A MATHEMATICAL AND COMPUTER SIMULATION MODEL OF THE RUNNING ATHLETE

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SIGNED: R. Hugh Morton — OCTOBER, 1984

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SAMPLE COMPUTER OUTPUTS IN SEPARATE BINDER

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ABSTRACT

This thesis describes the construction, computerisation and simulation of a mathematical model of the running athlete. The model is in part biomechanical and in part biochemical, in part theoretical and in part empirical.

A three variable (force, velocity and distance) Newtonian biomechanical model of Keller (1973, 1974) is examined, and extended to include a fourth variable, power developed. This model segment is feed-forward linked by equating mechanical power developed to bioenergetic power supplied. A proposed three component bioenergetic model of Margaria (1976) was examined in detail (Morton 1984), but found to be unsuitable. Thus an empirical three component model segment was developed. Bioenergetic power is supplied by three variables, oxygen uptake, and glycolytic and alactic body energy store depletions. The glycolytic process is of particular interest, since the accumulation of lactic acid in the working muscle can induce fatigue. A two compartment, working muscle and blood volume, physiological model segment originally developed by Freund and Zouloumian (1981a, b) for post exercise is examined for use during exercise. This segment takes as input the lactate produced by the glycolytic energy process and circulates it between compartments, or removes it by biochemical breakdown. Lactate concentrations in the two compartments are the modelled variables in this segment. Finally a negative feedback link in the whole model is provided by a fatigue equation, where the maximum muscular force exertable is constrained inversely by the lactate concentration increase in the working muscles. The athlete of course can by choice operate at a force below the constraining level, in which case the feedback link does not operate. Sooner or later however, except for the lower workloads, muscular lactate will build up to such a level as to invoke the feed-back.

Parameters of the whole model include initial body energy stores, maximal muscular strength, resistance to motion, diffusion constants for lactate circulation, bioenergetic parameters of the oxidative, glycolytic and alactic energy processes, biomechanical energy equivalents, body mass, fatigue coefficients, etc, twenty four in number. These parameters bind the relationships between the nine variables of the whole model, expressed as nine simultaneous differential equations with respect to time. Parameter values were in some cases obtained by estimation from data on exercising subjects, specially collected for that purpose. In other cases values determined experimentally by other researchers and published in the literature were utilised.

Simulation was performed using numerical integration methods provided by a computer programme from the NAG library of routines (NAG, 1983). Simulated results on all nine variables are realistic, conforming well to those observed in the laboratory on exercising subjects. There remains scope however for future refinements, in the main improving the theoretical content of the whole model and in the process extending it to include the recovery period after exercise.