DYNAMICAL SYSTEM OF PHYTOPLANKTON AND DISSOLVED NUTRIENT INTERACTION

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ABSTRACT

The first step in modelling the dynamics of phytoplankton and dissolved nutrient interaction made use of a simple model called The N-P model. This model was extracted from Busenberg *et al* [2]. Results from the analysis were obtained. Laboratory experiments were set up and from the observations the process of conversion of the dead phytoplankton occurred after a certain number of time delay. Moreover, from the data obtained the populations of these interactive components behaved in oscillatory mode. This was believed to happen due to the present of a third component called the *dead* phytoplankton. All these experimental results did not agree with the results obtained from the analysis. Thus a new model was then formulated, called the N-P-D model. A unit function with the delay-time and the dead phytoplankton (D) were included. Analysis were made and results compared.

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CHAPTER ONE
INTRODUCTION

Fishing industry in Brunei Darussalam is heavily active especially at the Brunei Bay as compared to various other places scattered at Brunei coastal areas. A majority of these fishing activities are due to private individuals themselves whom are mostly fishermen. So this becomes, partly, a growing concern to the Brunei Government in particular the Fishery Department Of Brunei in trying to control these activities. The main reason towards this consideration is to avoid over-fishing. Accompanying this problem, there is a rather awkward situation happening to Brunei coastal areas especially at the Brunei Bay every year. This situation is what we call the *red-tidal waves*. These incoming waves are very poisonous to the marine populations especially to the fish. Various studies of this phenomenon have been made. Henceforth, the fishery department of Brunei is presently working on a 'control' breeding of certain species of fishes, prawns and others. In responding to this objective the Universiti Brunei Darussalam especially the mathematics and biology departments are involved. The mathematics department in particular led by the head of the department itself, is developing certain models in studying minute marine populations i.e. planktons, of which are the main sources of foods to fishes and prawns. The development of these models are the initial steps taken.

Therefore it becomes the main aim of this writing to analyse the dynamical system of phytoplankton (P) and dissolved nutrient (N) interaction. The dynamics of this interaction is described as a system of first order nonlinear ordinary differential equation which will be written in the next chapter. As a form of guidance towards the develpoment of models, the model which will be used in this analysis is primarily extracted from the 3-component model introduced in the article Busenberg *et al.* The three interacting components studied there are phytoplankton , herbivorous zooplankton (Z) and dissolved nutrient . Assumptions are made throughout the analysis of these three components interaction. The two plankton densities are measured in terms of their nitrogen contents, N, as assumed that the nutrient is responsible for limiting the phytoplankton reproduction. The total nitrogen level is maintained to be constant throughout. Thus for the 3-component model, $P + Z + N = N_T =$ constant.

The first analysis of phytoplankton and dissolved nutrient interaction which is described in a 2-component model (N-P model) begins in chapter two. This consists of finding the steady-states and studying their stability. Various questions that need to be answered such as how these steady-states solutions behave under the change of a distinguished parameter ; in which region (the feasible region), does the biologically feasible solution stay in ; will also be included in this chapter.

Chapter three accounts a slight deviation of the N-P model studied in chapter two. Briefly, in the N-P model, dead phytoplankton decays automatically into nutrient . In other words, the source of N is supplied directly from dead P. It is thought that this conversion of dead P into N will take place after a certain number of time. So a delay -time factor is introduced into the N-P model. A third component, dead phytoplankton (D), will also enter the model. This new model (N-P-D model) provides a different scenario in terms of its dynamics when compared to the N-P model. The analysis will begin at finding the new steady-states, studying their stability, defining the feasible region, etc.

As far as the analysis concerns parameters entering both models, N-P and N-P-D, are defined. The definitions together with the dimensions of these parameters are given as follows

a maximal uptake rate for the phytoplankton $\{day^{-1}\}$

c phytoplankton death rate $\{day^{-1}\}$

k Michealis-Menton half saturation constant {μg atom NO_3 / dm^3}

Other constants also entering this analysis are defined as follows

N_T total concentration of nitrates { μg atom NO_3 / dm^3 }

τ delay-time dead-phytoplankton conversion into dissolved nutrient $\{day(s)\}$

The summary of the whole analysis is included in the final chapter of this writing. Several biological interpretations of the results obtained are also discussed. Numerical simulated results of the N-P-D model together with the program are given as appendices.