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Gaussian Process based Model Predictive Control

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Abstract

The performance of using Model Predictive Control (MPC) techniques is highly dependent on a model that is able to accurately represent the dynamical system. The data-driven modelling techniques are usually used as an alternative approach to obtain such a model when first principle techniques are not applicable. However, it is not easy to assess the quality of learnt models when using the traditional data-driven models, such as Artificial Neural Network (ANN) and Fuzzy Model (FM). This issue is addressed in this thesis by using probabilistic Gaussian Process (GP) models.

One key issue of using the GP models is accurately learning the hyperparameters. The Conjugate Gradient (CG) algorithms are conventionally used in the problem of maximizing the Log-Likelihood (LL) function to obtain these hyperparameters. In this thesis, we proposed a hybrid Particle Swarm Optimization (PSO) algorithm to cope with the problem of learning hyperparameters. In addition, we also explored using the Mean Squared Error (MSE) of outputs as the fitness function in the optimization problem. This will provide us a quality indication of intermediate solutions.

The GP based MPC approaches for unknown systems have been studied in the past decade. However, most of them are not generally formulated. In addition, the optimization solutions in existing GP based MPC algorithms are not clearly given or are computationally demanding. In this thesis, we first study the use of GP based MPC approaches in the unconstrained problems. Compared to the existing works, the proposed approach is generally formulated and the corresponding optimization problem is efficiently solved by using the analytical gradients of GP models w.r.t. outputs and control inputs. The GPMPC1 and GPMPC2 algorithms are subsequently proposed to handle the general constrained problems. In addition, through using the proposed basic and extended GP based local dynamical models, the constrained MPC problem is effectively solved in the GPMPC1 and GPMPC2 algorithms. The proposed algorithms are verified in the trajectory tracking problem of the quadrotor.

The issue of closed-loop stability in the proposed GPMPC algorithm is addressed by means of the terminal cost and constraint technique in this thesis. The stability guaranteed GPMPC algorithm is subsequently proposed for the constrained problem. By using the extended GP based local dynamical model, the corresponding MPC problem is effectively solved.

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List of Abbreviations

ANN Artificial Neural Network

BFGS Broyden-Fletcher-Goldfarb-Shanno

CG Conjugate Gradient

CGP Convolved Gaussian Process

DGP Dependent Gaussian Process

DMC Dynamic Matrix Control

DOF Degree-of-Freedom

FM Fuzzy Model

FP-SQP Feasibility-Perturbed Sequential Quadratic Programming

GA Genetic Algorithm

GMV Generalized Minimum Variance

GP Gaussian Process

GPC Generalized Predictive Control

GPDM Gaussian Process Dynamical Model

IAE Integral Absolute Error

IDC Inverse Dynamics Control

IGP Independent Gaussian Process

KKT Karush-Kahn-Tucker

LGP Local Gaussian Process

LL Log-Likelihood

LMC Linear Model of Coregionalization
LMI Linear Matrix Inequality
LQR Linear-Quadratic Regulator
LTV Linear Time-Varying
GP-LVM Gaussian Latent Variable Model
MAE Mean Absolute Error
MAP Maximizing A Posterior
MCMC Markov Chain Monte Carlo
MFAC Model-Free Adaptive Control
MIMO Multiple-Input Multiple-Output
MISO Multiple-Input Single-Output
ML Machine learning
MLE Maximum Likelihood Estimation
MPC Model Predictive Control
mp-QP Multi-Parametric Quadratic Programs
MSE Mean Squared Error
NLL Negative value of Log-Likelihood
NLTV Nonlinear Time-Varying
NMPC Nonlinear Model Predictive Control
PCA Principal Component Analysis
PFC Predictive Functional Control
PFDL Partial Form Dynamic Linearization
PSO Particle Swarm Optimization
QP Quadratic Programming
RBFN Radial Basis Function Network
SMPC Stochastic Model Predictive Control
SQP Sequential Quadratic Programming
UAV Unmanned Aerial Vehicle