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Word problems in teaching and learning algebra

A thesis presented in partial fulfilment of the requirements for the degree of

Master of Educational Studies
in
Mathematics

at Massey University, Palmerston North,
New Zealand.

Philip R. Bennett

2002

Abstract

This research seeks to examine student understanding of algebra and how teachers facilitate algebraic learning for the purpose of improving learning outcomes. Based on the earlier work of Nathan and Koedinger (2000a; 2000c; 2001), the role of word problems in particular is investigated in relation to student development of algebraic understanding and technique.

The year 10 students surveyed displayed particularly low levels of algebraic thinking and poor algebraic skill. The results show that as the structural complexity of problems increased, student understanding diminished and there was a clear shift in student choice of strategy. The use of calculators showed a significant increase in algebraic proficiency, supporting the view that beginning algebra students find it difficult to focus simultaneously on the algebraic and arithmetical aspects of problems. Story problems with result unknown and start unknown complexity solicited a greater proportion of informal strategies than equation problem counterparts. When students chose to use algebra, it was predominantly for problems in an equation format.

The results indicate a disparity between what is being taught and what is being learned. This may be explained in part by the apparent philosophical conflict in teacher beliefs, where importance is placed both on achieving success in algebraic technique, and also on encouraging student driven solution methods. In order to capture student interest, teachers endorse the use of informal strategies by students through advocating word problems as applications of the real world and promoting a goal oriented approach to problem solving. Findings from this study suggest that in order to promote algebraic thinking teachers should present problems for which algebraic means of finding a solution is both preferred and optimal. Students should be made explicitly aware of the purpose for a particular set task, such as word problems, and monitored carefully in their choice of strategy.

Acknowledgements

This research was supervised by Dr. Glenda Anthony, Head of Department for Technology, Science and Mathematics Education. I am grateful for her constant encouragement and expert guidance. Additional help, for which I am grateful, was provided by Dr Mitchell Nathan of the University of Colorado.

Approval for the surveys conducted as part of this research was granted by the Massey University Human Ethics Committee. My sincere appreciation goes to the students and teachers who generously gave up time to participate in this study.

Support for this research was provided through a Mathematics, Science and Technology Teachers Study Award funded by the Ministry of Education.

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