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MINIMAX APPROACHES TO ROBUST CONTROL

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Abstract

This Thesis is a fundamental investigation of minimax approaches to robust control. The minimax games considered here are for bounded classes of uncertain plant where the performance is measured by a quadratic cost function. These games are between the controller and a group of uncertainty, disturbance and measurement noise signals with the possible inclusion of the initial condition of the plant.

An H_∞ with transients problem is presented where a non zero initial condition and structured uncertainty are permitted. Necessary and sufficient conditions for the existence of controllers that solve this problem for state feedback and measurement feedback are given. The optimal solution to the state feedback problem may be found by a convex optimisation. These results represent an extension of [Khargonekar et al., 1991].

A state feedback minimax problem is presented where the initial condition is known and multiple channels of uncertainty, each satisfying an integral quadratic constraint, are permitted. Necessary and sufficient conditions for the existence of a minimax controller are given and the design is shown to be the result of a convex optimisation. These results are an extension of [Savkin and Petersen, 1995]. Similar measurement feedback problems are also discussed. Comparisons and special cases of the minimax and H_∞ with transients and structure problems are presented. Also, expressions for the worst case uncertainty, disturbance and measurement noise signals are given.

Finally, a set valued estimation problem is considered for closed loop uncertain plants. The initial condition of the plant is constrained to lie in an ellipsoid and the uncertainty is permitted to be structured and satisfies a type of integral quadratic constraint. Given the history of measurements from the initial time to the current time, a method for determining the set of possible current states is presented. This result represents an extension of [Savkin and Petersen, 1995a] and [Bertsekas and Rhodes, 1971] to permit structured uncertainty. It is also shown how the set valued estimator may be used as a model invalidator for models with bounded uncertainty.

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Notation

The following notation will be used throughout the Thesis: Signal norms will be represented as follows

$\ a(t)\ ^2$	$\int_0^\infty a(t)'a(t) dt$	A squared L_2 norm,
$\ a(t)\ _M^2$	$\int_0^\infty a(t)'Ma(t) dt$	A weighted squared L_2 norm,
$\ a(t)\ _{[b,c]M}^2$	$\int_b^c a(t)'Ma(t) dt$	A weighted time integral.

The L_2 induced norm, or infinity norm for a linear system, will be denoted using the same notation as for the L_2 norm of a signal;

$\ G\ $	$\sup_{w(t) \neq 0} \frac{\ G(w(t))\ }{\ w(t)\ }$	The L_2 induced norm.
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The following acronyms will also be used

ARE	Algebraic Ricatti Equation,
RDE	Ricatti Differential Equation,
LQR	Linear Quadratic Regulator,
LQG	Linear Quadratic Gaussian,
IQC	$\ w_i(t)\ ^2 \leq \ z_i(t)\ ^2 + d_i$ Integral Quadratic Constraint,

along with some additional notation

\dot{a}	$\frac{da}{dt}$	Time derivative,
(\cdot)	$\frac{d}{d\epsilon}(\cdot) _{\epsilon=0}$	A derivative of a scaling matrix τ , in some direction. See equations (3.24) to (3.27) for details.
\mathbf{R}^n		The set of real vectors of dimension $n \times 1$.