no place in a successful learning community, yet it may be the predominant type of talk as students learn to collaborate. As collaboration develops, cumulative talk can emerge. With this, ideas are accepted, repeated, and elaborated but with little evaluation. As cumulative talk becomes more exploratory, Mercer notes that a sense of joint purpose emerges involving active listening and questioning, with ideas built on or challenged. Inquiry-based classrooms provide a forum for ideas to be raised and reason to be publicly communicated (Hershkowitz, Tabach, & Dreyfus, 2017) supporting opportunities for students to learn argumentation, engage in mathematical explanations and justifications (Anthony & Walshaw, 2009; Hunter, 2008; Mullins, 2018).

However, again, there is a level of challenge and adjustment for both teachers and students (Bruce, 2007; Hunter, 2006; Mullins, 2018; Wood & Kalinec, 2012). Based on earlier learning experiences students may not naturally engage in mathematical argumentation, instead tending to anticipate what the teacher is looking for, rather than describing their own thinking (Sullivan et al., 2013). Moreover, mathematical argumentation may not be understood by all, and may be seen as confrontational (Hunter & Hunter, 2018). Research has found that teachers can scaffold specific strategies to promote mathematical discourse (Boaler, 2006a; Bruce, 2007; Hunter, 2007, 2008; Mullins, 2018; Wagganer, 2015), and themselves learn when to intervene and when to address misunderstandings (Anthony & Walshaw, 2009). Stein, Engle, Smith, and Hughes (2008) offered five practices for orchestrating discussions, as follows. Through anticipating how students will approach the mathematics and by monitoring students’ thinking as they work, teachers are able to select which thinking or solution strategies will be shared. This determines a sharing sequence which makes mathematical connections and big ideas explicit. Academically productive discourse also involves the repetition and revisiting of ideas and facts, giving many opportunities to hear key details or clarify interpretations. Used effectively, this *revoicing* can support learning (Chapin & Connor, 2007; Ferris, 2014; Wagganer, 2015).

Mathematical discussions become more meaningful if students not only understand why talk is important but are specifically taught how to listen and respond (Wagganer, 2015). Remedios, Clarke, and Hawthorne (2008) found that students placed a higher value on speaking than listening during collaborative work and caution against regarding verbal participation as collaborative and silence as a failure to collaborate. Whilst acknowledging participation as the key to collaboration, Remedios et al. argued that verbal participation is not equivalent to collaboration and advocated promoting listening as a collaborative skill; a silent participant may be actively listening, checking the group's thinking in order to add insights that could further the group's understanding.

In inquiry classrooms which require high levels of mathematical discourse it can be a challenge for teachers to support equitable outcomes for all students. Teachers must understand and overcome the possible barriers students may encounter (Hunter & Hunter, 2018). The creation of a learning ethos in which all contributions are valued can help develop a sense of belonging. Successful collaboration engenders pride, leading to students encouraging each other and respecting all contributions of the group (Gresham & Shannon, 2017).

2.3.4 Risk-taking

A safe, risk-taking environment with positive student-teacher relationships (Sharma, 2015) is essential for productive discourse and successful collaborations to occur. Through promoting discourse and allowing for interactions without right answers, teachers communicate that students are sense-makers and that their ideas are valued (Kazemi & Hintz, 2014). However, exposing one's thinking, questions, and misconceptions for all to hear involves risk taking. Bennett (2010) used the analogy of poker chips to describe risk-taking in a discourse-rich classroom environment. 'Chips' are earned through mathematical success. The more chips you have to begin with, the more willing you are to risk them. Bennett argued that students may not risk their chips unless they are assured success, so the more they experience success, the more willing they will be to risk chips. In this scenario, lack of mathematical success could lead to students not risking the chips and withdrawing from discussion.

For risk taking to be an accepted aspect of classroom culture students may need to be specifically taught how to take risks in their learning (Gresham & Shannon, 2017; Sharma, 2015). Students' concerns about revealing uncertainty by asking questions might be overcome in a context that focuses attention on learning rather than just obtaining correct answers.

2.3.5 Help-seeking

Help-seeking is a social behaviour shaped by people's experiences in particular contexts (Butler, 1998). When engaging in group-worthy problem-solving tasks it is inevitable that students will encounter situations where they will require help; yet research indicates that help is not always sought, even when it is needed (Newman, 1998; Ryan & Pintrich, 1997). While self-regulated learners have the cognitive, communicative, and social skills to know when and how to seek help, not all students have this self-regulation (Newman, 1998). Ryan and Pintrich described the three steps that must occur for help to be sought. Firstly, metacognition is required: the student must be aware that help is needed. Secondly, the student must be motivated to seek this help. Finally, the student must be able to implement strategies for engaging help.

Help sought by students with learning goals (mastery orientation) differs from those with performance goals. Those with learning goals are more likely to seek task-related information and seek feedback on work which helps lead to task mastery (Newman, 1998). Related to these two broad learning goals there are a number of factors which determine differing help-seeking behaviours. For example, a student who exhibits a personal need for autonomy–an *autonomous orientation* focused on independent mastery–will likely value perseverance as a coping strategy and thus be reluctant to seek assistance. On the other hand, avoiding help could be an attempt to mask perceived incompetence. Having this *ability-focused orientation* poses a dilemma: the inability to complete a classroom task indicates poor ability, whereas asking for help is also an indication of inability (Butler, 1998).

The *vulnerability hypothesis* states that a disposition to seek help will be lowest amongst low achieving students anxious to avoid further confirmation of low ability (Butler 1998; Ryan & Pintrich, 1997). In this situation, an *avoidant-covert* pattern of help-seeking (Butler, 1998) may be adopted, where a student is unlikely to ask for overt help, preferring more covert means such as copying answers or listening in on the conversations of others. Moreover, higher achieving students whose achievement goals are performance rather than learning-related may also avoid any risk of showing incompetence (Newman, 1998). These students may avoid challenge and difficulty in order to maintain their self-perception of ability and their high status in the perceptions of others.

In contrast to an ability orientation, a student with a strong personal need for autonomy may adopt *autonomous help-seeking* behaviours. These students may avoid seeking help because of a determination to persist on a problem and achieve independent mastery. Importantly, when these students seek help it is because they know that completing the problem will enhance their understanding and lead to subsequent mastery (Butler, 1998).

A third pattern of help-seeking is described as *executive help-seeking* (Butler, 1998). Unlike those with an autonomous orientation, these students do not want hints to help with their learning; they want answers and view help as a mechanism to complete the task (Newman, 1998). These students may prefer others to solve the problem rather than solving it themselves. If the teacher or classmates do not respond to such direct requests for help students may stop asking for help, knowing it will not result in task completion (Butler, 1998).

Given these scenarios, it is apparent that the degree of teacher control, the pattern of expectations set in the classroom, and the degree of support afforded to students are all ways a teacher can influence help-seeking behaviours (Newman & Schwager, 1993). A competitive environment can increase opportunities for students to compare their own performance to that of others, which may inhibit help-seeking. In contrast, effective collaboration helps students develop a sense of each other's needs, so help is often provided without it being sought (Gillies, 2003). Group members may be more aware than their teacher of what their peers do or do not understand and may be in a better position to respond to requests for help (Gillies, 2003).

2.4 Mathematical Practices

It is important that a strong mathematical focus is maintained as the learning community develops (Anthony & Walshaw, 2009b). In developing an ethic of care, teachers must be concerned with the development of students' mathematical proficiency and identity (Anthony & Walshaw, 2009b). As such, supporting collective participation in mathematical practices involves being responsive to students whilst holding them accountable to learning goals such as procedural fluency, strategic competence, adaptive reasoning, and productive dispositions (Kazemi, Franke & Lampert, 2009; Kilpatrick, Swafford & Findell, 2001). This requires a coherent conceptual framework of pedagogical strategies that supports the development of mathematical practices (Hunter, 2008; Selling, 2016).

The Standards for Mathematical Practice presented in the *Principles to Action* (National Council of Teachers of Mathematics, 2014) describe eight key mathematical processes–a framework with which teachers can guide students in learning how to use mathematics and think mathematically: problem-solving, reasoning, mathematical argumentation, mathematical modelling, using appropriate tools, attending to precision and looking for patterns. Ideally, these practices will become a habitual way of interacting with any mathematics encountered (Rutherford, 2015). A successful mathematics lesson may not attribute its success to one particular mathematical practice used well, but to "the synergy that was created from integrating the practices in a coherent way" (Smith, Bill, & Raith, 2018, p. 41).

Challenge within problematic tasks prompt the use of mathematical practices (Downton & Sullivan, 2017). Making sense of mathematical ideas embedded within tasks emphasises that mathematical learning is about thinking mathematically rather than just about finding a solution (Anthony & Walshaw, 2009b). High-level tasks give students opportunities to learn what mathematics is and how one does it (Downton & Sullivan, 2017; Suh, Graham, Ferranone, Kopeinig, & Bertholet, 2011). Teachers must balance allowing students to struggle with knowing when to ask leading questions and when to insert mathematical ideas (Gresham & Shannon, 2017). Careful questioning scaffolds student attempts at conjecture, disagreements and counterarguments (Anthony & Walshaw, 2009b). In this way, students learn how to use mathematical ideas and language and, while teachers ensure students' efforts will lead to the solution (Chapin & Connor 2007), attention is focused more on the thinking that leads to the solution (Anthony & Walshaw, 2009b).

In solving problems students experience opportunities to build connections between ideas, persevere, and develop a *growth mindset:* the belief that you can improve ability by working harder (Boaler, 2013). It is important that goals and learning are made explicit (Sullivan, 2011) and that teachers let students know when they see valuable ways of working (Boaler 2006a). A framework designed to understand how better to engage all students in mathematical practices uses eight *reprising moves* which make mathematical practices explicit without being prescriptive (Selling, 2016). These reprising moves include naming the mathematical practice, evaluating student engagement in the practice, explaining the goal rationale for engaging in the practice and self-assessing in respect of the mathematical practice. Students are thus able to develop their own meanings for the mathematical practices relative to their prior experiences and connect the mathematical practices beyond the problem at hand.

2.4.1 Group-worthy Tasks

In a mathematical inquiry community research has found that group-worthy tasks allow students to work and grow as mathematicians (Boaler, 2006a). Typically, group-worthy tasks are open-ended, requiring complex problem-solving. They have multiple entry points and multiple opportunities to show intellectual competence (Lotan, 2003). They require skills such as asking good questions, explaining, being logical, justifying work, considering answers, and using manipulatives. They have discipline-based, intellectually important content, and require positive interdependence in addition to individual accountability. In cases where they are close to authentic problems students' engagement with such tasks prepare them for dealing with real life uncertainties (Lotan, 2003).

Working to the premise that all students are capable of complex mathematical thinking, the use of group-worthy tasks can increase equity in the mathematics classroom (Boaler, 2015). Providing multiple ways of achieving success, means that more students are able to experience success. When mathematics is open to interpretation and challenge, students from a wide range of positions are able and willing to identify with it (Angier & Povey, 1999). As students share experiences, justify beliefs, analyse, synthesise and evaluate, discuss controversial issues or cause and effect, build consensus and draw conclusions, their opinions become part of the intellectual content (Lotan, 2003). Agency is increased as all students are able to contribute to and progress through learning experiences, allowing them to feel part of the class learning community (Sullivan, 2011).

2.5 Grouping Practices

The collaborative nature of learning within inquiry communities necessitates reconsideration of how to group students for optimal learning outcomes (Askew, 2012). Increasingly, research evidence contradicts the traditional practice of grouping students according to perceived ability, suggesting that if ability grouping were to be replaced by heterogeneous grouping, achievement and participation rates would improve significantly (Boaler, 2013).

Collaborative group work goes beyond students simply helping or motivating each other. Creating a situation in which group members need each other engenders a sense of purpose and commitment to the team and helps develop a group identity. When students are dependent upon one another for group success they are more likely to encourage one another to succeed; this interdependence is at the core of collaborative learning (Brown & Thomson, 2000; Slavin, 1996). Esmonde, Brodie, Dookie, and Takeuchi (2009) found the most influential factors determining which groups work well and when they do not were found to be interactional style, mathematical understanding, and friendships and relationships.

2.5.1 Group Composition

It is recognised that it takes time to establish collaborative groups, and group organisation is no guarantee that group work will take place (Askew, 2012; Edwards & Jones, 2003; Mercer & Sams, 2006; Mulryan, 1995). Optimal use should be made of complementary combinations of teacher-directed groupings, cooperative groups, structured peer interaction and individual work (Alton- Lee, 2003). Indeed, on occasions students may benefit from time working away from others in order to develop their own thinking (Anthony & Walshaw, 2009).

Group size can affect participation and learning. While some research suggests a group size of three to five learners is satisfactory, others argue that groups of five or six are liable to split into sub-groups, risking dominant members taking over while others withdraw, lose motivation or even opt-out (Brown & Thomson, 2000). As noted by Enu, Danso, and Awortwe (2015) optimal size is often dependent on the classroom/learning context. The level of cohesion within the class may prevent the use of larger groups and under some conditions, pairs would be advisable, such as while a class is being established or the complexities of effective group work are still to be learned (Brown & Thomson, 2000).

Paired tasks can be worked on by individuals with a move towards cooperative behaviours as individuals check, compare and listen to each other's answers (Hertz- Lazarowitz, 1995). Whilst working as a pair, social behaviours such as attentive listening, taking turns, timekeeping and relating to each other also develop. The next stage of collaboration would see students working in small groups but with low levels of collaboration, which would develop into higher levels of collaboration. Finally groups would become fully integrated. In general, smaller groups provide opportunities for discussion, reflection, and feedback, all of which consolidate learning, clarify understanding and allow for the exploration of ideas and concepts.

There are many factors to consider when forming groups. Two common approaches are for a teacher to strategically plan groups to create an optimal learning environment or to have students form self-selected groups (Liljedahl, 2014). Strategic groupings can be formed with either educational or social goals in mind. Education goals consider how groups could work together to benefit learning, maximise group productivity or maintain a calm working environment. Social goals may aim to integrate students into the community or make collaboration easier by utilising existing friendship groups. Diversity may be desired, perhaps to mix genders or to encourage collaboration with those whom students would not usually choose to socialise.

Liljedahl (2014) noted that a mismatch between grouping goals of the teacher and those of the students can create tension between classroom goals and the students' own goals. This, together with social barriers, can lead to problems. As strategic grouping is unlikely to satisfy everyone, Liljedahl suggested random grouping as a third way to structure groups. Randomly selecting group participants can create combinations not previously considered. It is seen as fair by students and can be good for community building, fostering the idea that everyone can work with everybody else. In Liljedahl's classroom research with Year 10 students in Canada, they found groups formed at the beginning of each lesson in a visibly random way, while met with initial resistance, was soon accepted. In as quickly as a few weeks social barriers were eliminated in the classroom and students become more agreeable with who they work with. Engagement and collaboration increased, reliance on the teacher diminished while reliance on themselves and others increased.

2.5.2 Heterogeneous Grouping

Using heterogeneous grouping, where "students are trained as academic and linguistic resources for one another" (Cohen & Lotan, 2014, p. 23), can help all students develop a greater understanding of their learning. They may become more able to recognise achievement and progress and can identify goals for improvement (ERO, 2018). ERO found that those students who were previously in 'lower' groups experienced increased confidence and enjoyment when working in mixed ability groups. Mixed ability groups also were seen to benefit more able mathematicians, who were required to think deeply about alternative solutions when working collaboratively.

A big plus for heterogeneous grouping is the lessening of students being publicly labelled as low achievers (Alton-Lee, Hunter, Sinnema, & Polegato-Diggins, 2011). Working in heterogeneous grouping enables opportunities to utilise strengths from all learners, allowing teachers to focus and build on what a student can do, rather than what they cannot do (Askew, 2012). Heterogeneous grouping also minimises the common experience of homogenous groups where students in lower groups can be disadvantaged due to a negative effect on teacher expectations with resultant exposure to tasks which present less challenge, acceptance of students' unwillingness to engage with the task, or further reduction of the cognitive task demand (Sullivan, 2011).

With heterogeneous grouping all students are exposed to the same cognitively challenging tasks so, rather than teaching 'to the middle' or to the 'lowest common denominator', the teacher can provide an intellectual challenge for all; a temporary lack of skills need not be a barrier (Cohen & Lotan, 2014). Such an environment affords all students the opportunity to develop the ability to make conjectures and engage in mathematical argumentation (Anthony & Walshaw, 2009a), which can be scaffolded and extended, whatever the level of prior knowledge (Alton-Lee, Hunter, Sinnema, & Polegato-Diggins, 2011).

Heterogeneous grouping can provide a supportive environment in which student agency can develop. If a lesson requires all students to produce a result, yet not all students have the prerequisite skills to achieve a result on their own, students must take ownership of their learning and be prepared to work with the group (Cohen & Lotan, 2014). When the group achieves, several individuals achieve, allowing more students to achieve mathematical success (Askew, 2012; Gibbs & Hunter, 2018; Carpenter & Lehrer, 1999).

2.5.3 Working as a Group

A group can be said to be working effectively when it has the capacity to complete tasks successfully, with each group member able to take on different roles within the group. These roles would vary according to the nature of the task and the composition of the group (Galton & Williamson, 2003) and may either evolve naturally or be deliberately assigned. Assigning roles can help the organisation of group work, helping students develop responsibility for the smooth functioning of the group, fostering interdependence, and helping develop teamwork skills. (Brown & Thomson, 2000). Moreover, assigning roles can supports delegation of authority to students group members responsible for checking for understanding and work completion, assigning duties, and ensuring all group members are involved (Ehrlich & Zack, 1997).

Galton and Williamson (2003) described roles, be they assigned or evolving naturally, within a group as either *instrumental* or *expressive*. *Instrumental* roles are those which help the group complete the task, whereas *expressive* roles provide social leadership. A key expressive role would involve breaking any tension that may arise, often due to disagreements regarding how to proceed on a task. Another way that Galton and Williamson categorise roles is *authority roles, negotiating roles,* or *supportive roles.* Group members taking on an authority role will organise tasks, define goals and offer solutions. Those with a negotiating role will initiate ideas, make suggestions, and keep things moving by asking questions. A negotiator will strive for decisions to be made, assess progress and encourage and involve others. A student in a supportive role would listen attentively and follow the group's progress and affirm agreement. A supportive group member may also relieve tension by telling jokes or taking on unpopular tasks. In contrast, group members may choose to take on a *non-cooperative r*ole. This group member would obstruct progress by reacting negatively to suggestions or raising problems, possibly seeking sympathy. Their off-task behaviours disrupt collaborative work whilst a refusal to participate actively deprives the group of their support and assistance.
Rewarding groups on their performance can be motivational, in that group members can attain personal goals through the success of the group (Slavin, 1996). When it is important to each individual that the group succeeds, individuals work hard to ensure the group’s success and encourage fellow group members to do likewise. However, group rewards must be based on the individual learning of all group members, rather than a single team product: that is, there must be individual accountability with the group to provide explanations, help and encourage one another (Brown & Thomson, 2000).

The social cohesiveness of the group can influence the effectiveness of collaborative learning (Slavin, 1996). The *social cohesion perspective* describes students helping one another learn and succeed not because it has an impact of their own achievement, but because they care about each other and want each other to succeed. Hunter and Hunter (2018), in the context of DMIC classrooms, have found that when students feel responsible for fellow group members’ achievement, they motivate each other to engage in cognitive processes. That is, when students value the success of the group they will encourage and help one another achieve individual success.

2.6 Summary

Research provides evidence that engaging in the practices of a community of inquiry helps support students develop a conceptual understanding of mathematics, a productive disposition and a positive mathematical identity. There is a requirement for teachers to establish learning environments which provide students with access to learning partnerships, challenging conversations, and responsive feedback (Ministry of Education, 2007). Teachers and students have an important role in constructing inclusive classrooms and both must adjust their perspectives on mathematical teaching and learning whilst developing habits of productive mathematical practices within an inquiry classroom (Hunter & Hunter, 2018)

Pedagogical changes present challenges for both teachers and students (Hunter, 2008; Selling, 2016). The ambitious teaching that takes place in a mathematics inquiry classroom places high expectation on the teacher to work dynamically with students whilst ensuring students are accountable for their own learning. Although teachers may understand and recognise the different aspects of ambitious teaching, implementation can be difficult (Kazemi, Franke, & Lampert, 2009). Students may lose confidence or become confused when the norms seem to have changed and when faced with new demands on how they are expected to participate and in what they are expected to do mathematically (Hunter & Hunter, 2018; Selling, 2016). As such, it is important that teachers build both the environment and student confidence so that all students can participate within this environment (Hunter & Hunter, 2018).

Although learning mathematics collaboratively is becoming widely accepted as best practice, some challenges remain: "Our field cannot fully understand (and thereby support) how students author, share, and debate mathematical ideas without taking into consideration how they negotiate relationships of power in the collaborative mathematics classroom" (Langer- Osuna, 2017 p. 238). Langer-Osuna argued that only by that understanding how students adapt to communities of inquiry, and specifically how intellectual authority is developed in a collaborative classroom, will greater understanding of learning in a collaborative mathematics context be achieved. Looking at how students experience being a participant and a learner within a developing community of mathematical inquiry, with the aim of understanding more about learning such an environment, is the focus for this study.

Chapter 3 Research Design

3.1 Introduction

This research used student voice to gain an understanding of how students experience and participate in a developing community of learning. The framing research questions were:

- How do students experience learning to engage and participate in a mathematical community and its associated mathematical practices?
- What do students understand about their role within a mathematical inquiry community?

The aim of this exploratory case study is to explore students' experiences within the first 8 months of learning to participate in a developing mathematical inquiry community. Situated within our school-based involvement in the professional learning imitative *Developing Mathematical Inquiry Communities* (DMIC), where both students and teachers were learning to enact ambitious ways of teaching and learning mathematics, a series of cogenerative dialogues (cogens)— partnership conversations with teacher and students—were used to support the development of effective group work. In terms of this research, cogens were viewed as particularly suitable to explore students' experiences in learning to participate and 'be' a mathematics learner.

3.2 Methodology

This study is situated within an authentic classroom context involving both the researcher and the student participants who collaborate to shape their learning. Placing the research within the socio-constructivist paradigm, a qualitative methodology is used. As Hoepfl (1997) describes, "phenomenological inquiry, or qualitative research, uses a naturalistic approach that seeks to understand phenomena in context-specific settings" (p. 48).

Qualitative research methods are useful for researching phenomena about which little is known, or as a means to discover new perspectives; they have an interpretive character, aimed at discovering the meaning events have for the individuals who experience them, and the interpretations of those meanings by the researcher. Rather than looking for causal determination, prediction, and generalization of findings, qualitative research seeks illumination, understanding, and extrapolation to similar situations (Golafshani, 2003). Using cogenerative dialogue to generate data places the researcher as the instrument of data collection.

As part of establishing a mathematical inquiry community, the teacher-researcher and a volunteer group of students participated in a series of cogenerative dialogues (cogens). In general, cogens are collaborative discussions which support participants to speak their minds, identify specific examples to illustrate where improvements can be made, and identify examples of exemplary practices or counterexamples of practices that need to change (Bondi, 2013). For this study, the data generated through a series of eight cogenerative dialogues were analysed using thematic analysis. Thematic analysis revealed several themes which will be outlined in Chapter 4 and discussed in more detail in Chapter 5.

3.3 Cogenerative Dialogues

In this study student voice, in the form of cogenerative dialogues (cogens) is used to generate local theory regarding the development of community. Cogens are a learner-centred pedagogy which use student voice to help students take control of their own learning environment, through giving students and teachers the opportunity to engage in a learning relationship in which all have a responsibility to guide processes and challenge power dynamics in the learning environment (Boss & Linder, 2016). For enactment of cogens, norms must be established so all voices are equal and both students and teacher take responsibility for improving future class sessions (Boss & Linder, 2016). A cogens environment thus develops respect, rapport, and inclusion for all stakeholders (Roth, Tobin, & Zimmerman, 2002).

In the study content, cogens took the form of open discussions involving a group of students and teachers, focused on both the mathematics lesson which had just taken place and on other recent mathematics lessons. As a result of each cogenerative dialogue, action steps for both students and teacher were identified for subsequent mathematics lessons.

Analysis of cogen discussions was used to track the trajectory of students' experiences as learners and as co-participants in developing a community of mathematical inquiry. Cogens are therefore reflection in practice, being both a curriculum evaluation and an improvement tool with a collectively assumed responsibility for evaluation and instigating change (Gunkel & Moore, 2005; Roth & Tobin, 2004). As such, the use of student voice in this study thus acted both a component of the developing student agency necessary to co-construct this community of learning and as a research tool.

3.3.1 Student Voice and Agency

Cogens privilege student voice as a research tool, a method that is becoming increasingly valued in educational research (Anthony, Kaur, Ohtani & Clarke, 2013; Hunter, 2006; Lee & Johnston-Wilder, 2013; McDonough & Sullivan, 2014). The use of student voice affords students the agency to co-construct their learning environment and offers the opportunity to give feedback within the community. To ensure authenticity, it was important that the students felt able to speak freely and honestly during the cogens, so it was crucial that an atmosphere of trust was developed from the outset. For this reason, participation norms for cogens were developed in the initial cogen.

Student voice is an integral part of student agency. Bray and McClaskey (2016) describe a continuum of voice; at its most basic level students can express themselves by offering opinions and answering questions. Then, as student voice develops, students can be involved in consultation and begin to participate in decision making. This can lead to activism, such as identifying problems and generating solutions. Finally, student voice leads to learners becoming leaders, able to guide groups, co-plan and make decisions. When looking at this continuum, it can be seen that both the research method and the context for learning provide many opportunities to develop student voice.

Unlike a focus interview, which focuses on specific questions set by the interviewer, cogenerative dialogues are dynamic and increase student agency. Teachers and students with shared classroom experiences cogenerate theory in order to improve teaching and learning practices; participants shape learning, rather than reacting to existing conditions (Roth, Tobin & Zimmermann, 2002; Saunders, Averill & McCrae, 2018).

Boss and Linder (2016) suggested that cogenerative dialogues can be a powerful companion tool for educators interested in creating a learner-centred classroom where an individual reluctance to take agency over learning can be addressed collectively. Cogens and the implementation of proposed action plans can increase student empowerment, motivation and participation (Emdin, 2008; Bondi, 2013). Students have increased agency as they become less dependent on the teacher as they navigate their own learning and become more dependent on each other as experts and supports (Bondi et al., 2016; Saunders, Averill & McCrae, 2018).

3.3.2 Role of the Researcher

The researcher is an experienced full-time primary school teacher with school leadership experience and additional experience as a specialist mathematics support teacher in accelerated learning programmes. The researcher was the teacher of the participants, and it is acknowledged that, as an inside researcher, there is the potential for subjectivity and bias within the research. However, the methodology supports this role. Cogens focus on the discussion of shared experiences, with participants selected from those participating in the given field (Tobin, 2014).

Greene (2014) described insider research as research conducted within a social group, organisation or culture to which the researcher belongs. Being an inside researcher with a good initial understanding of the case and an established rapport with the research participants, can foster greater trust, more honest responses, and more natural interactions than if the researcher were an outsider (Greene, 2014; Unluer, 2012). Indeed, tor cogens to be effective the researcher "needs to be part of the community and experience it as closely as the other participants' (Stith & Roth, 2006, p. 9). Rapport and knowledge of the participants could make it easier to interpret the data as intended; rather than corrupting the data, participation is a way of understanding it.

In taking an extreme position, Stith and Roth (2006) claim that it could be considered unethical to be a 'fly on the wall' in such a study: that is, being a non-participating observer who may not have the opportunity to fully understand what was being observed. However, one needs to acknowledge that disadvantages could include a lack of objectivity and possible bias. For example, over familiarity with the group could result in data which is more obvious to an outsider being overlooked (Unuler, 2012). As such, it was important to be aware of the possibility of bias and to ensure familiarity did not lead to a narrowed or subjective analysis.

3.3.3 Developing a Learning Community

Bondi (2013) described the many advantages of using cogens to build a positive learning community. Cogens can provide a process to better understand what is going on in the classroom, addressing classroom dynamics and helping resolve inequities. Through cogens, students are able to develop student agency as they have the opportunity to contribute to their learning and have an impact on class outcomes. Students can take time to think about how they learn best, considering their own needs in the context of the collective. This shift from individual to group needs can lead to a greater awareness amongst students of differing perspectives and needs, so peer support is developed. Tobin (2014) suggested that when rules of cogenerative dialogue are followed, with an even distribution of talk and respect amongst participants, a mutual focus and a feeling of solidarity can be established among participants. Thus, in addition to the practical suggestions for developing the community, engagement with the cogen process would likely in itself be beneficial for strengthening the learning community.

3.4 Research Setting

The research took place in a full primary school in a semi-rural district. At the time of the study, this decile 10 school, with a roll of approximately 150 students, was in the process of transitioning from single-cell class teaching to flexible learning spaces with co-teaching.
Students involved in the study belong to one of two parallel year 6/7/8 classes which still use a single cell classroom environment but utilise co-teaching practices The cogens took place in a withdrawal room and involved students from the researcher's own mathematics class.

3.4.1 Participants

Fourteen students, six boys and eight girls, accepted the invitation to participate in the cogen and represent 45% of the Year 6 and Year 7 2018 cohort. One student left the school at the end of 2018, after the first cogen discussion. The students involved in the cogens belonged to the same mathematics class.

Research participants are broadly representative of the cohort (see Tables 1-3). However, there are proportionally more Year 7 /8 students and those who identify as NZ Europeans. No student who identifies as NZ Maori elected to participate. Based on current achievement data, the group was representative of achievement levels across the cohort, although no boys who were not achieving at the expected level were among the participants. From the teacher/researcher assessment, all participants were students who were characterised as engaged learners, which is noted as a limitation to this study, Thirteen students participated in the first cogenerative dialogue: smaller groups of between eight and ten students participated in the subsequent seven cogens. It was originally anticipated that six to eight students would participate each time, a number which is common for cogenerative dialogues. However, to meet expectations of students who volunteered to be involved, each cogenerative dialogue involved slightly more students. By ensuring that cogen norms were adhered to and by monitoring student participation in each cogen, care was taken that the larger size of the group did not reduce a student's opportunity or willingness to participate. In practice, having a greater number of students able to participate across the eight cogenerative dialogues allowed for student absence or withdrawal from the research and gave a broader range of perspectives to the research.

3.5 Data Generation

Data were generated through a series of eight cogenerative dialogues, held at approximately fortnightly intervals from late-term 4, 2018 to mid-term 2, 2019. The cogenerative dialogue sessions lasted between 20 and 30 minutes and focused on the students' experience of both the previous mathematics lesson and other experiences in the mathematical classroom since the previous discussion. Discussions explored their experiences in terms of what is perceived to be beneficial or detrimental to their learning both in terms of their role, their peers, and the teacher actions. Students identified a focus of action/change the following mathematics lessons, and the teacher-researcher reflected on the session and used the data to inform subsequent lesson planning and enactment (see Appendix B, Table 4).

3.6 Data Analysis

The cogenerative dialogue sessions were audio-recorded, then transcribed verbatim for later analysis, supported by NVivo software. The exact categories used in the qualitative analysis were developed through thematic data analysis (Braun & Clarke, 2006), consistent with the exploratory methodology. Starting points for analysis and the selection criteria for critical excerpts included expressions of participation, evidence of collaboration and expressions of learning outcomes.

The dataset comprises eight transcribed cogenerative dialogues. As the data were collected over a period and reviewed after each cogen, data content was already familiar before analysis began. In this way, transcribing the cogenerative dialogues informed the earliest stages of analysis (Braun & Clarke, 2006).

The main purpose of conducting a qualitative analysis is to generate themes to address the research questions (Adu, 2016). In order to "place the raw data into logical, meaningful categories; to examine them in a holistic fashion; and to find a way to communicate this interpretation to others" (Hoepfl, 1997, p. 55), data were analysed using thematic coding. Thematic coding involves "searching across a data set… to find repeated patterns of meaning" (Braun & Clarke, 2006, p. 15). It can be described as a flexible way of analysing data with the capacity to provide a rich and detailed account of the data, resulting in themes which accurately reflect the whole data set (Braun & Clarke). Thematic coding involved the identification, coding and categorizing of themes found in the data.

Analysis began with the identification of initial themes. The word query function of QSR International's NVivo 12 qualitative data analysis software determined these (Fig.1; Table 5). Word frequency analysis enabled initial inductive coding of the raw data. Using word frequency, including similar words, as a starting point, 28 nodes were created (Table 6). These nodes comprised the 17 most frequently occurring plus 11 other words found in the top 100 most frequently used words appropriate to the research focus. References were coded to the relevant node.

The second stage of coding examined each data item coded to each node and created further classifications (Table 7). The third round of coding considered the themes which were emerging from the coding. For example, the most frequently occurring word was *group,* so the node group was created, and data items containing this word were coded to this node. These data items were then coded on to the new nodes *group composition*, *group dynamics* and *group work.* From here, a conceptual understanding was developed, and the following nodes created: people in our community, working as a group, and what is group work? Data items from across the dataset were coded to these nodes as appropriate (Table 8). Other conceptual nodes created were *being a learner*, *student voice,* and *mathematical learning.*

From these, three overarching categories were developed to provide the lens through which the data could be examined. The first, *The Collaborative Learning Environment,* encapsulates the data which allows closer examination of the community itself, enabling questions such as, "Who are the people in our community" and, "How can our community work well together?"

to be addressed. The second category, *Learning Mathematics Collaboratively,* brings together data which allows for exploration of questions like, "What it is like being a learner in this community?' and "How can our learners be supported when learning mathematics in this community?" A third category, *Developing Student Agency,* collates instances of student voice and the development of student agency which see our students learning how to participate in this learning community (Table 9)

3.6.1 Validity and Reliability

In this study, objectivity is necessary for reliability. Scott and Morrison (2005) argue that if a researcher is thought to bring their own set of values to a research project, as soon as any researcher analyses data, then they are exerting power; as soon as they interpret data collected from another person, they are, "violating the way another person sees themselves" (p.178). However, if the research is conducted with an open mind and discussions analysed with respect for the participants, then the power of the researcher over the participant is reduced. In the educational context of this study and with the researcher being the teacher of the participants, there were power dynamics to consider with the potential to affect research.

At the data generation stage, there was potential for power dynamics to affect participants' honesty or openness. It was important that students knew they could speak honestly without fear of impacting their relationship with their teacher or their future learning. Norms regarding how cogens were conducted were established in the opening discussion and adhered to throughout the cogen process.

Power dynamics can influence the consent stage. To address this, participation was voluntary, and the risk of coercion addressed. It was made clear that there would be no repercussions to those that did not consent to participate, nor additional benefits to those who did. Both factors add trustworthiness to the data.

Conducting cogens over a period of 8 months allowed for the analysis of a wide range of student voice as students experienced and affected change in their learning environment. The choice of methodology allowed student's developing experiences to be explored in depth. All cogenerative dialogues were fully transcribed to ensure that all information could be set in context and no information missed or lost. An awareness of the danger of bias and subjectivity also led to an inductive coding process, embedding the findings in the data.

This methodology is not without its disadvantages. The unstructured nature of cogens makes the research difficult to replicate, and thus claims of generalisation are limited. There is also the risk of researcher bias as leading questions may influence the data (Braun & Clarke, 2006). However, overall, the detailed qualitative data that is generated and the way that student voice is authentically used to co-construct the learning community makes this an appropriate methodology to answer the research question.

3.7 Ethical Considerations

The study was designed and conducted according to the Code of Ethical Conduct for Research, Teaching and Evaluations Involving Human Participants (Massey University, 2017). Ethical approval for the study was made by the Massey University Human Ethics Southern B Committee prior to the commencement of data collection (see Appendix D).

After informal discussions and agreement with the co-teacher, the researcher sought approval for the research project from the Board of Trustees and the Principal (see Appendices E & F) Upon receiving consent, the researcher formally approached students and their parents. In early term 4 2018 all current year 6 and 7 students who were continuing at the school in Term 1 2019 received parental and student information forms and invitations to participate (see Appendices G-J).

Consent forms completed prior to the first cogen, outlined participants' rights. Recordings were made on a password protected device and data were stored on the researcher's password-protected laptop. However, if a student wished his or her comment to be excluded from any research data, such as a direct quote in publication, they could discuss this after the cogen and this would be noted. Participants were encouraged to speak freely, with ground rules about respect and group confidentiality. Stress caused by participation in the research was monitored.

As part of the cogen process, feedback was provided to the whole class following each session. Feedback based on cogen discussions, involved decisions around improvements or changes to students' engagement and teacher pedagogy. Care was taken to attribute feedback action steps to the group rather than to individuals. Other students in the class were aware of who was involved with the cogenerative dialogues but in this thesis, pseudonyms are used with no identifiable information attributed to responses.

Ethical considerations concerning the teacher as a researcher were considered. It was made clear that there would be no disadvantage to those choosing not to participate. Students not participating in the research remained in the classroom with the class co-teacher. Cogen timing was organised so that participants were not excluded from any crucial learning time.

Within the co-teaching arrangement, the teacher/researcher had full responsibility for the planning, design, and teaching of the mathematical activities for these students. The non-involvement of the co-teacher in the mathematics activities of the student participants meant that the content of the cogenerative dialogues did not impact or concern his teaching.

As cogens have the dual purpose of being used to both conduct research and to improve teaching and learning (Boss & Linder, 2016; Bondi et al.,2016), understanding gained from this study will directly benefit the participants. Therefore, cogens have the purpose of benefiting participants not only in the future but also as research is enacted (Shady, 2014).

3.8 Summary

This study uses a qualitative case study methodology. Data were generated by a series of eight cogenerative dialogues and analysed using thematic analysis. Three main themes emerged from the data: *The collaborative learning environment*, comprising group work, group composition and group dynamics; *Learning mathematics collaboratively*, comprising mathematical learning and being a learner; and *Developing student agency*, comprising student agency and student voice.

Chapter 4 Findings

4.1 Introduction

Providing a forum for students to discuss their experiences and thoughts about their learning environment, the cogenerative dialogues (cogens) served to cogenerate understandings which would facilitate student and teacher learning within the emergent community of mathematical inquiry. However, these research findings are not concerned with how well these cogens worked or the action plan associated with the cogens, even though these are important. Rather, the cogen analysis focuses on examining students' perceptions of the collaborative learning environment.

This chapter describes the findings of this analysis. Findings suggested that negotiating the collaborative learning environment is of primary concern for these students. The data offer a glimpse of how these students experience and work within groups, how they go about their learning in groups, and how they feel groups should be organised. Findings also considered to what extent students develop student agency through collaborative work and being able to share student voice.

The chapter begins by describing how the data provided the lens through which to examine how students experience the collaborative learning environment. The issues concerning working as a group, group composition and group dynamics are described in Section 2. Section 3 explores issues around learning mathematics collaboratively: status, learning mathematics in a group, group-worthy tasks and providing support. Section 4 examines the role of student agency and Section 5 provides a summary of the findings.

4.2 Developing the Collaborative Learning Environment

Amongst the most frequently used words in the data were those related to collaborative learning. *Group, work, people, talk, together* and *questions* all appear within the 13 most commonly occurring words. Other high-frequency words which can be grouped thematically alongside these words are *confident, roles, friends, communicate, listen* and *explain*. From these words, the following themes relating to the collaborative learning environment were developed: *working as a group, people in our community,* and *learning mathematics.*

The word *group*, recurring most frequently across the cogenerative dialogues, can be categorised into three areas: group work, group composition and group dynamics. As the second most frequently used word in the dataset, *work* was categorised as either a specific task or as one of the verbs *to labour* or *to function*. Findings indicate that concerns with work, whether this be the nature of the work or the dynamics required to work effectively as a group, were central to the students' experience of the learning community.

4.2.1 Group Work

Group work requires group-worthy tasks, tasks which use the combined skills of a group. Working in this way can foster the development of both collaborative skills and more complex mathematical problem-solving skills (Boaler, 2015). From the findings, participant students showed awareness that effective group work is essential in a community of learning. They knew of the need to work collectively in order that everyone understands:

Charlotte *Yeah, but if someone doesn't understand you can't just keep working without them. You know it's meant to be a team effort.* (Cogen 2)

For some students, working together and solving problems as a group provided a sense of satisfaction. For many, a significant attribute of successful group work was knowing how to seek help in a way that allowed the collective capabilities of the group to help the individual achieve success beyond which they would be capable as an individual:

Ella *I feel more accomplished when I am in a group.* (Cogen 8)

Lucy *You can help each other and go on much harder problems than you normally could.* (Cogen 8)

Sam *I think my group works well because they aren't afraid to ask if they need help.* (Cogen 4)

However, findings revealed that students did not always feel successful. They may not have understood the mathematics and may have lacked the skills to gain understanding. Students noted that not knowing what to do, or a lack of understanding led to feelings of frustration and inadequacy:

Daniel *...but then they didn't reply. I asked them a few times later but Charlotte said, "Why didn't you listen, we're not going to tell you anymore." They gave me my* *chance. I decided not to do it because I wanted to find it out myself but then after that when I wanted to find out how to do it, they said they didn't want to help me.* (Cogen 2)

Notably, early in the process, the cogens revealed a degree of passivity in those who find mathematics difficult; the onus seemed to be on group members to provide explanations to those who do not understand. The perceived inefficacy of these explanations compounded feelings of powerlessness:

Lucy *Yeah, and when they try and explain it to you, they just say it in a few words and then continue.*

Daniel *And you still don't understand what they're saying.* (Cogen 1)

Conversely, students may have felt powerless because they were aware of the need to work as a group but were unable to help group members who did not understand:

Ella *And at the opposite end of the scale, if you're with people who don't understand it at all and you try to explain it five, six, seven times and they still don't understand so you're like, "I've explained it loads of times, why don't you go and ask the teacher?" and they're like, "I still don't understand it" and I'm like, "I've told you…"* (Cogen 1)

As working as a group requires contribution and effort from all group members, unequal levels of participation impacted on all students and was frequently referenced across the cogens. Some references hinted at developing anxieties, while other discussions centred on a sense of injustice:

Ella *I think everyone will agree that you see someone who is, like, really working hard and there are those people who are just sitting there and talking to their friends and not really doing anything at all.*

Charlotte *It's also really hard in group work when some people are doing lots of work and other people aren't really doing much at all.* (Cogen 1)

Across the cogens, it was evident that non-participation took various forms. Initially, there was much discussion regarding off-task behaviour resulting in noise levels that were detrimental to effective group work. Perhaps only half the group would be working, while others would sit back and take the opportunity to chat with their friends, allowing others to do the work for the group. Whatever the reason, lack of participation was a continued source of frustration for some; students who put a lot of effort into trying to engage and support their fellow group members appeared perplexed by a lack of participation:

Katie *No, but he's like...I don't think he knows how important maths is. It's sort of like he's just there to sit.* (Cogen 4)

Some students were keen to express their belief that the key to understanding is listening. The notion that if you listen, you will understand, suggested that a key requirement for successful group work is to pay attention:

Sam *People who don't understand, they should just really listen to try and understand.* (Cogen 2)

This belief added to the frustration for some students in their efforts to help others with their understanding:

Ella *Well, I read it out to everybody and one person listened and we started working on it and then I look up at the other two people and they're like sitting there, not doing anything and I'm like, "Do you guys understand?" and they're like, "We don't know what the question is," and I'm like, "I read it out to you, twice, and you don't understand, and you actually have to try," and they just sit there. So.* (Cogen 2)

Despite the challenges, a general enjoyment of group work, especially due to the social aspects of working collaboratively, was evident across the cogens. These students wanted group work to be successful and viewed the off-task behaviour or the non-participatory practices of others as a hindrance to successful group work:

Tane *Yeah, it's probably half of the people are doing the work in the group, and half of the people are like…*

Lucy *Some people go off topic and don't get the work done.* (Cogen 1)

Upon reflecting on our progress with establishing group work, the issue of "what constitutes working as a group" emerged. Do these students consider group work to be group work if group members are not always working together? Does breaking off, working independently for a while and then coming back together constitute group work? Although some students liked to be able to have time to think on their own and then come back together to share ideas, discussions explored whether those who 'opt out' of working in the group were not pulling their weight within the group. As Lucy explained:

Lucy *You should like to try and push on and be, like… I know that in our group there was only, like, two of us trying to do it, and we both didn't really know and we were just trying to work it out, and we were just having moments where we were just, like, we can't do this. We couldn't really do it by ourselves but we kinda have to because no one else will.* (Cogen 2)

While teacher observations noted that group work sometimes drew out the competitive side of students as groups vied for being the first to complete a problem, it was clear that as cogens progressed so did the recognition that the development of social and communication skills gained during group work can benefit learning:

Katie *I think it's good to work in groups, like it does help with communication skills and like the social side and stuff but it's also good to work independently and like, buddy work as well because then you can think for yourself sometimes.* (Cogen 8)

However, the final cogen confirmed that some participant students were still invested in more traditional ways of learning mathematics. As Ella noted, there are different ways of learning and she did not necessarily see group work as being able to fulfil all her learning requirements:

Ella *I think it's good to have a mix of this new modern learning style but also with our usual textbooks and just sit with a partner and do it, 'cause I would prefer to do that than sit in a group and work out a problem. Like, just write equations over and over because that's how I learn.* (Cogen 8)

There was also discussion around whether working as a group is more suited to some aspects of mathematics than others, and that it is difficult to make someone work within a group if they really do not want to:

Daniel *I think some people find it that with different equations they'd rather work independently or in groups.*

Ben *So maybe people could have a choice.*

Daniel *'Cause forcing people to act differently isn't going to work.* (Cogen 3)

Such comments indicated that these students were still transitioning from more individual work to working within a learning community. As the community was not yet functioning optimally, students appeared to be experiencing both frustration and success during group work. Until they can consistently feel the success of group work, they may retain an attachment for the less challenging environment of individual-based work.

4.2.2 Group Composition

One word featured frequently in the data is *people*. Our community was made up of a diverse range of people who needed to be able to work collaboratively. According to the data, these people can be smart, confident, or hard working. They may be good at mathematics and may have differing levels of motivation. They may be your friend, or perhaps not your friend but someone with whom you would associate. Some people may be prone to being easily distracted and others may or may not like a noisy environment. It was clear that the academically and socially diverse students in our community had different personalities, attitudes and preferences towards learning and this made working in groups challenging:

Lucy *Sometimes, some of the groups you are in, like the group me and Rachel were in, well, we just kind of wanted to be in a different group, and we had been working with the group for maybe three weeks, and we just didn't really like it.* (Cogen 4)

A strong and recurring discussion point in the cogens was that group composition impacts significantly on an individual's experience of collaborative learning. As such, organising groups in order to optimize the learning experience and foster collaborative skills became a priority focus.

There was much discussion around whether it was preferable, or indeed beneficial, to work with friends. Students were aware of the dynamics when working with friends and, although they saw some benefits for such grouping, the general feeling was that groups work better when they are not based on friendship. However, as Ella argued, there was a strong belief that there has to be some social connection for effective communication within group work:

Ella *Yes, but also on the opposite end if you're not with friends and you're supposed to be working in a group you end up doing it independently because you don't really connect with those people.* (Cogen 1)

In revisiting the definition of *friend.* Ella expanded this to mean someone with whom you can communicate, noting that working with such people does make discussion easier, increasing the inclination to contribute and take risks with thinking:

Ella *I think it might be the people you feel comfortable communicating with. You know like, you're supposed to go out of your comfort zone or whatever, but when I am with people that I often talk to, or I often tell jokes to or hang out with, I feel more comfortable talking to them and explaining my thinking.* (Cogen 7)

Disadvantages to working with those you would not normally choose to work with were also raised. While working with friends can encourage more open discussion, working with those you may not get on with sometimes made group work less productive:

Ella *Sometimes people in your group have grudges outside of maths with you and decide not to say anything.* (Cogen 8)

However, as the cogens progressed, students became more open to working with students other than their friends. Groups that may have struggled to work together initially began to develop collaborative skills and began to recognise each other's strengths. Indeed, diversity within groups, whether this be social or academic, was recognised as beneficial because an individual's strengths become important to the group as a whole:

Lucy *We have a good combination so everyone, like, they're not all good at the same thing so they might be good in different areas.* (Cogen 3)

Alex *I reckon our group that we have now, we weren't really working together as much but now, from what Lucy said, we're getting to know each other's strengths and things.* (Cogen 4)

Sam *I also think that some people are better at explaining than others so maybe pair people that you know are good at explaining with those who are not so good.* (Cogen 2)

Diversity in strengths enables mutual support but this can become more difficult when group members are working at different academic levels. Heterogeneous grouping presented challenge. Not only did students find it "*hard when people are at different levels"* (Cogen 1), but the perception that group members were less academically able than themselves led to some students feeling that they lacked support:

Anna *If you wanted help with something but the people in your group were really not, like, as good, it's harder 'cause they don't really know what you're talking about and it's like, oh well, how do I explain this, or like, if you do this, something like that...without any help*. (Cogen 1)

Having all group members at a similar academic level may have made it easier for some, reducing the frustration levels of those who experienced little success when helping group members. Some students reported the benefits of homogeneous grouping in terms of both reducing the likelihood of dominance within the group and raising the contribution levels of group members:

Rachel *I think our group worked really well together because we are all kind of at the same level at maths, and not one person just takes off and does all the maths.* (Cogen 7)

Students who worked in groups that were more homogeneous may have experienced group work differently, which may have made it easier, and for some this was perceived as unfair:

Ella *Yeah, but the problem is in your group you have the people who are motivated to learn their maths and they're good at maths and know that they are good at* *maths and not just the people who are unmotivated. So, it's an easier work environment for you and your group.* (Cogen 4)

Nevertheless, students appreciated that a range of levels within the group can help the group with the mathematics. It is interesting that year levels were mentioned, indicating perhaps that Year 7 students perceived themselves to be less able than their Year 8 classmates:

Teacher *Are you getting the explanations that you need?*

Rachel *Yes. Especially with two Year 8s!* (Cogen 4)

Lucy *Sometimes when you are in a group with older students, it's not the best. There was a really good group I was in ... we worked really well even though I would think that we wouldn't be able to, and we're all, like, the same year but... I think that it was a really good combination, sometimes when you're with older students it is good and sometimes it's not.* (Cogen 3)

In describing the optimum group, Sam suggested that those who found mathematics more challenging were prepared to work harder than those who found mathematics easier and thus proposed that groups should be comprised of both categories of students. He nicely justified the benefits as follows:

Sam *I have a feeling that if you put a group of smart people together and a group of not so smart people then the group of smart people, they won't all be working and they won't all be learning but all the not so smart people will be trying their best, so if you pair a smart person with a group of not so smart people it would help the smart person to teach them and to also help them actually learn from what they are doing.* (Cogen 4)

Gender diversity within groups was felt to be beneficial, although this was not the case initially when it was thought to inhibit group cohesiveness:

James *Liam is the only boy in his group. Maybe he feels kind of outnumbered.*

Teacher *... Do you think that gender makes a difference?*

Several *No.*

Alex *It's actually better two girls and two boys.*

Katie *Well, I don’t think Liam minds being with the girls, or boys, it doesn't really matter.* (Cogen 4)

While discussion on group composition evolved, it was evident that the composition of the group remained an ongoing concern for some, perhaps presenting as many difficulties as the mathematics itself:

Anna *Well, it wasn’t really the problem for me. It was just like, our group got completely dismantled so I was put in a completely different group.* (Cogen 7)

4.2.3 Group Dynamics

The findings provided a sense of how group dynamics affected a student’s experience of learning mathematics collaboratively.

Lucy *I think because two of the people in our group were just messing around and they weren’t even doing anything. They didn’t even want to listen and they were just messing around with everything they could play with, and we were just trying to explain it to them and they just wouldn't listen. I feel they should just be in a group that they want to work with them and they want to work with you.* (Cogen 2)

Teacher *What makes it harder in a group?*

James *Too many people to argue. I don't usually argue with myself.* (Cogen 8)

Over the period of the cogens, groups became more settled but even groups that usually worked well together had times when they did not function effectively:

Lucy *Normally our group works together but today it just fell apart!* (Cogen 6)

Even with careful consideration given to group composition, the task facing this learning community from the outset was to find a way to stay focussed:

Daniel *So once you’ve got all the good people…*

Teacher *So, you're talking about the groupings?*

Daniel *Yes, once you've done that, make sure they are actually on topic***.** (Cogen1)

Off-task behaviour caused difficulties for those trying to work collaboratively. From the first cogen to the last, non-participation remained an issue:

Katie *It's definitely different for different people. Some people who want to get it done, work hard. Some people are like, oh, it's just activities. It's not like, important*. (Cogen 1)

Katie *I think with my group now there's just no point in telling them to join in and stuff because one person just completely goes off-topic, completely works by himself and then, like today, well he was fine but another person joined the group and he was definitely off topic and…*

Teacher *Did that affect your maths learning?*

Katie *Yep, because me and Jess were constantly trying to get him to do his maths, and to, like, encourage him to read the question and we were like explaining it to him, so yeah.* (Cogen 8)

Responses indicated that differing levels of motivation and learning dispositions may affect group dynamics. Cogens explored whether levels of engagement were dependent on personality:

Ella *Yeah, but I also think it's a personality type. Like, I'm the kind of person who really wants to work hard, and there are some people that really don't care. They don't care what the teacher thinks.* (Cogen 1)

Ben *I don't think the other two want to solve the actual problem. I don't know how on task they are and if they want to solve it.* (Cogen 3)

Students also suggested that the mathematics itself could affect engagement, possibly because it was either too easy or too hard:

Ella *I also think that it depends on the topic, like if it's something you find really easy you just get down to it but if it's harder you might not have as much perseverance and just start talking to your friends and stuff like that.* (Cogen 1).

Ben *Yeah, some people might find it more engaging if it's harder, it'd be like "Yay, it's hard, now I can work on it really well. But if it's easy, they won't.* (Cogen 1)

When students were enjoying their work, they reported that they were more likely to be motivated and engaged, which aided collaboration:

Daniel *I think we worked pretty well together I think because most of us were enjoying the task. Most of the time, like, we kind of enjoyed it.* (Cogen 5)

In tackling the issue of engagement, cogens acknowledged the benefits of having a supportive group that works well together. Improved communication may have helped this positive dynamic:

Lucy *Our group is working well because we are all sort of on the same page and we all agree with each other, and kind of communicate better.*

Rachel *Um, our group seemed to, like, as Ella said, communicate well, and we've got like a mix of learners, around like, we're not all, "Oh, we all want to just to do his now and get it all finished," we're all like, sometimes we'll have a break and talk to each other, and then get back on work, instead of just going work, work, work, work, work.* (Cogen 4)

Interestingly, discussion in the initial cogen offered the suggestion of collaborative skill assessment as a way to ensure effective collaboration:

Sam *I think there should be a group test to see how well the group works together, not like of the questions but more of the group working.*

Teacher *So, do you think I should ask you questions about how you work on certain things so you can do some self-monitoring?*

Katie *So you can see how everyone is working together, so you can see you're not doing all the work.* (Cogen 1)

Early recognition that non-participation and poor collaboration caused problems also prompted discussion on whether assigning roles to group members would improve group dynamics. Possible roles identified as important were that of *encourager, listener* and *leader*. An *encourager* was thought necessary to increase the motivation of the group, especially when they were struggling with difficult tasks. The *listener*, who could also be the disagreer or challenger, was assigned to monitoring group discussions to ensure they remain mathematically robust. While leadership was recognised as an important role, later discussions indicated that it was not an easy role to fill, as it takes social and mathematical confidence to assume and maintain this role. Students posed the experience of having a natural leader in the group as one way to support collaboration:

Ella *Our group was… um… good because I think we switched the group around a bit and we had a natural leader in our group who is quite confident with her maths.* (Cogen 7)

Confident people had an impact on group dynamics. When confident people showed strong leadership and encouraged an inclusive work environment the impact was positive:

Teacher *Do you think you might have been reliant on the confident person?*

Anna *I don't think it's reliance, I think it's the fact that they can take leadership...*

Teacher *So, do you think losing somebody that's confident made a difference?*

Anna (Nods head)

Teacher *Why does it make a difference? Is it mathematical ability, or is it confidence?*

Anna *I think it's confidence, because you definitely need someone to encourage you as well as, like, being confident, and just like knows what to do and can just teach you.* (Cogen 7)

Charlotte *Caitlin is quite good in the respect that she sort of gave out jobs, and there wasn't one person who just wasn't doing anything, so everyone had to work as a team.* (Cogen 7)

Being confident does not always lead to dominance, but it appeared that sometimes mathematically strong group members may have reduced the need for others to contribute:

Ben *Usually Tane gets the problem pretty quick so we don't really have time to discuss it or think about it before he's got the answer so today, without Tane, we were talking about it more and working together a bit more.* (Cogen 4)

Despite discussion around assigning roles to group members to address non-participation, with the intention that roles be taken up as and when required, the final cogen revealed that non-participation remained a problem, albeit less of an issue than earlier. While group work was developing and becoming more focused, it was not yet known whether a general awareness of roles may filter into collaborative work and support more productive group dynamics going forward:

Teacher *So, are we getting any better at listening to each other's ideas?*

Ben *Kind of.*

James *Yeah, kind of.*

Sam *Yes.* (Cogen 5)

4.3 Experiencing the Collaborative Environment

There are two aspects to learning mathematics which our students were required to adapt to: learning to work collaboratively and learning mathematics through group-worthy tasks.

Thematic coding revealed the frequent use of the words *learning, understand, smart,* and *maths.* References to *maths* were sorted into three categories: *maths time*—the actual lesson itself; *specific maths*—references to descriptions of doing mathematics; and *being good at maths*. References containing the words *learning* and *understand* were combined with more general references to mathematics to form the category *maths learning.* References including the word *smart* were included in the category *being good at maths.* Other words which frequently occur in the data were *confident, easy, struggle, challenge, feel, enjoy, solve* and *problem.* Examining how these words were used in the data gave an insight into what it is like being a mathematics learner in this community, providing guidance on what can be done to support our learners with mathematical group work.

4.3.1 Status

Status and participation in the learning community appeared to be affected by whether or not a learner was perceived as being good at mathematics (Marks, 2014). Findings suggest that students assigned levels of competence to their peers which were often relative to perceptions of their own ability. Students described these levels as *good at maths*, *smart*, *not so smart*, or *unsmart.* For example, Sam defined an unsmart person as:

Sam *Someone who doesn't understand the difficult equations which you can easily work out.* (Cogen 4)

A student's perception of their own ability may also affect participation levels:

Rachel *I think our group is pretty good, it's just I would say I'm probably bottom ranks of smartness in there, so ...* (laughs) *it's… I dunno, I find it harder to get it as fast.* (Cogen 6)

Ella *But, also not just a strength in maths, like, but especially when it's complicated on the piece of paper if you struggle with reading or writing it's also really hard for you.* (Cogen 4)

Students affirmed that displaying abilities may not always be a comfortable or desirable thing to do. For example, Anna and Ella provide descriptors of mathematically modest scenarios:

Anna *So, well, say, for example, people who are really great at maths, they just back off and say they're really not.*

Ella *Yeah, people don't really like to say, like, I'm really good at maths, 'cause then you like, they might tease me, they like… copy.* (Cogen1)

Modesty could be due to an awareness that there are many aspects of mathematics that they do not find easy, so they cannot accept that they are as good as people believe them to be. On the other hand, as Ella suggested, there may be concerns that others will take advantage of this ability or see it as a negative rather than a positive attribute. Descriptors of being good at mathematics noted that ability could vary according to context:

Lucy *Sometimes it all depends on how your brain is and sometimes some people can find that some bits of maths are really easy and then the other bits of maths they don't understand at all. It really just depends on the question.* (Cogen 4)

The mathematical confidence that can develop through supportive group work feeds into further successful collaboration; collaboration benefits from those who are confident enough to show leadership and motivate others in their learning:

Katie *I think we've got a good confident person in our group but he's really good at asking questions, getting us to ask questions and explaining it to us, which helps.* (Cogen 7)

Ella *Well, I think we already had two natural leaders in our group, but the thing is, in our group nobody was particularly confident with their maths. And when we added the person that was very confident, we were like, oh yeah, we like, it was just like a clock.* (Cogen 6)

On the other hand, a lack of confidence may reduce the willingness to collaborate and take risks with mathematical thinking:

Maia *Well, in my old group no one was very confident so we just sat there doing it by ourselves, whereas last time was really good as we were just working together.* (Cogen 7)

Katie *There's, like confidence. For example, if you get it wrong you don't want people to make fun of you.* (Cogen 1)

Perhaps unsurprisingly, those who did not perceive themselves as good at mathematics typically expressed a lack of confidence, sometimes to the detriment of others wanting to support them:

Ella *Well, the thing is that, Daniel, we gave you the sheet and you read it and said, "I don't understand. I'm not good at maths. I'm not good at maths. I don't understand. I don't know what it is." And, it was just a bit like, well, whatever I say it is not going to help.* (Cogen 2.)

Thus, while the students affirmed that working as a group rather than as an individual could make problem-solving easier, working with others was tempered with perceptions of competence and status as evidenced across the cogen discussions.

4.3.2 Using Group-Worthy Tasks

While building our mathematical inquiry community there was a deliberate effort to use challenging group-worthy tasks to support students' development of problem-solving skills. Solving such tasks became easier as group members shared ideas and offered alternative solutions:

Anna *Yeah, it's just like, we always have one way of doing it, we're all on the same page, and it as kind of like that page was turned and all of a sudden we had to do something else, so it was kind of weird.* (Cogen 5)

However, for some students, trial and error remained a common approach to problem-solving, although there was evidence of more systematic thinking routines developing:

Sam *If I had got one answer, and I wasn't sure it was right, I would check whether it was right, and check through all the other possible answers in the 11 times table and see if I could get them as well and see what was probably the best answer.* (Cogen 3)

As part of the professional learning programme the role of launching a problem was highlighted. However, cogens highlighted students' concerns that, despite the teacher launch, not all students felt able to make enough sense of a task to be able to begin attempting the problem. It appeared that what was deemed well-explained by some was not explained well to others:

Katie *Like today, the question was quite well explained, like sometimes with the more harder questions they're not explained as well on the piece of paper, they sort of, sometimes people would think they are explained well but sometimes people, well, that's not their strength in maths. (*Cogen 4)

Interestingly, students also described how some problems were a better fit for them than others:

Ella *I just thought, like, me and Caitlin saw the problem and we're just like, oh, we know what to do. ... I think the way that I usually think was how we had to figure out the problem, so that was a really good problem for me, personally. I didn't find it that difficult.* (Cogen 6)

Group-worthy tasks were often used to revisit concepts in a different context, or to introduce new material, allowing students time and space to develop their own understanding of new concepts. This was appreciated by some students:

Ella *Yeah, I think what kinda made it easier was that the problems were based around the stuff that we were learning in class not just like random stuff.* (Cogen 4)

Expecting challenge, the discussion in Cogen 5 raised the surprise of students who assumed that all problems will require effort to solve. This may mean they overthink and miss more obvious solutions. Solving problems, whether difficult or not, gave a sense of satisfaction:

Alex *Our group were looking, as we got the equation we were looking into it too hard, like 8 times 7 divided by 3 times 2, something like that.*

Teacher *What were you trying to do at that point?*

Alex *We were looking into it too much, instead of just doing it easy, like Daniel did.*

Teacher *It was actually surprising how straightforward it was, wasn't it?*

Ben *I thought it was too simple, then I said, wait, that is right.*

Daniel *I was just going off what my first idea was. I never thought I'd actually get it right though.*

Teacher *You went just off your first idea? And it worked*

Daniel *Yes, and then I'm like, it's actually right.* (Cogen 5)

While whole class discussion concerned the different ways success can be achieved in problem-solving and drew the students' attention to the various ways they could contribute to group work, it was apparent that different group members had different expectations of what it means to be successful when solving group-worthy tasks. For some, finding the answer was important, perhaps even the sole objective of the exercise:

Katie *I think some people think that it's all about getting the answer but not really about talking with your group and going through it, I need to get the answer, otherwise I won't be good at maths.* (Cogen 4)

For others, the mathematical learning that happens during group-worthy tasks was recognised as an equally important aspect of learning:

Lucy *I think it's like, when you're working in groups it's more about working as a group and trying to find the answer than it is getting the answer*. (Cogen 4)

4.3.3 Learning Mathematics Collaboratively

Learning mathematics as a group invariably means that, with any task, there will be some group members who understand how to approach the task and others who do not. The language of the problem or the mathematics itself may present challenge, but even with consensus on what needs to be done to solve the problem, there will be times when the mathematics proves difficult for some, if not all, group members. Findings indicated that when everyone understood the problem, working as a group became easier:

Teacher *What sort of things helped you work as a group today?*

Alex *Everyone understood it.* (Cogen 4)

Moreover, it was recognised from the outset that working as a group was easier and more productive if the group worked well together:

Sam *If you have a hard question you all work together and it makes it relatively easier. If you don't have people you work well with then you will solely be on the difficult question.* (Cogen 1)

Working as a group also became easier when students were motivated. Different reports noted that it could be the task itself that is motivating, or the element of competition and the satisfaction in solving a problem quickly that propels the group:

Teacher *Your group did really well, Daniel. What do you think was different about your group today?*

Daniel *I don't know. Maybe just wanting to find out the answer*. (Cogen 5)

Tane *It's nice when you are working in a group and you beat a group to solving the problem*. (Cogen 8)

A lack of understanding tests collaboration skills. This may have been a contributing factor to a perceived lack of effort or focus, which caused frustration:

Daniel *No. But they're just sitting there and talking about things. Not doing enough.* (Cogen 6)

Rachel *Because some people just wouldn't listen.*

Lucy *And they weren't like interacting and trying to work it out.* (Cogen 2)

However, regardless of the difficulty of the problem, poor collaboration made learning mathematics more difficult:

Ella *I didn't think that the actual problem was hard, I just think our group was really dysfunctional so it took us a lot longer.* (Cogen 6)

Students also found it difficult to know what to do when fellow group members felt so frustrated they were unable to make progress:

Teacher *Have you ever come across people when you are working with them who may be feeling that and you're not?*

Daniel *Me. Sometimes*

Teacher *Are you able to empathise with anybody who may be feeling that in your group?*

Daniel *Yes.*

Ella *Try to. Not all the time though!*

Ben *Try to.*

Ella *I'm like, "Why don't you understand?"*

Teacher *Would that possibly make you more sympathetic to people who were struggling?*

Anna *Our team works really well together, but that kind of leads to if one person is frustrated, we all get frustrated.*

Teacher *What could you do to stop that?*

Anna *Try and explain, but if all of us don't know how to do it...?* (Cogen 5)

Students may also become frustrated in this learning environment because they prefer working individually. Collaborative work is a change for them and adapting to this may take time:

Daniel S*o I was like a kind of reluctant person in Ella's group and the reason I was, was because I wanted to figure it out for myself, yeah*. (Cogen 2)

However, for some, they were aware that working within a group offered a means of support, noting that peers' acceptance of differences increased their confidence:

Rachel *Our group were pretty good because we're all quite tight and we all communicated well with each other and… yeah.*

Teacher *So, did that naturally make you more confident?*

Rachel *Yeah.*

Teacher *Because you were already communicating well together?*

Rachel *Everyone was supporting each other.*

Teacher *When you say, "supporting each other'" what are you doing to support each other?*

Rachel *If you like, got it wrong, they encourage you to try again.*

Ella *And not actually laugh at you and say, "You suck at maths!"*

Teacher *Have you ever heard that?*

Ella *Yes, I've been told that, multiple times. Maybe when I was a Year 6, and it was the Year 8s.* (Cogen 7)

Discussion included different ways of approaching the mathematics problem as a group. Some students liked to have time on their own with the problem first, working individually but enjoying the security of being able to check in with the group afterwards. This check in allows for discussion, perhaps checking for accuracy or taking thinking along a different path.

In cogen 4 there was a robust discussion that clarified the distinction between agreement that the solution is correct, and that the whole group understands how the solution was derived:

Rachel *Yeah, I found like when we were doing it, we'd go off and then come back in and see if it's all right and if it's not then we'd start working better.*

Lucy *Yeah, if we got the right answers. If everyone agreed what the answer was and if everyone understood how we'd got it*. (Cogen 4)

In the following cogen, students talked about valuing individual working time within a group situation, with the collaborative aspect of the learning being the coming together and sharing work with frequent checking in to see if a potential solution has been found:

Katie *Well, I think the problem was really well explained and I think that our group got it and we sort of did like different ones. We did one big one together and that added up to something and then we split off and sort of did our own ones and met together again and then, like, sort of.* (Cogen 5)

Although individual work was again raised as an important aspect of learning mathematics, by the final cogen working well together could be summarised as group discussion, sharing ideas and working on the problem together, evidence of developing collaboration:

Alex *We talk to everybody about the problem, we work it together, share ideas and stuff.* (Cogen 8)

From teacher observations, it was noted that as groups became more used to collaborative work, students worked less as individuals and became more supportive of each other. Students also talked about *the group* rather than an individual having ideas and solving the problem, which provided evidence of a move to more collaborative practice:

Sam *Well, for a while we didn't really know what to do. Then our group had the idea to try and find the area of it so around the end of the time I think we were getting closer to the answer to the problem.* (Cogen 7)

4.3.4 Supporting Others

In a collaborative learning environment it is important that all students are supported by their fellow group members. Findings suggested that the challenging nature of group-worthy tasks means that there will always be students who struggle to make enough sense of a task to be able to begin solving the problem, creating an obstacle to group work:

Sam *Well, the people in my group aren't as good at maths as I am so I basically have to constantly keep explaining to them how to do the thing and most times they don't understand so I have to explain it better and take it slower so that I know they understand and they don't just not understand but say they do.* (Cogen 2)

For some students, feeling unable to keep up with others in the group caused anxiety:

Katie *Also, when the teacher asks if everybody knows in your group, like how to explain it, it's like, if you don't know they tell you really quickly. It's like, I don't actually don't get that...*

Rachel *And I've found previous times that I've worked in groups there's always someone that takes over and then doesn't tell you anything and just takes control and says, "Oh, you should have known that; we've been talking about this the whole time." But they're talking really fast, so you don't understand and they get annoyed with you.* (Cogen 2)

Providing or receiving peer support proved challenging. Supportive students found it frustrating if their efforts to support others were not appreciated:

Daniel *Err, we'll try explaining it to you, if you'd listen.* (Cogen 6)

Working with others requires patience, as not all group members will form understanding at the same time. Developing empathy must be developed over time: what may be easy for one person may not be so easy for someone else. Findings indicated that initial group discussions of the problems and any ensuing discourse were not always at a pace that others can follow:

Rachel *One thing I find is like, if someone is reading it* (the problem*) they'll be reading it really fast and then it will be really hard for them* (the group) *to understand and then they won't get it and they'll be talking about other stuff because they're trying to think about what you've said but they've got it all wrong.* (Cogen 2)

These instances of a lack of group support reduce students' enjoyment and participation. It was a challenge for students to remain motivated when they did not feel their contributions were appreciated:

Teacher *Did you enjoy that problem today?*

Rachel (Very quiet) *No. I just felt like I was being put down every time I gave an option and then it just made me think that there is no option.*

Teacher *Did anyone else have a situation where someone was telling you that your numbers didn't work?*

Lucy *I did. I found an answer which looked like it could have worked but she ignored it, and I pointed it out later, and she still sort of ignored it. Kind of.* (Cogen 3)

Supporting others was difficult when individuals were motivated and keen to progress with the problem but felt obliged to support others in their learning; this provided a new challenge for students:

Alex *Like, if somebody gets it, they don't just work on it themselves. The other two say, "Oh, we don't get that," and they say, "We'll tell you once we've finished." It's like, work through it while they're there.*

James *Yeah, and don't just speed up as soon as you have an idea, and saying random words. Make sure everyone understands.* (Cogen 2)

Ella *I think it was like the problem that you gave us. Um, I think once you start getting on a roll of something you just want to keep going and don't want to actually stop and explain it to your group so it's a little bit hard to stop.* (Cogen 5)

Cogen discussions concerning mathematical argumentation tended to focus more on the frustrations of peer reactions, with uncertainty about how successful they were at providing explanations that were helpful to others:

Katie *And sometimes, like when you're going through the problem, sometimes people just switch off their learning 'cause maybe it's taking too long for it to be explained or they don't understand it or something.* (Cogen 3)

Indeed, it was also frustrating for those unable to make sense of explanations given:

Rachel *Times before I find that people use really fancy maths words then I don't understand what they mean, and they say this is the median of blah blah blah and stuff and then I'll be like, "What does that mean" and they say, "Have you not been listening this whole time?" Now I get confused*. (Cogen 2)

Students appeared to appreciate the importance of whole group understanding and were beginning to recognise that if a verbal explanation is not effective, merely repeating it will not help. However, unless they are felt to be effective or appreciated, our students may be disinclined to invest time and energy into ensuring the understanding of others. The occurrences appeared to impede the development of collaborative practice:

Rachel *Maybe try different ways to explain, 'cause they're repeating*

Katie *...The person that's explaining, so maybe they give like, a picture.*

James *Yeah, but that could take too long and you could run out of time and you should just be working on it.* (Cogen 2)

Providing mathematical explanations can be beneficial to all parties. Having to explain things to others helped students develop and check their own understanding:

Sam *Because I have to help them a lot, but that helps me learn because I'm using more of my brain and I'm trying harder… I've realised it's helping me and my learning to be with other people that aren't as smart as I am.* (Cogen 4)

Charlotte *When you're trying to explain it to someone else who doesn't quite understand, then you might find something that you've made a mistake on, cause when you're rushing through it in your head it's all good but then when you actually have to explain it you realise that it's wrong.* (Cogen 6)

Students found it hard when fellow group members could or would not contribute to solving the task. Having one person in the group who is not engaged with learning affected others who were taking their learning more seriously:

Sam *Well, in our group we have one person who just goes off on their own and just distracts us and then three of us work together, and just have one person really distracting.*

Katie *And the person, he's definitely not very good at explaining it, um, like, we're like, do you, can you explain it to us, he's like, I've got the answer, and he makes this big scene about random things and he goes off topic.*

Lucy *Sometimes there's those days when you can't do anything.* (Cogen 6)

It was also difficult when students were unable to understand someone else's ideas about how to solve the problem. Working with others to ensure they are following the task became a key focus of group work; the willingness to ensure the group rather than the individual moves forward began developing:

Sam *Well, for me and my group, it was pretty easy for me to work out by myself and then as I was explaining to them it was easier for them to listen because I know I must have been explaining it better than I usually do.* (Cogen 4)

However, having someone in the group who did not understand but could ask questions was felt to be helpful:

Lucy *She asks questions and it makes us think about it more, and understand the problem more, I guess.*

Ella *And I guess lots of people say that the only way that you understand your maths is by explaining to someone so you know that you really understand how to do it.*

Katie *And sometimes the questions that people ask you, you wouldn't even have thought of that, and that can even help you with the maths problem and then like, yeah, understand it more.* (Cogen 6)

4.4 Developing Student Agency

Students become more active participants in their learning through participating in discourse. Findings suggested that the development of classroom discourse practices was matched by increased student awareness of the need for productive discourse to further their learning. Asking questions was focused on as a means of reducing the passivity of non-participating group members:

Ben	*Yeah, Julia was asking some pretty good questions.*
Teacher	*What sort of questions was she asking? Can you remember?*
Ben	*How does this work? How does that make this? Could you say that again more clearly?*
Teacher	*So you heard some good questions. Did you hear some good questions from your group, Sam?*
Sam	*Umm.. just the occasional 'How did you get that answer'*
Teacher	*What about your group, Anna. Did you hear some good questions from your group?*
Anna	*Yeah, like, how does this work, because if this is this, does it equal that? And like, how would you get that if the problem's something else? You know.* (Cogen 6)

As noted by Katie, question asking had become a deliberate strategy in her learning:

Katie	*Whenever we do group work I personally ask more questions, like, know how to ask more questions and...*
Teacher	*Even when it's not maths?*
Katie	*Yeah.* (Cogen 8)

From teacher observations, the incidence of off-task talk reduced and discourse became more productive as the study progressed. However, discussion in the cogens did not provide evidence of concern for or awareness of explicit mathematical argumentation. Rather the focus on discourse continued to be about participating well in groups through talking about their learning tasks with others, rather than relying on teacher support:

Teacher *What do you think I can see when I watch? What can I see that shows you're working well as a group?*

Maia *Well, we're kind of like talking a lot, together?* (Cogen 4)

Students expected to help each other with their learning, but some students found it difficult to articulate their learning needs:

Alex *For the two other people in our group, they weren't actually doing anything so I asked them if they understood but they didn't, so I was saying, "What don't you understand about it?' so I was like, "I can't help you if you don't tell me what you don't understand."* (Cogen 3)

As the study progressed, groups began to work well together with reduced teacher involvement, but discussions around leadership, as described earlier, indicated that most students need more time and experience before this aspect of student agency can develop.

In the early stages, some students expressed frustrations with group members who adopted a passive role. For example, Sam noted that some learners were unwilling to seek help as follows:

Sam *Yeah, 'cause in my group they're not as smart as I am, and they don't get the problems as well as I do, but I know basically all the time that they don't really understand the problem but they are not asking me for help so I can't really do anything to help them if they don't want help. If they don't want to solve the problem*. (Cogen 3)

This frustration was later expressed again by Daniel:

Daniel *The only question the other two in our group asked us was, "What's the answer."* (Cogen 6)

However, findings suggested that many students did take responsibility for ensuring their own understanding through making an effort to understand or working on the task with more focus:

Alex	*I had to say that I didn't understand and that I'd tried to understand and I understood in the end.* (Cogen 2)

Katie	*I think this time our group didn't get distracted. We all stayed on task.*
Teacher	*Why do you think that might have been?*
Katie	*Well, we were all doing the equation, and we didn't sort of have to, tell people, encourage them.* (Cogen 5)

Student agency can develop as students become more connected to their learning. Some students reported enjoyment when learning with others, from others and supporting others. They took their role as a group member seriously, understanding their role in the group's success:

Teacher	*What sort of questions were you hearing?*
Ben	*Like, we were explaining to them how the minutes, putting it into minutes and everything, and doing that and dividing it by ten, 'cause it's ten times faster,and they were asking, "How does this work?" and stuff like that, so were just trying to explain it again more clearly.* (Cogen 6)

Students who are beginning to develop commitment to their learning are progressing towards self-driven learning. Open-ended problem-solving tasks provide an ideal environment in which to develop the desire to take learning further. The development of this can be seen in some responses:

Daniel	*After we had done our first equation we were told to move on and try for adding all the numbers to see what the highest answer would be...was possible, and we decided to do that instead of coming up with another answer.*
Teacher	*So you were motivated to do some more learning?*
Daniel	*Yes.* (Cogen 5)

Ella *We're nearly there.*

Charlotte *We're so close.*

Ella *We're so, so close. We need to..., we already knew an answer, we just didn't write it down or anything, so we'll probably just review what we did and then we'll just continue on*. (Cogen 7)

As solving group-worthy tasks can be challenging, success not only requires a supportive group and a desire to learn but also a positive attitude. It is not only the group that can affect a student's engagement, but their own mindset:

Katie *Yes, you can't just put yourself down,' cause then you won't be able to do it*.

James *And you believe it*. (Cogen 2)

Findings suggested that these students are becoming more positive and seem more likely to maintain engagement when the mathematics is not too difficult. The mathematics became more enjoyable with a more accessible problem which students believed they could do, making students feel they could persevere and have a *continuous mindset:*

Ella *You can communicate more effectively, like you have a continuous mindset and that's really hard to have when it's a really hard problem.* (Cogen 4)

Teacher *... Maia?*

Maia *Um. I really liked it, it's like what Tane said, it wasn't impossible for our group.*

Alex *I'd say about the same as Anna, about 7 out of 10 because it was possible and it was quite fun as well.*

Ella *I'd probably say about a 6 or a 7 because the problem itself was really fun but my team kind of struggled to really do it as a group.* (Cogen 5)

However, although our students accepted they must persevere on problems, Findings suggested that they did not all enjoy struggling with a problem:

Ben *How would you rate that problem?*

James *Zero out of ten. I hated it*

Teacher *Why did you hate it*?

James *Because… it was frustrating.*

Teacher *Anna?*

Anna *I'd give it a 7 because I like my team and they encourage each other but… it's still fun watching people get frustrated.* (Laughter: inaudible comments.)

Sam *I'd say it was a big struggle, probably zero to a three.* (Cogen 5)

Findings revealed varying degrees of motivation amongst participant students. They liked problems that were difficult enough to engage, but not so difficult that they demotivated.

Tane *Um. I found that one really good*.

Teacher *Why?*

Tane *It was not impossible, not super hard but it wasn't super easy as well, and there was more than one solution so you've got ones who aren't bored.*

Ella *It's like a balance.* (Cogen 5)

Working as part of a group can be motivating. For students who do not have strong student agency, performance within and between groups may be more important than the learning. Thus, it could be the group environment, rather than a desire to learn, which initially provides motivation:

Tane *It makes you try harder and try and get it before the other group.* (Cogen 8)

However, a student with strong student agency will, have a love of learning without the need for extrinsic motivation. The early stages of this can be seen in the development of a growth mindset:

Teacher *As a teacher I was trying to find a problem that was accessible to everybody in the group. What do you think I mean by that?*

Sam *Everybody could pitch in and try.*

Alex *Being able to achieve.*

Daniel *Everyone can understand it properly.*

Ella *People could switch their mindsets from just fixed to growth mindset.* (Cogen 5)

There was an awareness of the need for a growth mindset when mathematics becomes difficult:

Teacher *What did you experience today that you don't often experience in your group?*

James *Having trouble with equations.*

Ella *Struggle.*

Daniel *Yeah, struggle.*

Anna *Yeah, struggle.*

Teacher *And what does having that struggle with the problem bring out in you?*

Anna *Frustration.*

Teacher *Frustration?*

Daniel *Not a growth mindset. Fixed. You don't want to do it. You're just like, grrr…*

Anna (Sings) *See what you can do.* (Cogen 5)

Struggle can be quite hard to deal with, especially when it is a novel experience, but working on group-worthy tasks required students to persevere in finding solutions, including continuing working when errors have been made and an alternative solution needs to be found. If a problem caused frustration, perseverance and focus were required, although some groups were not ready to take their learning beyond the minimum requirement:

Lucy *You get down, kind of, because you're like I can't do this, I'm never going to work it out, I just want to give up, but when it's kind of easier you can work as a group even more.* (Cogen 4)

Sam *I think my group wouldn't take it too well because they would have worked hard to get one and I wouldn't think it would be easy to get the next one because, like I tried to get a different number from 99 but it was pretty hard for me.* (Cogen 3)

Whilst some groups responses indicated developing perseverance, in some cases focus could be lost when tackling a difficult problem:

Teacher *I'm curious to know, James and Anna, what things you had to do in your group to try and get the answer, what sort of things were you trialling? What sort of things were being said?*

Anna *Just random equations, yelling out random equations…*

James (Laughs) *Yes, that was pretty much it!*

Anna *and…*

Teacher *So, random equations?*

Anna *Yeah, we were annoyed at BEDMAS because it was...annoying.*

Teacher *So, like just random trial and error?*

Anna *Yeah…*

(Some inaudible general chatter)

It was not even systematic! (Cogen 5)

Working as a group can be a positive experience, as group success may be more achievable than individual success. However, feeling unable to make worthwhile contributions to group work can reduce student agency and cause disengagement. As indicated by the student responses, this may have been due to either difficulties in working collaboratively or difficulties with the problem itself:

Teacher *So why do you think he's not contributing much, or anybody who's not contributing much?*

Alex *Probably because you only enjoy maths if you understand it, if you know that you can get it, but if you don't really understand it you know you don't get any point in this. I don't get the point so don't understand so I'll never be able to get the answer*. (Cogen 4)

As self-confidence developed, students became more positive about challenge, seeing it as a learning opportunity rather than something to be avoided. Perseverance and risk taking become established learning behaviours. These students accepted that mathematics is challenging. Those who face this challenge with positivity are developing agency:

Teacher *You know why it was deliberately hard, don't you?*

Maia *Yeah, because you are trying to make us think lots*. (Cogen 2)

Ella *It was actually a problem, not just "Let's go and put pieces of paper together."* (Cogen 7)

4.5 Summary

Cogens revealed that the move to collaborative group work was generally enjoyed by these students. However, their primary concerns centred on the nature of the group itself–who is in the group and how well the group works together–rather than the mathematical practices that can be developed or the learning than can be occasioned through collaborative work. Findings suggested that it takes time for students to develop the skills necessary for effective collaboration. These skills require student agency and it is the development of student agency that signifies the change for the teacher and students in this inquiry learning community.

Chapter 5 Co-constructing the Learning Environment

5.1 Introduction

Participating in cogens has allowed students to collaborate with their teacher and co-learners to co-construct their learning environment. Cogens have revealed that students' social interactions and experiences within their groups are complex and have a significant influence on learning. Findings discussed in Chapter 4 revealed two important issues in this developing community, both of which centre on the nature of group work. The first issue concerns who was in the group, and the second concerns how the group learned to work together.

Drawing on literature to further interpret these findings within the context of co-constructing the learning environment, this chapter firstly discusses the implications of the social aspects of grouping and group dynamics in the establishment of group work. Secondly, the chapter discusses evolving collaborative practices of participation, help seeking and supporting others within the community of learners.

5.2 Establishing group work

Managing the social environment is a crucial aspect of creating a problem-solving environment (Smith & Piggot, 2007). Group work, with its associated social and academic challenges, embodies one of the major changes for students working within a community of learning. In terms of fostering productive interactions, conditions that were discussed in the literature, such as optimal group size, the extent of group diversity, or physical organisation of groups when working (Dillenbourg, 1999), were all prominent for the students in this study.

As the cogens progressed, it became clear that it takes time to establish groups that work well together. Initially, off-task behaviours were given as a main reason for the ineffectiveness of group work. During cogens, off-task behaviours were explored in terms of a lack of engagement or an inability to participate due to mathematical difficulties. However, there is the possibility that these behaviours were more to do with status as students seek to establish identity and jostle for position in groups. Langar-Osana (2018) argued that participation constructs positionality, which affects further participation; thus being able to enact authority influences engagement and the development of mathematical identity. From this perspective, off-task conversations serve to resist domination by peers with higher academic status; those marginalised from engagement in mathematics can use off-task conversations to position themselves with more social power than they would achieve through on-task interaction (Langar-Osana, 2018). If this is the case in this community, off-task behaviours could be addressed by taking care to assign academic status to all students (Boaler, 2006a). This would require scaffolding small group interactions, scaffolding students to take a stance when engaged in mathematical argumentation, and ensuring all students are viewed as academically competent (Hunter, 2007).

For the teacher, building a classroom community involves letting go of some classroom control to open spaces for students to interact (Amindon & Trevathan, 2016). Organising the classroom so that all students were able to sit together as a group of four was the first step to creating this environment. However, findings indicated that this community was still developing the collaboration necessary to successfully work in groups of four, with students often preferring to split into smaller groups, or work as individuals before coming back together as a group. Working as a pair was perceived by students as a positive step towards working as a group (Askew, 2012) and, if followed by joining with another pair for consultation around the task, may be a useful way to gradually develop confidence around collaboration skills (Brown & Thomson, 2000). Although a larger group does have the advantage of providing a greater diversity of skills and perspectives, this diversity would not be beneficial if collaboration skills have not been established (Brown & Thomson, 2000).

The cogens explored whether working both individually and as a group constitutes group work:

Ella *Yes, because you're still working your individual maths skills and your group communication skills or whatever, it doesn't have to be one thing or the other.* (Cogen. 8)

Preferring to work alone at times may not indicate a lack of collaboration but may be indicative of growing personal autonomy. As Brown and Thomson (2000) noted, working within a supportive learning environment where you learn how to listen, challenge the ideas of others, and recognise alternative viewpoints can help learners develop the capacity to work alone with confidence and develop personal autonomy.

5.2.1 Social Grouping

Working effectively within groups can be seen as the very essence of collaboration, and it was evident from the initial cogen that students attributed the effectiveness of group work to group composition. Early cogen discussions revealed that students prefer working within their own social groupings, believing friendship with collaborators to be important (Edwards & Jones, 2003; Liljedahl, 2014). However, while the social nature of the group appeared to make the group work more enjoyable, it was soon recognised that working with friends, for some students, increased the likelihood to off-task behaviours (Le, Janssen & Wubbels, 2018). Based on the recognition that more controlled, balanced groups could potentially lead to better work habits (Brown & Thomson, 2000; Liljedahl, 2014) students recommended trialing new groupings made up of a mix of those you know that you get on with and others who you do not usually choose to work. The thought was that there would be enough social connection to propel discussion, but not so much that focus is lost.

Possibly in recognition of the value of learning in a group (Boaler, 2006a), discussions in the second cogen shifted from a focus on personal preference, towards consideration of how they worked as a group. Some suggested the group should take time to reflect on their group collaboration. For example:

Anna *At the end of the session ... just talk about what you did well and stuff, and if you did that you could stay together and if you didn't work well you should just split up.* (Cogen.2)

By the third cogen there was an awareness that not working with your friends is beneficial. That is, group composition that worked well in the short term–such as working with friends–was not necessarily viewed as optimal for achieving long term objectives (Galton & Williamson, 2003; Liljedahl, 2014). Thus, there was a sense of students and teacher requiring time for experimentation when developing effective groupings, beginning with ways to address classroom noise and off-task behaviour. After considering the social aspects of grouping, thoughts then turned to alternative ways of grouping.

5.2.2 Group Dynamics

In addressing the issue of how students can be grouped to facilitate productive collaboration, discussion then moved to the learning task at hand. In recognition that students need to have the opportunities to be positioned as productive group members (Amindon & Trevathan, 2016), students wanted groups to have a combination of people that would work well together– variously described as: *good people; smart people; lots of different people.*

The process of becoming a group was seen as dynamic. Group members adjusted according to the demands of the group, and the group itself adjusted to the needs of group members (Galton & Williamson, 2003). As the cogens progressed, observations indicated that groups became more settled and group work became more effective. However, some students who were comfortable working in a particular group expressed concern about expectations to change groups later in the term. Others continued to express anxiety around working in a group per se. As noted by Galton and Williamson (2003), such anxiety may be due to tension between the desire to protect personal identity and the need to help contribute to the developing social identity of the group. However, for some, being able to switch groups was favoured. For example, those students who had previously had less positive group work experiences welcomed modifications to groupings, noting that this helped collaboration.

The suggestion that we focus specifically on collaborative skills prompted a focus on how best to work as a group. However, a lack of student experience with collaborative skills created difficulties in identifying potential strategies. For example, rather than identifying what the group could do to improve collaboration, a frequently offered solution was to remove offending group members, rather than work with them to improve the situation: *Just kick them out of the group.* (Quiet laughter). *Well, they're not being helpful so you might as well* (Cogen. 3). The inability or reluctance to work well together, meant that for some groups it was difficult to achieve the momentum required to drive the problem-solving aspects of group work. In contrast, those groups that do work well may be able to motivate each other and persevere with the problem as a group.

This focus on grouping is perhaps not surprising when we consider that when students choose who they work with they are making strategic choices in order to achieve their goals for the lesson. In the same way, teachers spend time organising groups strategically in order to provide the optimal environment for achieving their goals (Liljedahl, 2014). Tension can arise when individual goals of the students differ from the classroom goals of the teacher. As is evidenced from this study, some students were dissatisfied when the grouping failed to meet their own individual goals. Liljedahl argued that one way to overcome this tension would be to remove all attempts at strategic grouping and group students randomly, stressing that this method of grouping must be *visibly random*: students need to see the process.

Given that many participant students were resistant to changes in group structure, even after they had conceded that it was not always best to work with friends, Liljedahl's' *visibly random grouping* approach may be useful. In Liljedahl's study he noted that while initially met with resistance visible random grouping is usually accepted within 3 to 4 weeks of implementation. After this relatively short time, students in his study became agreeable to work in any group; social barriers in the classroom were eliminated; the mobility of knowledge between students increased; reliance on answers from the teacher reduced while reliance on co-constructed intra and inter group answers increased. Liljedahl pointed out that by achieving the outcomes that strategic grouping aims for but often fails to achieve, *visibly random grouping* can increase student agency and help the class coalesce into a community.

Cogenerating an understanding of how we can help groups work more effectively helped our learning community gradually achieve more success when working collaboratively. Working through how students could work in groups effectively was an important first step for the community, and it was important that any problems with group dynamics were dealt with promptly: once groups can work effectively then learning mathematics can become the focus.

While not achieved in the timeframe of this study, v*isibly random grouping (*Liljedahl, 2014) may provide an environment in which collaboration can flourish.

5.3 Developing Collaborative Practice

If students are to benefit from collaboration they must adapt their behaviour and engagement to the demands of setting (Mulryan, 1995). Student voice across the cogens clearly suggested that adapting to this new setting takes time. Le, Janssen, and Wubbels (2018) identified four obstacles to collaborative learning, all of which were seen in this learning community: a lack of collaborative skills, non-participation, competence, and friendship. In their study, Le et al. found that both teachers and students recognised a lack of collaborative skills when group work first began, suggesting a need for continued active promotion and reflection from both teacher and students.

In the current study, while the cogens process affirmed that collaboration is valued by both the teacher and students, findings indicated that students needed a clearer idea of how to collaborate. Indeed, for me as the teacher/researcher the challenges associated with structuring collaborative activities were 'real'. In managing group work and establishing group norms, the likes of managing off-task behaviours and difficulties monitoring student performance and achievement required ongoing adaptations to my practices. Arguing that the problems that teachers encounter are likely to affect the effectiveness of the development of the inquiry community Le et al. (2018) note the need for both teacher and students to work through issues and difficulties together.

The importance of effective collaboration goes beyond developing a harmonious classroom. Effective group work is necessary in order to derive cognitive benefits (Mercer & Sams, 2006). Because inquiry communities provide a climate where students "are able to think, communicate, reflect upon and critique the mathematics they encounter; their classroom relationships become a resource for developing their mathematical competencies and identities" (Anthony & Walshaw, 2009b, p. 7). As part of thinking about how well our students are communicating and participating, we must also consider how well their mathematical practices are developing (Hunter 2008).

Although cogens suggested that peer collaboration is perceived as a useful way to learn mathematics (Newman, 1998) the shift towards mathematical inquiry learning communities, was challenging and took time. Indeed, students reported that poor collaboration made learning mathematics more difficult for the group as a whole:

Ella *I didn't think that the actual problem was hard, I just think our group was really dysfunctional so it took us a lot longer*. (Cogen. 6)

The study affirmed the necessity for students to be explicitly taught *how* to be active in their learning and develop student agency. This requirement for student agency is the big change in this learning environment: both how students participate in their learning and how the teacher facilitates this learning has changed. Through both the cogens process and the development of collaborative learning environment norms, I, as teacher/researcher, began to delegate authority to students, giving them the responsibility and freedom to work through tasks in their own way, to monitor their own understanding, to struggle and to make mistakes (Cohen & Lotan, 2014).

Student agency can be developed through participating in discourse. While findings suggested that the students in this inquiry community are developing discourse in the context of group work, observations and cogens suggested they were not yet proposing and defending mathematical ideas and conjectures and engaging in mathematical argumentation with their peers. It is essential that this community develops further as a discourse rich environment which stays focused on the mathematical ideas, ensuring all discussions are productive learning experiences for all students (Batista & Chapin, 2019). Making mathematical practices explicit by creating such a discourse rich environment will allow all students have access to the mathematics (Selling, 2016).

Engagement is an important aspect of student agency. Continued perseverance on a problem which has caused difficulty requires motivation and engagement (Attard, 2012). Non-participating students may have a fixed mindset: they may not believe themselves capable of success so fail to engage. The issues around non-participation and help seeking behaviours raise the question of who should take responsibility for learning. A key issue discussed in cogens was whether it was the responsibility of the individual or the group to ensure the mathematics was understood. A consensus view was that whilst the group was always prepared to support others in their learning, everyone was ultimately responsible for their own understanding. As such, individuals needed to be accountable for participating fully in group work, seeking help when required, and persevering with problems. Findings suggested that a lack of student agency prevented some students from participating in terms of actively seeking help, either passively accepting help or withdrawing participation altogether.

Another reason for a lack of effective collaboration could be that the tasks used in this learning community were too open-ended for students with this rudimentary level of collaborative skills. When tasks have clear, testable outcome students can test ideas and work towards a solution collaboratively but when tasks are open-ended students may be satisfied with the first solution offered and avoid debating the value of other solutions (Galton & Williamson, 2003).

5.3.1 Participation

If one of the main goals of collaborative learning is to empower students as mathematical thinkers (Coome & Lee, 2017) then non-participation is a concern Findings indicate that collaborative work in this community is hindered primarily by the non-participatory practices of some students:

Alex *For the two other people in our group, they weren't actually doing anything so I asked them if they understood but they didn't, so I was saying, 'What don't you understand about it?' so I was like, "I can't help you if you don't tell me what you don't understand."* (Cogen. 3)

It is important that teachers understand the challenges that non-participating students might face in order that these challenges can be addressed (Hunter & Hunter, 2018). Developing good collaboration involves *relational equity*–respect for other people's ideas, commitment to the learning of others and learned methods of communication and support (Boaler, 2008). As students become more collaborative and begin to treat each other more respectfully, we would expect to see more appreciation of contributions from all students (Hunter, 2007; Remedios, Clarke, & Hawthorne, 2008). While cogens and classroom observations indicated that this learning community was developing relational equity, a lack of contribution from some students limited collaboration.

Addressing reasons for non-participation is challenging but necessary. As Groff (2012) argued, students will be more engaged if they feel competent to do what is expected of them, have a clear sense of purpose, and perceive the environment as favourable to learning. In this study, changing the rules of participation may have affected levels of engagement. Motivation to contribute could also be linked to self-perceptions of competence. Mulryan (1995) argued that working in small groups can increase the likelihood of students comparing themselves with others which may influence behaviour in groups while the unpredictability of these group settings may also contribute to reduced involvement. As noted in Section 4.3.1, some students remained reluctant to expose their lack of knowledge.

Goal orientation may also affect levels of passivity. While students with mastery goals seek to understand content even when facing challenging tasks, those with performance goals may only be interested in whether they can perform the assigned task correctly, giving up quickly when encountering challenge (Sullivan, 2011). Positively, cogens revealed that some of the students had an expectation that anyone who does not understand needs to try to gain understanding. Students suggested that trying to understand involves communicating with the group, asking questions like *why?*, or *what or does this mean?* However, intentional passivity may be a coping mechanism (Mulryan, 1992), encouraged if group members do not actively involve passive students. To avoid the risk that passive behaviours become accepted within groups, cogens discussed how we could motivate students who are not contributing:

Katie *I'd probably say, like, if* (teacher*) came over and asked you to work it out, to explain it to her, you wouldn't know what to do, because it's not all about the answer. (*Cogen. 2)

Cogen discussions also highlighted concern that, in some cases, students' non-involvement in collaborative work was due to poor communication skills, resulting in a reduced ability to ask questions, explain reasoning, or reflect on solution processes (Hunter, 2007; Le et al., 2018; Kramarski & Meverech, 2003). Mercer and Sams (2006) argued that, with guidance and practice, language could be used more effectively as a tool for collaborative problem solving in addition to improving mathematical understanding. This community would benefit from the language skills essential for effective group work, such as being able to ask questions and articulate thinking, being constantly scaffolded and modelled by students and teacher alike.

Having more students contributing would also reduce dominance in the group. Those who dominate group talk may have their opinions valued more than those who contribute to a lesser extent, which could further marginalise less active group members (Langar-Osana, 2018).

Just as having individual accountability within groups can influence participation (Brown & Thomson, 2000), not feeling accountable to the group can also reduce effort, as cogens revealed was sometimes the case in this community. Individual accountability can be fostered if a culture of interdependence within the groups is created. As noted above (Section 5.2.1) students suggested that providing regular opportunities for group and individual reflection to monitor their group dynamics was a suggested positive action.

Participation may influence how a student feels about group work. Cogens revealed that some students prefer independent work to group work, whether for social or academic reasons, such as avoidance of the more challenging group-worthy tasks. Those who find it difficult to adjust to working in a group may lack the skills or desire to overcome these difficulties, preferring to opt-out and work independently.

Having identified that non-participation and underdeveloped collaborative skills hindered group work, assigning roles within a group was suggested to encourage participation and help develop interdependence and collaborative skills (Heck, Hamm, Dula, Hoover, & Hoffman, 2019; Boaler, 2006a). Cogens discussions exploring which roles were necessary to improve collaboration indicated concerns when students competed for the same role while other important roles were not filled. There was agreement that someone would need to take the lead to encourage collaboration, but challenges associated with who would take on leadership often resulted in groups deciding not to appoint a leader suggesting the role of *encourager* as an alternative. This term soon became *jokester* as students felt the need for a group member to reduce tensions when difficulty arose and motivation waned. Suggesting the role of *listener* showed the value placed on checking mathematical thinking. However, discussions indicated this may be due to a desire to 'get the answers right' rather than a desire to engage in more complex mathematical argumentation. Nevertheless, it provides evidence that there is an understanding that successful group work needs active participants.

An important role not identified in cogens is that of a facilitator. Having someone in this role would aid interaction and help keep the group functioning, but it is not a role that every group member would find easy (Ehrlich & Zack, 1997). The role a student can take on successfully may depend on personality, which may be why the study students were happy for roles to evolve naturally, being taken up as and when they were required by those most able to carry out the roles. While formalised roles help develop positive group dynamics, add strength and cohesion to a group and enhance group interdependence (Boaler, 2006a; Ehrlich & Zack, 1997), participant students enjoyed flexibility within their groups and felt that roles should evolve naturally:

James *I reckon it's better if they just come naturally*

Lucy *Yeah, instead of being forced.*

Daniel *'Cause forcing people to act differently isn't going to work.* (Cogen. 3)

In the timespan of the project, it seemed that although these students were aware that working collaboratively required group members to participate and contribute, their focus was more directed to cooperation rather than full collaboration. Students felt that by assigning roles to particular members, group work would become more structured and everyone would participate; this might ensure task completion, but would not necessarily aid collaboration (Boaler, 2006a; Ehrlich & Zack, 1997; Heck et al., 2019).

Specific role allocation was not intended to restrict students to particular roles, as this risks high status students taking leadership roles dominating others thus increasing the separation between high and low status students (Boaler, 2006a). Rather, collaborative roles need to be shared across all group members and be *mathematically meaningful* (Heck et al., 2019). When using such mathematically meaningful roles, students can practise using specific behaviours targeted by the role, such as asking clarifying questions and providing explanations.

5.3.2 Seeking Support

Help seeking is a self-regulatory skill, an essential component of student agency. Findings highlighted although there were varying levels of support offered within the groups, that both a lack of help-seeking behaviours, or inadequate help responses impacted on participation behaviours. Unsuccessful requests for help, a lack of rapport within the group or a reluctance to work with certain people may also inhibit help-seeking.

Whatever the reasons, those that do not seek help present concern (Newman & Schwager, 1993). For this reason, cogens addressed help-seeking behaviours as a way of addressing non-participation. Determining help-seeking orientations would be a good starting point for this community. Do we seek help simply to find solutions to tasks, or because we want to promote learning (Aleven, Stahl, Schworm, Fischer & Wallace, 2003; Butler, 1998)? Students may not seek help even though they know it will help learning, as they do not want to risk seeming incompetent (Newman & Schwager, 1993). When seeking help a potential helper must be identified, which involves several considerations, including the perceived competence of the helper and the expected sensitivity to the needs of the learner, without which students will not support each other (Newman, 1994). Openly discussing the importance of asking questions and ascertaining whether our students believe it is the 'dumb kids' or the 'smart kids' who seek help would make the value of questions overt, and normalise this as an academic behaviour. As students may not realise that asking questions is an important part of their sense-making, teachers must model the behaviours that will support students in this change (Hunter & Hunter, 2018)

In this study, not all students were seen as potential helpers. Indeed, this was regarded by one student as a drawback to heterogeneous grouping. The need to provide an environment in which all students have the status where they are seen as a potential source of help is important. However, a dilemma for these students was when to use others as a resource for help. The use of group-worthy tasks and the value placed on persistence may give the expectation that students must always persist in the face of difficulty: students may not be able to establish the point at which asking for help would be acceptable (Butler, 1998). Strengthening collaborative skills would help, as the more students believe they can help each other in their learning (Newman & Schwager, 1993) and the safer they feel within the learning community, the more they will feel able to seek support from each other.

In seeking support, later cogen discussions indicated that finding the solution to the problem was no longer seen as the primary goal for these students (see section 4.3.2). However, for some students there remained evidence that completing tasks quickly was a key motivational driver. If a group values the speed of task completion, regarding it as an important criterion for success, group members who feel time constraints may discourage students with negative self-efficacy from participating (Mulryan, 1995). As Maia explained:

Maia *It's also quite hard if you're the not so smart person when all the people race ahead because they find it easy*. (Cogen. 1).

To support student help seeking practices a focus on the *social cohesion perspectiv*e of collaborative learning, where group rather than individual success is the goal (Slavin, 1996), could be a next step for this community. This would involve the community working with specific tasks that depend upon group members noticing and responding to others' needs and ideas and developing collaboration towards a single goal.

5.3.3 Providing support

Being willing to support others is a basic requirement for collaborative learning (Gillies, 2003). Cogens reflected that many students were willing to help others and discussed various ways that support could be provided, particularly when not all group members understood the task/solution. Discussion focused on what other group members would do to ensure that individuals, and thus the group, developed understanding so that the group could progress with the problem:

Lucy *Maybe ask them what they are struggling with. Maybe try and help them, and maybe try and understand what they are struggling with, and try to explain it simpler than what is on the paper.* (Cogen. 4)

As the study progressed, the question of who should take responsibility when understanding is lost was explored in both the cogens and during class discussion. This helped develop a shared understanding of the expectations of individual effort and collaborative group work.

Findings indicated that providing help to peers was not always easy. As Wood and Kalinec (2012) note: "in order for students to gain from peer teaching, we must specifically teach students how to help one another" (p. 126). In particular, learning to make mathematical explanations is difficult; it requires effort from both those who are explaining and those are receiving explanation. Although research suggests that explainers learn from teaching others

(Gillies, 2003), students must move beyond simply explaining to justifying and engaging in mathematical argumentation, transforming learning mathematics into a social activity within a community of learners (Hunter & Hunter, 2018). Providing explanations is more than just helping all group members 'keep up' with the thinking of others or providing an understanding of how a solution was arrived at; it can highlight new perspectives, clarify understanding and construct new learning (Gillies, 2003).

Cogens noted frustrations when attempts to provide help were unsuccessful, which in turn led students to develop a negative view of group work. Anxiety around individual success being replaced by group success may also be a factor in students preferring to work individually.

Ella *I think that teachers shouldn't put as much pressure on you to work well in your group.* (Cogen. 2)

While receiving recognition as being helpful can raise self-efficacy (Gillies, 2003), Wood and Kalinec (2012) suggest that care must be taken when positioning students as peer teachers, as students who know more may become dominant and prevent others from engaging in mathematical talk. Galton and Williamson (2003) argued that the absolute and relative level of achievement of a student within a group predicted their interaction within the group. If we are to create a learning community in which all students feel able to give and receive support it is important that all students have equal status and well-developed self-efficacy.

5.4 Summary

Effective collaboration requires two elements: agency and equal status. The findings lead us to think about how we can empower students so they have agency within a collaborative learning environment and how we can raise the status of all students so they all have the space for all to participate.

Through being invited to co- participate with their teacher in the cogens, students were involved in iterative feedback and actions to drive the change process. As with previous research (Lee & Wilder, 2012), this co-research has proved significant in constructing ideas about how students learn mathematics effectively. Discussing how learning was experienced in a community of learners revealed how students perceived and experienced these changes in action, helping us to understand what is different for our students, how our students experience these differences and what impact these differences make. By examining students' perspectives and gaining understanding into the challenges experienced within the change process, this research considers how we might (or might not) create a better learning environment for students to have agency and learn mathematics effectively.

Chapter 6 Conclusion

6.1 Introduction

This aim of this study was to develop an understanding of how diverse students experience learning mathematics in a developing inquiry community. The study addressed the research questions:

- How do students experience learning to engage and participate in a mathematical inquiry community and its associated mathematical practices?
- What do students understand about their role within a mathematical inquiry community?

Findings provide insights into how students experienced transitioning to an inquiry-based learning community and into how students can develop effective communication and collaborative practices. Findings suggest that using student voice in the form of cogens is an effective way for a learning community to develop the shared understandings which are essential for the development of the relationships necessary for effective group work to occur. They help us understand what is different for our students, how our students experience these differences and what impact these differences make. These findings may be of interest to educators and researchers involved in mathematics reforms.

6.2 Developing a Mathematical Inquiry Community

Considering the research question, "How do students experience learning to engage and participate in a mathematical inquiry community and its associated mathematical practices?", led to the cogens being examined in order to understand how students experience collaborative learning. This study then reflected upon the nature of these experiences, and their impact on the developing community of learning.

Establishing effective group work was the initial focus of students' experience in learning to engage and participate in a mathematical community. Group work was generally enjoyed, which may have been due to the social aspects of working collaboratively. This enjoyment led to greater engagement, which aided collaboration. However, the impact that group composition had on an individual's experience of collaborative learning became a major focus for the community. As we worked together to implement group norms associated with group task activities in class, cogen discussions indicated that students became more accepting of the benefits of having social and academic diversity and a range of collaborative skills within a group. However, discussions also continued to reference how differing levels of motivation and learning dispositions affected group dynamics, particularly with reference to unequal levels of participation which hindered collaborative work. The focus for this learning community then centred on how to foster participation within collaborative groups.

Being considered good at mathematics raised the status of a learner in this community, while those who did not perceive themselves as good at mathematics typically lacked confidence. Although non-participation could have been caused by a lack of support from the group, with motivation diminishing if students did not feel their contributions were appreciated, the challenging nature of group-worthy tasks suggests that there will always be students who struggle to make enough sense of a task, creating an obstacle to group work.

The research question, “What do students understand about their role within a mathematical inquiry community?” focused the study on how students participate within the community. Participatory practices were examined, and the effects of participation, non-participation, and giving and receiving support within the learning community was explored.

In coming to understand their role, and the role of others, in group work, students who were more motivated and keen to progress with the problem were challenged by the obligation to support others in their learning. However, as students began to recognise that the learning journey towards a solution may be as important as the solution itself, adopting a supportive role became more relevant. Being able to request and provide explanations in order to develop understanding became a desirable skill within the learning community. Students found it hard when their fellow group members could not either contribute to solving the task or understand someone else’s ideas about how to solve the problem. Their lack of mathematical understanding tested collaboration practices and was probably a contributing factor to the perceived lack of effort or focus of some students.

As the project progressed, students became increasingly aware of the need for productive discourse to further their learning. Considering the various ways to contribute to group work framed the development of collaborative skills and helped raise contributions from previously unconfident students. Although students expected to support each other with their learning, not all students could articulate their learning needs: asking questions was focused on as a means of reducing the passivity of non-participating group members.

As the community developed, students began to take more responsibility for ensuring their own understanding. Groups began to work well together with reduced teacher involvement, and students began to take their role as a group member seriously, understanding the role each individual has in order to allow the group to achieve success. While these students are still transitioning from expectations around individual work to working within a learning community, evidence suggests that the students are moving to more collaborative practice.

6.3 Limitations of the Research

The exploratory nature of this single-case study necessitated clearly identified boundaries. (Punch & Oancea, 2014). These boundaries helped define what the case is not, therefore ensuring that the research question did not become too wide. Future studies could look more closely at how the teacher reacted to and learnt from the cogen process. In this study, although the cogen process did impact on teaching, this was not the focus of the study which instead focused on student voice as a way to explore student experience, and the value that can be gained through its use.

As this is a single-case study, findings are not generalisable, nor do they intend to provide advice or solutions to how best to develop such communities. Rather, the study has value in that it raises awareness of how students can struggle when learning to participate in an inquiry community, thus providing useful data to those who are also developing communities of inquiry. Further case studies, which could involve case study synthesis or cross-case comparisons, could also be undertaken, enabling the development of more robust data (Barth & Thomas, 2012)

6.4 Reflection on the Cogens

Bondi et al. (2016) suggested that cogens not only give teachers valuable information that can be used to support student needs but also allow students to hear how others experience learning; this moves discussion beyond content to student needs, the learning process, and responsibilities of community building. Talking to students individually after this series of cogens gave an insight into how beneficial they felt the cogen process to be.

As noted in previous research (Edwards & Jones, 2003,) all students felt that participating in the cogens had a positive effect on their mathematics learning. Many students enjoyed the opportunity to talk about their learning and felt that changes made to how the class was run encouraged collaboration. The benefits of the cogens included the opportunity to have student voice and to hear people's opinions outside of the classroom situation, with some students suggesting that cogens be implemented on a regular basis.

Through participating in cogens students recognised that through learning from both the teacher and other students the whole group learns together. Several students pointed out that the cogens helped them understand both how others work and how individuals feel about group work, prompting everyone to make changes in order that everyone can work well together. Cogens were also described as a good way to learn about yourself, making you think more deeply about mathematics and what needs to be done in order to "get it right".

Students discussed having increased confidence through participating in cogens, enabling the development of their communication skills which will help them contribute more effectively within a group. Students also discussed becoming more aware of individual strengths which can be brought to a group and began to appreciate that outwardly confident people are not necessarily confident in all aspects of mathematics.

These post-cogen conversations indicated that students are developing their understanding of what is needed to develop our class learning community. They recognise that it is important to ensure everyone in the group participates and that answers are justified. The cogens process has thus given our participant students co-ownership of developing classroom norms. For these students, participating and contributing is seen as the key to a successful learning community, alongside listening to others, asking questions, discussing the mathematics, justifying opinions and being flexible in group work, including the need to be prepared to take leadership. However, it is clear that to embed these norms there is a need for ongoing discussion and for norms to be revisited regularly.

Students recognised the benefits of participating in cogens to both themselves as individual learners and as learners within a learning community. They contributed to decision making by sharing ideas, identifying problems and co-generating solutions. They learned about themselves as learners and recognised learning differences within the community. Participation in cogens has helped develop the discourse, problem solving, and student agency required in an effective community of learning. The next steps for this learning community will be to continue to build on and consolidate the established norms. The use of cogens in the future could be used to continue to develop the learning community by developing a continuing partnership between all members of the learning community.

From the perspective of a teacher, the cogens have been invaluable, providing an opportunity to hear how individual students feel and experience mathematics within the classroom learning community. As Beltramo (2017) described, the cogens have helped me, as a teacher, develop deeper relationships with students, resulting in more informed and honest conversations about teaching and learning. In my own experience and sharing with other teachers, teaching mathematics through group work can sometimes lead teachers to be solely concerned with addressing students' immediate academic needs, which can often be seen as problematic when working with students of different attainments. However, although a student's academic needs remain paramount, participating in cogens has resulted in a greater appreciation for students' social and emotional needs when working within groups. Whilst students seem to work naturally together in many different situations, cogens has revealed that students' social interactions and experiences within these groups, whilst not always visible, have the potential to significantly influence learning outcomes.

6.5 Concluding Thoughts

Cogens promote equity by allowing all voices to be heard, providing all participants with the power to influence teaching and learning developments in the classroom. As Lee and Johnston-Wilder (2012) discussed, student voice has a vital part to play in the continuous improvement of teaching and learning and mathematics, and so is essential for effecting change. Through sharing intellectual authority between teacher and students, students can take ownership of ideas which can lead to greater intellectual understanding and greater identification with mathematics (Langer-Osuna, 2017). Taking the time to gather student voice gives an appreciation of the story that students tell and, in this case, helps with understanding the social as well as the academic aspects of learning. This may be a useful addition to the way we learn to teach.

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Appendices

Appendix A

Table 1
Participation by Gender and Year Group

	Year 6 2018 / Year 7 2019		Year 7 2018 /Year 8 2019	
	Number of Participants	% of invited group	Number of Participants	% of invited group
Boy	1	25%	5	50%
Girl	4	50%	4	57%
Total	5	41%	9	53%

Table 2
Participation by Ethnicity

	NZ European	NZ Maori	Pasifika	Other European
Invited Group	84%	10%	10%	13%
Cogen. Group	92%	0%	6%	15%

Note. Students may identify with more than one ethnicity

Table 3
Participation by Achievement and Engagement

	Above Expected Achievement	Expected Achievement	Below Expected Achievement
Above Expected Engagement	3	1	1
Expected Engagement	2	4	3
Below Expected Engagement	0	0	0

Appendix B

Table 4
Cogenerative Dialogues

Cogen	Discussion summary	Action Items
1	• Establish purpose of cogens • Establish norms for cogens • Classroom noise levels • On task/off task behaviour • Composition of groups	• Discussion and set classroom norms regarding noise levels • Share with class discussion about re-groupings. • Seek student voice on the composition of groups: act on this feedback. • Students to feed back on grouping at later cogens.
2	• Reminder of cogen norms. • Discuss redeveloped groups. • What is it like being a learner within a maths group. • Whose responsibility is it for not understanding? • The need to work as a team, so it is important that everyone understands.	• To discuss as a class and add to class norms: next steps for the group if you or a group member doesn't understand something- raise awareness that not everyone enjoys group work: what can we do collectively to help?
3	• What do we do when group members don't understand. How can we work better as a group when the maths is difficult or when group members make mistakes? • Difficult to help someone who doesn't want to help themselves. • Not everyone likes it when someone takes leadership and leads the work. • Do specific roles help with successful group work?	• How to ensure that all group members participate. Class discussion to address appropriate participatory practices.
4	• Encouraging people to participate, explaining more, making an effort to listen and keep groups focused. • Group work felt to be more effective. • The group dynamic seems to influence how students feel about the maths. Perceived	• Discuss possible group roles, continue being encouragers: Encourage people to make contributions. • Emphasise that there are multiple ways of being successful in a maths classroom. This will hopefully enable people to take risks and feel that their contributions can be valued.

	ability/maths status within groups discussed. • Group work easier when people are motivated to learn. Growth mindset is helped when working in a group situation. • Communication improved when in familiar groups • What does it mean to be successful in maths class?	
5	• Negative reaction towards creating group roles Having assigned roles in a group not happening naturally. Some thought they were unnecessary and would evolve naturally. • Status in the maths classroom raised again • Motivated to find the answer when the task is more accessible and enjoyable • Dong difficult problems may mean that all problems are assumed to be difficult • Recognition that problem solving requires flexible thinking and perseverance. Some satisfaction that group solved the problem, rather than it being solved by an individual.	• Discuss perseverance and growth mindset as a class. Cogen students to model and encourage this in group work. • Revisit the different roles required in collaborative work.
6	• Discussion of group roles being fluid, and people naturally moving between roles, • Still a problem with non-participation • Group work becoming more focused, Recognition that when a group works well it does make maths easier. • Discussion indicates that those with higher status in the maths class are assumed to be correct, or having a smart person can be a help or a hindrance • Optimal group size discussed. • Recognition that unequal contributions from group members hinders collaboration	• To again address levels of non-participation: ask the class as whole to focus on work better as a group and ensure ideas are part of class norms.

7	• When focusing on how we can work more effectively as a group, making asking questions a focus • Mathematical confidence is important for leadership in the group • Group dynamics can help problem solving	• Discuss with class issues with group dynamics: what is working well- can we have a focus on maintaining positivity • Students to contribute confidently to group work and talks leadership where necessary
8	• How we can work well as a group, regardless of who is in group. Make the group dynamic work for us. Think about being more confident in our group work • Enjoy the social aspect of working in a community_ this may lead to improved social skills in other aspects of school life • Importance of communicating • Collaborative skills still need development • How to define group work	• Focus on working well together, regardless of who is in the group: need to highlight successful examples of collaboration. • Develop confidence when working as a group: teacher needs to assign competence and show how much good collaborative practice, especially communication skills, is valued.

Appendix C

Figure 1

Word frequency Word Cloud: Full Dataset.

NVivo qualitative data analysis software; QSR International Pty Ltd. Version 12, 2018.

Table 5

Word Frequency Query including Similar Words: Full Dataset

NVivo qualitative data analysis software; QSR International Pty Ltd. Version 12, 2018.

Word	Length	Count	Weighted Percentage (%)	Similar Words
group	5	472	4.74	group, grouped, grouping, groupings, groups
work	4	344	3.46	work, worked, working, works
think	5	274	2.75	think, thinking, thinks
people	6	232	2.33	people, people'
problem	7	143	1.44	problem, problems
understand	10	107	1.08	understand, understand', understanding, understands
know	4	105	1.06	know, knowing, knows
good	4	102	1.02	good
maths	5	97	0.97	maths
talking	7	95	0.95	talk, talked, talking, talks
questions	9	81	0.81	question, questioner, questions
together	8	80	0.80	together, together'
answer	6	79	0.79	answer, answer', answering, answers
different	9	77	0.77	difference, different, differently
time	4	76	0.76	time, times
explain	7	75	0.75	explain, explained, explaining
helps	5	71	0.71	help, helped, helpful, helping, helps
need	4	67	0.67	need, needed, needs
actually	8	66	0.66	actual, actually
things	6	64	0.64	thing, things

Table 6

Nodes Version 1

NVivo qualitative data analysis software; QSR International Pty Ltd. Version 12, 2018.

Name	Files	References	Created On	Created By	Modified On
Answer	7	47	26/07/2019 10:49 AM	FR	26/07/2019 3:10 PM
Challenge	6	17	26/07/2019 3:47 PM	FR	26/07/2019 4:45 PM
Confident	5	23	24/07/2019 3:54 PM	FR	25/07/2019 2:34 PM
Different	8	61	26/07/2019 11:04 AM	FR	26/07/2019 3:10 PM
Easy	6	25	26/07/2019 3:56 PM	FR	26/07/2019 4:51 PM
Enjoy	6	15	26/07/2019 4:05 PM	FR	26/07/2019 5:04 PM
Explain	8	52	26/07/2019 11:53 AM	FR	26/07/2019 6:06 PM
Feel	7	33	26/07/2019 3:03 PM	FR	26/07/2019 4:50 PM
Friends	5	14	26/07/2019 2:32 PM	FR	26/07/2019 5:17 PM
Good	8	79	26/07/2019 8:34 AM	FR	26/07/2019 8:54 AM
Group	14	270	24/07/2019 3:53 PM	FR	26/07/2019 5:06 PM
Helps	8	58	26/07/2019 12:08 PM	FR	26/07/2019 3:10 PM
Know	8	76	25/07/2019 2:29 PM	FR	25/07/2019 3:07 PM
Learning	7	28	26/07/2019 2:03 PM	FR	26/07/2019 4:49 PM
Listen	6	18	26/07/2019 4:21 PM	FR	26/07/2019 4:28 PM
Maths	8	66	25/07/2019 1:32 PM	FR	26/07/2019 8:33 AM
People	8	150	24/07/2019 3:53 PM	FR	25/07/2019 2:34 PM
Problem	10	115	24/07/2019 3:54 PM	FR	26/07/2019 2:19 PM
Questions	8	52	26/07/2019 9:32 AM	FR	26/07/2019 3:10 PM
Roles	5	19	26/07/2019 2:43 PM	FR	26/07/2019 4:59 PM
Smart	5	12	26/07/2019 4:16 PM	FR	26/07/2019 5:05 PM
Solve	6	20	26/07/2019 2:17 PM	FR	26/07/2019 3:10 PM
Struggle	8	83	26/07/2019 4:31 PM	FR	26/07/2019 5:18 PM
Talk	9	46	25/07/2019 10:11 AM	FR	26/07/2019 4:39 PM
Think	9	205	25/07/2019 10:12 AM	FR	26/07/2019 11:35 AM
Time	8	60	26/07/2019 11:26 AM	FR	26/07/2019 3:10 PM
Together	8	69	26/07/2019 10:23 AM	FR	26/07/2019 3:10 PM
Understand	8	67	25/07/2019 2:09 PM	FR	25/07/2019 2:34 PM
Work	8	224	25/07/2019 9:45 AM	FR	25/07/2019 2:34 PM

Table 7

Nodes Version 2: Developing Themes
NVivo qualitative data analysis software; QSR International Pty Ltd. Version 12, 2018.

Name	Files	Created On	Created By	Modified On
Group	14	24/07/2019 3:53 PM	FR	31/07/2019 8:23 AM
Group composition	15	28/07/2019 1:31 PM	FR	30/07/2019 4:51 PM
Group dynamics	15	28/07/2019 12:59 PM	FR	30/07/2019 4:52 PM
Group work	15	28/07/2019 12:59 PM	FR	30/07/2019 4:45 PM
Participate	6	28/07/2019 12:00 PM	FR	31/07/2019 8:35 AM
Friends	5	26/07/2019 2:32 PM	FR	31/07/2019 8:35 AM
Roles	5	26/07/2019 2:43 PM	FR	31/07/2019 8:35 AM
Together	9	26/07/2019 10:23 AM	FR	31/07/2019 8:35 AM
Put together	1	28/07/2019 3:07 PM	FR	28/07/2019 3:11 PM
Working together	9	28/07/2019 3:06 PM	FR	28/07/2019 3:21 PM
People	8	24/07/2019 3:53 PM	FR	31/07/2019 8:35 AM
People in the community	8	28/07/2019 3:48 PM	FR	28/07/2019 4:33 PM
General people	8	28/07/2019 3:47 PM	FR	28/07/2019 4:34 PM
Types of people	7	28/07/2019 3:47 PM	FR	30/07/2019 4:43 PM
Think	9	25/07/2019 10:12 AM	FR	31/07/2019 8:35 AM
Affirming	4	28/07/2019 2:20 PM	FR	31/07/2019 8:06 AM
Offer opinion	4	28/07/2019 2:20 PM	FR	31/07/2019 8:06 AM
Seek opinion	4	28/07/2019 2:20 PM	FR	31/07/2019 8:06 AM
Thinking	6	28/07/2019 2:26 PM	FR	31/07/2019 8:06 AM
Know	8	25/07/2019 2:29 PM	FR	31/07/2019 8:35 AM
Understand	7	28/07/2019 6:08 PM	FR	28/07/2019 6:23 PM
Have knowledge	8	28/07/2019 6:07 PM	FR	28/07/2019 6:25 PM
I realise	3	28/07/2019 6:05 PM	FR	28/07/2019 6:23 PM
You know connecting to listener	4	28/07/2019 6:04 PM	FR	28/07/2019 6:23 PM
Unable to explain thinking or provide answer	6	28/07/2019 6:04 PM	FR	28/07/2019 6:25 PM
Talk	9	25/07/2019 10:11 AM	FR	31/07/2019 8:35 AM
Referring to	1	28/07/2019 3:02 PM	FR	28/07/2019 3:04 PM
Do or don't talk to	2	28/07/2019 3:00 PM	FR	28/07/2019 4:30 PM
Discussion	2	28/07/2019 2:58 PM	FR	28/07/2019 3:33 PM
Rules around talk	2	28/07/2019 2:56 PM	FR	28/07/2019 2:57 PM
Invitation to talk or participate	0	28/07/2019 2:55 PM	FR	28/07/2019 2:55 PM
Off task talk	1	28/07/2019 2:53 PM	FR	28/07/2019 3:03 PM
On task talk	5	28/07/2019 2:53 PM	FR	28/07/2019 4:10 PM
Communicate	5	28/07/2019 12:14 PM	FR	31/07/2019 8:35 AM
Listen	6	26/07/2019 4:21 PM	FR	31/07/2019 8:35 AM
Explain	8	26/07/2019 11:53 AM	FR	31/07/2019 8:35 AM
Learning	7	26/07/2019 2:03 PM	FR	31/07/2019 8:35 AM
Understand	9	25/07/2019 2:09 PM	FR	31/07/2019 8:35 AM
Maths	8	25/07/2019 1:32 PM	FR	31/07/2019 8:35 AM

Learning maths	16	28/07/2019 6:34 PM	FR	30/07/2019 4:49 PM
Maths time	4	28/07/2019 6:33 PM	FR	29/07/2019 6:32 AM
Being good at maths	9	28/07/2019 3:50 PM	FR	30/07/2019 4:56 PM
Specific maths	13	28/07/2019 1:25 PM	FR	30/07/2019 4:38 PM
Smart	7	26/07/2019 4:16 PM	FR	31/07/2019 8:35 AM
Confident	5	24/07/2019 3:54 PM	FR	31/07/2019 8:35 AM
Status	8	28/07/2019 2:10 PM	FR	30/07/2019 4:56 PM
Confidence	5	28/07/2019 2:10 PM	FR	30/07/2019 4:56 PM
Easy	7	26/07/2019 3:56 PM	FR	31/07/2019 8:35 AM
Struggle	10	26/07/2019 4:31 PM	FR	31/07/2019 8:34 AM
Challenge	6	26/07/2019 3:47 PM	FR	31/07/2019 8:34 AM
To challenge	2	29/07/2019 7:47 AM	FR	29/07/2019 7:48 AM
Barrier to effective group work	14	28/07/2019 1:00 PM	FR	30/07/2019 4:51 PM
Difficult to do	12	28/07/2019 12:58 PM	FR	30/07/2019 4:45 PM
Barrier to learning	14	28/07/2019 12:58 PM	FR	30/07/2019 4:36 PM
Feel	7	26/07/2019 3:03 PM	FR	31/07/2019 8:34 AM
I think	2	29/07/2019 8:12 AM	FR	29/07/2019 8:14 AM
Feelings	9	29/07/2019 8:12 AM	FR	30/07/2019 4:56 PM
Enjoy	6	26/07/2019 4:05 PM	FR	31/07/2019 8:34 AM
Work	8	25/07/2019 9:45 AM	FR	31/07/2019 8:34 AM
Work labour	4	28/07/2019 12:52 PM	FR	28/07/2019 1:36 PM
Work task	2	28/07/2019 12:51 PM	FR	28/07/2019 1:20 PM
Work function	4	28/07/2019 12:51 PM	FR	28/07/2019 1:36 PM
Questions	9	26/07/2019 9:32 AM	FR	31/07/2019 8:34 AM
Group work questions	8	29/07/2019 6:34 AM	FR	29/07/2019 7:40 AM
Cogen	2	29/07/2019 6:33 AM	FR	29/07/2019 7:37 AM
Solve	10	26/07/2019 2:17 PM	FR	31/07/2019 8:34 AM
Problem	10	24/07/2019 3:54 PM	FR	31/07/2019 8:34 AM
Problem in other context	3	28/07/2019 3:26 PM	FR	28/07/2019 4:26 PM
Maths task	15	28/07/2019 3:25 PM	FR	30/07/2019 2:02 PM
General problem	6	28/07/2019 3:25 PM	FR	28/07/2019 4:35 PM
Helps	8	26/07/2019 12:08 PM	FR	31/07/2019 8:32 AM
Time	8	26/07/2019 11:26 AM	FR	31/07/2019 8:27 AM
Different	8	26/07/2019 11:04 AM	FR	31/07/2019 8:27 AM
Answer	7	26/07/2019 10:49 AM	FR	31/07/2019 8:27 AM
Good	9	26/07/2019 8:34 AM	FR	31/07/2019 8:27 AM

Table 8

Nodes version 3: Sorting the Nodes into Categories

NVivo qualitative data analysis software; QSR International Pty Ltd. Version 12, 2018.

Name	Files	References
Work	8	224
Work function	4	17
Work labour	4	43
Work task	2	7
Group	14	271
Group composition	15	72
People in our community	8	78
Group dynamics	15	171
Working as a group	16	279
Group work	15	164
What is group work?	15	213
Participate	6	10
Friends	5	14
Roles	5	19
Together	9	70
Put together	1	5
Working together	9	53
People	8	150
People in the community	8	78
General people	8	35
Types of people	7	19
Think	9	205
Affirming	4	7
Offer opinion	4	32
Seek opinion	4	40
Thinking	6	15
Know	8	76
Understand	7	21
Have knowledge	8	38
I realise	3	3
You know connecting to listener	4	4
Unable to explain thinking or provide answer	6	11
Talk	9	46
Referring to	1	3
Do or don't talk to	2	3
Discussion	2	4
Rules around talk	2	4
Invitation to talk or participate	0	0

Off task talk	1	5
On task talk	5	8
Communicate	5	9
Listen	6	18
Explain	8	52
Learning	7	28
Understand	9	68
Maths	8	66
Learning maths	16	156
Mathematical Learning	16	355
Problem in other context	3	9
Maths task	15	103
General problem	6	8
Maths time	4	8
Being good at maths	11	33
Specific maths	13	54
Smart	7	14
Confident	5	23
Status	8	15
Confidence	5	19
Easy	7	26
Struggle	10	85
Challenge	6	17
To challenge	2	6
Barrier to effective group work	14	99
Difficult to do	12	48
Barrier to learning	14	88
Feel	7	33
I think	2	4
Feelings	9	56
Being a learner	16	229
Enjoy	6	16
Questions	9	53
Cogen	2	3
Student voice	16	159
Group work questions	8	27
Solve	10	38
Problem	10	115
Problem in other context	3	9
Maths task	15	103
General problem	6	8
Helps	8	58
Time	8	60
Different	8	61

Answer	7	47
Good	9	80

Table 9
Developing categories into concepts

Work	To function	What is group work?	How do learners learn in this community?	The collaborative learning environment
	Labour			
	Task			
Group	Group work			
	Group composition	People in our community	Who is our community?	
	Group dynamics	Working as a group	How can our community work well together? What are the barriers to effective group work?	
Participate				
Friends				
Roles				
Communicate				
Listen				
Explain				
Talk	Do/don’t talk to			
	On task talk			
	Off task talk			
Confident	Status			
	Confidence			
Together	Working together			
People	Types of people	People in our community	Who is our community?	
	People in the community			

<table>
<tr><td>Think</td><td>Thinking</td><td rowspan="9">Mathematical learning</td><td rowspan="9">What is it like learning maths in our learning community?

What can be done to support learners with group work?</td><td rowspan="21">Learning mathematics collaboratively</td></tr>
<tr><td>Learning</td><td rowspan="4">Maths Learning</td></tr>
<tr><td>Questions</td></tr>
<tr><td>Understand</td></tr>
<tr><td rowspan="4">Maths</td></tr>
<tr><td>Maths time</td></tr>
<tr><td>Specific maths</td></tr>
<tr><td rowspan="2">Being good at maths</td></tr>
<tr><td>Smart</td></tr>
<tr><td rowspan="2">Confident</td><td>Status</td><td rowspan="12">What does it feel like to be a learner in this environment?</td><td rowspan="12">What are the challenges for a learner in this environment?

What are the barriers to our learning?</td></tr>
<tr><td>Confidence</td></tr>
<tr><td>People</td><td>Types of people</td></tr>
<tr><td>Easy</td><td rowspan="3">What is difficult and what is easy?</td></tr>
<tr><td>Struggle</td></tr>
<tr><td rowspan="2">Challenge</td></tr>
<tr><td>To challenge</td></tr>
<tr><td>Problem</td><td rowspan="2">Maths task</td></tr>
<tr><td>Solve</td></tr>
<tr><td rowspan="2">Feel</td><td>I think</td></tr>
<tr><td rowspan="2">I feel</td></tr>
<tr><td>Enjoy</td></tr>
</table>

Questions	Cogens related	Student voice What do students say about their learning?	What aspects of student agency are students developing? How does having student agency impact on disposition or learning?	Developing student agency
Think	Seek opinion			
	Offer opinion			
	Affirm			
Know	You know as a connection to the listener			
	I am unable to explain			
	I realise			
	Have knowledge			
Talk	Discuss			
	Invitation to talk			
	Referring to			
Feel	I think			
Together	Put together	Code on		
Problem	General problem			
	Problem in other contexts			
People	General people	Code on		
Time, Answer Different, Good, Helps	Coded on			

Appendix D

MASSEY UNIVERSITY
TE KUNENGA KI PŪREHUROA
UNIVERSITY OF NEW ZEALAND

Date: 03 October 2018

Dear Fiona Rice

Re: Ethics Notification - **SOB 18/50 - Using student voice to explore participation in a co-constructed Mathematical Inquiry Community**

Thank you for the above application that was considered by the Massey University Human Ethics Committee: **<u>Human Ethics Southern B Committee</u>** at their meeting held on **<u>Wednesday, 3 October,</u>**

Approval is for three years. If this project has not been completed within three years from the date of this letter, reapproval must be requested.

If the nature, content, location, procedures or personnel of your approved application change, please advise the Secretary of the Committee.

Yours sincerely

Professor Craig Johnson
Chair, Human Ethics Chairs' Committee and Director (Research Ethics)

Research Ethics Office, Research and Enterprise
Massey University, Private Bag 11 222, Palmerston North, 4442, New Zealand **T** 06 350 5573; 06 350 5575 **F** 06 355 7973
E humanethics@massey.ac.nz **W** http://humanethics.massey.ac.nz

Appendix E

MASSEY UNIVERSITY
TE KUNENGA KI PŪREHUROA
UNIVERSITY OF NEW ZEALAND

Institute of Education
Massey University, Te Kunenga ki Purehuroa, New Zealand

Using student voice to explore participation in a co-constructed

Mathematical Inquiry Community

CONSENT FORM: BOARD OF TRUSTEES AND PRINCIPAL

This consent form will be held for a period of five years.

PRINCIPAL CONSENT

- I have read the information sheet and have had the details of the research study explained to me.
- My questions have been answered to my satisfaction and I understand that I can ask further questions at any time.

- **I agree** to ____participating in this research study under the conditions set out on the attached information sheet.

Principal's name (printed) _________________________ Signed _____________________

Date _________________________ Email ______________________

BOARD OF TRUSTEES CONSENT

- I have read the information sheet and have had the details of the research study explained to me.
- My questions have been answered to my satisfaction and I understand that I can ask further questions at any time.

- **I agree** to ____participating in this research study under the conditions set out on the attached information sheet.

Chair of Board of Trustee's name: (printed) _________________ Signed ______________

Date _________________________ Email ______________________

Please return the completed consent form to Fiona Rice by Monday 12th November.

Appendix F

Institute of Education
Massey University, Te Kunenga ki Purehuroa, New Zealand

Using student voice to explore participation in a co-constructed

Mathematical Inquiry Community

BOARD OF TRUSTEES/PRINCIPAL INFORMATION SHEET

Dear [redacted] Board of Trustees,

In 2015, when I had the role of Mathematics Support Teacher (MST) at our school, I began postgraduate study through Massey University, a requirement for MST teachers. Since 2015 I have completed the following papers: Making Mathematics Accessible; Mathematics Education; Qualitative Research Methods and Quantitative Research Methods. I am currently undertaking a research thesis in order to complete my Master of Education (Mathematics Education). For my research, I plan to explore student experiences and opportunities to learn mathematics in mathematical inquiry communities. This aligns with the Developing Mathematical Inquiry Communities focus for our current professional development within our Community of Learning (CoI).

My research will involve a group of volunteer year 6 and 7 students participating in a series of teacher/student cogenerative dialogues—group discussions designed to develop a shared understanding about our ongoing experiences of community of mathematical inquiry activities within the mathematic lesson. Importantly, student voice in the discussions will be used to inform further subsequent teaching and learning and will help students develop student agency.

The discussions will take place at a time mutually agreed by myself, my co-teacher and participants, ideally as soon as possible following a mathematics lesson. We will ensure that participation in these discussions will not result in students being deprived of crucial learning or breaktime. Six students will be involved in each discussion, with discussions being approximately 30 minutes in duration. Consenting students will not necessarily participate in every discussion. Discussion will take place in a private space and will be audio recorded for teacher reflection and analysis.

I am seeking your consent for this research to be conducted within our school. The research would begin in term 4, 2018 and conclude in term 1, 2019. All data gathered would be used for the purposes of this study. Given my role as teacher/researcher/author of the thesis, information about the study will be in the public domain, in that the school will be identifiable to those in the community. With the increased move to teacher inquiries and promulgation of findings, identification of school is now common practice. However, in written reports efforts to address confidentiality of individual student participants will be strengthened by use of pseudonyms and avoidance of any identifying personal information.

Please note that participation in this research study is voluntary and gives you the following rights:

- to ask questions at any stage of the research study
- be given access to a summary of the research study findings

This project has been reviewed and approved by the Massey University Human Ethics Committee.
If you have any questions regarding this research study, please contact me:

- email: [redacted] ph. [redacted].

Alternatively, contact my supervisors at Massey University (Palmerston North).

- Professor Glenda Anthony
  email: gjanthony@massey.ac.nz ph.06 356 9909 ext. 84406
- Ms Raewyn Eden
  email: R.Eden@massey.ac.nz ph. (06) 356 9099 ext. 86252

Fiona Rice

Appendix G

MASSEY UNIVERSITY
TE KUNENGA KI PŪREHUROA
UNIVERSITY OF NEW ZEALAND

Institute of Education
Massey University, Te Kunenga ki Purehuroa, New Zealand

Using student voice to explore participation in a co-constructed

Mathematical Inquiry Community

CONSENT FORM: PARENT/CAREGIVER OF STUDENT PARTICIPANT

This consent form will be held for a period of five years.

PARENT/ CAREGIVER CONSENT

- I have read the information sheet and have had the details of the research study explained to me.
- My questions have been answered to my satisfaction and I understand that I can ask further questions at any time.

- **I agree** to my child_______________________________________ participating in this research study under the conditions set out on the attached information sheet.

Parent/Caregiver Name: (printed) __________________________ Signed ______________

Date: __________________________

Please return the completed consent form to Fiona Rice by Monday 12th November

Appendix H

Institute of Education
Massey University, Te Kunenga ki Purehuroa, New Zealand

Using student voice to explore participation in a co-constructed

Mathematical Inquiry Community

PARENT/CAREGIVER INFORMATION SHEET

Dear Parents,

I am currently undertaking a research project through Massey University in order to complete my Master of Education (Mathematics Education). Currently our school is working to incorporate a greater use of collaborative group activities in maths lessons, and the research will seek to explore students' views about how they are experiencing and learning during these activities. This research has been approved by both the Board of Trustees and the Principal.

My research will involve a group of volunteer year 6 and 7 students participating in a series of teacher/student group discussions designed to develop a shared understanding about our ongoing experiences of community of mathematical inquiry activities within the mathematics lesson. Importantly, student voice in the discussions will be used to inform further subsequent teaching and learning and will help students develop agency in their learning.

I would like to invite your child to participate in this study. This will involve your child participating in these small group discussions about the preceding mathematics lesson. Drawing on their experiences in the lesson, the students and I will talk about what is working, what is not, and how we can improve learning. These group discussions will be audio recorded, allowing me to review and reflect on responses as part of the research.

The discussions will take place at a time mutually agreed by myself, my co-teacher and participants, ideally as soon as possible following a mathematics lesson. We will ensure that participation in these discussions will not result in students being deprived of crucial learning or breaktime. Six students will be involved in each discussion, with discussions being approximately 30 minutes in duration. Consenting students will not necessarily participate in every discussion. Discussion will take place in a private space and will be audio recorded for teacher reflection and analysis.

I am seeking your consent for your child to participate in this research, which would begin in term 4, 2018 and conclude in term 1, 2019. Students will also be asked for their consent. All data gathered would be used for the purposes of this study. Given my role as teacher/researcher/author of the thesis, information about the study will be in the public domain, in that the school will be identifiable to those in the community. However, in written reports efforts to address confidentiality of individual student participants will be strengthened by use of pseudonyms and avoidance of any identifying personal information.

Please note that participation in this research study is voluntary and gives you the following rights:

- to withdraw your child from involvement in the study
- to ask questions at any stage of the research study
- be given access to a summary of the research study findings

This project has been reviewed and approved by the Massey University Human Ethics Committee. If you have any questions regarding this research study, please contact me:

- email: [redacted] ph. [redacted].

Alternatively, contact my supervisors at Massey University (Palmerston North)

- Professor Glenda Anthony
  email: gjanthony@massey.ac.nz ph.06 356 9909 ext. 84406
- Ms Raewyn Eden
  email: R.Eden@massey.ac.nz ph. (06) 356 9099 ext. 86252

Thank you for considering allowing your child to take part in this research.

Fiona Rice

Appendix I

MASSEY UNIVERSITY
TE KUNENGA KI PŪREHUROA
UNIVERSITY OF NEW ZEALAND

Institute of Education
Massey University, Te Kunenga ki Purehuroa, New Zealand

Using student voice to explore participation in a co-constructed

Mathematical Inquiry Community

CONSENT FORM: STUDENT PARTICIPANT

This consent form will be held for a period of five years.

STUDENT CONSENT

- I have read the information sheet and have had the details of the research study explained to me.
- My questions have been answered to my satisfaction and I understand that I can ask further questions at any time.
- **I agree** to participating in this research study under the conditions set out on the attached information sheet.

Student's Name: (printed) ________________________ Signed ________________

Date: _________________________

Please return the completed consent form to Fiona Rice by Monday 12th November

Appendix J

Institute of Education
Massey University, Te Kunenga ki Purehuroa, New Zealand

Using student voice to explore participation in a co- constructed

Mathematical Inquiry Community

STUDENT INFORMATION SHEET

Dear

This year I have the opportunity to be involved in research through Massey University to complete my Master of Education (Mathematics Education). My study will explore how we are going with developing our new mathematical inquiry community. I am interested in how you as a student feel about learning and participating in group activities and sharing your solutions in maths lessons.

During term 4 this year and term 1 next year we will organise a series of group discussions with those students who choose to participate. During the discussions we will discuss the maths lesson that has taken place that day. If you choose to participate, you will have the opportunity to discuss what the lesson was like for you and your group, what went well with your learning, what didn't go so well, and what we can all do to make the next lesson work well for you.

In deciding the best time to meet I will ensure that you will not miss out on any important learning or break time. Discussions will be no more than 30 minutes long. They will be audio recorded so that I am able to review the discussion and what the group felt we needed to do next. When I write about the results of this study I will not use your name, so that whatever you say in these discussions cannot be identified to you personally.

I would like to invite you to participate in my study into student experiences and opportunities to learn in our mathematical inquiry community. However, you are not obliged to take part and there will be no disadvantage to you if you choose not to participate. Please note that as part of this research you have the following rights:

- to say you do not wish to participate in the study
- to withdraw from the study
- to ask questions at any stage throughout the study
- to participate knowing you will not be identified in written reports
- to ask for specific comments made by you in the discussion to be removed from the research data
- to be given a summary of the study findings

This project has been reviewed and approved by the Massey University Human Ethics Committee. If you have any questions about this research study, please contact me:

- email: [redacted] ph. [redacted].

Alternatively, contact my supervisors at Massey University (Palmerston North)

- Professor Glenda Anthony
  email: gjanthony@massey.ac.nz ph.06 356 9909 ext. 84406
- Ms Raewyn Eden
  email: R.Eden@massey.ac.nz ph. (06) 356 9099 ext. 86252

Thank you for thinking about participating in this research.

Fiona Rice

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