A STUDY OF FACTORS AFFECTING TEST DAY RECORDS OF DAIRY CATTLE

A thesis presented in partial fulfilment of
the requirements for the degree of
Master of Agricultural Science
in Animal Science at
Massey University

by Brian Walter Wickham

AUGUST 1972

AUGUST, 1972

I give permission for my thesis, entitled A STUDY OF FACTORS AFFECTING TEST DAY RECORDS OF DAIRY CATTLE to be made available to readers in the Library under the conditions determined by the Librarian.

I agree to my thesis, if asked for by another institution, being sent away on temporary loan under conditions determined by the Librarian.

I also agree that my thesis may be microfilmed for Library use.

Signed:

B W Wickham

B.W. WICKHAM.

ACKNOWLEDGEMENTS

It is a pleasure to thank Prof. A.L. Rae for his assistance and guidance in all aspects of this study. Dr. D.A. Evan's, of Dalgetys N.Z. Ltd, help with the development of models and estimation procedures is gratefully acknowledged. For assistance with programming and computing I sincerely thank Mr R.P. Irving and Mrs S. Shilton of the New Zealand Dairy Board.

The opportunity to use the extensive records and computing facilities of the New Zealand Dairy Board has been much appreciated. Without the generous financial assistance of the Farm Production Division of the New Zealand Dairy Board this study would not have been possible.

TABLE OF CONTENTS

INTRODUCTION

Dairyfarming in New Zealand is fundamentally concerned with obtaining an income from the sale of milk and milk fat produced by ones cows. A dairy farmer's income can be increased by either reducing the costs of production and/or increasing the value of milk and milk fat sold. One method by which income can be increased is the culling of low producers and selection of high producing replacements.

In New Zealand Herd Improvement Associations provide three systems whereby the production of individual cows is measured monthly, bimonthly or twice yearly. In using these records for culling and selective breeding there are two main groups of influences that have to be taken into account - environmental influences and genetic influences. In selective breeding the major objective is in fact to separate the environmental and genetic influences so that the genetically "best" animals can be used to produce the next generation.

Any attempt to evaluate either the genetic or environmental influences on test records must take into account the other. For example in bull selection the daughters of the bulls being compared may be of different ages and milked in different herds. The relative genetic value of the bulls can only be evaluated after the influence of age and herd, on daughter production, has been taken into account. Likewise in evaluating the influence of age on production the cows in each age class may be the daughters of different sires. Only by taking account of these genetic differences can the affect of the environmental factor be evaluated.

For the purposes of selective breeding environmental factors can only be taken into account if they fulfill two requirements - the environmental factor can be measured and - the influence of the environmental factor on production is known or can be readily estimated. Many environmental factors, such as level of feeding and climatic conditions, are not readily quantified under New Zealand conditions.

While others, such as year and geographical region are readily quantifiable but their affect on production cannot be predicted and is known only in retrospect.

Age at calving, herd and days in milk when tested are known for each test day record collected in New Zealand. These environmental factors are known to have a major influence on test day production. The affect of herd as measured by test day herd average is a combination of genetic and environmental influences. Days in milk is an influence which is the combination of the influence of stage of lactation and season of calving. Just how important season of calving is, is not known and cannot be determined because it is always completely confounded with stage of lactation in its affect on test day production.

The objective in adjusting test day records for these environmental factors is to estimate what the production would have been had the cow been mature, lactating for some standard number of days when tested and milked in an average herd. All other environmental and genetic influences are assumed constant. If selection is only on a within herd basis then herd need not be taken into account. A perfect adjustment procedure would result in the adjusted records having the same genetic variance but a reduced environmental variance when compared with the unadjusted records. The amount by which the environmental variance is reduced would be equal to the environmental variance accounted for by these environmental influences in the uncorrected data.

Techniques are available for the simultaneous estimation of environmental effects and the prediction of sire effects from test day records. These procedures however require very large computing facilities the like of which are not available at present. For some time to come it seems likely that environmental effects will be taken into account by estimating their influence in retrospect and using these estimates as the basis for adjusting subsequently collected records. Since 1963 test day records collected in herds on twice yearly testing have been adjusted for age and days in milk using factors developed from records collected in 1956 to 1958.

The factors used to adjust records for days in milk are based on the assumption that season of calving does not contribute significantly to the influence of days in milk on test day production. Also the factors used to adjust for age may contain biases due to sire selection. It is as a result of these two facts that the present study was initiated.

The objective of this study is to use the technique of best linear unbiased estimation (BLUE) to obtain estimates of the affect of age on test day production free of biases due to sire selection and to estimate the affect of days in milk and days in milk squared on test day production. An attempt will be made to evaluate the effectiveness of correction factors based on these estimates.

REVIEW OF LITERATURE

A. FACTORS INFLUENCING TEST DAY PRODUCTION

In discussing factors which influence test day production consideration will be given to methods of estimating the effects and of removing the variance controlled by these effects from test day records.

1. AGE AT CALVING

Estimates of the affect of age at calving on test day production are used primarily to derive age correction factors. Searle and Henderson (1960) in a discussion of the use of age correction factors state, "... The purpose of such factors is to correct a record for age alone, assuming all other environmental factors remain the same. The corrected record is then an estimate of what the cow would have produced under exactly the same conditions had she been older ..."

Age per se is not the factor which is being corrected for. Rather, it is those affects of maturity of the mammary gland and of lactational ability for which age is a convenient highly correlated measure. In fact a number of alternatives to age alone have been investigated. Lactation number has been considered by Sanders (1928a) who concluded that age is more closely related to stage of maturity than lactation number. Fimland et al (1972) consider age within lactation number and find it accounts for approximately 3% of the, first and second lactation, variation in milk yield and fat corrected milk yield. Miller R.H., et al (1970) and Ronningen (1967) found that a larger percentage of the total variation in milk yield was accounted for by a combination of age and lactation number than age or lactation number alone. Syrstad (1965) concluded that the specific affect of lactation number (i.e. affect independant of age) on milk yield and fat percentage was of minor importance. Other authors (e.g. Clark and Touchberry (1962) and Miller and Hooven (1969)) have investigated the use of a combination of body weight and lactation number. As none of these measures are perfectly correlated with maturity of lactation performance they are not ideal measures of maturity. Because age at calving information is readily available for most cows, is accurate and as useful or more useful than lactation number or body weight it is the preferred measure of maturity.

Estimating the affect of age:

Estimates of the affect of age, on total lactation and part lactation yields, have traditionally been based on records collected under commercial farming conditions. Kay and M'Candlish (1929) review some of the earlier estimates which were obtained using what has subsequently been termed the gross comparison method. This method uses age class averages, pooled over years, as the basis for comparing age groups. Sanders (1928a) and Kay and M'Candlish (loc cit), recognizing that the age class average for older age classes would be inflated by culling, used the paired comparison method. The paired comparison method is based on the average difference between records made by the same cow at different ages. Lush and Schrode (1950) show that both of these methods give biased estimates of the affect of age on lactation yield when culling is based on production. A study reported by Miller (1964) in which gross and paired comparison age factors are compared illustrates the difference between these two methods. Gross comparison gave consistently higher estimates of age effects than did paired comparison.

As shown by Henderson (1949) best linear unbiased estimation (BLUE, called maximum likelihood in Hendersons paper) procedures are appropriate to estimation in situations where repeated observations are collected and selection on the basis of the magnitude of earlier performance is practised. That cow culling on the basis of previous performance is practised has been shown by Hinks (1966a), 1966b) for Ayrshire and Friesian populations in the United Kingdom, and by Allaire and Henderson (1966a, 1966b) for a Holstein population in the United States. Sire selection may also cause biases in estimates of age effects. Rendel and Robertson (1950) predict genetic trends due to sire selection. Such a trend can be observed as an increase in average sire rating in the New Zealand Dairy Board's Artificial Breeding Scheme (N.Z.Dairy Board (h)).

Miller et al (1966) used the BLUE method to determine age effects in a sample of 24,636 lactations collected over an eight year period. As predicted the estimates of age effects obtained using BLUE were between those obtained by paired and gross comparison. To obtain unbiased estimates of age effects using BLUE the ratio of certain variance components must be known without error. The magnitude of the bias resulting from the use of incorrect ratios is not clear. Henderson (1958) found that for each 0.01 by which repeatability (function of variance component ratio) exceeds true repeatability the bias in environmental trend is - 0.08 lb fat/year. Estimates of age effects obtained using BLUE have been reported by Miller and Henderson (1968), Miller P.D. et al (1970) and Wunder and McGilliard (1971).

Where variance component ratios are not known Cunningham and Henderson (1968) suggest an iterative procedure for estimating variance components and fixed effects. With the modifications suggested by Thompson (1969) this method gives unbiased estimates of variance components and fixed effects. As Searle (1971) points out the iterative procedure is only practicable for models involving a single random classification. This method is discussed in greater detail on page 26 .

Estimates of the affect of age on test day milk and fat yields have been obtained using only the gross comparison method. Searle (1961a) developed factors which corrected test day fat records for age and first month on tests simultaneously. Van Vleck and Henderson (1961c) developed factors for adjusting monthly test day records for age and season of calving. Because these estimates are obtained by gross comparison it is reasonable to suggest they will be biased to the extent that monthly production is a component of the "culling variate".

Factors influencing the affect of age:

Searle and Henderson's (1960) statement of the purpose of age correction factors implies that different sets of factors are needed for different environments in as much as the environment affects rate of maturity. For any scheme where age correction is to be used it is thus highly desirable to know what environmental factors influence rate of maturity.

(i) Herd environment:

Herd differences can cause large differences in the production of cows genetically identical as was shown by Brumby (1961) when identical twins were distributed to high and low producing herds. Hickman and Henderson (1955) found a negative relationship between herd level and increase in age corrected production from first to second lactation. That this negative relationship was due to a positive relationship between herd level and actual increase from first to second lactation was pointed out by Hickman (1962). The fault being that the age correction factors used tended to overcorrect records in low herds and undercorrect records made in high herds. Thus producing a relationship between herd level and age for corrected records which is reverse in sign to that for uncorrected records. Searle and Henderson (1959) using uncorrected records, from New York Holstein herds, determined regressions of age factors on level of herd production. The regressions were all significantly positive. These results were confirmed for New Zealand records by Searle (1962) using a simplified method of determining herd level age factors (Searle (1960)). However, Hickman (1957), Searle (1962) and Searle and Henderson (1960) found little advantage in using individual herd age factors compared with the use of general multiplicative age correction factors. Lee and Hickman (1967) observed that records corrected using individual herd level factors had zero regression of yield on age compared with a non zero regression for records corrected by a method which ignores herd differences.

(ii) Season of calving:

In dairy regions where year round calving is common practice it has been shown that the influence of age on lactation production interacts with season of calving. For lactation milk production significant interactions have been found by Lee and Hickman (1967), McDaniel and Corley (1966), McDaniel et al (1967b), Miller and Henderson (1968), Miller et al (1966), Miller P.D. et al (1970), and Wunder and McGilliard (1971). McDaniel and Corley (loc cit) also found a significant interaction for lactation fat yield. Miller and Henderson (loc cit) explain part of the interaction found, by using gross or paired comparison methods, as being due to a confounding of level of herd production with season-age of calving effects. Miller P.D. et al (loc cit) confirm these findings but suggest that the real interaction is of sufficient magnitude to justify separate age factors for each month of calving subclass. The major component of the season by age interaction is that young cows show a smaller reduction in lactation yield under poorer conditions than do older cows. This is what would be expected if poorer seasonal conditions were analogous to poorer herd environments.

Significant interactions between age and season of calving for test day production have been found by Van Vleck and Henderson (1961c) and Miller et al (1967). In both of these studies the method adopted was to compute ratio factors (by gross comparison) for correcting records made at the same stage of lactation by cows calving in the same season. These factors were then classified and using a three way analysis of variance significant effects were determined.

Under New Zealand conditions where calving occurs over a restricted period of time Searle (1961a) found that month of first test and age at calving interact in their affects on lactation fat yield. Because the length of lactation is related to the month of first test the reason for this interaction is not clear. It may be due either, to a simple month of first test by age interaction, or, an interaction between age and lactation curve shape. An interaction between month of first test and age effects on test day production can be observed in Searle's results.

Table 1 shows this interaction. October first test cows have larger age correction factors than do cows first tested in August or September.

Table 1

First Test	Month of Lactation 1	2	3	4
August	1.36	1.30	1.27	1.23
September	1.32	1.30	1.27	1.27
October	1.34	1.34	1.32	1.32

Test day age correction factors for cows two years old at calving according to month of first test and stage of lactation (from Searle (1961a)).

In general later calving young cows show a smaller increase in test day production than do later calving mature cows when compared with earlier calving young and mature cows respectively. This is the same as is observed for lactation production. Searle (1961a) derives age factors which correct records made at the same stage of lactation for age and month of first test simultaneously. To correct records made on the same test day a consideration of the interaction of stage of lactation with age is required (see page 11). In this situation season of calving is completely confounded with stage of lactation.

(iii) Geographical region:

If season of calving and herd environment influence rate at which maturity, in milk production, is reached it would seem reasonable that similar differences may occur between geographical regions. In the United States McDaniel et al (1967b) found substantial differences between geographical regions when developing age correction factors for adjusting Dairy Herd Improvement Association lactation records. Further evidence is provided by Miller (1964) and McDaniel and Corley (1966). More recently Miller, R.H. et al (1970) have shown that the factors developed by McDaniel et al (1967b) do not take adequate account of regional differences in age effects.

In New Zealand age correction factors used differ according to geographical region (N.Z.Dairy Board (d) and Herd Improvement Assoc. Manual (1970)). That failure of age correction factors to take into account differences due to geographical region can result in errors in sire ranking has been shown by Miller, P.D. et al (1970). Just how much of the affect of region on rate of maturity could be accounted for by herd and season of calving effects is not known.

(iv) Year:

Koh and Henderson (1964) report a year by age interaction in New York dairy records. Searle (1961a) considered the difference between test day age factors in two successive years to be non-significant. If year x age interactions do exist, which seems likely, they may cause biases in sire evaluation when sires are compared between years, and within years when sires have different number of daughters in each age class. However present methods do not permit simultaneous estimation of age effects and prediction of sire effects. Thus, at present, age correction must be based on the assumption of non-significant year by age interactions.

(v) Breed - sire - genotype:

In a herd where both Ayrshires and Holsteins were milked Hickman (1957) observed that the Ayrshires were quicker maturing in 180 day milk yield than the Holsteins. Other reports have suggested rate of maturity differs between breeds (Kendrick (1955), McDaniel and Corley (1966), McDaniel et al (1967b), Herd Improvement Assoc. Manual (1970)). These estimates are confounded with the affects of herd environment. Having different sets of factors as derived by McDaniel et al (loc cit) will not cause serious errors providing all comparisons are made within breed. In New Zealand the breed structure of the dairy population is undergoing rapid change (N.Z. Dairy Board (g)) and between breed comparisons (Jersey with Jersey x Friesian) are being made within herds. The affect of breed per se on the affect of age needs to be known for unbiased comparisons.

Hillers and Freeman (1965) found significant differences between sires in the regression of age on first lactation production. Ronningen (1967) reports a heritability of 0.06 - .02 for increase in yield from first to second lactation. Ward and Campbell (1938) suggest a positive correlation between level of production in first lactation and the increase in fat yield from first to second lactation. Robertson and Khishin (1958) found zero values for the regression of increase from 1st to 2nd lactation and 2nd to 3rd lactation, on first lactation contemporary comparison. Only if there were a negative genetic relationship between first lactation and increase to subsequent lactations would age correction need to take into account cow genotype. Sire would need to be considered in age correction if age factors were highly heritable and present evidence does not suggest this.

(vi) Stage of lactation:

In New Zealand all cows in a herd are tested on the same calendar day and will thus be at different stages of lactation. It has been clearly established that the effect of age on test day production is strongly influenced by stage of lactation (for test day milk fat yield Searle (1961a), (1961c) and (1963), for test day milk yield Sanders (1923), (1928a), Madden et al (1956), Lennon and Mixner (1958), Lamb and McGilliard (1959), (1960), Van Vleck and Henderson (1961c), Smith and Legates (1962a), McDaniel et al (1967a), Spike and Freeman (1967), Appleman et al (1969) and Wood (1969) and for both test day milk yield and fat yield Madden et al (1955), Lamb and McGilliard (1967b) and Miller et al (1967)). This interaction is mainly due to the greater persistency, of milk and fat yield, in first lactations compared with later lactations. Thus to age correct the test day records of young cows different age factors are needed for different stages of lactation. Such factors have been developed from New Zealand data by Searle (1961c), (1963) and Castle (pers comm).

(vii) Trait:

Many studies have shown that whole and part lactation milk yields and fat yields are affected differently by advancing maturity (e.g. Kay and M'Candlish (1929), Hickman and Henderson (1955), Clark and Touchberry (1962), McDaniel and Corley (1966), Miller et al (1966), and McDaniel et al (1967b)). Milk yield and fat yield require different age correction factors because it appears that milk yield increases with age while milk fat percentage declines slowly with age (Mahadaven (1951)) so that fat yield increases with maturity but at a rate slower than that for milk yield.

2. STAGE OF LACTATION AS A FACTOR AFFECTING TEST DAY PRODUCTION

The form of the trend in milk yield, fat yield and milk fat percentage, with time from parturition has been the subject of numerous studies. When all cows in a herd are tested on the same calendar day it is inevitable that they will not all be at the same stage of lactation. That stage of lactation is an important source of variation in test day production has been shown, for New Zealand conditions, by Searle (1961c) and Castle (pers comm). Within a seasonal supply herd approximately 80% of cows calve within a ninety day period (N.Z. Dairy Board (b)). This variation will be much greater in herds maintaining year round production.

Describing the lactation curve:

(i) Curve shape

In early studies of the lactation curve Brody and Ragsdale (1923a) derived an equation to describe the curve. The equation had the general form: $Y_t = A_e^{-kt}$ where Y_t is average daily yield t units of time after parturition, A and k are constants describing peak yield and rate of decline respectively. This equation did not give a good fit and Brody et al (1924) modified it to

$$Y_t = Ae^{-k_1 t} - Be^{-k_2 t}$$

to explain the initial rise as well as the gradual decline as lactation progresses. Brody attempted to give these equations biological interpretations. Other measures of curve shape have included:

- Shape factor = $\frac{\text{total lact yld}}{\text{max wkly yld}} \times \frac{\text{av. max. wkly yld}}{\text{av. total lact yld}}$

as used by Sanders (1923)

- Average percentage decline for each month after peak lactation was used by Turner (1926)
- Ludwick and Peterson (1943) used a weighted decline after peak lactation
- Wood (1967) fitted the model

$$Y_t = at^b e^{-ct}$$ to the lactation curve

where a, b, and c are curve constants and Y_t is the average daily yield in the t'th week of lactation.

Ronningen (1967) gives a brief review of measures of persistency. By using these measures of curve shape it is then possible to test the significance of factors affecting curve shape.

(ii) Ratios and regressions of whole lactation on part lactation:

Many North American workers have used either ratios or regressions of whole on part to predict whole lactation and to study factors affecting the lactation curve. A good example of the use of ratios is provided by the study reported by McDaniel et al (1967a). In this study the ratio of lactation production to monthly and accumulative monthly production were computed for the classifications, age, yield trait, season of calving and month of lactation. The average ratios for each age-trait-season of calving subclass were used as individual observations and analysed using a factorial design. From the results it was concluded that breed, age, season of calving and yield trait have significant affects on the ratio factors for total to part yield.

Factors affecting lactation curve shape:

The objective in correcting test day records for stage of lactation is to reduce this source of variation in test day records. The corrected records will give a more accurate prediction of cow genotypes than the uncorrected record. If the affect of stage of lactation is a function of cow genotype then correction may not increase prediction accuracy.

(i) Breed-sire-genotype

There are numerous reports of differences in lactation curves due to breed of cow (for milk yield Lamb and McGilliard (1959), (1960), (1967a), Fritz et al (1960), McDaniel et al (1967a), for fat yield Lamb and McGilliard (1967a), and for milk fat percentage Korkman (1950) and Erb et al (1953)). However few studies have separated the affect of breed per se from other confounding factors such as herd. Fletcher (1960) found significant differences in ratio extension factors for Jersey and Jersey x Sindhi cows milked in the same herd.

Smith and Legates (1962a) used the ratio of production for the last 215 days to the production for the first 90 days of the 305-day lactation as a measure of persistency. Persistency in first lactation records had a heritability of 0.33 and for second and later lactations was -ve. Wood (1970) concluded that sire explained less than 5% of the variation in lactation curve shape. Wood also found a low repeatability of curve shape. Although based on relatively small numbers of observations these two studies tend to indicate that genotype is a minor source of variation in curve shape. This being so the use of correction factors for stage of lactation seem likely to increase the accuracy of test day genotype prediction.

(ii) Age at calving:

The interaction between age and stage of lactation has been discussed on page 11 .

(iii) Season of calving:

The early studies made by Hammond and Sanders (1923) and Sanders (1927a) revealed distinctly different shaped lactation curves for spring and autumn calvers in the United Kingdom. These findings were confirmed in the U.S.A., by Turner (1923) and more recently by many workers (Hickman (1960), Fletcher (1960), Lamb and McGilliard (1959), (1960) and (1967b), McDaniel et al (1967a) and Appleman et al (1969)). Season of calving can influence the shape of the lactation curve and different sets of stage of lactation factors would be needed for each distinct season of calving.

Under New Zealand conditions approximately 80 percent (N.Z. Dairy Board (b)) of the cows in seasonal supply herds, calve over a three month period. Should this be considered as one season of calving or do cows calving in early August have distinctly different curves from those calving in late October? Sanders (1930) considered month of calving caused significant differences in persistency. In the U.K. February, March, and April calvers have different shaped lactation curves (M.M.B. (1961)). Wilson (1964) found that 8, 9 and 10 pairs of identical twins differing in average date of calving by 49 days, 52 days and 82 days respectively had very similar shaped lactation curves (eye appraisel). Wilson also plotted average milk yields according to month of calving of Jersey cows three years of age and older milked in the Massey University herd. These curves indicate shape differences. Later calvers were less persistent and reach peak yield sooner than earlier calving cows. Searle (1961a), working with a much larger sample than Wilson, used percentage of cows tested still in milk six months later as a measure of persistency. Searle showed that, within age class, cows first tested in August were more persistent than cows first tested in September or October. Searle also noticed that peak of production is in the third month of lactation for cows first tested in August and in the second month of lactation for cows first tested in September and October. In developing factors for correcting test day records for age and stage of lactation Castle (pers comm), using the same data as Searle (loc cit), assumed the same shaped curve for all months of calving.

To correct test day records, for stage of lactation, knowledge of the full lactation curve is required only if cows at the beginning and the end of lactation are being tested on the same test day. This is rarely the case in New Zealand seasonal supply herds. In fact what is required is a knowledge of the affect of days in milk (which is fully confounded with stage of lactation) on test day production for each test period (month in New Zealand). Using this knowledge it would then be possible each test day to bring all cows to effectively the same stage of lactation (same date of calving) and is the method used by Searle (1963). Factors which affect the influence of days in milk would be expected to be the same as those affecting the affect of stage of lactation.

(iv) Herd:

Cannon et al (1942) found the average persistency, of milk yield, in the Iowa College herd was significantly different from the average persistency of USDHIA records. Wood (1970) found that herd accounted for 5.4% of the variation in lactation curve constants for a sample of 1567 lactations of 336 Freisian cows in 10 herds. Van Vleck and Henderson (1961b) calculate regression factors for extending various part lactation milk records to complete lactation records. The method of extension takes into account herd by expressing the age and season corrected test day record for the ith month of lactation as a deviation from the herd mean for the ith month of lactation before multiplying by the regression factor. Van Vleck and Henderson (1961e) show that this procedure is always more efficient than when herd is ignored. However they suggest that the extra efficiency is not sufficient to offset the extra computation involved. Lamb and McGilliard (1959) concluded that herd did not significantly affect ratios of part- to whole- lactation records. Fritz et al (1960) found a significant herd effect for milk yield regression extension factors and a non-significant effect for fat yield regression extension factors.

Herd does influence lactation curve shape but because taking herd into account is computationally expensive it is usually ignored. Age and season of calving are much more important sources of variation than herd.

(v) Yield trait:

Milk yield and fat yield have different shaped curves due to the fat percentage curve being almost a mirror image of the milk yield curve.

(vi) Gestation:

The affect of gestation is non-significant until approximately 140 days after conception. After 140 days gestating cows show significantly lower persistency of milk yield than non-gestating herd mates (Brody et al (1923b), Sanders (1927b) and Erb et al (1952)). Both Korkman (1950) and Erb et al (1967) found that gestation of 180 days or greater causes a more rapid increase in fat percentage than in non-gestating herd mates. For cows calving at yearly intervals gestation is not an important factor affecting curve shape up to 240 days after parturition.

(vii) Dry period:

The influence of the length of dry period on curve shape Sanders (1928b) found to be small. In New Zealand, where drying-off applies to all cows in the herd after a fixed date, length of dry period would not be expected to be short ($<$ 30 days) but no information on this matter could be found. Because length of dry period is likely to be confounded with genetic merit (see Searle 1961a) attempts to remove any affects of dry period may reduce genetic variation.

3. HERD-YEAR AS A FACTOR AFFECTING TEST DAY PRODUCTION:

That a major part of the difference between herds in average lactation production is environmental has been shown by Robertson and McArthur (1955), Pirchner and Lush (1959), Roberston et al (1960), Brumby (1961) and Van Vleck (1963). About 80% of the differences are environmental the remaining 20% being genetic. Expressing lactation records as a deviation from a contemporary average reduces the genetic- and herd-variances without altering the error component (N.Z. Dairy Board(k)). In New Zealand to prevent this reduction of genetic variance in sire proofs the genetic level of contemporaries (estimated from male ancestor proofs) is corrected for (Shannon (1971)). There is little information available relating to the affect of herd-year on test day production. Since lactation production is an aggregate of test day productions it may be concluded that on average herd-year has a similar affect on both.

B. CORRECTING TEST DAY RECORDS

Herd level- and multiplicative- correction factors have been used to correct test day records for age and days in milk or season of calving. In all cases the correction factors have combined correction for the two environmental factors involved. Searle (1961c) used herd level factors to correct test day records made on the same test day for age and month of first test. The herd level correction factor is a linear function of herd average corrected test day production and is added to the uncorrected record. Searle and Henderson (1959) have shown that the affect of age is a function of herd-level. However Searle's factors assume that the affect of month of first test is also a function of herd level. This has not been demonstrated.

Castle (pers comm) and Searle (1963) developed multiplicative factors to correct test day records for age and days in milk. Using these factors the corrected record is the product of the correction factor and the uncorrected record. Searle and Henderson (1960) showed that multiplicative factors are capable of taking into account between herd differences in age effects. The use of multiplicative factors implies also that the effect of days in milk is a function of test day production. The evidence of Madden et al (1959) suggests that level of production and curve shape are independant. Madden et al arrived at this conclusion by comparing cows milked twice daily with cows milked thrice daily. On the other hand Appleman et al (1969) concluded that higher producing cows show lower persistency than lower producing cows. This can be explained by Appleman's et al's method of distinguishing high and low producers. They used peak lactation yield to distinguish high and low producers.

Peak lactation yield is negatively phenotypically correlated with persistency and selection on this basis would lead to a negative relationship between persistency and peak yield.

A more recent development has been the correction of age and days in milk adjusted test day records for "herd level" (N.Z. Dairy Board (f)). This is attempted by expressing the age, days in milk adjusted test day record as a ratio of the herd average. It is suggested that this ratio accumulated over test days will provide a valuable aid to cow culling. The use of a ratio rather than a deviation was due to the ability of a ratio to remove differences in within herd variance of yield between herds (Stichbury (pers comm)). This accumulated ratio has a smaller herd by sire interaction (as % of total variance) than do accumulated deviations (N.Z. Dairy Board (k)). For the purposes of sire selection a ratio may give smaller herd by sire interactions by taking into account the small non-linear regression of daughter records on herd mates (Van Vleck (1963)).

In this study an attempt will be made to separate corrections for age and days in milk and to further evaluate the value of correcting for herd.

C. PARAMETERS OF TEST DAY RECORDS:

The literature contains several studies where parameters of test day records, made at the same stage of lactation, have been estimated (Madden et al (1955), Searle (1961b), Van Vleck and Henderson (1961a), Lamb and McGilliard (1967b), and Keown and Van Vleck (1971)). The only estimates for records made on the same test day are those of Searle (1963). Searle used records which had been corrected for age and stage of lactation using multiplicative factors. The heritability of monthly fat yield for the first 6 test months (August to January) ranged from .23 to .28 and .09 to .11 when paternal half sib correlations and within herd daughter-dam regressions were used to estimate heritability respectively. The corresponding figures for lactation yield were 0.36 to 0.25 respectively. Searle also attempted to estimate the genetic correlation of test day with lactation records but very high standard errors made the results meaningless. However in the studies, using lactation month records, almost without exception high genetic correlations between test day record and lactation record have been reported. Some of the studies (Searle (1961b), and Keown and Van Vleck (1971)) suggested lower genetic correlations for early and late stages of the lactation. These genetic correlations support the hypothesis that different genes are involved in influencing level of production at different stages of lactation. The overall level of genetic correlation suggests that the influence of these "part lacation" genes on test day production is relatively minor.

Searle (1961b) and Everett et al (1968a) in estimating variance components show that the between-and within-herd variance of test day milk and milk fat decline as the lactation proceeds.

D. THE NEW ZEALAND RANKING INDEX:

As a result of a study conducted by the Farm Production Division of the New Zealand Dairy Board (N.Z. Dairy Board (c), N.Z. Dairy Board (e), Searle (1961a), (1961b) and (1961c)) it was shown that selection based on as few as two tests during the middle months of lactation would result in only slightly slower genetic improvement in lactation yield than selection based on regular monthly tests for the whole lactation. Table 2 is an extract from table 4 of N.Z. Dairy Board (e).

Table 2. Estimates of heritability, genetic correlation with lactation yield, and relative selection efficiency.

Combination of months	Heritability	Genetic correlation	Relative Selection efficiency *
Aug. + Sept.	.34	.80	78%
Aug. + Oct.	.34	.84	82%
Sept. + Oct.	.33	.91	87%
Sept. + Nov.	.32	.94	89%
Oct. + Dec.	.30	.97	89%

* Rate of progress in improving additive genetic merit for lactation yield, when selecting on monthly combinations, compared with selecting on lactation yield itself.

As a result of these findings the "Production Ranking Test" was introduced as an alternative to the "Alternate Month Test" and the "Monthly Test" in the 1963-64 season. Stichbury (1963) describes how the system operates:

> Each herd will be tested twice during the season ... after the second test has been carried out ranking information will be supplied for each cow ... The ranking information will be in the form of an age corrected lactation butterfat yield ... adjusted to a common length for each cow. It will be known as the Ranking Index ...

In the 1968-69 season all farmers who tested their herds were supplied with a ranking index for each cow after each test. The method of calculating this index is described in N.Z. Dairy Board (f): (1969 method)

> The index (Ranking Index) is based on a "within test-day" comparison of each cow with her herd mates. Each test-day record is corrected for stage of lactation at which the test was made, and records of immature cows are also corrected for the age and breed of the cow. Each cow's record is then expressed as a percentage of the herd average of all corrected records for that test day. ...Up to the first 240 days the values are averaged after each test, the average being adjusted (regressed) to take into account the number of test days on which it is based. The average is the ranking index shown in the farmers testing return after each test ...

These two definitions of ranking index differ not only in their method of calculation but also in the type of correction factors used. The correction factors used for the "Production Ranking Test" are described in N.Z. Dairy Board (e). A different set of factors is used each month to correct for age and days in milk. Each test month these factors adjust the test day record to a mature equivalent standard days in milk record. The correction factors used in the 1969 method as found in the H.I.A. Manual (1970) are the same for all test months. These factors convert the test day record to an estimated mature equivalent lactation record and in this respect are analogous to the ratio extension factors of McDaniel et al (1967a). These procedures for correcting test day records have in common the assumptions that the affect of days in milk is the same for all months of calving and the affect of days in milk is the same for all months of test. Also it can be shown that these factors give the same ranking within a herd on a single test day.

Variance components of the ranking index (calculated by the 1969 method) have been reported (N.Z. Dairy Board (k)) along with variance components of several other measures of production. Table 3 is an extract of table 1 from N.Z. Dairy Board (k).

Table 3. Estimated Variance Components for various Production Traits. Component as percentage of total variance given in parentheses.

Trait	Age	Herd	Sire	HxS	Residual	Within herd heritability
Herd mate fat	2 year	478 (9.1)	253 (4.8)	377 (7.2)	4132 (78.9)	.21
	mature	139 (3.1)	125 (2.8)	136 (3.0)	4118 (91.1)	.11
Ranking index	2 year	29 (9.0)	15 (4.5)	12 (3.6)	284 (83.0)	.20
	mature	8 (3.0)	10 (3.9)	-3 (-1.1)	248 (94.0)	.16

These are the only known estimates of the variance components of the ranking index. It is the aim of the present investigation to obtain correction factors using BLUE and to evaluate by using variance component estimates different methods of expressing test day records.

METHOD OF ANALYSIS

ESTIMATION OF THE AFFECT OF AGE AND DAYS IN MILK ON TEST DAY PRODUCTION:

The model:

Within each month of testing the following model was fitted to the data

$$y_{ijk} = \mu + a_i + hs_j + b_{i1} \cdot X_{ijk} + b_{i2} \cdot X^2_{ijk} + e_{ijk}$$

where y_{ijk} is the test day record of the k th cow, in the j th herd-sire subclass, in the i th age group at calving. In this model,

μ is a general mean,

hs_j is an element common to all records in the j th herd-sire subclass,

a_i is an affect attributable to the i th age,

X_{ijk} is the number of days in milk when tested, for the k th cow, in j th herd-sire subclass, in the i th age group at calving,

b_{i1} and b_{i2} are partial linear regression coefficients of days in milk on test day production and days in milk squared on test day production for records made by cows in the i th age class at calving,

e_{ijk} is an element peculiar to the k th cow, in the j th herd-sire subclass, in the i th age group at calving.

The e_{ijk} and hs_j are assumed to be normally and independently distributed random variables with zero means and variances σ_e^2 and σ_{hs}^2. Also it is assumed that the e_{ijk} and hs_j are not correlated with each other.

A further assumption implicit in the model is that there is no interaction between ages and herd-sires. If interactions between age and herd-sires exist then the applicability of the estimates would be affected. By converting age effects to multiplicative factors or herd level factors any real interaction between age and herd-sires will be partly offset.

The model may be written in matrix form as

$$Y = X^*b^* + Zu + E$$

where Y is a N x 1 vector of test day production, X* is a known N x q matrix, b* is an unknown q x 1 vector of fixed affects and regression coefficients,

$$b^{*'} = (\mu, a_1, \ldots, a_4, b_{11}, b_{12}, \ldots, b_{41}, b_{42}),$$

Z is a known N x t matrix of rank t, u is a t x 1 unknown vector of random elements, and E is a unknown N x 1 vector of error elements. To facilitate the obtaining of estimates the first column of X* and row of b*, for the general mean, μ, are deleted to give X and b.

<u>A general solution for the model</u>:

We assume that in the model

$$Y = Xb + Zu + E \qquad \text{- - - - (1)}$$

the rank of X, = r, with b representing q-1 = r fixed effects and u, in representing the random effects contains t affects for just one random factor, having variance σ_u^2. As a result, Z has full column rank, t, with its columns summing to 1. By the nature of Z, the matrix Z'Z is diagonal, of order t, with $(Z'Z)^{-1}$ existing and being readily determined.

The BLUE solution for b and u is obtained by maximising the log of the joint distribution of y and u with respect to b and u. The solution is:

$$\begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z + \sigma_e^2/\sigma_u^2 . I \end{bmatrix} \begin{bmatrix} b \\ u \end{bmatrix} = \begin{bmatrix} X'Y \\ Z'Y \end{bmatrix} \qquad \text{- - - - (2)}$$

where σ_e^2 is the variance of e_{ijk} in the model and I is an identity matrix of order, t. Because Z'Z is generally a large matrix a solution for b and u may involve tedious calculations. A solution involving less tedious calculations is obtained in the following manner.

From (2) and letting $P = Z'Z + \sigma_e^2/\sigma_u^2.I$

$$X'Xb + X'Zu = X'Y \qquad ---- (3)$$

and $$Z'Xb + Pu = Z'Y \qquad ---- (4)$$

From (3) and (4)

$$\hat{b} = (X'X - X'Z\,P^{-1}\,Z'X)^{-1}\,(X'Y - X'Z\,P^{-1}Z'Y) \qquad ---- (5)$$

and $$\hat{u} = P^{-1}(Z'Y - Z'Xb) \qquad ---- (6)$$

This solution involves the inverse of the diagonal matrix P and the inverse of $(X'X - X'Z\,P^{-1}\,Z'X)$ which has rank r. However, it is assumed that the variance component ratio, σ_e^2/σ_u^2 is known. When this ratio is not known, Cunningham and Henderson (1968) suggest an iterative procedure for estimating the fixed effects and variance components.

Cunningham and Henderson's method is:

1. Make a prior estimate of k $(k = \sigma_e^2/\sigma_u^2)$
2. Estimate b as $\hat{b}$ in (5)
3. Calculate R(b, u)* and R(u/b)** where

$$R(b,u) = b'\,(X'Y - X'Z\,P^{-1}\,Z'Y) + Y'Z\,P^{-1}\,Z'Y \qquad ---- (7)$$

$$R(u/b) = R(b,u) - Y'X\,(X'X)^{-1}\,X'Y \qquad ---- (8)$$

4. Estimate σ_e^2 and σ_u^2 by equating calculated values of, (Y'Y - R(b,u)) and R(u/b) to their expectations and solving for σ_e^2 and σ_u^2.

---

* R(b,u) = reduction in sums of squares due to fitting b & u
= R(b/u) + R(u)

where $R(b/u) = b'\,(X'Y - X'ZP^{-1}\,Z'Y)$

and $R(u) = Y'ZP^{-1}\,Z'Y$

** R(u/b) = reduction due to u eliminating b
= R(b,u) - R(b)

where $R(b) = Y'X\,(X'X)^{-1}\,X'Y$

The expectations Cunningham and Henderson used or, implied in these estimates were:

$$E\ (Y'Y - R(b,u)) = (N - (t + r+1))\ \sigma_e^2 \qquad - - - - (9)$$

$$E\ (R(u/b)) = (r + 1)\ \sigma_e^2 + (tr\ (Z'Z - Z'X(X'X)^{-1}X'Z))\ \sigma_u^2 \qquad - - - - (10)$$

5. Calculate k and iterate from stage 2.

It was soon realized that Cunningham and Henderson's expectations contained serious errors. This was first pointed out by Thompson (1969) who derived the expectation of R(u/b) as,

$$E\ (R(u/b)) = E\ (Y'SZTZ'SY) \qquad - - - - (11)$$

$$= (\text{rank}\ (T) - k\ tr\ T)\ \sigma_e^2 + (tr\ Z'SZ - k \cdot \text{rank}\ (T) + k^2\ tr\ T)\ \sigma_u^2 \qquad - - - - (12)$$

where $S = I - X(X'X)^{-1}X'$

and $T = (kI + Z'Z - Z'X(X'X)^{-1}\ X'Z)^{-1}$.

Evans (1970) derives the same expectation for R(u/b) as given in (12). Evans also shows that

$$E\ (Y'Y - R(b,u)) = (N - r - t + k\ tr\ T)\ \sigma_e^2 + (tk - k^2\ tr\ T)\ \sigma_u^2 \qquad - - - - (13)$$

which is different from the same expectation in (9).

A solution for σ_e^2 and σ_u^2 is not readily available from (12) and (13) because T is the inverse of a nondiagonal matrix of rank t. t is usually large.

Thompson (1968) shows however that by equating k to σ_e^2/σ_u^2 (12) becomes:

$$E\ (R\ (u/b)) = (tr\ Z'SZ)\ \sigma_u^2 \qquad - - - - (14)$$

and (13) becomes

$$E\ (Y'Y - R(b,u)) = (N - t)\ \sigma_e^2 \qquad - - - - (15)$$

Using (14) and (15) as the basis of an iterative procedure provides unbiased estimates of σ_e^2 and σ_u^2 when k settles. This procedure was used in this study to obtain estimates of σ_e^2, σ_u^2, and b. σ_u^2 in representing the variance of the random element in the model is equal to σ_{hs}^2 where σ_{hs}^2 is the sum of herd, sire and interaction variances.

A program - RIANAL (appendix A) was written for a IBM 360/30 computer to obtain estimates of b, σ_e^2 and σ_{hs}^2 using Thompson's method. Estimates were obtained for test day-milk yield, - fat yield and - milk fat percentage for each of three test months in the 1970-71 season.

CORRECTION OF TEST DAY RECORDS:

(i) For days in milk:

If lactation curve shape and lactation production are independent then additive factors will suffice to reduce the variation controlled by days in milk.

Each test month all records were corrected to a standard number of days in milk and days in milk squared. The standards for the f th test month are dim_f and dim_f^2 - the average number of days in milk and average days in milk squared for all cows tested in the f th test month. These averages are used so as to minimize extrapolation errors. dim_f and dim_f^2 are obtained as a product of the RIANAL program. The additive correction factor for a record made in the f th test month by a cow in the i th age class which has been in milk x days is

$$df_{if} = \hat{b}_{i1f} \; (dim_f - x) + \hat{b}_{i2f} \; ((dim_f^2) - (x)^2) \qquad \text{- - - - (16)}$$

Where $\hat{b}_{i1f}$ and $\hat{b}_{i2f}$ are estimates of the partial regression coefficients of test day yield on days in milk and days in milk squared respectively, for cows in the i th age class at calving whose records are made in the f th test month.

(ii) For age at calving:

Multiplicative correction factors were derived for each test month. The factor for a record made in the f th test month by a cow in the i th age class at calving was derived as

$$A_{if} = \frac{\mu + a_4 + b_{41f} \cdot dim_f + b_{42f} dim_f^2}{\mu + a_i + b_{i1f} \cdot dim_f + b_{i2f} dim_f^2} \quad - - - - (17)$$

The factor for cows in age class 4 (5-9 year olds) is 1 for all test months.

If y_{ijkf} is a record made in the f th test month by the k th daughter, in the i th age at calving class, of the j th herd-sire subclass then the corrected record is

$$Z_{ijkf} = (y_{ijkf} + \hat{b}_{i1f} (dim_f - X_{ijkf}) + \hat{b}_{i2f} ((dim_f^2) - X_{ijkf}^2)) A_{if} \quad - - - - (18)$$

Using this method records were adjusted using factors derived from 1970-71 records.

THE CORRECTED RECORDS:

For each herd the average of corrected test day records was calculated for all test months as

$$h_f = \sum_i \sum_j \sum_k Z_{ijkf}/n_f$$

where h_f is the herd average for the f th month and n_f is the number of cows tested in the f th month. Z_{ijkf} is the corrected test day record as given by (18).

For each test day record the following were calculated for milk yield, fat yield and milk fat percentage:

(i) days in milk and age corrected test day record; Z_{ijkf}, using (18).

(ii) total corrected production to date (tp_{ijkf}).

Method of calculation for first record was to multiply the corrected test day record by the average number of days in milk (dim_f) for all cows in the month of first test. At each subsequent test the quantity,

$$\tfrac{1}{2} (dim_f - dim_{f-1}) (Z_{ijkf} + Z_{ijkf-1})$$

where f is the month of test and f-1 the month of the previous test, was added. This is the same procedure as is used to accumulate uncorrected records in the test interval method which is presently used by the New Zealand Dairy Board (Herd Improvement Assoc. Manual (1970)). The procedure used here differs in that the records are first corrected to the same stage of lactation then accumulated.

e.g. The total corrected production to date for a cow with two test day records, one in month 1 and the other in month 3, will be

$$tp_{ijk3} = dim_1 \times Z_{ijk1} + \tfrac{1}{2}(dim_3 - dim_1)(Z_{ijk3} + Z_{ijk1})$$

(iii) the ratio of corrected test day record to corrected test day herd average: $R_{ijkf} = Z_{ijkf}/h_f \times 100$.

(iv) the average of the ratio, R_{ijkf}, to date, weighted for number of test days contained in the average. This quantity is calculated as,

$$RI_{ijkf} = (\sum_{f=1}^{f} R_{ijkf}/C - 100)\, W + 100$$

where $\sum_{f=1}^{f}$ is the sum of the ratios R_{ijkf} for all test months, where there is a test day record, up to and including the f th test month. C is the number of test day records made, up to and including the f th test month. The weighting factor W is given by the expression

$$W = \frac{C}{1 + (C - 1)R} \times \frac{1 + (d - 1)R}{d} \qquad - - - - (20)$$

where d is the maximum number of records available for monthly testing in a 305 day lactation (d is equal to 10). The value of R, the correlation between test day ratios, was assumed to be 0.5 for milk and fat and 0.6 for milk fat percentage. The weighting factor is in fact the regression coefficient of the mean of C correlated observations on the mean of d correlated observations where $C \leq d$ and the d observations include all the C observations.

This quantity (RI_{ijkf}) is the "Ranking Index" as given in N.Z. Dairy Board (f), with the difference in correction procedure used.

The program A-DAY CORRECTION (see appendix A) was used to correct test day records and calculate the quantities: Z_{ijkf}, tp_{ijkf}, R_{ijkf}, RI_{ijkf} for each test day record.

ANALYSES OF CORRECTED RECORDS:

Two analyses were conducted to determine variance components of corrected test day records.

1. Using Henderson's (1953) method 1 and assuming the model

$$y_{ijk} = \mu + s_i + h_j + (hs)_{ij} + e_{ijk} \qquad ---- (21)$$

where μ is a general mean, s_i an affect due to the i th sire, h_j an affect peculiar to the j th herd, $(hs)_j$ an interaction affect due to the i th sire in the j th herd and e_{ijk} is a random error associated with the k th daughter of the i th sire in the j th herd. Assuming the model is fully random the variance components σ_s^2, σ_h^2, σ_{hxs}^2, and σ_e^2 were estimated for each test month for the four quantities Z_{ijkf}, R_{ijkf}, RI_{ijkf} and tp_{ijkf} for milk, fat, and milk fat percentage. This was achieved by the program ANOVA 2 (see appendix A) using the method described by Searle (1971, p 480).

2. For the fully random model

$$y_{ijkl} = \mu + S_i + h_j + (hs)_{ij} + C_{ijk} + e_{ijkl} \qquad ---- (22)$$

where the elements are the same as in (21) except for the inclusion of C_{ijk} a random element for the k th daughter of the i th sire in the j th herd. e_{ijkl} is a random error peculiar to the l th test day record of the k th daughter of the i th sire in the j th herd. Estimates of σ_s^2, σ_h^2, σ_{hxs}^2, σ_c^2, and σ_e^2 were obtained using method 1 of Henderson (1953). Estimates of the variance components were obtained for Z_{ijkf} and R_{ijkf} for milk, fat, and milk fat percentage. A program was written for this analysis - ANOVA 1 (see appendix A) using the method given in appendix B.

DATA

SOURCE OF DATA:

All herds, with herd code number in the range 2000000 - 2002999 using monthly, or alternate monthly testing, in the 1970-71 season were screened. Only cows which fulfilled the following criteria were selected:

- Jersey breed,
- sire identified by a Jersey sire code,
- calved between 31st of May and 1st of December,
- age at calving in the range 2 to 9 years inclusive,
- lactation of 100 days or greater.

For each cow selected, test day records were selected if they fulfilled the following criteria:

- made 5 days or more after calving,
- were not classified as abnormal*.

The selected records were arranged on magnetic tape. On this file, test day production records consisted of the following:

- date of test, as number of days from 1st June in the year of calving,
- month of test, June = 1, July = 2, August = 3 etc,
- days in milk when tested,

---

* A test day record is classified as abnormal if any one of the following occurred:

1. cow missed a test,
2. sample mixed, spilt, bottle broken, etc.,
3. cow in season,
4. cow held milk at p.m. and/or a.m. milking,
5. cow sick, mastitis, lame,
6. p.m. milk weight 25% or more greater than a.m. milk and total milk at least 20 lbs,
7. cow running with calves.

- total test day milk* to nearest pound,
- test day milk fat percentage, to nearest single decimal,
- test day milk fat yield, to nearest 1/100th of a pound.

Each test day record was identified with a herd code, sire code, cow number, year of calving, age at calving (4 age classes, 2 year, 3 year, 4 year and 5-9 year) and date of calving (in days from 1st June 1970).

Table 1 shows the number of test day records in the four age classes meeting these requirements for each month of test. The small number of records in June and July precluded further use of these months. The lower number of records in December than in November or January is due to the practise of beginning December testing on or about the 25th of November. This practise allows testing officers to have a Xmas holiday! Thus the November records will contain a number of pairs of records made by the same cow.

Table 2 shows the average number of days in milk for each age class in each test month. The 2 year olds are on average in milk 1.7 to 6.7 days longer than the 3 year olds. These ages representing the extremes.

Appendix tables 1, 2 and 3 give the average, milk yield, fat yield and milk fat percentage respectively for each age class each test month. These averages as well as the contents of tables 1 and 2 were determined using the X'X and

---

* "The weight of milk produced, on the test day, by each cow is recorded at both milkings by a "milk meter" attached to the pipeline ... . The milk meter as well as recording the weight of milk takes a proportionate sample. Samples from the evening and morning milking are combined ... and tested for milk fat percentage on the farm by the Gerber method". (N.Z. Dairy Board (a)).

Table 1.

The number of test day records in each age class for the test months June 1970 to May 1971.

	Age class				
Month	2	3	4	5 - 9	Total
June	14	1	1	3	19
July	49	36	28	69	182
August	1407	933	746	1758	4844
September	2380	1588	1224	2874	8066
October	2556	1790	1412	3411	9169
November	3574	2596	1944	4688	12802
December	1975	1440	1129	2776	7320
January	2820	2062	1548	3777	10207
February	2609	1972	1522	3747	9850
March	2329	1735	1376	3335	8775
April	1832	1459	1194	3024	7509
May	666	548	477	1261	2952
	22211	16160	12601	30723	81695

Table 2.

The average number of days in milk for each age class when tested in the months August to May.

	Age class				
Month	2	3	4	5 - 9	All ages
August	23.5	21.8	22.4	21.9	22.4
September	43.8	40.9	41.8	41.6	42.2
October	68.2	64.4	65.6	64.2	65.5
November	101.6	95.7	97.6	96.3	97.8
December	126.8	122.4	123.3	121.2	123.3
January	160.4	153.9	155.9	154.8	156.3
February	189.8	184.4	185.8	184.5	186.1
March	217.0	210.5	213.1	211.1	212.9
April	246.2	241.1	243.0	241.0	242.6
May	268.8	262.1	264.2	262.1	263.9

X'Y matrices which were obtained as a by-product of the program RIANAL.

This sample of test day records for Jersey cattle milked in the 1970-71 season is representative of the Auckland Herd Improvement Association Jersey population. Using the herd codes as a means of obtaining a sample of herds is unlikely to result in a grouping of herds according to geographical region. Herd codes in general are allocated according to sequence of application for herd testing services. For this reason this sample may contain a higher than average proportion of pedigree herds because of the tendency for pedigree herds to have been testing for a longer period of time than non pedigree herds. The requirement for sire identification may result again in a further disproportion of pedigree cows in the sample compared with the Auckland Jersey population. This however is not likely to be a serious bias since the estimates obtained will only be applied to the herd tested population.

RESULTS

ESTIMATES OF FIXED EFFECTS AND REGRESSION COEFFICIENTS

The program RIANAL was used to estimate the vector b under three different models. As has already been described an iterative procedure was used to obtain BLUE estimates of b and unbiased estimates of the variance components σ_e^2 and σ_{hs}^2. These estimates are obtained on the assumption that the herd-sire effects are normally and independantly distributed about a mean of zero with variance σ_{hs}^2. An initial ratio of σ_e^2 to σ_{hs}^2 of zero was chosen for August test day milk yield, fat yield and milk fat percentage. Choosing this initial k value of zero means that the matrix, $(X'X-X'ZP^{-1}Z'X)$ does not have an inverse due to the columns of the matrix, (X'X, X'Z) summing to the same quantity as the columns of the matrix (Z'X, Z'Z). However, due to rounding errors, the program MINV is able to determine an inverse and thus provide a value for k for the 2nd iteration. After this first iteration the matrix, $(X'X-X'ZP^{-1}Z'X)$ does have an inverse due to the addition of a non zero quantity k to the diagonal matrix Z'Z. Each iteration for the 2139 August herd-sire subclasses takes 6½ minutes of computer time. Thus to complete nine iterations for each of the three August test day traits would take 3 hours. For this reason only limited use of this procedure was made.

Figure 1 shows the value of k obtained at the end of each iteration for the three August test day yield traits. The ratios for milk yield and fat yield have settled by the 6'th iteration while for milk fat percentage a settled value has not been achieved by the 9th iteration. Appendix figures 1(a), 1(b) and 1(c) show the actual variance component estimates obtained at the end of each iteration for the three yield traits. From these figures it appears that the variance component estimates for milk fat percentage have settled at the same rate as those for milk yield and fat yield. The reason for milk fat percentage k ratio not apparently settling being due to the actual k being large so that small changes in variance components cause relatively large changes in k.

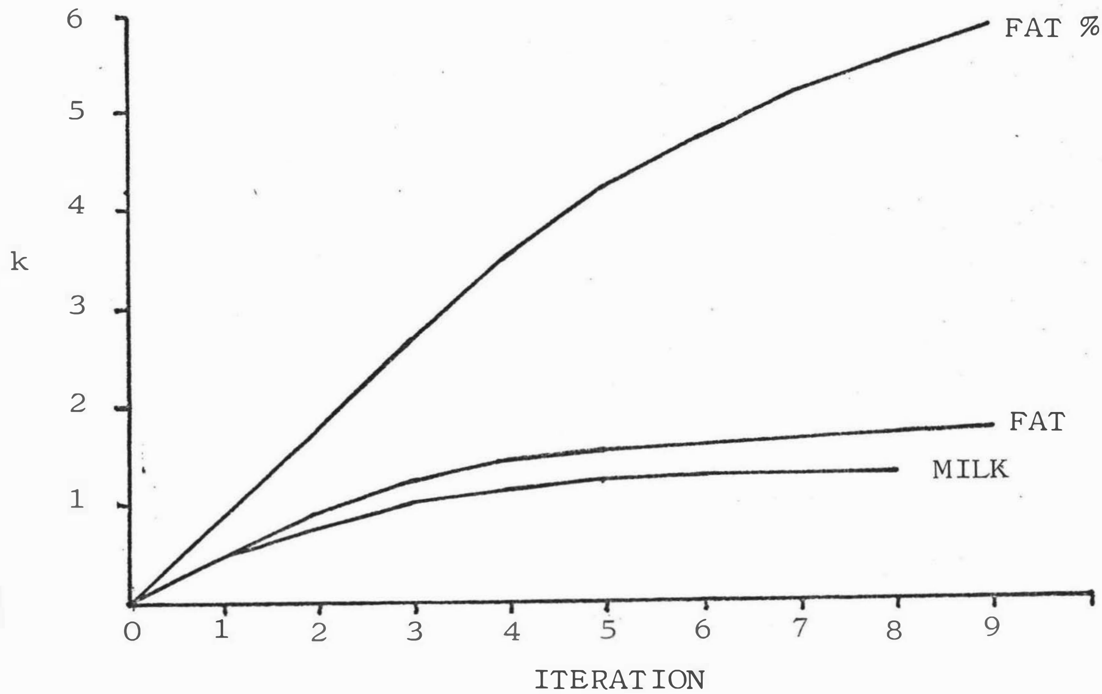

Figure 1. Estimated k ratios obtained at the end of each iteration for the three August test day yield traits.

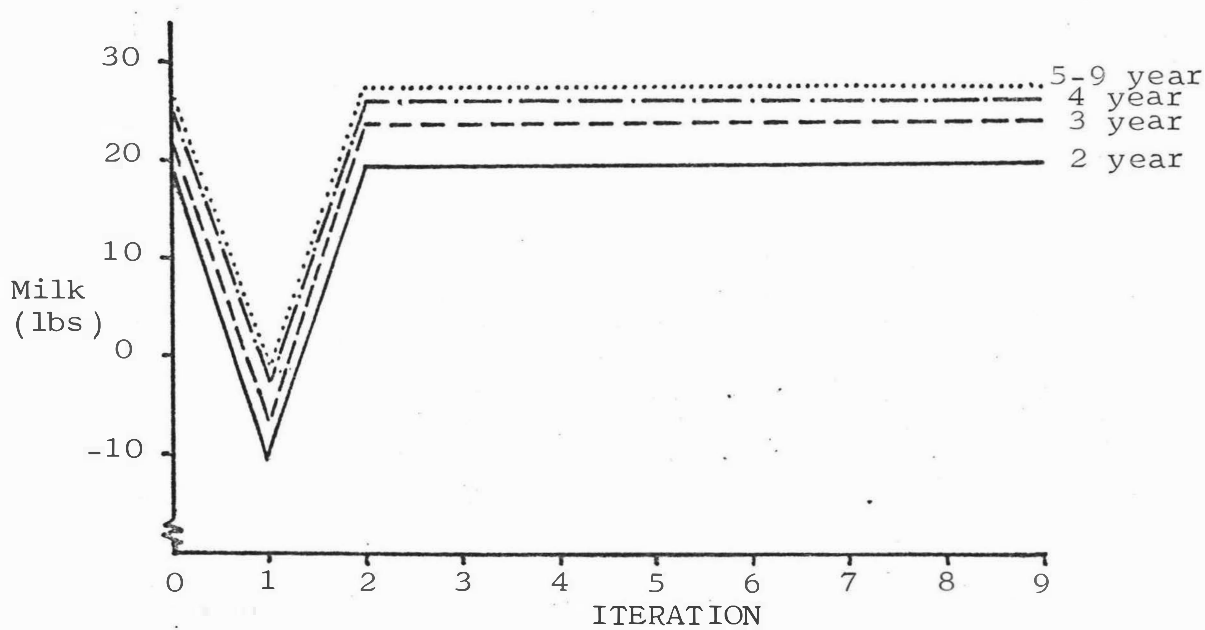

Figure 2. Estimated value of the fixed effects $\mu + a_i$ for August test day milk yield for the four age classes as obtained at the end of each iteration.

Figures 2, 3 and 4 show how the estimates of the elements of b changed from one iteration to the next. The values of the elements of b for iteration one have no particular significance due to their existence as a solution being solely due to rounding errors. However for all succeeding iterations the estimates are conditionally unbiased for the k value used. The estimates of the elements of b for iteration zero were obtained assuming a fixed model in which herd-sire was ignored. These estimates (LSQE) are ordinary least squares estimates obtained as,

$$\hat{b} = \left[X'X\right]^{-1} X'Y \qquad - - - - (23)$$

where terminology is identical with that on page 26.

The third model used for estimating b was the fixed model in which the affect of the j th herd-sire subclass is assumed fixed. A solution for this model is obtained in exactly the same manner as the first iteration for BLUE with an initial k value of zero. However to obtain a solution the restriction that the last herd-sire subclass effect is zero was imposed. This was simply achieved by deleting the last row and column for the last herd-sire subclass from the normal equations. Estimates assuming this model were obtained for only August test-day records.

Table 3 shows the BLUE estimates $\hat{b}$ obtained from the last iteration for the three test months August, September and October. Standard errors were obtained for these estimates by using the relationship,

$$Sb_i = \sqrt{\hat{\sigma}_e^2 \times g_{ii}} \qquad - - - - (24)$$

where g_{ii} is the i th diagonal element of the inverse matrix $(X'X - X'ZP^{-1} Z'X)^{-1}$, $\hat{\sigma}_e^2$ is the estimated error variance and Sb_i is the standard error of the i th element of b. To test is regression coefficients are significantly different from zero the ratio $t = \hat{b}_i/Sb_i$ was computed with an infinite number of degrees of freedom.

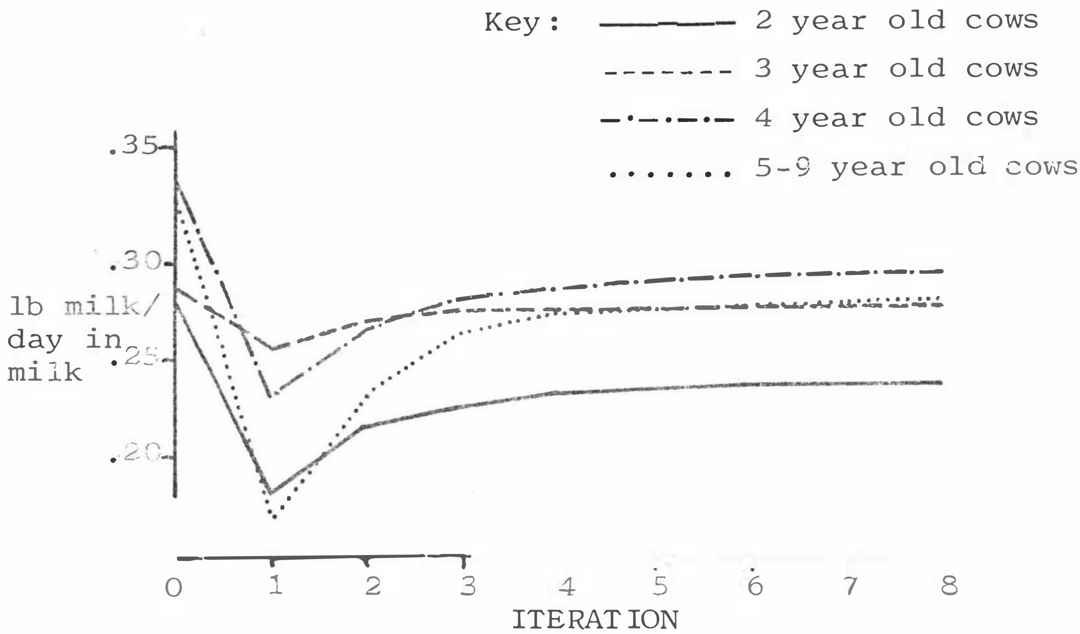

Figure 3. Estimates of the partial regression coefficient, b_{i1}, the regression of days in milk on August test day milk yield, as obtained at the end of each iteration.

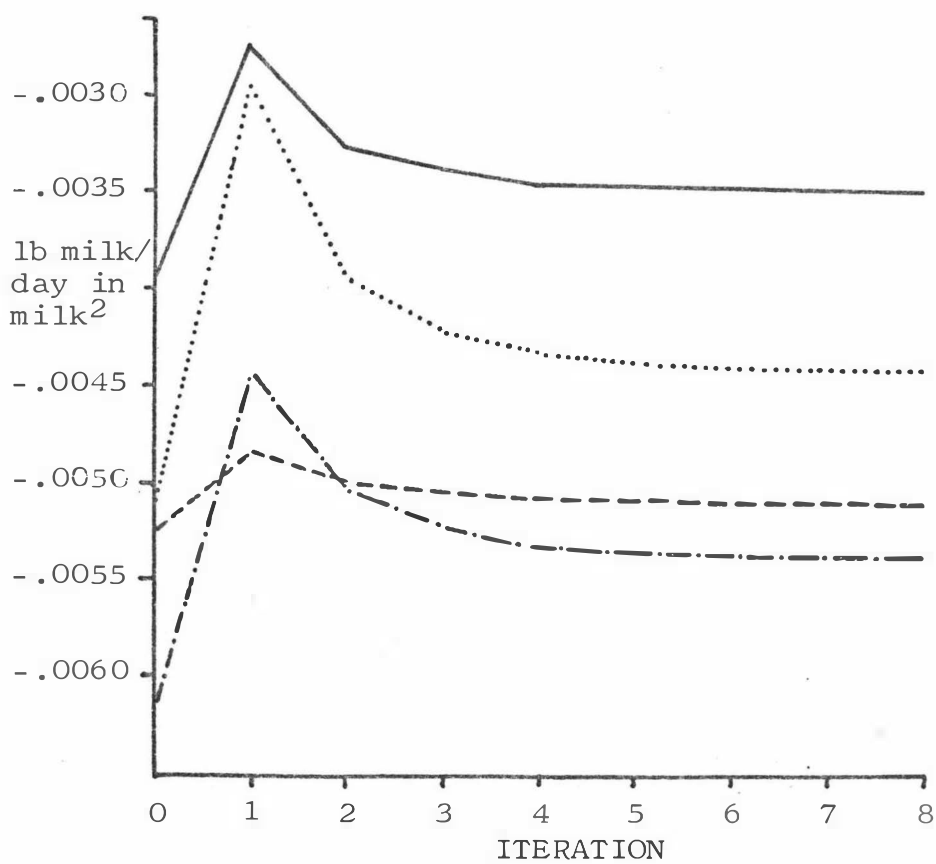

Figure 4. Estimates of the partial regression coefficient, b_{i2}, the regression of days in milk squared on August test day milk yield, as obtained at the end of each iteration.

Table 3.

BLUE estimates of the affect of age and days in milk and days in milk squared on test day milk yield, fat yield and milk fat percentage.

(a) Test day milk yield (lbs).

Month	Age	$\mu + a_i \pm SE$	$b_{i1} \pm SE$	$b_{i2} \pm SE$
August	2	19.4 ± 0.5	0.24 ± .04**	-.0035 ± .0006**
	3	24.0 ± 0.5	0.28 ± .05**	-.0051 ± .0008**
	4	26.1 ± 0.5	0.29 ± .05**	-.0054 ± .0009**
	5-9	27.2 ± 0.4	0.28 ± .04**	-.0044 ± .0006**
September	2	21.1 ± 0.6	0.15 ± .03**	-.0016 ± .0003**
	3	27.2 ± 0.6	0.14 ± .03**	-.0022 ± .0003**
	4	30.5 ± 0.7	0.12 ± .03**	-.0018 ± .0004**
	5-9	30.9 ± 0.5	0.16 ± .02**	-.0021 ± .0002**
October	2	24.4 ± 0.6	-.0006 ± .02	-.00017 ± .0001
	3	30.6 ± 0.6	-.013 ± .02	-.00045 ± .0001**
	4	32.4 ± 0.7	0.028 ± .02	-.00076 ± .0002**
	5-9	34.3 ± 0.5	0.016 ± .02	-.00061 ± .0001**

(b) Test day fat yield (lbs).

Month	Age			
August	2	0.87 ± .03	0.018 ± .002**	-.00025 ± .00004**
	3	1.12 ± .04	0.016 ± .003**	-.00025 ± .00005**
	4	1.27 ± .04	0.013 ± .003**	-.00028 ± .00005**
	5-9	1.24 ± .03	0.019 ± .002**	-.00028 ± .00004**
September	2	0.96 ± .03	0.010 ± .001**	-.000093 ± .00002**
	3	1.19 ± .04	0.011 ± .002**	-.00012 ± .00002**
	4	1.37 ± .04	0.010 ± .002**	-.00012 ± .00002**
	5-9	1.36 ± .03	0.012 ± .001**	-.00013 ± .00002**
October	2	1.14 ± .04	0.0021 ± .001	-.000015 ± .00001
	3	1.38 ± .04	0.0028 ± .001*	-.000031 ± .00001**
	4	1.52 ± .04	0.0041 ± .001**	-.000044 ± .00001**
	5-9	1.51 ± .03	0.0064 ± .001**	-.000057 ± .00001**

* Sig. different from 0.0 at .5% level.

** Sig. different from 0.0 at .1% level.

Table 3 cont'd.

(c) Test day fat percentage.

Month	Age	$\mu + a_i \pm SE$	$b_{i1} \pm SE$	$b_{i2} \pm SE$
August	2	4.5 ± 0.1	0.030 ± .005**	-.00039 ± .00008**
	3	4.7 ± 0.1	0.005 ± .007	0.00004 ± .0001
	4	4.9 ± 0.1	-.008 ± .007	0.00023 ± .0001
	5-9	4.6 ± 0.1	0.018 ± .005**	-.00022 ± .00008**
September	2	4.6 ± 0.1	0.012 ± .003**	-.000055 ± .00003
	3	4.5 ± 0.1	0.011 ± .004*	0.000002 ± .00004
	4	4.5 ± 0.1	0.011 ± .004*	-.000054 ± .00005
	5-9	4.5 ± 0.1	0.013 ± .003**	-.000054 ± .00003
October	2	4.7 ± 0.1	0.0084 ± .003*	-.000018 ± .00002
	3	4.5 ± 0.1	0.0113 ± .003**	-.000019 ± .00003
	4	4.7 ± 0.1	0.0079 ± .003	-.000006 ± .00003
	5-9	4.5 ± 0.1	0.0142 ± .002*	-.000057 ± .00002*

Table 4.

Estimates of the affect of age, days in milk and days in milk squared on August test day yield traits. These estimates were obtained assuming the affect of herd-sire is fixed.

Trait	Age	$\mu + a_i$	b_{i1}	b_{i2}	A_i
Milk	2	21.30	0.1926	-.00292	1.35
	3	25.44	0.2571	-.00484	1.14
	4	28.37	0.2314	-.00446	1.04
	5-9	29.80	0.1821	-.00306	1.00
Fat	2	0.91	0.0166	-.000231	1.35
	3	1.17	0.0400	-.000233	1.15
	4	1.37	0.0099	-.000184	1.05
	5-9	1.32	0.0162	-.000236	1.00
Fat %	2	4.23	0.0349	-.000436	0.99
	3	4.57	0.0024	0.000079	1.00
	4	4.81	-.0088	0.000235	0.98
	5-9	4.34	0.0235	-.000274	1.00

Appendix tables 4, 5 and 6 give the LSQE estimates of the elements of b for milk yield, fat yield and milk fat percentage respectively. These were obtained using a modified form of the program RIANAL. The machine time required to produce these estimates was about 1/15th of that required to produce BLUE estimates. Standard errors were not estimated because estimates of σ_e^2 under this model were not available.

Table 4 shows the estimates of b obtained assuming the affect of herd-sire is fixed. These estimates were determined for August test day records only.

ADJUSTING TEST-DAY RECORDS FOR AGE AND DAYS IN MILK:

The test months August, September, October and November 1970 were chosen to evaluate the different methods of expressing test day production. Estimates of the partial regression coefficients of days in milk and days in milk squared on test day yield showed consistant but small differences when estimated using BLUE compared with LSQE. The LSQE estimates generally gave a curve which contained a larger quadratic influence than did the BLUE estimates. Over the range of days in milk encountered, each test month, these differences were small. BLUE estimates were used in this study as the basis for additive adjustment factors in August, September and October. These partial regression estimates are given in table 3. Since BLUE estimates were not available for November, LSQE estimates given in appendix tables 4, 5 and 6 were used for this month.

Multiplicative age adjustment factors, were derived using equation (17) and the averages of days in milk and days in milk squared given in table 5.

Table 5.

Average number of days in milk (dim) and days in milk squared (dim^2) for the test months August to May.

Month	dim	Range	dim^2
August	22	5-84	654
September	42	5-110	2104
October	65	5-144	4872
November	98	5-182	10353
December	123	31-201	15945
January	156	65-233	25173
February	186	94-272	35381
March	213	122-292	46049
April	243	154-307	59865
May	264	185-308	70340

These factors were derived from both the BLUE and LSQE estimates of b as well as for the August estimates for the fully fixed model. The factors are given in table 4 for the fully fixed model, table 6 for BLUE and LSQE and in appendix tables 4, 5 and 6 for LSQE for all test months. The adjustment factors derived from the three sets of estimates are very similar. In fact rounding errors at the second decimal place account for most of the differences. For this sample of data ignoring the affect of herd-sire (LSQE) gives estimates of the fixed effects and regression coefficients similar to those obtained by considering the effect of herd-sire as random (BLUE) or fixed.

Also given in table 6 is the top line of equation (17) for each age class, trait, method of estimation combination. These averages could be considered as points on a standard lactation curve for each age class.

The age factors used to adjust days in milk corrected test day records are those given in table 6 based on LSQE estimates.

Table 6.

Age adjustment factors and day in milk adjusted averages for milk, fat and milk fat percentages.

Month	Method	Age	Age factors (A_i) Milk	Fat	Fat%	Adjusted Averages Milk(lb)	Fat(lb)	Fat%
August	BLUE	2	1.36	1.36	.99	22.5	1.11	4.92
		3	1.13	1.15	1.00	26.9	1.29	4.84
		4	1.05	1.05	1.00	29.1	1.41	4.87
		5-9	1.00	1.00	1.00	30.6	1.48	4.86
	LSQE	2	1.35	1.34	0.99	22.7	1.12	4.95
		3	1.14	1.14	1.00	27.0	1.32	4.89
		4	1.05	1.04	0.99	29.4	1.44	4.92
		5-9	1.00	1.00	1.00	30.8	1.50	4.88
September	BLUE	2	1.41	1.37	.98	23.4	1.12	4.83
		3	1.14	1.14	1.00	28.9	1.36	4.75
		4	1.03	1.02	1.01	32.0	1.51	4.71
		5-9	1.00	1.00	1.00	33.1	1.54	4.76
	LSQE	2	1.39	1.37	.98	23.9	1.19	4.95
		3	1.16	1.15	.99	28.7	1.41	4.94
		4	1.05	1.04	.99	31.7	1.56	4.91
		5-9	1.00	1.00	1.00	33.3	1.63	4.89
October	BLUE	2	1.37	1.36	0.99	23.5	1.20	5.18
		3	1.17	1.16	0.99	27.6	1.41	5.14
		4	1.06	1.05	0.99	30.5	1.57	5.17
		5-9	1.00	1.00	1.00	32.4	1.64	5.10
	LSQE	2	1.38	1.36	0.99	23.6	1.22	5.19
		3	1.17	1.16	0.99	27.8	1.43	5.17
		4	1.06	1.05	0.98	30.7	1.59	5.21
		5-9	1.00	1.00	1.00	32.6	1.67	5.13
November	LSQE	2	1.36	1.35	0.99	22.9	1.22	5.34
		3	1.18	1.17	0.99	26.3	1.41	5.38
		4	1.07	1.05	0.98	29.2	1.57	5.40
		5-9	1.00	1.00	1.00	31.1	1.64	5.31

The program A DAY CORRECTION was used to adjust all test day records in the original sample using "nil" factors. These "nil" factors consisted of zero for all the regression coefficients and unity for all age adjustment factors. The test day averages for each age class and month of test of these adjusted records was the same as those obtained from the elements of the X'X and X'Y matrices as is given in appendix tables 1, 2 and 3. From these nil adjusted (NA) records variance components were estimated assuming the models given by equations (21) and (22).

Again using the program A DAY CORRECTION the age, days in milk and days in milk squared adjustment factors already described were applied. The test month averages of these adjusted records are given in table 7. These adjusted averages show no consistent trend with age. Variance components assuming the models given by equations (21) and (22) were estimated for these adjusted records.

Table 7.

Test day averages of records adjusted for days in milk, days in milk squared and age.

		Month of test			
Trait	Age	August	September	October	November
Milk	2	30.7	33.8	32.4	31.4
	3	30.4	32.6	32.5	30.8
	4	30.7	32.6	32.6	30.6
	5-9	30.6	33.3	32.6	31.1
Fat	2	1.52	1.63	1.66	1.67
	3	1.51	1.61	1.66	1.65
	4	1.51	1.58	1.67	1.64
	5-9	1.49	1.62	1.67	1.64
Fat %	2	4.89	4.85	5.09	5.34
	3	4.89	4.94	5.12	5.28
	4	4.92	4.96	5.14	5.35
	5-9	4.88	4.88	5.13	5.31

VARIANCE COMPONENT ESTIMATES

The variance components given in table 8 for adjusted and nil adjusted test day records assuming the model given by equation (21) were determined using the entire sample of test day records. It is obvious that the sire component of variance has been overestimated especially for the nil adjusted records. These variance components show that adjusting records for age and days in milk reduce the sire component of variance. This is probably due to a confounding of the affect of age and days in milk with herd and sire. However it is doubtful if much can be concluded from variance components which are obviously seriously biased. For this reason the variance components of the adjusted records were re-estimated using only those records with an A.B. (Artificial Breeding) sire code. This limitation should help ensure that each sire is represented by records in more than one herd and perhaps reduce biases in the variance component estimates. Table 10 shows the class and subclass sizes for the sample including all records and for the sample including records with only AB sire codes. The average number of herds in which each sire is represented has been increased from an average of less than 2 herds to greater than 4 herds each test month by excluding non A.B. sire coded records.

The variance component estimates for this limited sample of the adjusted records is given in table 9. The corresponding variance components for test day milk yield and milk fat percentage are given in appendix tables 7 and 8. Within herd heritability estimated as $4 \times \hat{\sigma}_s^2/(\hat{\sigma}_s^2 + \hat{\sigma}_e^2)$ is given in tables 8 and 9 and appendix table 7 and 8 for the corresponding variance components. The heritability estimates for the sample of A.B. sire coded records show an increase for adjusted test day yield (Z_{ijkf}) and accumulative test day yield (tp_{ijkf}) from August to later test months. This trend is not apparent for the heritability of test day ranking (R_{ijkf}). Test day ranking and accumulative ranking contain

Table 8.

Variance component estimates assuming the model given by equation (21) for test day fat yield measures which have (AD) and have not (NA) been adjusted for age, days in milk and days in milk squared.

Measure	Month		$\hat{\sigma}_h^2$	$\hat{\sigma}_s^2$	$\hat{\sigma}_{hxs}^2$	σ_e^2	h^2
Z_{ijkf}	August	NA	0.042	0.025	-.0027	0.083	.93
		AD	0.049	0.008	-.0007	0.089	.33
	September	NA	0.046	0.035	-.011	0.096	1.07
		AD	0.051	0.017	-.012	0.106	.55
	October	NA	0.044	0.043	-.017	0.098	1.22
		AD	0.050	0.023	-.015	0.105	.72
	November	NA	0.032	0.034	-.004	0.086	1.13
		AD	0.037	0.014	-.002	0.090	.54
tp_{ijkf}	August	NA	37.9	22.6	-3.0	74.9	.93
		AD	23.9	3.9	0.1	43.5	.33
	September	NA	138.8	107.8	-22.5	300.6	1.05
		AD	75.6	23.1	-9.9	160.2	.50
	October	NA	302.9	301.8	-109	654.1	1.25
		AD	175.1	81.3	-50.1	366.7	.72
	November	NA	536.7	488.5	-296	924.7	1.40
		AD	406.1	148.4	-9.2	623.3	.78
R_{ijkf}	August	NA	-35.3	68.3	75.3	457.7	.52
		AD	-17.6	11.7	37.0	400.7	.11
	September	NA	-23.9	90.8	35.3	469.7	.65
		AD	-7.7	28.9	-4.0	417.7	.26
	October	NA	-22.7	87.9	45.1	449.9	.65
		AD	-7.7	20.7	16.3	388.7	.20
	November	NA	-2.3	100.3	46.9	404.1	.79
		AD	10.9	33.2	14.2	340.0	.36
RI_{ijkf}	August	NA	-7.3	22.5	22.2	133.7	.58
		AD	-1.9	5.4	10.5	116.6	.18
	September	NA	-5.6	38.2	13.0	158.3	.78
		AD	1.7	8.2	3.1	95.5	.31
	October	NA	-7.4	50.6	20.4	149.4	1.01
		AD	0.43	15.9	0.36	148.6	.38
	November	NA	8.8	71.1	25.7	174.1	1.16
		AD	16.3	25.9	6.5	140.9	.62

Table 9.

Variance component estimates for test day fat yield records adjusted for age, days in milk and days in milk squared. These estimates were obtained from a sample of test day records with A.B. sire codes.

Measure	Month	$\hat{\sigma}_h^2$	$\hat{\sigma}_s^2$	$\hat{\sigma}_{hxs}^2$	$\hat{\sigma}_e^2$	$\hat{h}^2$
Z_{ijkf}	August	0.0443	0.0052	0.0062	0.0901	.22
	September	0.0434	0.0085	0.0102	0.1081	.29
	October	0.0425	0.0145	-0.0109	0.1027	.49
	November	0.0315	0.0089	0.0038	0.0831	.38
tp_{ijkf}	August	21.4	2.45	3.05	43.43	.21
	September	66.3	15.33	-15.58	180.16	.31
	October	135.6	52.38	-30.23	379.8	.48
	November	297.1	86.25	75.1	624.1	.48
R_{ijkf}	August	-22.2	29.3	20.8	435.4	.25
	September	-4.7	26.6	-27.7	441.1	.23
	October	-6.2	22.7	2.9	398.1	.22
	November	-1.5	20.5	35.2	314.9	.24
RI_{ijkf}	August	-4.21	11.4	3.8	131.5	.32
	September	1.4	14.3	-6.7	141.9	.37
	October	1.0	16.9	-1.7	148.0	.41
	November	2.5	15.1	24.8	127.5	.42

Table 10.

Numbers of classes in the classifications used for analyses of variance assuming the model given by equation (21).

(a) Including all records.

Month	Herds	Sires	Interactions	Total obs.
August	178	1250	2139	4844
September	185	1538	3051	8066
October	190	1664	3332	9168
November	193	1713	3587	12802

(b) Including only records with an A.B. sire code.

Month	Herds	Sires	Interactions	Total obs.
August	111	358	1172	1958
September	125	445	1825	2924
October	130	466	1970	3107
November	136	487	2174	4435

Table 11.

Estimates variance components assuming the model given by equation (22) for two measures of milk yield, fat yield and milk fat percentage. Estimates for a sample of all test day records adjusted (AD) and not adjusted (NA) for age, days in milk and days in milk squared.

		Measure of production			
Trait	Comp.	NA Z_{ijkf}	AD Z_{ijkf}	NA R_{ijkf}	AD R_{ijkf}
Milk	Herd	9.3	14.0	47.7	64.2
	Sire	11.6	6.7	147.5	76.8
	HxS	-1.5	-3.8	-15.0	-47.0
	Cow	18.6	17.3	246.0	183.0
	Error	10.3	8.3	61.0	53.8
	Total	48.3	42.5	487.2	330.8
Fat	Herd	0.036	0.041	49.2	63.3
	Sire	0.036	0.016	145.5	81.3
	HxS	-.012	-.009	-24.4	-53.7
	Cow	0.046	0.043	221.8	173.8
	Error	0.048	0.056	188.1	178.3
	Total	0.154	0.147	580.2	444.0
Fat %	Herd	0.149	0.15	68.0	68.2
	Sire	0.140	0.14	73.8	74.0
	HxS	-.106	-.11	-59.0	-59.9
	Cow	0.110	0.11	53.8	47.4
	Error	0.336	0.31	81.6	80.7
	Total	0.629	0.60	218.2	210.4

Table 12.

Estimated variance components assuming the model given by equation (22) for the sample of records with A.B. sire codes.

Trait	Measure	$\hat{\sigma}_h^2$	$\hat{\sigma}_s^2$	$\hat{\sigma}_{hxs}^2$	$\hat{\sigma}_c^2$	$\hat{\sigma}_e^2$
Milk yield	Z_{ijkf}	10.43	3.70	-2.2	17.6	13.1
	R_{ijkf}	15.0	29.3	-10.4	189.6	86.8
Fat yield	Z_{ijkf}	0.0345	0.0114	-.0085	0.041	0.0676
	R_{ijkf}	12.98	31.23	-13.8	175.7	214.6
Milk fat %	Z_{ijkf}	0.1084	0.1005	-.077	0.107	0.336
	R_{ijkf}	15.33	26.04	-16.7	50.27	109.7

a small variance component due to herd. This is no doubt due to the expressing of test day production relative to the herd average. November records for all measures contain a relatively large positive herd x sire interaction variance component.

Assuming the model given by equation (22), variance components were estimated from the complete sample of adjusted and nil adjusted records. The results of these analyses are shown in table 11. The analysis was repeated on the adjusted records with AB sire codes. The results of this analysis are given in table 12. Table 13 gives the number of classes and subclasses for each of the samples. Estimates of within herd heritability were calculated as $4\ \hat{\sigma}_s^2 / (\hat{\sigma}_s^2 + \hat{\sigma}_c^2 + \hat{\sigma}_e^2)$ and the within herd, within year repeatability was calculated as $\hat{\sigma}_c^2 / (\hat{\sigma}_c^2 + \hat{\sigma}_e^2)$. Table 14 gives these heritability and repeatability estimates.

Table 13.

Number of classes and subclasses for analyses of variance assuming the model given by equation (22). Numbers for analyses including all records (ALL) and including those with AB sire codes only (AB).

Analysis	Herds	Sires	HxS	Cows	Records
ALL	214	1806	4099	11510	34883
AB	154	519	2607	4440	11932

Table 14.

Repeatability and heritability of Z_{ijkf} and R_{ijkf} for the four test months August to November inclusive for adjusted records.

		Z_{ijkf}		R_{ijkf}	
Analysis	Trait	r	h^2	r	h^2
ALL	Milk yield	0.67	0.82	0.77	0.97
	Fat yield	0.43	0.55	0.54	0.75
	Milk fat %	0.35	1.00	0.40	1.50
AB	Milk yield	0.57	0.43	0.68	0.38
	Fat yield	0.38	0.38	0.45	0.29
	Milk fat %	0.24	0.74	0.31	0.56

For all traits heritability is greater for adjusted test day yield (Z_{ijkf}) than test day ranking (R_{ijkf}) when the A.B. analysis is used. The analysis using all test day records gives inflated heritability values. The within herd, within year repeatability of test day ranking is greater than for the adjusted test day record for all traits.

DISCUSSION

ESTIMATION PROCEDURES

For the model given on page 24 BLUE will estimate age effects free of biases due to sire selection. The BLUE estimates as obtained in this study may contain biases due to cow culling. However these could be removed by a slight modification of the model and by fitting it over several years. The modified model suggest is given by

$$y_{ijk} = \mu + a_i + c_j + b_{i1} \times X_{ijk} + b_{i2} \times X_{ijk}^2 + e_{ijk} \quad \text{- - - - (25)}$$

where all elements except c_j and e_{ijk} are as defined on page 24. c_j is a random affect peculiar to a record in the j th herd - sire - cow subclass and is normally and independantly distributed with mean zero and variance σ_c^2. e_{ijk} is an error element peculiar to the k th record made by the j th cow. The estimates of $\mu + a_i$ obtained under this model would be free of the biases due to cow culling and sire selection.

Obtaining BLUE estimates of fixed effects and regression coefficients using the method described by Thompson (1969) has proven to be computationally expensive when compared least squares estimates assuming a simpler model. Figures 1, 2 and 3 show that the estimates are relatively insensitive to changes in the value of the ratio k with succeeding iterations. These estimates obtained at the end of each iteration are not BLUE but are conditionally (conditional upon the value of k) unbiased. From figure 2 a change in the value of k from 0.8 (for iteration 2) to 1.3 (for iteration 9) results in virtually no change in the estimates of $\mu + a_i$. This indicates that including herd-sire in the model as a random element has little influence on the fixed effect estimates. This is confirmed when herd-sire is assumed fixed and the estimates in table 4 are obtained. The affect of age when expressed as the ratio given by equation (17) being identical with those given in table 6. Thus if herd-sire is considered fixed or random makes little difference to the resulting estimates of fixed age effects.

If considering herd-sire as fixed versus considering herd-sire as random has little influence on the estimates of age effect what happens if herd-sire is ignored? The LSQE estimates given in appendix tables 4, 5 and 6 are based on a model which ignores herd-sire. The age factors given in table 6 clearly show that these are very similar to those obtained using BLUE. For the sample of data used in this study it thus appears that estimates of the affect of age on test day production obtained by ignoring the influence of herd-sire are equivalent to BLUE estimates in the biases they contain (if any). This immediately raises the question, if there has been a rapid improvement in the "productive ability" of bulls used in standard AB service over recent years, why are the LSQE estimates not biased?

Table 13 indicates that approximately 1/3rd of the records in the sample had AB sire codes. This fact alone would tend to dilute any affects of AB sire improvement. However by the nature of the age classes used cow culling may have cancelled the affect of AB sire improvement. This is due to many of the older AB sires (and thus supposedly inferior in productive ability) being represented by records in only the 5-9 year old age class. These 5-9 year olds will not be a representative sample of the daughters of these sires and as a result this selection will inflate the estimated average "productive ability" of the 5-9 year olds so as to cancel the superiority of the sires of the younger cows. The nett affect being that in this sample the average herd-sire effect for each age class is the same and not larger for the younger animals as expected.

A difference of perhaps some importance between the estimates obtained assuming the three models is in the regression coefficients. The LSQE estimates consistantly gave a more sharply parabolic curve than did the BLUE estimates. The fully fixed model gives the least sharply parabolic curves for August test day records. The actual differences are small over the range of days in milk for each test month. An explanation for this phenomena could not be found.

Before adopting LSQE as a procedure for estimating the affect of age and days in milk the model given by equation (25) should be fitted assuming c_j is random, fixed or ignored. BLUE estimates will be expensive to obtain but because they are "optional" they provide a good standard for evaluating less expensive estimates.

THE ESTIMATES

The affect of days in milk and days in milk squared on test day yield is influenced by test month. This conclusion is reached despite a confounding of test month with days in milk. For example referring to table 3 the affects of days in milk on test day fat yield show large (relative to standard errors) differences in linear and quadratic components between August and September test months. The most logical explanation being that the environment preceeding the August test day is considerably different from that preceeding the September test day record. So that a cow in milk 10 days in August will be yielding relatively less than a herd mate in milk 30 days in August than when the same comparison is made in September.

In the later test months for which LSQE estimates only are available an attempt was made to estimate standard errors of the regression coefficients. This involved assuming a value for σ_e^2 and calculating the standard error as

$$Sb_i = \sqrt{\sigma_e^2 \ p_{ii}} \qquad \text{- - - - (26)}$$

where p_{ii} is the i th diagonal element of the inverse matrix $[X'X]^{-1}$ and Sb_i is the standard error of the i th element of b. Table 15 shows the assumed σ_e^2 and approximate standard errors for all age classes.

Table 15.

Approximate standard errors for linear (Sb_1) and quadratic (Sb_2) regression coefs.

Month	Milk			Fat			Fat %		
	σ_e^2	Sb_1	Sb_2	σ_e^2	Sb_1	Sb_2	σ_e^2	Sb_1	Sb_2
November	20	.014	.0001	.10	.001	.00001	.40	.002	.00001
December	18	.023	.0001	.09	.002	.00001	.40	.003	.00001
January	16	.026	.0001	.08	.002	.00001	.40	.003	.00001
February	14	.032	.0001	.08	.002	.00001	.40	.005	.00001
March	12	.043	.0001	.07	.003	.00001	.40	.008	.00002
April	10	.046	.0001	.07	.004	.00001	.40	.009	.00002
May	8	.067	.0001	.06	.006	.00001	.40	.015	.00003

(Yield trait)

When these standard errors are considered in conjunction with the estimates given in appendix tables 4, 5 and 6 it can be seen that the regressions for the months January to May rarely reach significance (i.e. $t > 3.0$). This may be expected as the range of days in milk is limited (see table 5) for any single test month and in the later months many cows are approaching the end of their lactation. More information with accurate standard errors is needed for these later test months before it can be decided if the affect of days in milk is significant or not.

The regressions for August and September do not show significant differences between age classes. However for October the 2 year olds show greater persistency than the older age classes. This is in general agreement with the greater persistency for two year olds found previously.

Figure 5 compares the affect of days in milk found in this study with the affect of stage of lactation found by Castle (pers comm). The curves are for 2 year olds only and illustrate that a single set of stage of lactation factors are not likely to take account of the affect of days in milk observed in this study. If an attempt is to be made to adjust test day records for days in milk then month of test should be taken into account in deciding if and to what extent adjustment factors should be used. Because test month is

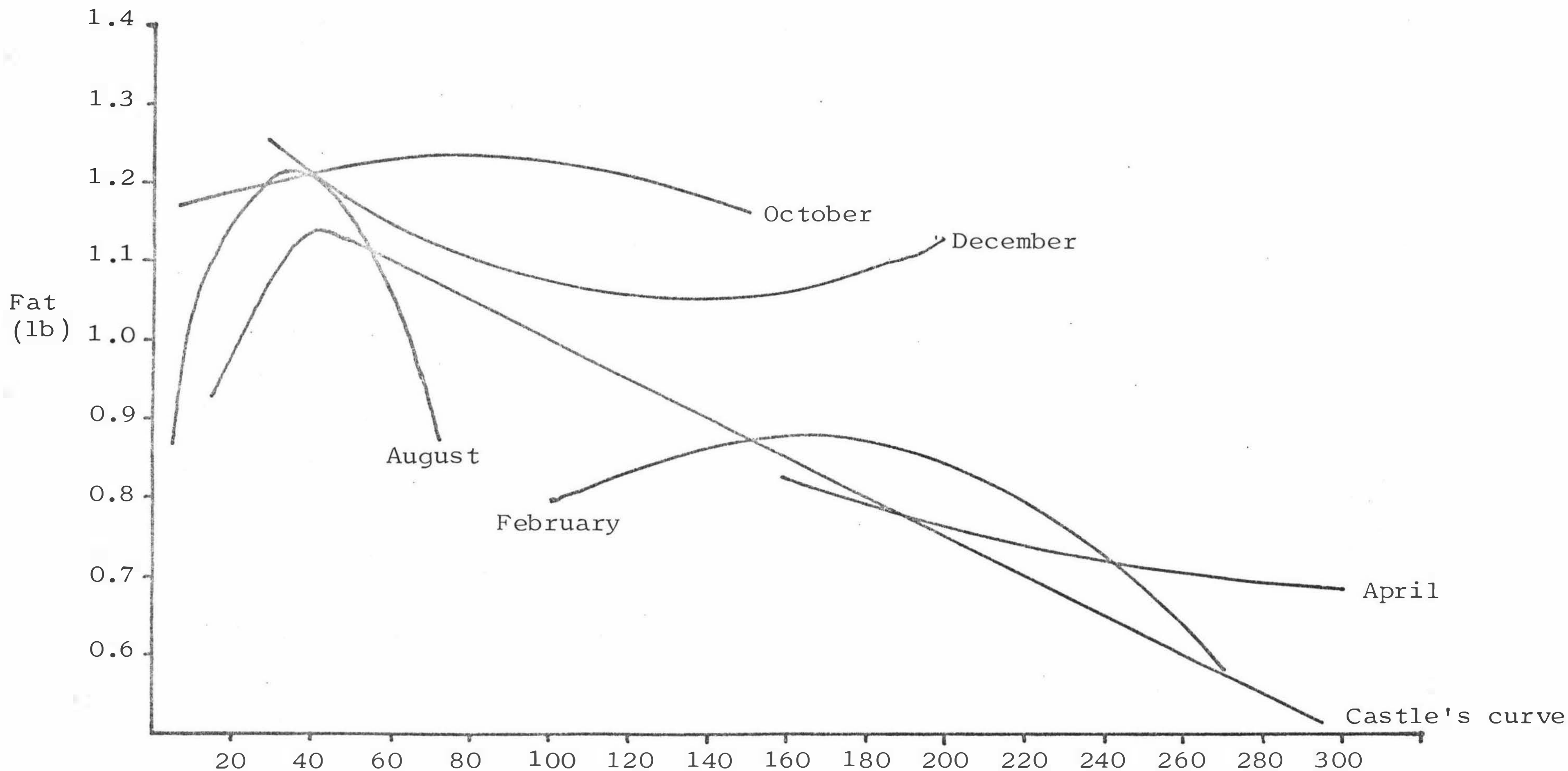

Figure 5. Affect of days in milk on 2 year old test day fat yield for alternate test months and as estimated by Castle (pers comm).

partly confounded with stage of lactation and partly with feed supplies it seems reasonable to suggest that the affect of days in milk and days in milk squared may vary from year to year for the same test month. If this is the case then factors derived in one year may be of little value in subsequent years.

Adjustment for days in milk and days in milk squared in this study has been based on a simple additive factor which is determined by the amount by which a cows number of days in milk deviates from the test month average. These factors are based on the assumption that the linear and quadratic effects of days in milk are independant of level of production. If higher producing cows show a different response to days in milk from low producing cows then factors related to level of production may well be justified. However the evidence presently available (Wood (1970) and Smith and Legates (1962a)) is insufficient to indicate a relationship or lack of it between level of production and the affect of days in milk.

The age factors given in tables 4, 6, appendix tables 4, 5 and 6 are calculated on a days in milk and days in milk squared adjusted basis. To calculate factors similar to those of Searle (1961a) it was assumed that cows first tested in August would have been in milk an average of 10 days in August and plus 30 days for each subsequent month. Likewise cows first tested in September are assumed to have been in milk 10 days when tested in September and plus 30 days for each subsequent month. On this basis age factors were calculated using equation (17) for cows first tested in August, September and October and are given in table 16 for two year old cows. In table 16 the factors estimated by Searle are also reproduced for two year olds. In both cases 5-9 year olds each test month, month of first test combination have an age factor of 1.00.

The two sets of factors show similar trends - decline with later test months and larger factors for later calving cows. However by May there is a large difference between the two sets of factors with Searle's being considerably smaller.

Table 16.

Age factors derived using equation (17) and comparable figures derived by Searle (1961a).

Month of test	Present study: Month of first test Aug.	Sept.	Oct.	Searle (1961a): Month of first test Aug.	Sept.	Oct.
August	1.37			1.36		
September	1.37	1.41		1.30	1.32	
October	1.38	1.39	1.35	1.27	1.30	1.34
November	1.35	1.38	1.38	1.23	1.27	1.34
December	1.36	1.40	1.42	1.21	1.27	1.32
January	1.32	1.34	1.39	1.20	1.23	1.32
February	1.31	1.35	1.44	1.18	1.22	1.29
March	1.26	1.32	1.47	1.15	1.20	1.28
April	1.23	1.26	1.28	1.06	1.14	1.23
May	1.25	1.29	1.26	1.00	1.09	1.17

If one uses the May test day averages given in appendix table 2 a gross comparison factor of 1.20 is obtained. It is apparent that this difference in age factors for the sample used in this study and the sample collected by Searle 14 years ago from the same population represents a change which could not have been predicted. This may be due to a changing in the feeding and management of the young cows or more importantly may indicate year to year variation in age factors. If variation of this magnitude occurs between geographical regions and between years then it would be very difficult to develop a general set of factors which would be unbiased from one year to the next. In any case factors developed from data collected 14 years ago are not appropriate as a basis for adjusting 1970-71 season test day records for age.

The age factors developed in this study show a consistent but small difference between test day milk yield and test day fat yield. The factors for fat yield being slightly smaller than the corresponding factor for milk yield due to the slight but consistent reduction in fat percentage with age for all test months.

VARIANCE COMPONENTS

With unbalanced data where the number of observations in a class is correlated with the affect of that class Harville (1968) has shown that variance component estimates may be biased. The sample of data used initially for variance component estimation possesses this characteristic due to the practise of selecting only the best bulls for extensive use through AB. Restricting estimation to records with AB sire codes will partially remove this source of bias. The variance component estimates given in tables 8 and 11 for nil adjusted test day records show what appear to be large biases. The sire component in many cases accounting for up to 20% of total variance. These "biases" seem to be considerably reduced when the records are adjusted for age and days in milk. This suggests that the sire component estimate is inflated by a confounding of sire with age and days in milk influences. If this is the case then the advantages of adjusting for age and days in milk are obvious.

When the sample is restricted to records with AB sire codes the sire component is further reduced as would be expected when the range of sires being considered is reduced. However a further affect of, and the main justification for, restricting the sample to records with AB sire codes is the reduction of the confounding of herd and sire influences. This confounding is due to a large number of naturally used sires having daughters in one herd only and a number of herds with cows whose sires are used in no other herds. The removal of this confounding has an unpredictable effect on the resulting variance components.

The variance components of within test month records adjusted for days in milk and age have not been reported previously. Searle (1961b) reports variance components for month of lactation. The components in the present study for four test months for milk and fat yield show an increase in total variance up to October and then a decline to November. These trends showing general agreement with those reported by Searle who found the 2nd and 3rd months of lactation to be most variable for fat yield.

MEASURES OF PRODUCTION

Two measures of test day production are being considered, Z_{ijkf} which is production adjusted for days in milk and age and R_{ijkf} which is the ratio of Z_{ijkf} to the herd average Z_{ijkf}. These two measures represent two broad classes of productive measure (a) measures of absolute production (b) measures of relative production. The accumulative form of these two measures are tp_{ijkf} and RI_{ijkf} as defined previously. In this study a method of evaluating the relative value of these measures for cow culling and sire selection has not been developed. However certain heritability values have been obtained. These show a lower heritability for R_{ijkf} for test day milk yield and fat yield than for Z_{ijkf}. On the other hand the difference between the heritability of these two measures for fat percentage is small and generally in favour of R_{ijkf}. Z_{ijkf} heritability for milk yield, fat yield and fat percentage shows an increase from August to later test months. The heritability of R_{ijkf} shows this trend only for fat percentage.

The reason for R_{ijkf} having a lower heritability an Z_{ijkf} seems to lie in the way sire influences production in herds at different environmental levels. If the affect of sire is additive then Z_{ijkf} will have a higher heritability because more of the variation between records is due to sire than for R_{ijkf}. This is made clearer by a simple illustration. Consider three herds in which the daughters of three bulls are milked and have Z_{ijkf} and R_{ijkf} as shown below assuming the affect of sire is additive to herd average

Herd / Sire	Av.	1 Z_{ijkf}	1 R_{ijkf}	2 Z_{ijkf}	2 R_{ijkf}	3 Z_{ijkf}	3 R_{ijkf}
A	+ 30lb + 10.8%	230	115	330	110	430	107.5
B	0 + 0%	200	100	300	100	400	100
C	- 30lb - 10.8%	170	85	270	90	370	92.5
	Av.	200	100	300	100	400	100

Now for this example it is obvious that sire explains more of the total variation in Z_{ijkf} than it does in R_{ijkf} and would thus give a higher heritability. If on the other hand the affect of sire was proportional to herd average the same sires would add respectively 10%, 0% and -10% of herd average and Z_{ijkf} and R_{ijkf} would be as below

Sire \ Herd		1		2		3	
		Z_{ijkf}	R_{ijkf}	Z_{ijkf}	R_{ijkf}	Z_{ijkf}	R_{ijkf}
A	+ 10%	220	110	330	110	440	110
	+ 30lb						
B	+ 0%	200	100	300	100	400	100
	+ 0lb						
	- 10%	180	90	270	90	360	90
	- 30lb						
Herd Av.		200	100	300	100	400	100

In this case sire explains more of the variation in R_{ijkf} than does sire in Z_{ijkf}. If the affect of sire was thus proportionate then heritability of R_{ijkf} would be greater than heritability of Z_{ijkf}.

From this reasoning it is suggested that in general the affect of sire is mainly additive to herd average for milk yield and fat yield and is partly additive and partly proportionate for milk fat percentage. Where the affect of sire is additive, Z_{ijkf} (or some function of it) would be the more accurate method of distinguishing genetic differences between sires.

From the within year repeatability estimates it can be seen that R_{ijkf} is more repeatable than is Z_{ijkf}. This suggests that from one test month to the next R_{ijkf} is more consistent than Z_{ijkf}. In other words the affect of cow is proportionate to test day herd average rather than additive to it.

The heritabilities of the two accumulative measures show differences similar to those for the test day measures for all test months except August. The heritability of August RI_{ijk}

is higher for all traits than the heritability of tp_{ijkf}. This is a result of regressing the accumulative RI_{ijkf} measure for number of records used in its calculation. By September the higher heritability of September Z_{ijkf} records when accumulated as tp_{ijkf} means that tp_{ijkf} is as highly heritable as the RI_{ijkf} measure.

Ideally different measures should be evaluated in terms of the annual gains which can be achieved using the records for culling and selective mating. However the results of these variance component estimations do indicate that the affect of sire on test day production and accumulative test day production is more additive to herd average than proportional. Thus suggesting that ratios to herd average would be inferior for selective breeding purposes.

AREAS FOR FURTHER STUDY

Having written programs to obtain LSQE estimates of the affect of age and days in milk these should be used to obtain estimates for different geographical regions, breed of cow and year. These estimates will enable a more accurate evaluation of the variation that exists between regions, breeds and years in the affect of age and days in milk.

The model described by equation (25) when fitted over years for the three estimation procedures c_j random, c_j fixed and c_j ignored will provide information on the biases in LSQE estimates due to cow culling and sire selection.

Ideally adjustment factors should be tested on data other than those from which they are derived. If there are large differences between years in the affect of age and days in milk on test day yield then this exercise would be futile. In this case investigation into the possibility of estimating factors from the data to which they are to be applied could well be justified.

Methods of evaluating measures of production need further evaluation.

CONCLUSIONS

1). Estimates of the affect of age and linear and quadratic effects of days in milk obtained assuming the affect of herd-sire as random, fixed or ignored are all very similar.

2). The affect of days in milk and days in milk squared on test day milk yield, fat yield and milk fat percentage is influenced by month of test.

3). In some test months 2 year old cows show a significantly different response to days in milk from older cows. The 2 year old cows showing a slower decline in milk and fat yield as days in milk increases.

4). Expressing test day records as an adjusted test day record provides more accurate genetic information than expressing the records as a ratio of herd average.

5). No good basis for comparing different expressions of test day production has been established.

SUMMARY

A sample of 81,695 test day records made by Jersey cows in the 1970-71 season were used to estimate the affect of age and the linear and quadratic regressions, within age class, of days in milk on test day milk yield, fat yield and milk fat percentage. Estimates were obtained assuming models which treated the affect of herd-sire as fixed, random and ignored. Additive factors for adjusting test day records for days and days in milk squared, within test month, were developed and used in conjunction with multiplicative age factors to adjust the August, September, October and November test day records. The adjusted records were also expressed as a ratio of the herd average of the adjusted records. Each test day the adjusted records were accumulated using the test interval method and the ratios were accumulated as a weighted average. Variance components were estimated by Henderson's (1953) method 1 for two different models.

Estimates of the affects of age and days in milk for all three models were very similar. The regressions of days in milk and days in milk squared on test day yield were significantly different from zero for the four age classes in August and September for milk and fat. Fat percentage regressions reached significance in some of the test months. The regressions differed between age classes in October with the 2 year olds showing a slower decline with increasing days in milk than the older cows. The differences in the regression coefficients between test months show the need for adjustment factors to differ according to test month.

The multiplicative age factors showed a decline in the test months January to May for the 2 year olds and 3 year olds. The age factors for these test months were consistantly different from those presently used in New Zealand.

Test day ratios within herd heritability was consistantly lower than that for adjusted test day record. Reasons for this difference are discussed. The within year, within herd repeatability of test day ratios were higher than the corresponding repeatabilities of the adjusted test day records for all yield traits.

BIBLIOGRAPHY

Appleman, R.D., Musgrave, S.D. and Morrison, R.D. (1969). Extending incomplete lactation records of Holstein cows with varying levels of production. J. Dairy Sci. 52, 360.

Allaire, F.R. and Henderson, C.R. (1966a). Selection practised among dairy cows. I. Single lactation traits. J. Dairy Sci. 49, 1426.

Allaire, F.R. and Henderson, C.R. (1966b). Selection practised among dairy cows. II. Total production over several lactations. J. Dairy Sci. 49, 1435.

Brody, S., Ragsdale, A.C. and Turner, C.W. (1923a). The rate of decline of milk secretion with the advance of the period of lactation. J. General Physiology 5, 441.

Brody, S., Ragsdale, A.C. and Turner, C.W. (1923b). The effect of gestation on the rate of decline of milk secretion with the advance of the period of lactation. J. General Physiology 5, 777.

Brody, S., Turner, C.W. and Ragsdale, A.C. (1924). The relation between the initial rise and the subsequent decline of milk secretion following parturition. J. General Physiology 6, 541.

Brumby, P.J. (1961). The causes of differences in production between dairy herds. Animal Production 3, 277.

Cannon, C.Y., Frye, J.B. and Sims, J.A. (1942). Predicting 305 day yields from short-time records. J. Dairy Sci. 25, 991.

Castle, O.M. and Searle, S.R. (1957). Repeatability of dairy cow butterfat records in New Zealand. J. Dairy Sci. 40, 1277.

Clark, D.R. and Touchberry, R.W. (1962). Effect of body weight and age at calving on milk production in Holstein cattle. J. Dairy Sci. 45, 1500.

Cunningham, E.P. and Henderson, C.R. (1968). An iterative procedure for estimating fixed effects and variance components in mixed model situations. Biometrics 24, 13.

Erb, R.E., Goodwin, M.M., Morrison, R.A. and Shaw, A.O. (1952). Lactation studies I. Effect of gestation. J. Dairy Sci. 35, 224.

Erb, R.E., Goodwin, M.M., McCaw, W.N., Morrison, R.A. and Shaw, A.O. (1953). Lactation studies V. Causes of variation in fat percentage. Washington A.E.S. Stations Circular 229, Sept. 1953.

Evans, D.A. (1970). An approach to maximum likelihood estimation of fixed effects and variance components in the unbalanced mixed model. Unpublished manuscript.

Everett, R.W., Carter, H.W. and Burke, J.D. (1968). Evaluation of the Dairy Herd Improvement Association record system. J. Dairy Sci. 51, 153.

Fimland, E.M., Bar-Anan, R. and Harvey, W.R. (1972). Studies on dairy records from Israeli-Fresian cattle. I. Influence of some environmental effects. Acta Agriculturae Scandinavica 22,34.

Fletcher, J.L. (1960). Factors for extending incomplete lactations of Jersey and Jersey x Sindhi cows. (abs.) J. Dairy Sci. 43, 446.

Fritz, G.R., McGilliard, L.D. and Madden, D.E. (1960). Environmental influences on regression factors for estimating 305-day production from part lactation records. J. Dairy Sci. 43, 1108.

Hammond, J. and Sanders, H.G. (1923). Some factors affecting milk yield. J. Agricultural Science 13, 74.

Harville, D.A. (1968). Statistical dependance between subclass means and the numbers of observations in the subclasses for the two-way completely random classification. American Stat. Assoc. J. 63,1484.

Henderson, C.R. (1949). Estimation of changes in herd environment. J. Dairy Sci. 32, 706.

Henderson, C.R. (1953). Estimation of variance and covariance components. Biometrics 9, 226.

Henderson, C.R. (1958). Estimates of environmental trends and biases resulting from errors in age factors and repeatability. (abs.) J. Dairy Sci. 41, 747.

Herd Improvement Association Manual for processing data on the New Zealand Dairy Board I.B.M. 360/30 computer. Revised edition 1970.

Hickman, C.G. (1957). Production studies on experimental farms dairy herds. I. Age correction of 180 day yield. Canadian J. of Agricultural Science 36, 123.

Hickman, C.G. (1960). Production studies on experimental farms dairy herds. II. Use of 180 day yield for culling. Canadaian J. of Genetics and Cytology 2, 189.

Hickman, C.G. (1962). Effect of level of herd environment. I. Relationship between yield and age. J. Dairy Sci. 45, 861.

Hickman, C.G. and Henderson, C.R. (1955). Components of the relationship between level of production and rate of maturity in dairy cattle. J. Dairy Sci. 38, 883.

Hillers, J.K. and Freeman, A.E. (1965). Differences between sires in rate of maturity of their daughters. J. Dairy Sci. 48, 1680.

Hinks, C.J.M. (1966a). Selection practises in dairy herds. I. First lactation performance and survival for second. Animal Production 8, 467.

Hinks, C.J.M. (1966b). Selection practises in dairy herds. II. Selection patterns in later lactations. Animal Production 8, 481.

Kay, R.R. and M'Candlish, A.C. (1929). Factors affecting the yield and quality of milk. J. of Agricultural Science 19, 343.

Kendrick, J.F. (1955). Standardizing Dairy-Herd-Improvement-Association records in proving sires. USDA, ARS-52-1 (January 1955).

Keown, J.F. and Van Vleck, L.D. (1971). Selection on test-day fat percentage and milk production. J. Dairy Sci. 54, 199.

Koh, Y.O. and Henderson, C.R. (1964). Year by age interaction of New York dairy records. (abs.) J. Animal Science 23, 851.

Korkman, N. (1950). Effect of advancing lactation on the butterfat percentage of the milk. Sonderdruck,, Ztschr.f. Tierzuchtung u. Zuchtungsbiologie", Bd 59, H.1.

Lamb, R.C. and McGilliard, L.D. (1959). Variables affecting the ratio of total milk produced in 305 days to monthly milk production. (abs.) J. Dairy Sci. 42, 924.

Lamb, R.C. and McGilliard, L.D. (1960). Variables affecting ratio factors for estimating 305-day production from part lactation production. J. Dairy Sci. 43, 519.

Lamb, R.C. and McGilliard, L.D. (1967a). Ratio factors to estimate 305-day production from lactation records in progress. J. Dairy Sci. 50, 1101.

Lamb, R.C. and McGilliard, L.D. (1967b). Usefulness of part records to estimate the breeding values of dairy cattle. J. Dairy Sci. 50, 1458.

Lee, A.J. and Hickman, C.G. (1967). Yield - age regressions in age-corrected records. J. Dairy Sci. 50, 987.

Lennon, H.D. and Mixner, J.P. (1958). Relation of lactation milk production in dairy cows to maximum initial yield and persistency of lactation. J. Dairy Sci. 41, 969.

Ludwick, T.M., and Peterson, W.E. (1943). A measure of persistency of lactation in dairy cattle. J. Dairy Sci. 26, 439.

Lush, J.L. and Schrode, R.R. (1950). Changes in milk production with age and milking frequency. J. Dairy Sci. 33, 338.

Madden, D.E., Lush, J.L. and McGilliard, L.D. (1955). Relations between parts of lactation and production ability in Holstein cows. J. Dairy Sci. 38, 1264.

Madden, D.E., McGilliard, L.D. and Ralston, N.P. (1956). Relations between monthly test-day milk production of Holstein-Fresian cows. J. Dairy Sci. 39, 932.

Madden, D.E., McGilliard, L.D. and Ralston, N.P. (1959). Relations between test-day milk production of Holstein cows. J. Dairy Sci. 42, 319.

Mahadaven, P (1951). The effect of environment and heredity on lactation. III. Butterfat percentage. J. Agricultural Science 41, 94.

McDaniel, B.T. and Corley, E.L. (1966). Environmental influences on age correction factors. J. Dairy Sci. 49, 736.

McDaniel, B.T., Miller, R.H. and Corley, E.L. (1967a). Sources of variation in ratios of total to part yield. J. Dairy Sci. 50, 1917.

McDaniel, B.T., Miller, R.H., Corley, E.L. and Plowman, R.D. (1967b). Dairy-Herd-Improvement-Association age adjustment factors for standardizing lactations to a nature basis. Dairy Herd Improvement Letter (Feb. 1967) ARS-44-188 Vol.43, No. 1.

Milk Marketing Board Production Division Report No. 11 (1961). Seasonality of milk output (supplementary diagrams).

Miller, R.G. (1964). Biases in the estimation of the regression of milk production on age. J. Dairy Sci. 47, 855.

Miller, R.H., Harvey, W.R., Tabler, K.A., McDaniel, B.T. and Corley, E.L. (1966). Maximum likelihood estimates of age effects. J. Dairy Sci. 49, 65.

Miller, R.H., and Hooven, N.W. (1969). Factors affecting the whole - and part - lactation milk yield and fat percent in Holstein cows. J. Dairy Sci. 52, 1588.

Miller, R.H., McDaniel, B.T. and Corley, E.L. (1967). Variation in the ratio factors for age-adjusting part lactation records. J. Dairy Sci. 50, 1819.

Miller, R.H., McDaniel, B.T. and Dickinson, F.N. (1970). Regression of mature - equivalent production on age at calving. J. Dairy Sci. 53, 453.

Miller, P.D. and Henderson, C.R. (1968). Seasonal age correction factors by maximum likelihood. (abs.) J. Dairy Sci. 51, 958.

Miller, P.D., Lentz, W.E. and Henderson, C.R. (1970). Joint influence of month and age of calving on milk yield. J. Dairy Sci. 53, 351.

New Zealand Dairy Board (a). "Dairy Herd Improvement in New Zealand". Pub. Herd Improvement Council 1971.

New Zealand Dairy Board (b). 27th Annual Report of Dairy Herd Testing in New Zealand. 1950-51 season. Table VI pg.78. Investigational and research work.

New Zealand Dairy Board (c). 36th Annual Report Herd Improvement Section. 1959-60 season pg.49. Studies in the population genetics of dairy cattle.

New Zealand Dairy Board (d). 37th Annual Report Herd Improvement Section. Table 3 pg.108.

New Zealand Dairy Board (e). 38th Farm Production Report. 1961-62 season, pg. 55. Herd improvement through using test-day records instead of lactation yields.

New Zealand Dairy Board (f). 45th Farm Production Report. 1968-69 season, pg.43. Measures of productive ability.

New Zealand Dairy Board (g). 46th Farm Production Report. 1969-70 season. Tables 14 and 15 pg.48.

New Zealand Dairy Board (h). ibid. Table 32 pg. 60.

New Zealand Dairy Board (i). ibid. Table 5 pg. 64.

New Zealand Dairy Board (j). ibid. pg. 82.

New Zealand Dairy Board (k). ibid. pg. 83.

Pirchner, F. and Lush, J.L. (1959). Genetic and environmental portions of the variation among herds in butterfat production. J. Dairy Sci. 42, 115.

Rendel, J.M. and Robertson, A. (1950). Estimation of genetic gain in milk yield by selection in a closed herd of dairy cattle. J. Genetics 50, 1.

Robertson, A. and Khishin, S.S. (1958). The effect of selection for heifer milk yield on the production level of mature cows. J. Agricultural Science 50, 12.

Robertson, A. and McArthur, A.T.G. (1955). Genetic differences between bull breeding herds. Proceedings of the British Society of Animal Production 1955, 94.

Robertson, A., O'Connor, L. and Edmonds, G. (1960). Progeny testing dairy bulls at different management levels. Animal Production 2, 141.

Ronningen, K. (1967). Phenotypic and genetic parameters for characters related to milk production in cattle. Acta Agriculturae Scandinavica 17, 83.

Sanders, H.G. (1923). The shape of the lactation curve. J. Agricultural Science 13, 169.

Sanders, H.G. (1927a). Variation in milk yield caused by I. Season of the year. J. Agricultural Science 17, 339.

Sanders, H.G. (1927b). Variations in milk yield caused by II. Service. J. Agricultural Science 17, 502.

Sanders, H.G. (1928a). Variations in milk yield caused by III. Age. J. Agricultural Science 18, 46.

Sanders, H.G. (1928b). Variations in milk yield caused by IV. Dry period and standardization of yields. J. Agricultural Science 18, 209.

Sanders, H.G. (1930). The analysis of the lactation curve into maximum yield and persistency. J. Agricultural Science 20, 145.

Searle, S.R. (1960). Simplified herd-level age-correction factors. J. Dairy Sci. 43, 821.

Searle, S.R. (1961a). Part lactations I. Age correction factors for monthly milk fat yields. J. Dairy Sci. 44, 104.

Searle, S.R. (1961b). Part lactations II. Genetic and phenotypic studies of monthly milk fat yield. J. Dairy Sci. 44, 282.

Searle, S.R. (1961c). Part lactations III. Progeny testing with part lactations. J. Dairy Sci. 44, 921.

Searle, S.R. (1962). Age and herd effects in New Zealand dairy cow production. J. Dairy Sci. 45, 82.

Searle, S.R. (1963). Genetic studies of dairy production early in lactation. J. Dairy Sci. 46, 1266.

Searle, S.R. (1971). "Linear Models". John Wiley & Sons, Inc. 1971.

Searle, S.R. and Henderson, C.R. (1959). Establishing age -correction factors related to the level of herd production. J. Dairy Sci. 42, 824.

Searle, S.R. and Henderson, C.R. (1960). Judging the effectiveness of age correction factors. J. Dairy Sci. 43, 966.

Shannon, P. (1971). Mimeo dated 1/11/71.

Stichbury, J.W. (1963). Simplified testing for culling. Dairy-Farming Annual 1971, 31. Pub. by Massey University, Palmerston North, New Zealand.

Stichbury, J.W. (1970). Directors review of the 1969-70 season. New Zealand Dairy Board. 46th Farm Production Report. 1969-70 season pg.6.

Stichbury, J.W. (1971). Directors review of the 1970-71 season. New Zealand Dairy Board. 47th Farm Production Report. 1970-71 season pg.6.

Smith, J.W. and Legates, J.E. (1962a). Factors affecting persistency and its importance in 305-day lactation production. J. Dairy Sci. 45, 676.

Smith, J.W. and Legates, J.E. (1962b). Relation of days-open and days-dry to lactation milk and fat yields. J. Dairy Sci. 45, 1192.

Spike, P.W. and Freeman, A.E. (1967). Environmental influences on monthly variations in milk. J. Dairy Sci. 50, 1897.

Swanson, E.W. and Hinton, S.A. (1964). Effect of seriously restricted growth upon lactation. J. Dairy Sci. 47, 267.

Syrstad, O. (1965). Studies on dairy herd records. II. Affect of age and season of calving. Acta Agriculturae Scandinavica 15, 1.

Thompson, R. (1969). Iterative estimation of variance components for non-orthogonal data. Biometrics 25, 767.

Thompson, N.R., Stone, W.K., Graf, G.C., Kramer, C.Y. and Freund, R.J. (1960). Errors in estimation of lactational yields of milk, fat and solids-not-fat from individual cows. J. Dairy Sci. 43, 951.

Turner, C.W. (1923). Seasonal variations in milk and fat production. J. Dairy Sci. 6, 198.

Turner, C.W. (1926). A quantitative form of expressing persistency of milk or fat secretion. J. Dairy Sci. 9, 203.

Van Vleck, L.D. (1963). Regression of records on herd-mate averages. J. Dairy Sci. 46, 846.

Van Vleck, L.D. and Henderson, C.R. (1961a). Estimates of genetic parameters of some functions of part lactation milk records. J. Dairy Sci. 44, 1073.

Van Vleck, L.D. and Henderson, C.R. (1961b). Regression factors for extending part lactation milk records. J. Dairy Sci. 44, 1085.

Van Vleck, L.D. and Henderson, C.R. (1961c). Ratio factors for adjusting monthly test-day data for age and season of calving and ratio factors for extending part lactation records. J. Dairy Sci. 44, 1093.

Van Vleck, L.D. and Henderson, C.R. (1961d). Regression factor for predicting a succeeding complete lactation milk record from part lactation records. J. Dairy Sci. 44, 1322.

Van Vleck, L.D. and Henderson, C.R. (1961e). Extending part lactation milk records by regression ignoring herd effects. J. Dairy Sci. 44, 1519.

Ward, A.H. and Campbell, J.T. (1938). The practical application of age conversion factors to dairy cattle production (butterfat) records. J. Agricultural Sci. 18, 33.

Wilson, G.F. (1964). The influence of date of calving on milk production. New Zealand J. of Agricultural Research 7, 80.

Wood, P.D.P. (1967). An algebraic model of the lactation curve in cattle. Nature 216, 164.

Wood, P.D.P. (1968). Factors affecting persistency of lactation in cattle. Nature 218, 894.

Wood, P.D.P. (1969). Factors affecting the shape of the lactation curve in cattle. Animal Production 11, 307.

Wood, P.D.P. (1970). A note on the repeatability of parameters of the lactation curve in cattle. Animal Production 12, 535.

Wunder, W.W. and McGilliard, L.D. (1971). Season of calving: age, management and genetic differences for milk. J. Dairy Sci. 54, 1652.

Appendix A

Programs used on the IBM 360/30

The programs used for computations on the electronic computer, the IBM 360/30, are listed below with their names and brief descriptions. Five were written by the author, and one is an IBM Scientific Subroutine. All input and output from and to disc or tape was achieved through ASSEMBLER language subroutines written by R. Irving of the Electronic Data Processing Department of the New Zealand Dairy Board. All other programs and subroutines are in FFORTRAN.

EDIT: From a file of all Auckland herds on monthly or bimonthly testing a file is created for all test day records meeting the selection criteria (p 32). This file (DATA TAPE) becomes the input for RIANAL and A-DAY CORRECTION.

RIANAL: The DATA TAPE sorted by month of test, sire and herd, is used as input. The Z'Z, X'Z and Z'Y matrices (of p 25) are written on to disc for milk yield, fat yield, and test, for a single test month. Initial values of k are supplied on cards. For one trait at a time the initial value of k is used and the first iteration is completed. The new value of k is then used for the next iteration. Either nine iterations or a change in k of .025 from one iteration to the next - which ever occurs first - terminates the iterations. The next trait is treated similarly and so on.

A-DAY CORRECTION: Using the DATA TAPE sorted by herd and cow as input the test day records are corrected for days in milk and age. The quantities given on page 29 are computed and the output of this program is a tape (CORRECTED DATA) and a printed summary of herds containing more than 600 test day records. Where a herd exceeds 600 test day records it is treated as two separate herds - the first having 600 test day records and the second having the remainder.

<u>ANOVA 1</u>: Estimates variance components (see appendix B) of corrected records. The input is the CORRECTED DATA tape sorted by sire, herd and cow.

<u>ANOVA 2</u>: Estimates variance components of corrected records. The input is the CORRECTED DATA tape sorted by month of test, sire and herd.

<u>MINV</u> (IBM Scientific Subroutine): Matrix inverter using the standard Gauss-Jordan method. Used for matrix inversion in RIANAL, ANOVA 1 and ANOVA 2.

Appendix B

<u>Estimation of variance components for the model given by equation (22)</u>

Using method 1 of Henderson (1953) the sums of squares computed were

$$(T) = \begin{bmatrix} T_t \\ T_s \\ T_h \\ T_{hxs} \\ T_c \\ T_o \end{bmatrix} = \begin{bmatrix} \sum_i \sum_j \sum_k \sum_l y_{ijkl}^2 \\ \sum_i y_{i...}^2 / n_{i..} \\ \sum_j y_{.j..}^2 / n_{.j.} \\ \sum_i \sum_j y_{ij..}^2 / n_{ij.} \\ \sum_i \sum_j \sum_k y_{ijk.}^2 / n_{ijk} \\ y_{....}^2 / N \end{bmatrix}$$

where N = total observations. These sums of squares were equated to their expectations under the model and a solution for the variance components obtained.

The expectation of the vector of sums of squares (T) is

$$E(T) = \begin{bmatrix} N & N & N & N & N & N \\ N & N & \sum_i \sum_j \frac{n_{ij}^2}{n_{i.}} & \sum_i \sum_j \frac{n_{ij}^2}{n_{i.}} & \sum_i \sum_j \sum_k \frac{n_{ijk}^2}{n_{i.}} & S \\ N & \sum_j \sum_i \frac{n_{ij}^2}{n_{.j}} & N & \sum_j \sum_i \frac{n_{ij}^2}{n_{.j}} & \sum_j \sum_i \sum_k \frac{n_{ijk}^2}{n_{.j}} & h \\ N & N & N & N & \sum_i \sum_j \sum_k \frac{n_{ijk}^2}{n_{ij}} & hs \\ N & N & N & N & N & c \\ N & \sum_i \frac{n_{.i}^2}{N} & \sum_j \frac{n_{.j}^2}{N} & \sum_i \sum_j \frac{n_{ij}^2}{N} & \sum_i \sum_j \sum_k \frac{n_{ijk}^2}{N} & 1 \end{bmatrix} \begin{bmatrix} \mu^2 \\ \sigma_s^2 \\ \sigma_h^2 \\ \sigma_{hxs}^2 \\ \sigma_c^2 \\ \sigma_e^2 \end{bmatrix}$$

where s and h are the number of sires and herds respectively, sh is the number of herd-sire subclasses with 1 or more observation and c is the number of cows.

$\hat{\sigma}_e^2$ was obtained as

$$\hat{\sigma}_e^2 = (T_t - T_c)/(N - c)$$

Estimates of the other four variance components were found by computing the vector T* where

$$T^* = \begin{bmatrix} T_s - T_o - (h - 1)\ \hat{\sigma}_e^2 \\ T_h - T_o - (s - 1)\ \hat{\sigma}_e^2 \\ T_{hs} - T_s - T_h + T_o - (hs-s-h + 1)\ \hat{\sigma}_e^2 \\ T_c - T_{hs} - (c - hs)\ \hat{\sigma}_e^2 \end{bmatrix}$$

and the matrix C* where E (T*) = C* and,

$$C^* = \begin{bmatrix} N-C_{6,2} & C_{2,3}-C_{6,3} & C_{2,4}-C_{6,4} & C_{2,5}-C_{6,5} \\ C_{3,2}-C_{6,2} & N-C_{6,3} & C_{3,4}-C_{6,4} & C_{3,5}-C_{6,5} \\ C_{6,2}-C_{3,2} & C_{6,3}-C_{2,3} & N-C_{2,4}-C_{3,4} + C_{6,4} & C_{4,5}-C_{3,5}-C_{2,5} + C_{6,5} \\ 0 & 0 & 0 & N-C_{4,5} \end{bmatrix}$$

and $C_{i,j}$ is the element of the i th row and j th column of the expectation coefficient matrix.

The solution was thus

$$\begin{bmatrix} \hat{\sigma}_s^2 \\ \hat{\sigma}_h^2 \\ \hat{\sigma}_{hxs}^2 \\ \hat{\sigma}_c^2 \end{bmatrix} = C^{*-1} T^* .$$

Appendix table 1.

Average test day milk yield (lb) for the four age classes in the test months August 1970 to May 1971.

	Age class				
Month	2	3	4	5 - 9	Av. all ages
August	22.7	27.0	29.4	30.8	27.5
September	24.0	28.7	31.7	33.3	29.4
October	23.6	27.8	30.7	32.7	28.9
November	22.8	26.5	29.2	31.3	27.6
December	20.2	23.5	26.1	28.0	24.7
January	18.3	21.1	23.3	24.7	22.0
February	15.1	17.3	18.9	20.3	18.1
March	13.4	15.1	16.7	17.7	15.9
April	11.2	12.2	13.0	14.1	12.9
May	10.6	11.6	12.0	12.9	12.0

Appendix table 2.

Average test day fat yield (lb) for the four age classes in the test months August 1970 to May 1971.

	Age class				
Month	2	3	4	5 - 9	Av. all ages
August	1.12	1.32	1.45	1.50	1.35
September	1.19	1.41	1.55	1.63	1.44
October	1.22	1.43	1.59	1.67	1.49
November	1.22	1.41	1.56	1.65	1.47
December	1.07	1.26	1.40	1.47	1.31
January	0.99	1.15	1.27	1.32	1.18
February	0.84	0.97	1.06	1.12	1.01
March	0.79	0.90	1.00	1.04	0.94
April	0.72	0.79	0.85	0.89	0.83
May	0.71	0.78	0.80	0.85	0.80

Appendix table 3.

Average test day fat percentage for the four age classes in the test months August 1970 to May 1971.

	Age class				
Month	2	3	4	5 - 9	Av. all ages
August	4.95	4.89	4.92	4.88	4.91
September	4.97	4.92	4.91	4.88	4.91
October	5.21	5.16	5.20	5.12	5.16
November	5.36	5.37	5.40	5.30	5.34
December	5.34	5.37	5.40	5.28	5.33
January	5.43	5.49	5.45	5.34	5.41
February	5.61	5.67	5.63	5.57	5.61
March	5.99	6.08	6.09	5.98	6.02
April	6.64	6.69	6.71	6.50	6.60
May	6.92	6.83	6.82	6.67	6.78

Appendix table 4.

LSQE of the affect of age, days in milk and days in milk squared on test day milk yield.

Month	Age	$\mu + a_i$	b_{i1}	b_{i2}	A_i
August	2	18.9	0.2807	-.00394	1.35
	3	24.0	0.2898	-.00530	1.14
	4	25.6	0.3485	-.00616	1.05
	5-9	26.7	0.3318	-.00512	1.00
September	2	20.6	0.1662	-.00174	1.39
	3	27.1	0.1551	-.00236	1.16
	4	30.0	0.1424	-.00207	1.05
	5-9	30.4	0.1880	-.00239	1.00
October	2	24.6	-.0085	-.00007	1.38
	3	29.7	0.0203	-.00066	1.17
	4	32.2	0.0300	-.00071	1.06
	5-9	34.4	0.0131	-.00054	1.00
November	2	29.8	-.1026	0.00031	1.36
	3	33.2	-.0669	-.00003	1.18
	4	38.2	-.0979	0.00005	1.07
	5-9	41.2	-.1262	0.00022	1.00
December	2	31.7	-.1628	0.00054	1.37
	3	35.3	-.1397	0.00034	1.18
	4	34.6	-.1723	0.00002	1.07
	5-9	42.8	-.1800	0.00045	1.00

Appendix table 4 cont'd.

Month	Age	$\mu + a_i$	b_{i1}	b_{i2}	A_i
January	2	31.7	-.1439	0.00037	1.33
	3	26.6	-.0182	-.00011	1.17
	4	28.3	0.0007	-.00020	1.06
	5-9	40.9	-.1509	-.00028	1.00
February	2	19.2	-.0046	-.00009	1.32
	3	21.9	-.0050	-.00011	1.17
	4	25.3	-.0131	-.00011	1.07
	5-9	30.8	-.0507	-.00004	1.00
March	2	18.6	-.0194	-.00002	1.30
	3	43.7	-.2450	0.00051	1.18
	4	39.4	-.1697	0.00029	1.06
	5-9	31.0	-.0694	0.00003	1.00
April	2	4.58	0.0847	-.00023	1.25
	3	-11.21	0.2375	-.00057	1.15
	4	-142.4	1.386	-.00303	1.09
	5-9	53.8	-.3026	0.00056	1.00
May	2	39.4	-.2045	0.00036	1.21
	3	22.9	-.0595	0.00006	1.11
	4	33.4	-.1388	0.00022	1.08
	5-9	-8.8	0.2026	-.00045	1.00

Appendix table 5.

LSQE of the affect of age, days in milk and days in milk squared on test day fat yield.

Month	Age	$\mu + a_i$	b_{i1}	b_{i2}	A_i
August	2	0.86	0.0194	-.00027	1.34
	3	1.13	0.0163	-.00027	1.14
	4	1.25	0.0159	-.00026	1.04
	5-9	1.23	0.0209	-.00031	1.00
September	2	0.94	0.0109	-.00010	1.37
	3	1.20	0.0114	-.00013	1.15
	4	1.35	0.0115	-.00013	1.04
	5-9	1.34	0.0138	-.00014	1.00

Appendix table 5 cont'd.

Month	Age	$\mu + a_i$	b_{i1}	b_{i2}	A_i
October	2	1.15	0.0020	-.000013	1.36
	3	1.34	0.0048	-.000047	1.16
	4	1.51	0.0052	-.000052	1.05
	5-9	1.52	0.0064	-.000056	1.00
November	2	1.38	-.0026	0.000009	1.35
	3	1.50	0.0005	-.000014	1.17
	4	1.73	0.0002	-.000018	1.05
	5-9	1.87	-.0018	-.000005	1.00
December	2	1.39	-.0048	0.000018	1.36
	3	1.57	-.0038	0.000009	1.17
	4	1.43	0.0023	-.000020	1.05
	5-9	1.88	-.0043	0.000008	1.00
January	2	1.13	-.0007	-.0000006	1.32
	3	1.16	0.0019	-.000012	1.14
	4	1.19	0.0038	-.000021	1.04
	5-9	1.82	-.0039	0.000004	1.00
February	2	0.249	0.0079	-.000025	1.32
	3	0.696	0.0046	-.000016	1.15
	4	0.973	0.0032	-.000014	1.05
	5-9	1.20	0.0018	-.000012	1.00
March	2	-.253	0.0112	-.000029	1.31
	3	0.395	0.0068	-.000020	1.15
	4	0.0292	0.0122	-.000035	1.03
	5-9	1.56	-.0023	-.0000007	1.00
April	2	1.23	-.0033	0.000005	1.22
	3	0.31	0.0059	-.000016	1.12
	4	-4.24	0.0460	-.000102	1.04
	5-9	1.58	-.0038	0.000004	1.00
May	2	1.59	-.0063	0.000011	1.19
	3	0.744	0.0017	-.000059	1.09
	4	1.58	-.0050	0.000008	1.05
	5-9	-.087	0.0090	-.000020	1.00

Appendix table 6.

LSQE of the affect of age, days in milk and days in milk squared on test day fat percentage.

Month	Age	$\mu + a_i$	b_{i1}	b_{i2}	A_i
August	2	4.54	0.0297	-.000383	0.99
	3	4.74	0.0062	0.000016	1.00
	4	4.94	-.0074	0.000219	0.99
	5-9	4.63	0.0179	-.000224	1.00
September	2	4.56	0.0123	-.000059	0.98
	3	4.49	0.0105	0.0	0.99
	4	4.55	0.0117	-.000064	0.99
	5-9	4.46	0.0126	-.000052	1.00
October	2	4.73	0.0092	-.000028	0.99
	3	4.51	0.0125	-.000034	0.99
	4	4.67	0.0109	-.000037	0.98
	5-9	4.47	0.0149	-.000067	1.00
November	2	4.67	0.0080	-.000011	0.99
	3	4.48	0.0128	-.000034	0.99
	4	4.51	0.0139	-.000045	0.98
	5-9	4.55	0.0106	-.000026	1.00
December	2	4.08	0.0158	-.000044	0.99
	3	4.21	0.0124	-.000022	0.98
	4	3.99	0.0197	-.000063	0.98
	5-9	4.19	0.0139	-.000038	1.00
January	2	2.52	0.0348	-.000102	0.99
	3	4.50	0.0074	-.000006	0.97
	4	4.07	0.0149	-.000038	0.98
	5-9	4.17	0.0123	-.000029	1.00
February	2	2.05	0.0331	-.000074	1.00
	3	3.73	0.0143	-.000020	0.98
	4	4.08	0.0111	-.000015	0.99
	5-9	3.83	0.0136	-.000022	1.00
March	2	-1.76	0.0694	-.000153	1.00
	3	-0.303	0.0573	-.000126	0.98
	4	-6.069	0.1155	-.000269	0.98
	5-9	4.62	0.0084	-.000009	1.00

Appendix table 6 cont'd.

Month	Age	$\mu + a_i$	b_{i1}	b_{i2}	A_i
April	2	9.41	-.0317	0.000082	0.99
	3	13.7	-.0662	0.000151	0.97
	4	39.6	-.2907	0.000631	0.96
	5-9	1.8	0.0354	-.000065	1.00
May	2	-1.07	0.0536	-.000088	0.98
	3	0.522	0.0462	-.000083	0.97
	4	2.10	0.0309	-.000049	0.98
	5-9	9.69	-.0273	0.000059	1.00

Appendix table 7.

Variance component estimates for test day milk yield records adjusted for age, days in milk and days in milk squared. These estimates were obtained from a sample of test day records with AB sire codes.

Measure	Month	$\hat{\sigma}_h^2$	$\hat{\sigma}_s^2$	$\hat{\sigma}_{hxs}^2$	$\hat{\sigma}_e^2$	$\hat{h}^2$
Z_{ijkf}	August	14.83	1.05	4.15	24.1	.17
	September	13.95	3.22	-2.89	31.3	.37
	October	13.85	4.57	-2.91	27.4	.57
	November	8.61	3.32	2.10	21.8	.53
tp_{ijkf}	August	7182.3	520.5	1996.2	11,675	.17
	September	20351.0	4919.4	-3483.4	54,072	.33
	October	48822.9	13609.9	-41959.4	118,049	.41
	November	91113.1	26547.2	34045.0	202,452	.46
R_{ijkf}	August	-14.9	20.98	34.44	286.6	.27
	September	-0.98	13.99	-10.02	300.0	.18
	October	-6.8	18.97	4.47	270.2	.26
	November	0.05	20.87	39.1	231.2	.33
RI_{ijkf}	August	-1.94	8.94	7.86	86.4	.37
	September	2.60	9.20	-1.2	100.1	.34
	October	2.37	11.51	2.5	116.5	.36
	November	3.26	12.6	25.3	110.8	.41

Appendix table 8.

Variance component estimates for test day milk fat percentage adjusted for age, days in milk and days in milk squared. These estimates were obtained from a sample of test day records with AB sire codes.

Measure	Month	$\hat{\sigma}_h^2$	$\hat{\sigma}_s^2$	$\hat{\sigma}_{hxs}^2$	$\hat{\sigma}_e^2$	$\hat{h}^2$
Z_{ijkf}	August	0.074	0.022	-.029	0.463	0.18
	September	0.053	0.044	-.021	0.405	0.39
	October	0.058	0.037	-.0098	0.482	0.28
	November	0.047	0.041	0.036	0.356	0.41
tp_{ijkf}	August	35.13	10.24	-13.6	225.1	0.17
	September	106.30	90.29	-69.9	666.4	0.48
	October	204.3	183.4	-147.3	1592.1	0.41
	November	446.4	444.2	131.9	2305.0	0.65
R_{ijkf}	August	-5.36	9.16	-12.8	198.1	0.18
	September	-0.48	19.73	-7.6	166.0	0.42
	October	-1.59	15.23	-2.9	183.7	0.31
	November	2.58	17.03	11.9	124.0	0.48
RI_{ijkf}	August	0.10	6.04	-7.5	80.8	0.28
	September	2.97	11.71	-6.9	66.7	0.59
	October	2.59	10.35	-3.9	68.4	0.53
	November	4.04	12.41	4.7	49.8	0.79

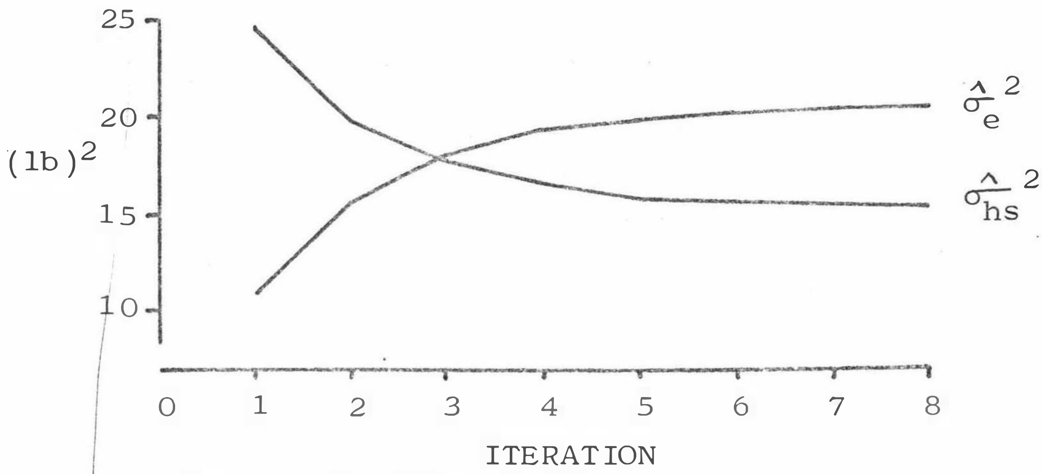

Fig. 1 (a) Milk yield

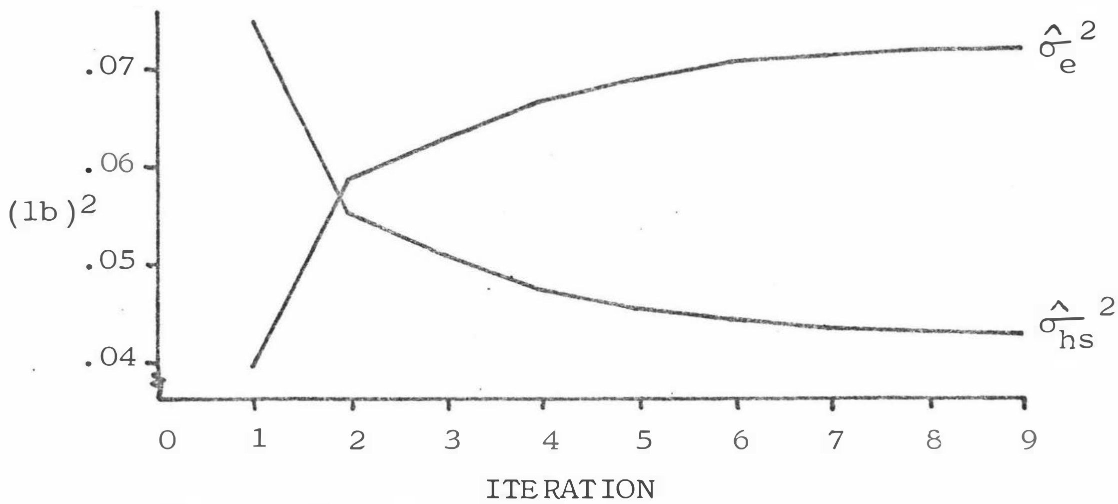

Fig. 1 (b) Fat yield

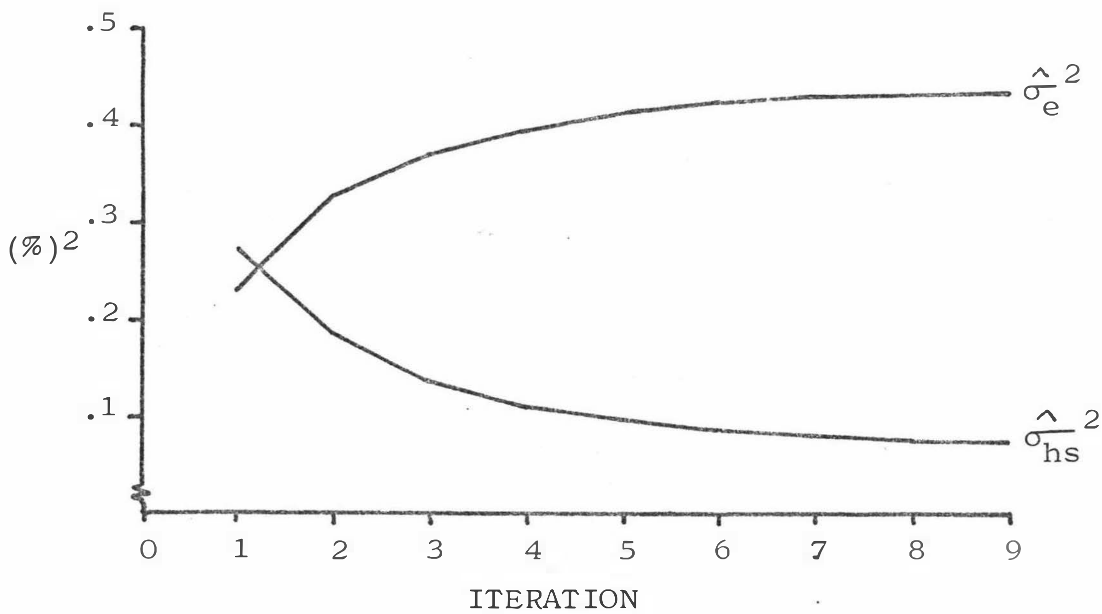

Fig. 1 (c) Milk fat percentage

Appendix figure 1. Variance component estimates obtained at the end of each iteration for August test day records.