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**Reforming Teaching, Learning and Assessment of Mathematics:
Middle School Students' and Teachers' Perspectives**

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the requirements for the degree of

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Abstract

All students can learn mathematics to a proficient level when the mathematics curriculum is designed with equity and discourse in mind. It is through teacher actions that every student can engage in collaborative mathematical discourse using a range of mathematical practices as an essential component for sense-making. Denying groups of students, access to more demanding and rigorous work at the primary school level as a result of ability grouping and a heavy focus on procedural teaching is contributing to uneven and inequitable outcomes for many students. In order to engage primary aged students in mathematics that focuses on developing skills in argumentation needed at the secondary level, value needs to be given to the quality of mathematical thinking and opportunities for talking as a means for sense making. In recent times, efforts to make significant changes to the pedagogy of mathematics both in New Zealand and internationally have focused on reforming instructional practice to include the explicit teaching and learning of mathematical practices through a socio-cultural context. Within a collaborative mathematics community, developing skills is experienced through an inquiry approach. By explicitly teaching students how to engage with respectful exchanges of ideas, the normal way of how students operate in mathematics class is disrupted. As a result, all students are afforded opportunities to access rich learning tasks.

Central to developing shifts in mathematics classrooms is the assumption that mathematics reform undertaken without also reforming mathematics assessment appears unlikely to succeed. International researchers highlight studies showing that assessment is the engine of curriculum reform, or the principal impediment.

This study offers a glimpse inside a Years Five and Six New Zealand classroom through the perceptions and actions of both students and teachers as they embarked on a journey to learn and teach mathematics equitably. Under consideration is the inclusion of mathematical practices, teacher pedagogical actions and the key role assessment plays in implementing change.

A classroom based qualitative research approach involving a teaching experiment approach was used to support a collaborative teacher-researcher partnership. Throughout terms three and four 2019, data collected from one-to-one student interviews, classroom observations and teacher focus group discussions was coded, analysed, and triangulated. Three salient themes emerged from the data: teacher actions that promoted mathematical and discursive practices, changes in student views on the mathematical learning process and the role of assessment in a reform classroom. Students identified teachers' actions such as facilitating group norms to support group work a key factor in expanding their thinking about what success looks like in mathematics. Initial attempts at including assessment of mathematical practices as well as content helped students and teachers to reflect on mathematical practices as well as content as part of the learning process in mathematics. In addition, assessment was found to alter attempts to reform mathematical instructional focus.

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Chapter One Introduction

1.1 Introduction

Both internationally and within New Zealand, educational systems produce starkly uneven and inequitable outcomes. Nowhere is this more evident and with such negative consequences for both individuals and society than in mathematics (Hattie, 2017; Stinson, 2004). High numbers of students are dropping out of the discipline as mathematics becomes increasingly difficult for many students advancing through the school system (Jorgensen, Gates, & Roper, 2014). Consequently, there is a mathematics revolution happening transforming the way that mathematics is taught. Challenging traditional ways of teaching mathematics is a social turn in mathematics research which sheds light on the intersection of individual conceptual understanding and the development of collective, community practices (Enyedy, 2003). As a result, teachers are changing their instructional focus from an agenda of deficiency in mathematics instruction to one of proficiency (Hunter, Hunter, Jorgensen & Choy, 2016) and equity (Anthony & Hunter, 2017; Askew, 2012; Boaler, 2002; Hodge, 2008; Hunter & Hunter, 2018; Jorgensen et al., 2014; Rand, 2003; Selling, 2016; Shah & Crespo, 2018; Stinson, 2004).

To be proficient in mathematics is to engage with discourse practices such as speech acts (Enyedy, 2003) which are embedded within mathematical practices (Moschkovich, 2003; Spelling, 2016). Being able to justify mathematical claims, use symbolic notation, make mathematical generalizations, describe patterns, use abstract thinking, and engage in argumentation are all examples of mathematical practices. Mathematical practices are the foundation for mathematical thinking (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and are a lever for increasing access to more complex mathematical thinking for students advancing through the school system. However, unequal access for students to participate in complex mathematical discussions that develop mathematical thinking is currently the norm in most New Zealand primary school classrooms.

Many students, including an over representation of students from marginalised groups have been categorised as 'low achieving'. Students who have been sorted into low achieving groups have fewer opportunities throughout their journey through the school

system to participate in mathematical conversations that develop skills in argumentation (Rand, 2003). The ‘bottom group’ in mathematics have more likely experienced a one-dimensional mathematics class (Boaler, 2016) with deficit teaching that encourages procedural mathematics instruction focusing on drilling skills (Selling, 2016), learning facts and parroting some mathematical phrase or faithfully following procedures (Enyedy, 2003).

Excluding students from gaining proficiency in complex mathematical thinking and discourse skills at the primary school level leaves groups of students lacking foundations in mathematical thinking needed to participate in the National Certificate of Education Achievement (NCEA) mathematics thus gaining basic secondary mathematics qualifications. This tragedy of unequal access to education resulting in lost potential and opportunities for young New Zealanders has profound implications as achievement in mathematics is seen as a gateway for economic access, full citizenship, and higher education (Stinson, 2004).

1.2 Background and Rationale for the Study

As a result of the introduction of the NCEA in 2002 as the main secondary school qualification, Jones (2011) identified significant changes that were made to the New Zealand Mathematics and Statistics Curriculum (NZMSC) at the secondary level. The mathematics standards writers used a framework based on the Structure of Observable Learning Outcomes (SOLO) thinking taxonomy (Biggs & Collis, 1982) to express the qualitative differences in the achievement standards. The standards used for achievement, merit and excellence awarded in mathematics and statistics are framed using the multi-structural, relational and extended abstract levels of cognitive complexity (Hook, Gravett, Howard & John, 2014). Prior to this development achievement in the learning process was quantified, as it still is at the primary level.

Despite these changes made at NCEA where students are awarded achievement, merit or excellence based on the complexity of thinking and mathematical practices, assessments in the learning process at primary level currently are not sensitive to student cognitive development. Highly influenced by the New Zealand Numeracy Development Project (NZNDP), assessment in primary schools is dominated by a focus on content (in the form of facts) and skills (associated with computational techniques). The issue according to

Serow, Callingham and Tout (2016) is whether this type of assessment provides a basis for making judgements about students' ability to deal with mathematical problems of twenty-first century daily life? Asking questions and being able to explain and justify mathematically is currently transforming pedagogical practices. As a result, the aforementioned authors ask the question, are we assessing what we really need to assess? This question has major implications to the teaching and learning of mathematics in the classroom as assessment in the mathematics classroom has the potential to alter the experiences of students in our schools at every level (Serow et al.) because testing drives instructional practice (Schoenfeld, 2015). If we are serious about challenging traditional ways of teaching mathematics at the primary level in New Zealand it is imperative to start conversations about also challenging current assessment practices to incorporate mathematical practices. As Barnes, Clarke and Stephens (2000) claim, attempts at curriculum reform are likely to be futile unless it is accompanied by matching assessment reform.

The Common Core State Standards for Mathematics (CCSSM) in the United States of America (USA) have standards for both mathematical content and mathematical practices. In contrast, the New Zealand equivalent to the CCSSM, the Achievement Objectives (AOs) only have AOs for mathematical content, not mathematical practices. Hence, there are no learning objectives to support primary school teachers in New Zealand to assess the use of mathematical practices outside of NCEA levels. The assessment of achievement standards and CCSSM which determine the complexity of thinking, drives requirements for teachers of NCEA in New Zealand and CCSSM in the USA to focus on mathematical practices for instruction and assessment. The omission of explicit references to mathematical practices in the NZMSC calls attention to many aspects of mathematical proficiency that are often left implicit in instruction (Rand, 2003) and assessed only by a few. As a result, prior to mathematics instruction and assessment at the NCEA level in New Zealand, the teaching focus has been on mathematical content and the method of learning is the rote learning of procedures.

Given that many teachers themselves did not experience how to use mathematical practices in their own schooling, they often have little experience of the process of teaching and learning mathematical practices (Selling, 2016). This is a challenging goal for teachers and as a result, problematic for students as they progress through the

schooling system. Without exposure to learning mathematical practices, students are missing opportunities to develop conceptual understanding in mathematics (Selling). Teachers' inability to develop mathematical programmes and assessments that include mathematical practices alongside content at the primary level leaves students ill prepared for mathematics at the secondary level. Extra cognitive demands occur for students at NCEA level as mathematics assessments place student reasoning and argumentation at the heart of mathematics activities. This highlights a significant mismatch between skills needed to be able to ask questions and argue mathematically at the NCEA level and objectives and assessments of conventional mathematics being taught at primary school. Therefore, reform of assessment in mathematics prior to NCEA is a necessary condition for any reform in mathematics curriculum (Pegg, 2003).

The challenge for teachers when considering the teaching, learning and assessment of mathematical practices alongside mathematical content is to make the implicit process of teaching mathematical practices more explicit without making the learning and teaching of mathematical practices prescriptive (Selling, 2016). To do otherwise could parallel what happened in the latter stages of the NZNDP (Hunter et al., 2016); where prescriptive teaching and assessment resulted in missed opportunities for the development of mathematical thinking and left students without voice or autonomy.

1.3 **Research Objectives**

The purpose of this study was to document the journey of both teachers and students from two Year Five and Six primary school classrooms where the teachers changed the instructional focus from one of deficiency to one of proficiency and equity. The perspectives of both students and teachers are recorded along with observations of teacher actions that supported student's engagement in a multidimensional mathematics class. Boaler (2016) describes multidimensional instruction as being a mathematics class where teachers think of *all* the ways for students to work mathematically. For example, the teachers in this study explored using mathematical practices alongside mathematical content during mathematics instruction and assessment.

The research questions are:

- What are the changes in students' thinking regarding the learning process in mathematics as a result of the transformation of the classroom culture to one of inquiry?
- What pedagogical strategies do the teachers employ to support students to communicate and participate in a multidimensional mathematics classroom?
- How does assessment affect the teaching and learning process in a multidimensional mathematics class?

1.4 Thesis Overview

Chapter Two explores how and why mathematical practices should be included alongside mathematics content in classroom instruction. The importance of developing a shared understanding to define and describe mathematical practices is examined through a socio-cultural lens. Current support and guidelines for teachers to grow mathematical practices is discussed. Finally, a number of questions are generated when challenges and incompatibilities of ideologies and philosophies occur with the complex process of measuring and reporting the quality of a student's mathematical understanding. Central to the debate is the role of assessment in a reform classroom. As a result, an empirically-based perspective that is sensitive to student cognitive development that underpins assessment initiatives (Pegg, 2003) warrants attention. This chapter concludes by addressing this issue by considering one such approach, Solo Taxonomy.

Chapter Three describes the research design used in this research. This includes the research setting, study sample, data collection and analysis, research schedule and ethical considerations. Chapter Four integrates the findings and discussion presenting key themes that emerged from the data. Shifts in students' thinking in regard to their mathematical learning process are identified and discussed. Observations from group interactions are analysed demonstrating the dynamics within groups that allow equal access to tasks. In addition, the effect assessment has on classroom instruction is highlighted. Finally, Chapter Five completes this thesis. The conclusions, implications and recommendations for future research are presented.

Chapter Two: Literature Review

2.1 Introduction

The previous chapter provided the context and rationale for the study. New Zealand students are expected to show proficiency in engaging with mathematical practices at the National Certificate of Education Achievement (NCEA) level but lack opportunities to engage mindfully with reasoning and argumentation at the primary level. This mismatch may be one of the reasons why many students drop out of the discipline as mathematics becomes increasingly difficult. Presently, instructional practices in many primary schools in New Zealand reflect a one-dimensional teaching approach to mathematics education. This one-dimensional approach to instructional practice based on content and skills in computational techniques narrows pathways for students to achieve success in mathematics. Furthermore, as assessment often drives instructional practice, it is essential to also reform assessment to align with an agenda of pedagogy focusing on proficiency and equity. Achievement at primary school level is presently quantified by measuring content and computational techniques. By including measurement of the qualitative differences of students' mathematical thinking and the development of conceptual development broadens the definition of the mathematical learning process reflecting the multidimensionality of mathematics.

The purpose of this review is to synthesise current literature regarding the teaching, learning and assessment of mathematics when mathematical practices are included alongside mathematical content for instruction. Section 2.2 examines the importance of developing a shared understanding to define and describe mathematical practices. Next, this section explores why mathematical practices should be included alongside mathematical content as part of mathematics instruction. Section 2.3 discusses the socio-cultural environment where mathematical practices originate and develop. In addition, the concepts of socio-cultural and mathematical norms are defined. Section 2.4 explores the teaching, learning and assessment of mathematical practices. The challenges that emerge when consideration is given to the assessment of mathematical practices is also examined.

2.2 Definitions of Mathematical Practices

There are many mathematical practices and these are described differently by different individuals. For example, the Common Core State Standards for Mathematics (CCSSM) describe the mathematical practice of justification as creating viable arguments and critiquing the reasoning of others (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010); the National Council of Teachers of Mathematics (2000) describes justification as part of the process standard of reasoning and proof. Haylock (2010) calls what is commonly called mathematical practices, key processes. As mathematical practices evolve and are transformed within the communities where they are developed (Hunter, 2007) it is important that the definitions of mathematical practices are based on a shared understanding with students as a result of classroom activities and discussions. To do otherwise leads to confusion (Cioe, King, Ostien, Pansa & Staples, 2015). For example, appropriating a mathematical practice through interaction depends on developing a shared understanding of the meaning of the language and symbols used in that practice and shared goals for engaging in particular practices (Selling, 2016).

2.2.1 Descriptions of Mathematical Practices

In this section clarification of mathematical practices definitions are discussed.

2.2.1.1 Explanations

Mathematical explanations are a critical mathematical practice. They are the basis of a mathematical argument and are amongst one of the practices that are the foundations for mathematical thinking. Being able to construct an explanation based on conceptual understanding rather than procedural understanding is at the core of reform mathematics. Explanations are statements which start as well reasoned conjectures (Whitenack & Yackel, 2002). The criteria includes the need for the explainer to make explanations explicit, relevant and experientially real for the audience (Hunter, 2007). The explainer uses representations to explain their reasoning such as practical materials, drawing pictures, creating diagrams, describing in words, and using symbols to help explain solutions (Askew, 2016). According to Hook et al. (2014) this level of understanding is quantitative as several aspects of the task are known and provided in the form of an explanation. However, there is no justification. Being able to provide explanatory

reasoning is an important precursor for students to engage in explanatory justification and argumentation (Hunter, 2007).

2.2.1.2 Justification – Moving Students to the Why

Having students share and defend their explanations and reasoning to show why something is true is the process of justification. Explanations become explanatory justifications when explainers provide further evidence to justify one’s ideas to themselves or someone else (Hunter, 2007). In order for a student to give a mathematical justification the student gives an explanation to address a classmate’s challenge to an idea (Whitenack & Yackel, 2002). Encouraging students to share their reasoning and explain how they know something is true is the process of justification (Cioe et al., 2015). To be able to explain causes for the explanation, according to Hook et al. (2014) demonstrates a qualitative shift in the complexity of thinking. At this relational level of understanding the student is able to make connections between aspects of the task, thus contributing to a deeper level of understanding. To illustrate the subtle differences between explanations and justifications the following example of a hexagon question (see Figure 2.1) and student responses to the question (see Table 2.1) from Cioe and colleagues (2015) is shown.

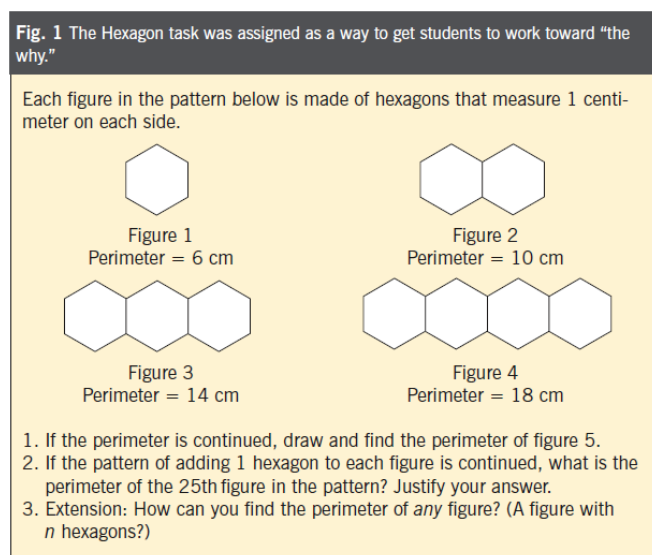


Figure 2.1 Hexagon Task From Cioe et al. (2015, p. 486)

Table 2.1 Student responses to Hexagon Task. The red show revisions of explanations to include justifications

Explanation , Not Justification		Justifications (Partial or Full)
Students articulate methods for finding the perimeter without explaining why the method is appropriate or correct.	Students give evidence that a relationship holds, but do not explain why the relationship must hold.	Students offer a mathematical reason for why their method is correct.
A. To find the perimeter, take away 2 from the figure number and multiply by 4. Then you add in 10 for the 2 you took away. So for figure 25, we do 23×4 plus 10, which is 102.	D. We saw it's always 4 times the figure number plus 2 because every time you take a figure number and multiply it by 4, and add 2, you get the perimeter. We tested it on all the values we had. So the perimeter of figure 25 is $4(25) + 2 = 102$.	F. (A, revised) To find the perimeter, take away 2 from the figure number and multiply by 4. You do this because each of the "interior" hexagons gives you 4 toward the perimeter. Then you add in 10 for the 2 you took away, because each of those 2 end hexagons give 5 each to the perimeter. So for figure 25, we do 23×4 plus 10, which is 102.
B. In our table, we saw that it goes up by 4 every time. So to get the 25th figure, you find 20 times 4, which is 80, and add it to 22 cm to get 102 cm,	E. We got $4n + 2$ because we noticed it went up by 4 each time, but $4n$ didn't work. It was always 2 too low. For example, figure 3 was 14, but $4(3) = 12 \neq 14$. So we	G. (D, revised) We saw it's always 4 times the figure number plus 2 because every time you take a figure number and multiply it by 4, and add 2, you get the perimeter. We know multiplying by 4 every time is right because for

<p>which is the perimeter of figure 5.</p>	<p>added 2 and got $4n + 2$, which always worked. So $25(4) + 2 = 102$</p>	<p>every hexagon, you have 4 sides—the 2 tops and the 2 bottoms—that are part of the perimeter. We know you have to add 2 in the end because there are 2 sides—the left end and the right end—that are not counted by the tops and bottoms and are part of the perimeter. So the perimeter of figure 25 is $4(25) + 2 = 102$.</p>
<p>C. Figure 5 has a perimeter of 22 cm. To find the perimeter of figure 25, you multiply figure 5 by 5. So $22 \times 5 = 110$ cm, which is the perimeter of figure 25</p>		
<p>From Cioe et al. (2015, p. 487)</p>		

As can be seen by this example the explanation becomes a justification when the word *because* is used to start the sentence. In order for the student to shift between the two practices (Hunter, 2007) to move from ‘the how’ to ‘the why’ (Cioe et al.) requires the teacher to facilitate discussions in which the student talks about their mathematical ideas (Whitenack & Yackel, 2002). In this way mathematical practices originate through social interaction. A common scenario in classrooms is for students to give an explanation rather than a justification to give evidence in solving a problem. Teaching to support students being able to justify is cognitively demanding. As Cioe et al. (2015) found in their study of seventh grade students in the states, they would press students for ‘the why’ when explaining their thinking but students would only give them ‘the how’. In this study a team of teachers and researchers worked together for two years to better understand what it takes to support middle school students’ engagement in justification in mathematics classrooms. The study provided justification tasks built on and developed from students’ thinking, even when it did not match how the teachers or researchers were thinking, or what they thought was the ‘best’ approach to solve tasks. Cioe et al. argue that this was the most effective way to facilitate students to develop their own ideas towards a more complete justification. The findings showed that to develop skills in justification required careful listening by the teacher and deliberate efforts to develop students’ ideas. As justification is about reasoning, it cannot be the teacher’s reasoning; it has to be the

students' reasoning, this is difficult for teachers. The commitment to building on students' thinking involves managing mistakes, finding what is productive in what students do and figuring out how much to support students, but not override, their thinking (Cioe et al.).

2.2.1.3 Generalising

Mathematical practices are interrelated social practices. In order for students to be able to generalise they need to first do some surface thinking to explain their ideas and support their reasoning using representations. Next they need to defend their explanation with deeper thinking to justify why they believe something to be true or not, thus developing their conceptual understanding further. At the extended abstract level, the new understanding at the relational level is rethought at another conceptual level, looked at in a new way, and used as a basis for prediction, generalisation, reflection, or creation of new understanding (Hook & Mills, as cited in Hook et al., 2014, p. 5). Inherent in generalisations are the processes, procedures, patterns and relationships in mathematics (Hunter, 2007). To make a generalisation is to make a statement observing that something is always true, or always the case (Haylock, 2010).

“Generalising moves students' awareness from the direct object of an activity to an indirect object. Through cycles of reflective and evaluative reasoning existing concepts are further extended, developed, tested, evaluated and justified. Teacher questions like “ ‘why’; ‘does it work for all cases’; ‘can you know for sure’ nudge students to search for patterns and to consider underlying mathematical generalised mathematical structure”

(Hunter, 2007, p. 70).

In order to understand how students create viable arguments, critique the reasoning of others and conceptualise generalisations needs an examination of the ways in which individual and social processes are mutually constitutive. By promoting whole class mathematical discussions within the community students are both actively involved in their own learning process and in the shaping of the practices of the community. Student activities and understandings are shaped by their participation in social configurations. Mathematical practices are developed for the communicative purpose of settling disputes (Enyedy, 2003). In order to settle mathematical disputes students need opportunities in joint discourse practices to argue, reason, present conjectures and represent their thinking

to convey their argument. These mathematical practices explored in the next section are levers for students to engage in complex mathematical thinking.

2.2.1.4 Reasoning, Representation, Argumentation and Conjectures

Reasoning helps with generalising. Reasoning is stated as a proficiency to be developed in students and is defined as being the capacity for logical thought and actions, such as explaining, analysing, proving, evaluating, inferring, justifying and generalising (Loong et al., 2018). Mathematicians prove their ideas by reasoning and making logical arguments and connections between ideas (Boaler, 2017). A core activity here is conjecture, a conjecture is a yet-to-be proved statement rather than a definitive statement of the solution or answer (Askew, 2016). Through interactive discourse conjectures and arguments are trialed, tested and challenged. By reasoning, students are convincing others who may be skeptical (Askew). Representations such as words, symbols, and diagrams can be used to support conjectures and justifications (Lepak, 2014). The next section explores why including mathematical practices alongside mathematical content offers a multidimensional and equitable approach to teaching mathematics.

2.2.2 Including Mathematical Practices Alongside Content

Increasingly mathematical practices have become positioned as a key aspect of learning and doing mathematics. A large body of researchers (e.g., Askew, 2016; Boaler, 2016; Hunter & Hunter, 2018; Moschkovich, 2013; Rand, 2003; Schoenfeld, 2011; Selling 2016) highlight the fact that mathematical practices are an essential component for the learning of mathematics with understanding. As a result, researchers (e.g., Hunter, 2007; Moschkovich) are concerned with discussing how mathematical practices appear in classrooms. Under investigation is the support teachers need to transform their classroom practice to develop a multidimensional mathematics class where teachers think of all ways to be mathematical (Boaler, 2016). Mathematicians perform calculations, but they also ask questions, propose ideas, connect different methods, use a variety of representations and develop skills in reasoning (Boaler). In short, they develop a classroom environment that is conducive for the teaching of mathematical practices

The Rand study (2003) identified three key reasons for focusing on mathematical practices. Firstly, they argue the need to confront the damaging cultural belief that only some people can learn mathematics. This brings up questions of equity, and the negative

effects on children who are taught in low-ability classes (Slavin, 1990; Oakes, 1995). As Stinson (2004) explains, it is no longer acceptable that some groups are excluded from the full range of opportunities which those who have access to advanced mathematics have. All students have the ability to learn complex mathematics (Boaler 2016; Hunter & Hunter, 2018) if they have equitable access to mathematical discourse (Hunter & Anthony, 2011). The inclusion of mathematical practices could be a lever for increasing access to more complex mathematical thinking for marginalised groups who are more likely to have experienced procedural mathematics instruction focusing on remembering facts and learning computational techniques (Selling, 2016). Students who do well with mathematics have developed a set of well-coordinated mathematical practices and engage in them flexibly and skillfully. Whether students acquire mathematical practices is part of what differentiates those who are successful with mathematics from those who are not (Rand). As a result of ability labelling in primary schools, assumptions about 'low achieving' learners and their fixed ability potential to be able to acquire mathematical practices leaves them with reduced mathematical experiences (Marks, 2014).

Secondly, Stinson (2004) argues that everyone should have access to advanced mathematics education as it is the key or gate to economic access, full citizenship, and higher education. As the Rand group (2003) explains, mathematics is increasingly needed for interpretation and analysis of information and is critical to functional citizenry. Such knowledge and use requires proficiency in mathematical practices. Students who fail to gain basic secondary mathematics qualifications, can have, as a result, more limited work and life choices (Darragh, 2013).

The last reason the Rand group outlines is the need to provide support and resources for teachers who are working towards more-complex educational goals. Without support and resources ambitious agendas for improvements in mathematics are unlikely to succeed, resulting in teachers who are discouraged and lack belief in their students.

What it would take for all students and teachers to achieve such ambitious goals has not been adequately examined. Consequently, despite greater expectations and important new goals, student performance may not improve..... when a teacher who has never before asked her students to explain their thinking suddenly asks those to justify their solutions, she is likely to be greeted with silence.

(Rand, 2003, p. 35).

In order for students to develop these social practices that are essential for the development of mathematical discourse, consideration needs to be given to the social and cultural nature of student learning (Hunter, 2007). The following section explores how teachers can develop communities of mathematical inquiry by creating an environment where everyone is supported and accountable for participating in mathematical discourse inherent in mathematical practices.

2.3 Socio-Cultural Context

In order to use mathematical practices to address a classmate's challenge or explain one's thinking involves being able to participate in mathematical discussions. Therefore, the origin of learning mathematical practices is through social interactions. Based on social theory developed under the influence of Vygotsky (Daniels, 2001) an extensive body of research (e.g., Boaler, 2002; Enyedy, 2003; Hunter, 2007; Jorgensen, et al., 2014; Rogoff, 1990) draws on the conceptualization of mathematical practices from a sociocultural perspective. From a Vygotskian perspective, all mathematics are a socio-cultural phenomena in the sense that they are higher order intellectual activities that originate through social interactions (Moschkovich, 2013). Moschkovich further states that once appropriated through socio-cultural interactions the practice can be accomplished alone. If we assume that mathematical practices are not socio-cultural in origin and are individually appropriated then we will continue to see some learners as 'deficient' because they haven't yet developed proficiency in the practices and see others as 'proficient' as they have supposedly developed the practices all on their own. Developing a deep understanding of mathematical concepts is linked to participation in these social discourses and as a result practices of mathematics (Enyedy).

Teachers have a key role to play in the construction and use of mathematical practices in the classroom (Hunter, 2007), as teachers facilitate the mathematical discourse within social interactions (Boaler, 2002; Hunter & Hunter, 2018; Selling, 2016). For teachers to understand mathematical practices and how they are learned greatly enhances their impact to create an environment where reasoned discourse can evolve. This applies especially among low-achieving students who may have fewer opportunities to develop mathematical practices (Rand, 2003). For many diverse learners of mathematics, active listening needs deeper exploration to shift student beliefs beyond assuming that they can learn through merely listening and looking, rather than active sense-making. Hunter and

Hunter (2018) report on a large scale project involving over four hundred teachers in thirty two schools. They describe how teachers supported low achieving students both socially and culturally to contribute to mathematical discourse by developing a safe, supportive and inclusive learning environment. In order for the students to feel confident to take risks to offer explanations for their reasoning and to respond to the reasoning of others by actively listening, the teachers co-constructed classroom, social and mathematical norms. By supporting student engagement in respectful exchanges of ideas resulted in increased inquiry and argumentation discourse. Similarly, Boaler (2016) found developing group norms of respect and listening helped students work well together and contributed to access to more complex mathematics and equitable results. She found that when groups have learnt to work well together the conversations within the group rise to the level of the highest-thinking students

The social and mathematical norms refer to a set of obligations and expectations which influence and regulate interactions in the classroom (Hunter & Hunter, 2018). Students must first learn to bring their social interactions under control through the use of culturally developed sign signals (Enyedy, 2003). These social norms provide guidelines for acceptable ways to participate in and communicate mathematical reasoning. The need for accountability for one's own listening was essential in Hunter and Hunter's study as the teachers explored active listening as key to sense making with the students. The mathematical norms relate to being able to construct mathematical explanations, representations, justification and generalisations (Hunter & Hunter). Both social and mathematical norms are important to the way in which students consider and use mathematical practices (Hunter, 2008, 2014).

Mathematical practices are first constructed interpersonally through mathematical dialogue premised within inquiry and argumentation (Hunter & Hunter, 2018) and then appropriated to become part of the repertoire of practices that an individual can use (Enyedy, 2003; Moschkovich, 2013). For example, developing the skills to use representations to explain and justify mathematical thinking to others (Askew, 2016). Conceptual understanding cannot be simply explained, it needs to come about through joint activity (Askew; Boaler, 2016; Enyedy; Henningsen & Stein, 1997; Mueller, Yankelewitz & Maher, 2011). Askew suggests through joint activity artifacts start their life in the class as a *model of* representations, such as number lines to explain thinking to

others. With support and practice, over time children begin to use the *model for* themselves, transforming the number line from a *model of* what the children did to a *model for* students to use themselves to explain, justify and generalise their thinking within a learning community.

By establishing a mathematical inquiry community within a classroom students are provided with access to the social conditions in which to practise discourse in mathematical practices (Moschkovich, 2013). A shift from a classroom being a collection of individuals to seeing mathematics classrooms as learning communities (Hodge, 2008) requires a change in how we perceive mathematics classrooms. In the next section, challenges with changing the current practice of teaching, learning and the assessment of mathematics to reflect a more multidimensional approach is examined.

2.4 Teaching, Learning and Assessing Mathematical Practices

Reform researchers such as (e.g., Boaler, 2016; Hunter, 2007, 2014; Selling, 2016; Schoenfeld, 2015) offer enlightenment for frustrations that are beginning to arise from New Zealand primary school teachers who believe they can teach mathematics better and more equitably than they currently are. More educators are beginning to see the boundaries imposed on mathematics for both teachers and students as an unintended consequence of the Numeracy Project. A multidimensional approach providing opportunities to achieve success from a broader spectrum that accommodates more ways to think mathematically (Boaler, 2016) is being explored in depth by the aforementioned researchers. As a consequence, support and guidance in how to implement a multidimensional teaching approach for teaching mathematics that changes the focus of pedagogy from one of deficiency in mathematics to one of proficiency is beginning to emerge for teachers in New Zealand.

2.4.1 Teaching and Learning Mathematical Practices

In order for a community of learners to develop skills in mathematical practices it is essential that all members participate and communicate mathematically within the community. The inclusion of explicit teaching of mathematical practices as part of mathematical pedagogy demands a significant shift for teachers from typical classroom practice (Jacobs et al., 2006; Kitke, 2015 as cited in Selling, 2016, p. 506) to approach teaching in a way that develops more advanced mathematical understandings. Many

teachers can be pedagogically challenged when required to enact classroom practices where the expectation is that student explanations extend to inquiry, justification and generalisation of mathematical thinking (Cobb & Jackson, 2011; Hunter & Hunter, 2018). Space needs to be opened up for student-promoted dialogue to engage with the discourse inherent in mathematical practices. Up until recently, there was little guidance for teachers to explicitly teach mathematical practices in terms of curriculum documents and resources.

Forty-five states in the United States now base their mathematics curricula on the Common Core State Standards for Mathematics (CCSSM). A hallmark of the CCSSM is the inclusion of the Common Core Standards of Mathematical Practices (CCSMP) alongside mathematics content standards to develop mathematical understanding. This recognises that mathematical practices are the foundation for mathematical thinking and that they are assessable (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). As a result, teachers in the US are now accountable for promoting and supporting students in attending to mathematical practices and developers of textbooks and tests are currently striving to support and assess mathematical practices (Otten, Keazer, & Karaman, 2019). Included in the theoretical introductory chapters in the CCSSM are explicit statements highlighting the importance of connecting mathematical content and mathematical practices. In contrast, the New Zealand Mathematics Curriculum Document (NZCD) (Ministry of Education, 2007) states that “students will be engaged in thinking mathematically and statistically” (np). However, even though there are a few references to mathematical practices, unlike the CCSSM, there are no definitions in the NZCD or descriptions of any of the mathematical practices. Furthermore, there are no explicit connections stated regarding the vital connections between content and practices.

Unlike the CCSSM that have standards for both mathematical content and standards for mathematical practices the New Zealand equivalent of the Common Core standards, the Achievement Objectives (AOs) only include mathematical content, not mathematical practices. Hence there are no learning objectives to support teachers. The omission of explicit references to mathematical practices in the NZCD may suggest why the instructional focus in most primary school classrooms has been on content and procedural learning. As a result of current research in New Zealand (e.g., Hunter & Anthony, 2011;

Hunter, 2007, 2014; Hunter & Hunter 2018) support for teachers to focus on proficiency rather than deficiency is emerging. Challenging current practice in the primary sector is the idea that conceptual understanding and procedural fluency, the main focus of instruction, are not enough. Mathematical proficiency also includes developing a positive disposition towards mathematics (Schoenfeld, 2015) including adaptive reasoning and strategic competence.

One tool that was designed to scaffold teachers to support students to connect mathematical practices to mathematical content, thus bringing the practices of mathematicians into the classroom is the Communication and Participation Framework (CPF) (Hunter, 2007) (see Appendix A). Hunter developed a smart tool: the CPF to provide guidance to help teachers modify their classroom into a Mathematics Inquiry Community. Within a Developing Mathematics Inquiry Community (DMIC) classroom teaching practices support students to engage in mathematical dialogue premised within argumentation and inquiry (Hunter & Hunter, 2018). The CPF describes actions that teachers can use adaptively and flexibly to support students to engage in mathematical talk, inquiry, and mathematical practices (Hunter).

The CPF supports teachers to gradually transition students to more complex participation and discourse patterns (Hunter & Hunter, 2018) which are inherent when using increasingly proficient mathematical practices. For example, Hunter & Anthony's (2011) study explored how four teachers developed classroom cultures of mathematical inquiry and argumentation by using the CPF to structure student engagement in collective reasoned discourse. The findings include: the ability of teachers to provide more opportunities for collaborative group activities; dramatic changes in the students roles from passive receivers to active learners; new ways for students to think about mathematics, including their role in mathematics and their relationship with mathematics; the cultural, social and mathematics well-being of all students resulted in positive outcomes for diverse learners. Another study (Hunter, 2007) illustrates how two teachers developed teaching practices that proved highly effective for diverse learners such as Māori and Pasifika students. Furthermore, the pedagogy that accelerated the mathematics achievement for these students has implications for all students in primary, intermediate, and lower secondary school. Through the use of CPF smart tool, the teachers were able to transform the ways in which their students interacted and participated. The effect sizes

for the gains in both classes were very large: $d = 2.39$ and $d = 2.53$ for each class. This progress represents the equivalent of several years' progress that occurred in just one school year as a result of this transformative practice. Professor Courtney Cazden of Harvard University described this intervention as *among the best you would find anywhere in the world* (Ministry of Education, 2013).

Within a DMIC classroom teachers establish learning environments that provide students with access to learning partnerships, group worthy tasks, challenging conversations, and responsive feedback (Hunter & Hunter, 2018) from interactions with other students rather than just the teacher. Success with facilitating appropriate behaviours in a DMIC classroom where the focus is on equity requires the teacher to use strategies drawn from complex instruction (CI) (Cohen & Lotan, 2014). CI is a pedagogical approach, designed to make group work equal (Cohen & Lotan) by disrupting status hierarchies inherent in classrooms (Shah & Crespo, 2018). There are three main components which support the CI strategies and which disrupt inequitable classroom interactions. First is to have multi-ability tasks where differentiation of ability is catered for within heterogeneous grouping. In mixed ability groups, group worthy tasks are open-ended allowing students from diverse backgrounds and varying levels of ability to be able to make meaningful contributions to group and classroom discussions. By using low floor, high ceiling mathematics tasks (Boaler, 2016) allows for mathematical discourse with students with varying capabilities.

The second component which supports CI strategies is to establish social and mathematical norms. If students are left to their own devices they are not likely to develop productive interactive norms (Boaler, 2016). In order for the respectful exchange of ideas to occur to develop mathematical practices within groups, students need to make changes to how they might normally operate in a group. Social norms need to be co-constructed to provide group members with guidelines for acceptable ways to participate and communicate mathematical reasoning. Mathematical norms refer to being able to construct mathematically acceptable explanations, representations, justifications and generalisations (Hunter & Hunter, 2018). Constant references and reflections in regards to the socio-mathematical norms needs to become part of the mathematics programme.

The third component is for teachers to learn to recognise and treat status problems by ensuring equal access and participation. Sociological research demonstrates that in CI, the more that students talk and work together, the more they learn (Cohen & Lotan, 2014). However, for various reasons besides ability, such as anxiety (Boaler, 2016; Hill et al., 2016), cultural dissonance (Hunter & Hunter, 2018; Shah & Crespo, 2018), lower socio-economic status (Stinson, 2004), pressure and low self-esteem in mathematics and the belief that a classroom norm of speed is more. The teacher who has no tools for the planning of group work is likely to run into trouble trying out new pedagogical tools (Cohen & Lotan, 2014) as some students will dominate, some will sit back and relax and some will not have social status with other students (Boaler, 2016) to allow their voice to be considered. Small groups tend to develop hierarchies, where some members are more influential and active than others resulting in inequalities in interaction (Cohen & Lotan, 2014). A key role for the teacher in promoting participation using CI is to raise the status of low status students by changing the perception of their status within heterogeneous groupings by disrupting control using what's called 'talk moves' (Shah & Crespo, 2018) or 'teacher moves' (Selling, 2016) to redistribute discourse patterns. Another strategy is to disrupt the culture of competitiveness by demonstrating a focus on thinking and developing mathematical practices rather than getting a problem correct. This behaviour can be reinforced in the social norms.

Selling (2016) suggests that teachers need to make mathematical practices explicit in classroom instruction. However, explicitness can become problematic if instruction turns complex disciplinary practices into prescriptions. Separating content and mathematical practices robs students of key opportunities to engage mindfully with mathematical problems of twenty-first century daily life. Selling presents a framework of eight types of teacher moves (see Table 2.2) that make mathematical practices explicit and argues that they did so without turning practices into prescription or reduced students' opportunities to engage with them.

Table 2.2 Framework of eight types of teacher moves (Selling, 2016)

Types of teacher moves
<ul style="list-style-type: none">● Naming the mathematical practice (or practices) in which students just engaged in● Highlighting aspects of student engagement in mathematical practices● Evaluating student engagement in mathematical practices● Explaining the goal or rationale for engaging in a mathematical practice● Connecting different students' engagement in mathematical practices● Framing student engagement in mathematical practices expansively● Eliciting self-assessment with respect to mathematical practices● Referring to a teaching narrative about mathematical practices
From Selling (2016, p.524)

Making the teaching of mathematical practices explicit without being prescriptive is particularly important for supporting students in transitioning from more traditional direct instruction classrooms to environments that ask them to take a more active role. Hunter (2007) and Selling's (2016) transformative teacher pedagogical strategies draw attention to the need for a change of a joint teacher and students' activity system (Engeström, 2001) promoting new forms of engaging with mathematics that are more equitable than more traditional methods of teaching. Through these approaches students have opportunities to study mathematics as a dynamic, exploratory, evolving discipline rather than as an absolute, rigid closed body of facts and rules to be memorized. However, a number of researchers (e.g., Barnes, et al., 2000; Bell & Burkhardt, 2001; Schoenfeld 2015) argue that attempts at curriculum reform are likely to be ineffectual unless accompanied by matching assessment reform. The following section explores challenges involved when attempting to align reform mathematics pedagogy with assessment.

2.4.2 Assessment in a Reform Classroom

A general consensus regarding the need to reform mathematics assessment is evident in research (e.g., Barnes et al., 2000; Boaler, 2016; Fry & Makar, 2012; Pegg, 2003; Rand, 2003; Schoenfeld 2015). Pegg (2003) highlights the debate and controversy that has surrounded assessment practices in the 21st century and argues for a genuine need for new assessment practices to compliment more traditional widely used techniques. This section

critically examines two key issues identified by researchers when alternative mathematics assessment practices attempt to align with reform instruction. Firstly, the assumption that mathematics reform undertaken without corresponding assessment reform appears unlikely to succeed (Barnes et al., 2000) as teachers teach to the test (Schoenfeld, 2015). Secondly, the assertion that the challenge for assessment is the question whether it is possible to assess student performance of such practices in ways that are reliable and valid? (Schoenfeld, 2015) without instruction becoming prescriptive (Hunter et al., 2016; Selling, 2016).

A number of studies (e.g., Barnes et al., 2000; Boaler, 2016; Fry & Maker, 2012; Hattie, 2012; Hipkins & Cameron, 2018; Pegg, 2003; Rand, 2003; Schoenfeld, 2015; Serow et al., 2016) recognise that assessments can be positive levers for change as learning, teaching, and assessment are inextricably linked. When using assessment for learning students become knowledgeable about their own learning process; what they know, what they need to know and ways to close the gap between the two (Boaler, 2016). For example, a study by Boaler (2006) found one way teachers nurtured a feeling of responsibility for students' learning was through the assessment system. Boaler (2016) refers to research by Black, Harrison, Lee, Marshall and Wiliam, (2002) and Black and Wiliam (1998a, 1998b) that found assessment for learning is so powerful that if teachers shifted their practice and used it, it would raise the mathematical achievement of a country (Boaler, 2016).

It is important to note that Boaler (2016) suggests reform assessment includes the form of diagnostic feedback of the quality of mathematical thinking to influence grades. This becomes an interesting aspect of reform mathematics as assessment practices are traditionally based on procedural performance and must satisfy community demands for accountability, which is currently the case in New Zealand and most other places around the world. Therefore assessment reform recognising all ways to be mathematical must also change beyond the classroom at school leadership level and Ministry of Education level to become the norm in New Zealand primary school classrooms.

“Many teachers believe in the breadth of mathematics and may value multidimensional mathematics in the classroom, but assess students only on whether they get correct answers to procedural questions. The best teachers I have worked with who had to give grades have used students' mathematical work rather

than test performance – recording, for example, whether they ask questions, show mathematics in different ways, reason, justify, or build on each other’s thinking. In other words, they assess the multidimensionality of math. When students are assessed on many ways of working in mathematics, many more students are successful”

(Boaler, 2016, p. 168)

Boaler’s (2006) study showed that teachers graded the work of a group of students by rating the quality of the mathematical conversations that they participated in. Similarly, Hunter and Anthony (2011) showed that students’ mathematical explanations provided the teachers with opportunities to make on-the-spot assessments of students’ current reasoning and to suggest ways the reasoning could be extended. The move from grades to diagnostic comments allows teachers to give students the gift of their knowledge and insights about how to improve conceptual development (Boaler, 2016). These insights and diagnostic comments Boaler suggests, propel students onto paths of higher achievement. Examples from Boaler’s work with the Vista School district showed that some teachers stopped grading content, instead they used a rubric with mathematical statements showing students what they could do in regards to mathematical practices such as asking questions, describing thinking processes and using representations. Likewise Lepak’s (2014) study highlights the benefits for using a rubric to empower eighth grade students to become autonomous learners. She illustrates strategies a teacher used to develop mathematical argumentation with students who were low achieving in mathematics. In particular, she wanted to communicate how to create strong arguments by using representations. She provided explicit instructions on how to justify a claim using a rubric that listed criteria for a task with different levels of complexity of mathematical thinking. By reinforcing instruction with the rubrics, Lepak provided a standard in which students could assess and evaluate their own and each other’s written arguments. Similarly, Loong et al. (2018) supported teachers to use a detailed rubric to assess students’ reasoning and to provide feedback. The rubric supported teachers in two ways. Firstly, for teachers to be aware of reasoning actions to further develop reasoning through the regular planning of tasks. Secondly, for teachers to be able to elicit a variety of reasoning actions that went beyond being able to give an explanation.

Currently in most primary schools in New Zealand mathematics assessment is aligned with procedural teaching. However there are possibilities of change starting to emerge to support some pre-service New Zealand teachers to consider a broader view of assessment in mathematics. One such example currently used at Massey University is a pre-service teacher mathematics course book that is available in the United States of America to support the teaching, learning and assessment of mathematics through the implementation of the Common Core Standards. Included as part of creating assessments for learning in this resource are ideas regarding using rubrics to support the equitable analysis of students' work (Van de Walle, Karp & Bay-Williams, 2019). Included as part of teaching through a problem solving approach is the recommendation to use a generic rubric which includes performance indicators such as: being able to explain strategies, justify answers, use logical reasoning, and the demonstration of perseverance and resilience.

In order for changes in instruction to include mathematical practices, changes in assessment as described by Boaler (2016) above needs to be included as part of instruction. Barnes et al. (2000) argue that the performances required for assessments should be precisely those performances that align with the goal of the curriculum. Biggs' (1998) sentiments are similar, suggesting that it is important to align instruction and assessment and consequently teach the intended curriculum. By asking teachers to reform their mathematics class and not reform their assessment to reflect the expectations in instruction is sending teachers contradictory messages about what the system expects of them in the teaching and assessment of mathematics (Barnes et al., 2000). Barnes et al. continue to highlight the fact that if a system's expectations are not well aligned with the means by which its outcomes are assessed, confusing messages are sent to teachers and students about what is truly valued and what is expected of them. Similarly Van de Walle et al. (2019) contend that what gets graded by teachers is what gets valued by students. Assessment needs to change to reflect a broader criterion for success. Therefore mathematical practices such as asking questions, describing thinking processes and using representations should be included in assessments.

An Australian study by Barnes et al. (2000) found that assessment that is not aligned with pedagogy can inhibit innovation in mathematics teaching. The findings in their study show that many teachers were reluctant to embrace new assessment and instructional practices unless new instructional practices were included in high-stakes assessment.

Even though Barnes et al.'s study was set in the secondary sector, it has relevance to current study for a number of reasons. Firstly, the study provides strong evidence that changes in assessment empowers and leverages change in curriculum. Secondly, assessment practices that include a broader mathematics curricula have a direct and profound impact on the teaching of mathematics not only in the higher levels of secondary school but also had a washback effect on the preceding years of schooling. Thirdly, attempts at curriculum reform are likely to be futile unless accompanied by matching assessment reform as assessment is the engine of curriculum reform, or the principal impediment. Schoenfeld's (2015) work investigating assessment being used for positive leverages for change in instructional practice in the states confirms these findings. High stakes testing drives instructional practice of procedural teaching which results in teachers teaching to the test (Schoenfeld).

One of the dangers with including mathematical practices in the assessment process is that the teaching of mathematical practices could become prescriptive rather than be allowed to develop organically. The implementation of making mathematical practices visible in this way could result in lessons being scripted by teachers similar to what happened as a result of the New Zealand Numeracy Development Project (NZNDP). The expectation for the NZNDP was for teachers to record clear learning intentions and, with the students, establish success criteria thus providing the students with a clear message about who is driving the learning (Hunter et al., 2016). In order not to make the teaching of mathematical practices prescriptive, teachers in Selling's (2016) research made mathematical practices explicit primarily after students had participated in them. Similarly Askew (2016) suggests that mathematical practices are best left implicit. He states that children do not need to be taught how to explain, justify and reason. That argumentation should be in a form of external thinking through talk, which subsequently becomes a post hoc reconstruction.

Perhaps consensus in this debate can be found by merging pedagogy and assessment in a non-prescriptive way. Mathematical statements could be used to describe student's mathematical thinking as suggested by Boaler (2016) and made explicit after students have participated in them as suggested by Selling (2016) and Askew (2016). Central to the debate is whether students and teachers will give the same value to the progress of the quality of thinking implicit in mathematical practices if success is only measured by

the progress of solving a problem through the assessment of ‘show me’ moderated tasks. The principle of learning to learn in the NZ Curriculum Document states that students should “reflect on their own learning processes and to learn how to learn” (Ministry of Education, 2007, p. 9). If reform mathematics includes mathematical practices as a learning process for mathematics, then students need to reflect on the progress of the quality of the conversations they engage in as well as content evident in moderated tasks. The danger of not including mathematical practices alongside content in a school system’s assessment expectations undermines teachers’ ability to transform the way they teach mathematics.

One such assessment approach that compliments more traditional assessment practices is SOLO Taxonomy. Pegg (2003) suggests that using SOLO helps teachers avoid teaching drills and procedures and the dissection of learning into individual targets. The levels in SOLO are broad markers along a developmental journey. While the characteristics of each level remain constant, the detail of the student response varies. Through a group worthy task accessed by all students, some students may offer an explanation, another may argue a justification and some may be able to come to conceptual understanding. SOLO levels form a coherent cycle of learning, where learners are encouraged to acquire a relational understanding in any particular learning cycle (Pegg).

SOLO is a powerful tool for aligning curriculum and assessment (Biggs & Collis, 1982). As mentioned in the introduction SOLO is currently being used in New Zealand to express the qualitative differences in achievement standards in NCEA for measuring student reasoning and argumentation skills. The power of SOLO lies in the ease with which it can differentiate surface, deep and conceptual levels of tasks and outcomes. SOLO provides a visible structure of a learning outcome enabling students and teachers a shared language to explicitly describe the cognitive complexity of learning tasks (Hook, Ipsen & McCombe, 2015). Such descriptions can be used to support students and teachers to recognise the subtle differences between mathematical practices such as explanations and justifications. Such statements entice good teachers to work collaboratively to internalise the differences, definitions, and descriptions. SOLO classifies learning outcomes in terms of their complexity, thus enabling an assessment of students’ work in terms of its quality, not on how many bits of a task they got right (Hattie, 2017). What differentiates SOLO from other attempts to evaluate quality such as Blooms Taxonomy

is the fact that SOLO can evaluate open-ended responses to questions, thus it can be used as an instructional model in an inquiry classroom (Biggs & Collis, 1982) among students working in heterogeneous groups working on the same task.

SOLO has the potential to structure the pedagogy that underpins the CPF in order to differentiate surface, deep and conceptual levels of a task and evaluate open-ended responses to questions. Within the same rubric the mathematical practices and mathematical content can make up a holistic impression of a student's progress. By using SOLO for both formative and summative assessments, teachers and students can monitor students' progress and next steps in their mathematical practices. For example, "students need to be confident to explain and represent their mathematical reasoning before they are expected to justify and generalize it" (Hunter & Hunter, 2018, p.6).

2.5 Summary

In the USA mathematical practices have recently been included alongside mathematical content for instructional practices across all levels of schooling. However, students in New Zealand are expected to show proficiency in engaging with mathematical practices at NCEA level but lack opportunities to engage mindfully with reasoning and argumentation at the primary level. In order to do so students need to first gain a shared understanding of what mathematical practices are. Next, within a socio-cultural environment the norms of how the group operates are necessary to enable the development of mathematical practices. Teachers play a key role in orchestrating mathematical discussions eliciting mathematical practices. Influencing this multidimensional way of teaching and learning mathematics is assessment.

The following chapter (Chapter 3) presents the methodological design for the study.

Chapter Three: Research Design

3.1 Introduction

This chapter outlines the research design and methodology used to investigate the journey of teaching, learning and assessment of mathematical practices with students between the ages of nine to eleven years old. Section 3.2 presents justification for the use of a teaching experiment design with a qualitative approach to data collection. Section 3.3 discusses the role of the researcher. Section 3.4 outlines the data collection methods used in this study, including interviews and classroom observations. Section 3.5 explores the research setting, the participants, and the research schedule. Section 3.6 details the data analysis used in the study. Section 3.7 describes measures taken to ensure the research findings were valid, reliable, and trustworthy. Finally section 3.8 summarises how ethical standards were upheld throughout the study.

3.2 Justification for Methodology

A qualitative case study of two classrooms was used to observe and document events as the teachers and students embarked on a journey of multidimensional teaching, learning, and assessment of mathematics. A case study is an exploration of a case over time through in-depth data collection involving multiple sources of detailed information and rich context (Merriam, 1998). It is used in this context to firstly describe changes in students' thinking in relation to their learning of mathematics as a result of the transformation of the classroom culture to one of inquiry. Secondly, classroom observations and interviews with the teachers are described to highlight the pedagogical strategies the teachers took to reform their mathematics instruction. A qualitative approach to research looks at why and how something happens (Buckley, 2015). It is described by Punch and Oancea (2014) as naturalistic, as a major feature of it is to study people, things and events in their natural setting. Using this methodology allowed observations made in the classrooms to be supported by the salient points from the interviews with both students and teachers.

The focus for the qualitative research questions were threefold. Firstly, to investigate whether students' thinking about their own learning process in mathematics changed as a result of reform teaching. Secondly, if changes did occur, what teacher actions contributed to students' rethinking their own learning process? Finally, to notice what ways

assessment influenced pedagogy. Evidence was collated from both students' and teachers' perspectives and classroom observations.

The research questions were:

- What were the changes in students' thinking regarding the learning process in mathematics as a result of the transformation of the classroom culture to one of inquiry?
- What pedagogical strategies did the teachers employ to support students to communicate and participate in a multidimensional mathematics classroom?
- How did assessment affect the teaching and learning process in a multidimensional mathematics class?

A collaborative teaching design experiment was deemed the most appropriate approach for this particular study in order for the researcher to support teachers as it is a complex process for teachers to reform their mathematical practice (Boaler, 2016; Hunter & Hunter, 2018; Selling, 2016). Using a teaching experiment approach allowed the researcher the opportunity to influence the mathematics community's pedagogical approach to mathematics teaching, learning, and assessment which Steff and Thompson (2000) describe as the greatest strength of a teaching experiment design.

By enabling teachers and researchers to collaboratively design instructional sequence, test it in classrooms, analyse the learning and the instructional sequence, and make revisions on the instructional sequence by retrospective analysis connects teaching with research and practice and theory (Özdemir, 2017). Moreover, the teaching experiment was designed for the purpose of eliminating the separation between the practice of research and the practice of teaching (Steff & Thompson, 2000).

3.3 Researcher Role

Using a teaching experiment approach meant that the researcher was involved in an intensive and continuous experience with the students and teachers as a participant observer. During participant observation the researcher interacted with the participants in their learning environment while collecting information. Participant observation is a unique method for investigating enormously rich, complex, and diverse experiences, thoughts, feelings, and activities of human beings (Jorgensen, 2015). As this study involved qualitative research, the researcher held the role of the primary instrument for

data collection and analysis. This role highlights the importance for the researcher to be open and honest about potential bias and judgements (Hammersley & Atkinson, 1983; Merriam & Tisdell, 2016). Unlike quantitative research, the research cannot be wholly objective and impartial about the study's findings as the researcher brings a certain set of experiences, values, judgements, and assumptions. Absolon and Willett (2005) contend that neutrality and objectivity do not exist and that all research is observed through human epistemological lenses. In order to develop an honest relationship with the reader and display ethical practice, Wilson (1994) suggests readers be provided with enough information to develop their own reality from research. Therefore location, where the researcher briefly explains their background in relation to the study is an essential component to help the reader understand the connection between the researcher and the study.

The researcher in this study was an experienced primary school teacher and senior leader who previously held the role of mathematics lead teacher for fifteen years in various schools. This familiarity meant that they had an in-depth understanding of mathematical outcomes and pedagogy. The researcher had previously completed Postgraduate Masters papers on current mathematics pedagogy including Developing Mathematical Inquiry Communities (DMIC) pedagogy and had implemented this pedagogy in their classroom prior to the study. The increased levels in communication and participation observed by all students when using a DMIC approach determined the focus for this study to further explore the teaching, learning and assessment of teaching mathematics for understanding.

In the initial stages of the teaching experiment the researcher was actively involved in the making of resources and supporting teachers' development of explicitly teaching mathematical practices alongside mathematical content. As the study progressed and the teachers became more familiar with the aims of the research the researcher gradually withdrew and took on more of an 'observer as participant' role, which Punch and Oancea (2014) describe as allowing for fuller recording of research 'notes'.

3.4 Data Collection

Design based research blends empirical educational research with the theory-driven design of learning environments which is an important methodology for understanding why, when and how educational innovations work in practice (The Design-Based

Research Collective, 2003). As a result, data collection techniques for qualitative research are determined by the data that is needed and by the research questions.

Data collection involved semi-structured interviews with both students and teachers and classroom observations. Data collection was a cyclic process involving the teacher participants in continuous cycles of design, enactment, analysis and redesign as an iterative process where conjectures were formulated, discussed and modified (The Design-Based Research Collective, 2003). A relationship of reflexivity was formed as data collection and analysis shaped the ongoing design of the research.

3.4.1 Interviews

The rationale for the inclusion of semi-structured interviews was to determine perceptions about teaching, learning and assessment of mathematics from both students and teachers. Firstly, all students ($n = 24$) were interviewed individually twice during the study, both before and after the intervention. The students were asked the same three main questions at both interviews. The reason for asking the same questions at both interviews was to compare changes in students' thinking as a result of the intervention. The students were asked: *What are you learning? How is it going? What do you need to do in order to improve?* The purpose for these questions was to determine the focus of mathematics learning from the students' perspective. White and Frederiksen's (as cited in Boaler, 2016, p. 150) research study concluded that students' low achievement was not a result of lack of ability but from the fact that students did not know what they should be focusing on. The most powerful learners are those who are reflective, who engage in metacognition to take control of their learning. This theme, reiterated in The New Zealand Curriculum Document (Ministry of Education, 2007) as the principle of *Learning to Learn* encourages students to reflect on their own learning processes and to learn how to learn. The principles should be the foundation for planning for teachers (Ministry of Education). The inclusion of the principle of, *learning how to learn* in this study provided opportunities to demonstrate students' thoughts on their own learning process both before and after the intervention. According to Hook et al. (2014) asking students: *what are you learning?* can demonstrate their understanding of the complexity of their thinking, in this specific study, mathematical thinking. Asking: *how well is it going?* allowed for differentiation of the quality and complexity of thinking among students and within groups. Allowing for differentiation to monitor progress within a mixed ability group fits

well with complex instruction (CI). As within heterogeneous grouping, each member may learn different aspects of mathematics from the same task. Asking: *what do you need to do in order to improve?* provided differentiation for the complexity of thinking about mathematics and also develops metacognition skills by challenging students to think of their next steps in their learning process within the group. The next steps can be both individual and collective goals, as well as both social and mathematical goals thus working well with reform mathematics and mixed ability grouping. These questions allowed for qualitative differences to differentiate three levels of understanding, surface, deep, and conceptual.

The students were asked additional questions to probe their thinking further. Firstly, they were asked an additional question at both interviews if they replied to the second question that their learning was going well: *Why do you think your learning is going well?* This question was asked to capture changes in their thoughts in their understanding of the mathematical learning process. Secondly, at the final interview only, the students were asked: *What were the reasons for the change in your thinking about the learning process in mathematics from your previous interview?* This question was asked to gain an understanding from the students' perspective as to what teacher actions caused them to broaden their view of the learning process of mathematics. It also linked the first two research questions by relating teacher actions to changes in student thinking.

A group interview was conducted with both teachers mid-way through the teaching intervention. The interview focused on the teachers' thinking about the teaching and learning of mathematical practices. Mid-way through the intervention was deemed a good time to gauge teachers' perceptions as transforming teaching practice requires a significant change in the division of labour in the classroom for both the students and the teacher. The teachers' role changes from being the mathematical authority to being a facilitator who skillfully draws and builds on student thinking with a goal to advance the understanding of all students in the learning community (Hunter et al., 2016). The data received at this time was an important contribution for the iterative process in this study consistent with a teaching experiment design shaping the focus for the remainder of the teaching experiment. The choice to use interviews as a form of data collection provided a supporting data source for triangulating and interpreting the results with the classroom observations.

3.4.2 Observations

Participant observation allows researchers to check terms that participants used in the interviews, observe events that informants discuss and to describe a holistic understanding of the phenomena under study (Kawulich, 2005). With consent from both teachers and students a sequence of five audio recorded observations from both classrooms of the interactive discourse activities that occurred in groups and whole class situations were recorded and analysed. It provided the researcher with opportunities to record instances of teacher pedagogical strategies used to elicit mathematical practices and students' reactions. The observations made in groups and whole class situations provide researchers with ways to determine who interacts with whom, to be able to grasp how many participants communicate with each other, and check for details about the interactions involved with activities (Schmuck, 1997). The research data collected during this phase included audio recordings of classroom and group discussions and strategies used by the teachers to draw out explanations, justifications and generalisations. Field notes and researcher recorded observations of how the resources were also used.

3.5 The Research Setting and Sample

Participants in this study included students from two classrooms of thirty Years Five and Six (nine to eleven years old). The school was a contributing (Years 1-6) primary school of three hundred and fifty students set in an independent urban community in the South Island of New Zealand. At the time of the study, the school had identified the need to transform current mathematics pedagogy and was starting to undergo a mathematics delivery review influenced by reform mathematics pedagogical ideologies consistent with this study.

The study took place in the third and fourth terms of the school year. Parallel case studies of two classes were conducted to afford studying a range of ways teachers with varying teaching experience and expertise might use pedagogical strategies to support access to mathematical practices. Sixty information and consent forms were sent home and twenty-four were returned signed by both students and parents. These twenty-four students became the participating students in the study. The responses from the student interviews from both classes were combined into one set of data for the findings. However, having two case studies allowed for perspectives from two teachers of varying teaching experiences.

The setting for the two case studies in this research drew on DMIC pedagogical practices. Within this environment students are in mixed ability groups, socially constructed by the teacher based on student's working relationships to promote respectful discussion as opposed to the more traditional ability groups (see chapter two, section 2.4.1). Hunter (2007) draws attention to a large body of research undertaken with primary aged children that outlines criteria for what is accepted as age appropriate well-structured mathematical practices. It is this criteria that teachers in this study used as a starting point to base classroom discussions in the development of a shared definition.

The two participating teachers were not representative of the larger population of middle school mathematics teachers. They were a purposive sample (Creswell & Clark, 2018) approached as a result of prior mathematical professional development and successful use of SOLO taxonomy as a tool for encouraging students to reflect on their own learning processes and to learn how to learn. Teacher A was the more experienced mathematics teacher of the two. She had completed postgraduate papers in mathematics and was a trained Mathematics Support Teacher who was committed to issues of equity in the mathematics classroom by teaching through Complex Instruction (CI). Her mathematical pedagogical practice reflected the belief that all students can succeed in mathematics given the right environment and support. Teacher B was a third year teacher. She was described by the principal of the school as being a highly innovative teacher who showed interest in reform mathematics. Both teachers had extensive experience of using SOLO in other curriculum areas except mathematics.

As the Ministry of Education used SOLO taxonomy as a framework to base the National Certificate of Educational Achievement (NCEA) mathematics it seemed logical to trial using the SOLO framework to align primary mathematics with NCEA in this research for assessment. Using SOLO provided an opportunity to express qualitative differences in teaching, learning and assessment at the primary level as opposed to the current quantified approach. Trialing these outcomes based assessments with markers along a developmental journey was welcomed by both teachers and the researcher and fitted well with the school's mathematics curriculum delivery plan.

3.5.2 The Research Study Schedule

The classroom teaching experiment project consisted of five phases conducted over a three month period involving twenty lessons (see Table 3.1). The results from each stage were used to inform the design.

Table 3.1: Timeline of study schedule and data collection

2019 Term 3	
Week 1 Sept	Information sheets and consent forms were provided and collected for students, parents/caregivers and teachers. The participant teachers and the researcher drew on DMIC pedagogical practices using the Communication and Participation Framework (CPF) to collaboratively plan a teaching unit.
Week 2	Resources such as talk moves resources and rubrics were developed collaboratively by the teacher participants and the researcher. Time during the weekly meeting was given to designing a rubric with mathematical statements based on the SOLO framework to provide the mathematical structure necessary for evaluating and learning how to engage with mathematical practices, content and numeracy holistically. As a result of discussions with the teachers two formats of rubrics (see Figures 3.1 & 3.2) with the same content were developed and trialed. The SOLO levels represent both functioning skills (mathematical practices), acquisition and declarative knowledge (knowledge based on content) outcomes.
Week 3	The researcher individually interviewed all students ($n=24$). The interviews were audio taped and fully transcribed. The interviews sought to investigate and compare changes in students' thinking from the start of the intervention about what they were learning, how it was going and what they needed to do in order to progress. In particular, the interview was used to probe the thinking behind any changes to students' perceptions of the mathematical learning process.
Week 4 Oct	The teaching unit commenced. Group meeting with teachers occurred.

Week 5&6	Observational data collection occurred in both classrooms.
Week 6	<p>Group interviews with teachers half way through the study sought to investigate the teachers' reflections and issues that occurred for them in line with what the research about these issues says. As a result, subsequent iterations of planning influenced the rest of the study. In particular an awareness of prescriptive teaching influenced subsequent planning.</p> <p>Through reflections during the iterative stages of the teaching experiment the teacher participants and researcher became more mindful of the issues discussed in the previous chapter. Firstly of separating the mathematical practices from content and numeracy and secondly prescriptive teaching of mathematical practices. Therefore two versions of the rubric were developed and trialed in each classroom (see Figures 3.1 and 3.2). Teachers modified how the rubrics were used.</p>
2019 Term 4	
Weeks 7-11	Observational data collection in both classrooms occurred.
Oct – Nov	Weekly group meetings took place with both teachers.
Week 11	Sequence of 20 lessons completed
Week 12	<p>All students in the study ($n = 24$) were individually interviewed again using the same questions as the initial interviews at the start of the study. In addition, the students were invited to reread their initial interview responses and reflect on teacher actions that were instrumental to changing their thinking about their mathematical learning process.</p>

Two rubrics were developed by the two teachers and the researcher during the planning phase. The rubrics were designed with the understanding that they were first attempts at aligning content, computation, and mathematical practices. It was acknowledged that the rubrics were far from perfect as all the mathematical practices (such as reasoning, using representations, engaging in representations and using conjectures) were not mentioned

on the rubric. Therefore the rubrics were thought of as a ‘first attempt or draft’ at aligning assessment of multidimensional practice with assessment to support an understanding of the trajectory of the quality of mathematical conceptual development. It was felt that as both the students and teachers became more proficient in using mathematical practices consideration could be given to developing the rubrics further as a result of the findings of the study. The first rubric (see Figure 3.1) shows a more traditional version of a rubric, where the trajectory is laid out in a horizontal design. It includes detail from Lepak (2014) where descriptors for words, symbols and pictures are related to surface, deep and conceptual levels of understanding.





	 Building knowledge Unistructural	 Building knowledge Multistructural	 Making Meaning Relational	 Apply Knowledge Extended abstract
Mathematical Practices	I can explain one of the steps I took to solve a problem.	I can use the context of the problem when giving an explanation. I talk about each step I took to solve the problem. <i>first I ... then I</i>	I am able to justify my explanation by offering a mathematical reason for why my method is correct. I use the words <i>because....., so if....., then.....</i>	I am able to make a generalisation. I can say if the solution or rule works for similar problems.
Words	I have written part of an explanation	an explanation is given in words but does explain <i>why</i> it works	"because" statements explaining <i>why</i> are provided for each step.	For all problems with area and perimeter every time the this happens for all problems
Symbols	an equation is attempted showing parts of how the problem was solved	an equation is used showing how the problem was solved	each part of the equation is labelled with what it represents	A rule is represented with letters symbols and numbers
Picture	a picture is attempted showing some parts of the explanation	a picture is given to support the explanation	the picture is labelled showing and matches the description of words and/or equation	The rule can be linked to a picture
Mathematics Content	I can measure the length of a rectangle. I can measure the width of a rectangle.	I know that to find the perimeter I need to find the length of the four sides and add them up. I know that to find the area of a rectangle I need to find the size of the surface area within the shape. I can do this using addition or multiplication.	I am able to investigate the relationship between area and perimeter.	I am able to make generalisations about the relationship between area and perimeter.
Numeracy strategies used to solve area and perimeter problems				
	Curriculum Level	Numeracy Level	Addition strategies	Multiplication Strategies
	Level 1	1-4	Counting	Counting all or skip counting (5, 10, 15)
	Level 2	E5 L5	18+6= 46+29=	5X3= 4X6=
	Level 3	E6 L6	158+54= 363+682=	3x6= 4x17=
	Level 4	E7 L7	5.87+6.5= 0.32+2.69+9.42=	8X24= 241x5=

Figure 3.1: SOLO rubric showing a more traditional form of rubric linking content with mathematical practices and integrating number. Rubric adapted from Hook (2012) and Lepak (2014)

The second rubric that was developed showed the trajectory of learning mathematics in a more holistic design (see Figure 3.2).

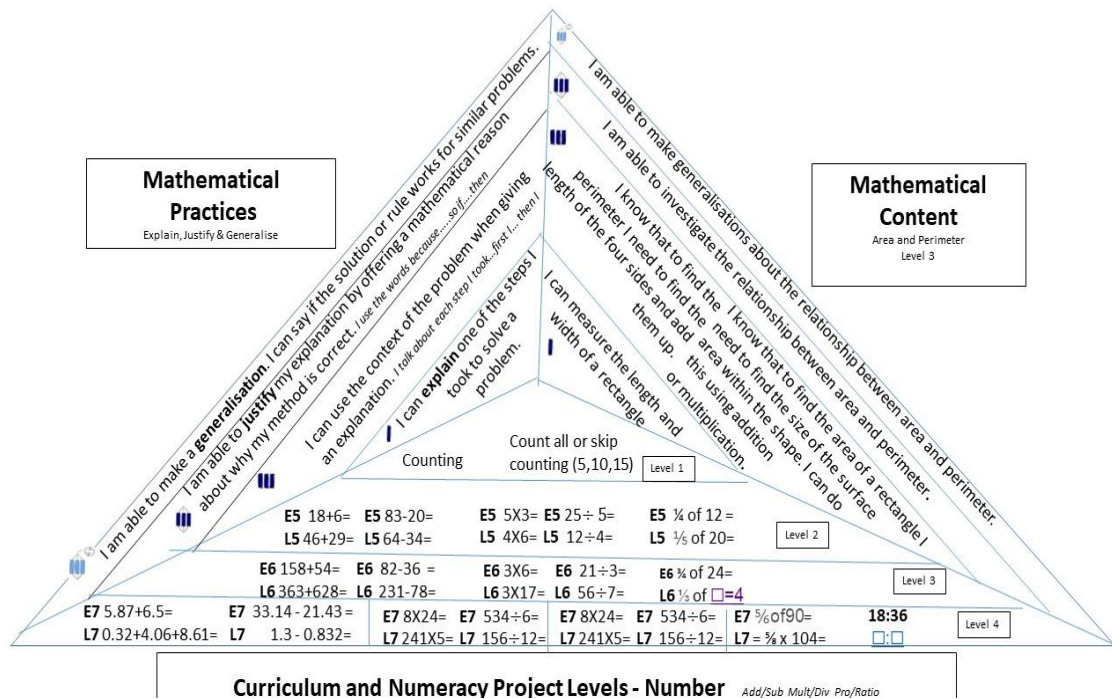


Figure 3.2: This rubric shows a holistic view of learning, linking content with mathematical practices and integrating number. Spidergram adapted from Hook, Booth, Fobister, & Price (2019)

3.6 Data Analysis

One of the features of design-based research is that data collection and analysis occur concurrently (Denscombe, 2010). Three phases of retrospective data analysis were used in this study. Firstly, comparisons were made as a result of the individual interviews with students both before and after the intervention. Secondly, observations from both classrooms during classroom instruction were analysed. The information gained from this analysis was fed back into weekly discussions with teachers. Thirdly, data was gained from interviews with both teachers.

A thematic analysis was used to analyse the interview data and a discourse analysis was used to analyse the mathematical discourse that occurred in groups. A content analysis method of qualitative data analysis was used to analyse data from the interviews. However, the data from the interviews with students was coded in two ways depending on whether questions asked afforded students to respond with multiple ideas. When open

questions were asked that afforded multiple responses from an individual student, each response was coded (see Table 3.2).

Table 3.2

An example of how responses were coded when all students ($n = 24$) were asked, *what are you learning in mathematics* in the final interviews. All responses were coded and percentages were calculated from the total number of responses given ($n = 179$).

Frequency Table December Data - Final Interviews					
Q1	<i>What are you learning in mathematics?</i>				
	Students from Teacher classroom	A	Students from Teacher classroom	B	% of total responses
Content	30		32	62	0.346369 35%
Context	3		1	4	0.022346 2%
Definition	2		0	2	0.011173 1%
Group work	12		8	20	0.111732 11%
MP	20		24	44	0.223464 22%
Norms	15		17	32	0.178771 19%
Problem Solving	2		1	3	0.01676 3%
Representations	5		1	6	0.03352 3%
Rubric	3		1	4	0.022346 2%
Skill	1			1	0.005587 1%
Stay on task	1			1	0.005587 1%
Total				179	0.977654

Audio recordings were fully transcribed to capture all the ideas from each student. Some of the ideas were single words and some were short sentences. Responses were recorded onto an excel sheet. A new line was inserted for each idea that each participant gave. To begin open coding was used. In order to refine and reduce initial themes a constant comparison (Kay, 2016) was performed in order to check that the coding was consistent. Whilst carefully reviewing the codes, patterns emerged as the development of theoretical ideas occurs alongside the analysis of data (Creswell & Clarke, 2018). Mathematical and social norms were both coded under one theme called norms. The reason for this was

because it wasn't deemed purposeful to attempt to separate them into mathematical norms and social norms for the purpose of this study. Table 3.3 presents an example of how responses were coded when a child was asked *what are you learning in mathematics?*

Table 3.3

An example of how responses to open ended questions were coded when asked *what are you learning in mathematics?*

Response	Code	Coding Value
<i>I have been learning area and perimeter and how to measure the area of a right angled triangle. Also I have been learning to use the rubric to help me justify and explain and give reasons why it is correct or wrong</i>	Area = content Perimeter = content Right angles = content Rubric = rubric Justify = mathematical practices Explain = mathematical practices	Content = 3 Rubric = 1 Mathematical practices = 2

The total of the coding value was analysed as a percentage of total responses given from all students. The number of total responses varied depending on the question asked. For example, the total number of responses given from all students when asked after the initial intervention for the question *what are you learning in mathematics* was ($n = 179$) (see Table 3.2). It was important to code this way to capture the variety of responses from all the students to reflect an increasingly multidimensional view of mathematics. For example some students' were able to identify that they were learning two mathematical practices (see Table 3.3) and some could identify learning more or less mathematical practices.

In contrast when students ($n = 24$) were asked *how are you going with your learning?* Each response was able to be coded under one heading. In this case results show the number of students who responded with an idea (see Table 3.4)

Table 3.4 When students were asked *how is your learning going?* Only one response was necessary.

Frequency Table December Data - Final Interviews				
Q.2	How are you going with your learning?			
	Students from	Students from		
	Teacher	Teacher	Teacher	
	classroom	classroom	classroom	Total
Good	11	11		22
Not Good		1		1
Improving		1		1
I don't know				0
Total				24

Weekly meetings occurred where the researcher and participant teachers' reflected on the retrospective data and as a result were able to focus attention on further instructional decisions and modification of the learning trajectory. This ongoing data analysis shaped the study and involved the researcher and two teachers working collaboratively to ensure the focus of the lessons were meeting the intentions of the study, were adjusted accordingly and to discuss and modify resources used.

Data gathered from interviews was triangulated with data from the observations. The interviews were audio-taped and wholly transcribed to; first notice concepts, and second to collect quotes that show examples of the concepts, and finally to analyse the concepts in order to find commonalities, differences, patterns and structures. Transcriptions and data from the interviews were analysed by the researcher to identify the discourse that students used when talking about solving mathematics problems.

3.7 Validity and Reliability

Validity serves the purpose of checking on the quality of the data, the results, and the author's interpretation of the data results (Creswell, 2018). Validity applies to the inference made from observations. A question to ask is how reliable and consistent are inferences made in relation to the concept it purports to measure (Punch & Oancea,

2014)? In the current study the data received from the observations and artefacts provided evidence for inferences made as a result of the data received from the interviews.

Strategies available to determine validity and used in the current study included what Cohen, Manion, and Morrison (2003) and Creswell and Clarke (2018) refer to as triangulation and member checking. By using a variety of data sources, reliability, trustworthiness and authenticity can be established through data triangulation. At the weekly meetings during this study, analysis of all data was shared with the two teachers. Member-checking involved the researcher taking summaries of observations of the findings back to the teachers and asking them whether the findings were an accurate reflection of their experiences. Confirmation of the accuracy of experiences happened through conversations with the teachers which focused on the ongoing results of the observations. The analysis of the data was discussed and confirmed with the teachers who were present in the classrooms during all observational and artefact data collection sessions, thus contributing to the reliability of the data. Using multiple sources of data and including the teachers in the conversations about interpretation of observational data contributed to internal validity, the credibility of the findings and the mitigation of bias. Angrosino (2012) states that working with multiple observers helps develop theories thus providing validation and reliability for the observations.

External validity or transferability of the findings refers to the generalisability of the research findings to a wider population (Punch, 2014; Scott & Usher, 1999). Generalisations present a challenge to determine to what extent the results can be generalized as the variable effects of teachers, community of learners and classrooms are impossible to control. However, the provision of in-depth descriptions, examples of resources and rich data enables readers of the research to determine if transferability and conclusions drawn from this study are possible to replicate in another setting.

3.8 Ethical Considerations

The research was conducted in accordance with the Massey University code of ethical conduct for research, teaching and evaluations involving human participants (Massey University, 2017). The key principles of the code include respect for persons, minimisation of risk to participants, respect for privacy and confidentiality, truthfulness and social and cultural sensitivity. Ethics approval was sought and obtained prior to data

collection. Consent was sought and obtained from the principal, the Board of Trustees of the school, participant teachers and the students. As the research involved students under the age of fifteen years old signed consent from the students' parents or caregivers was also obtained. All participants were provided with relevant information on which to base their decision (Appendices B, C, & D).

As the researcher is also a registered teacher and a member of the New Zealand Educational Institute, the research was conducted in accordance with the Education Council code of ethics for certificated teachers (Education Council, 2015). In accordance with the core principles of the code, the research was conducted in a way that supported the education and welfare of children and teachers.

The particular ethical issues arising from the current study concerned the position of the researcher as a teacher (on study leave) and participant confidentiality. Ethical dilemmas anticipated with the change in role from colleague to researcher were minimised by clear communication of plans within the research to value the teachers' perspective throughout the process. Participant teachers were invited to collaborate on the analysis of data. Planning and reflective meetings were held at times and settings of the teachers' choice. The researcher ensured that the participating teachers were informed every step of the way throughout the research process by offering to share all draft copies of the literature review, research design and findings. No evaluations of teaching and learning programmes were made other than those grounded in the context of the study. The anonymity of the teachers was maintained at all times and care was taken not to make evaluative judgements of the teacher or instructional programmes in the classroom.

As well as consent given to participate in interviews, verbal consent was sought before each interview. Students were informed that the interviews would be audio recorded, and verbal consent for recording to occur was sought. Anonymity within the classroom during observations was difficult as the participants were known to each other. Therefore, full confidentiality of the participants could not be fully guaranteed. However, steps were taken to maintain the anonymity of the teachers and students by the absence of identifying information within any written reports. Confidentiality was upheld, through the anonymisation of data. All names were withheld, and data sheets were identified by a participant identification number or letter. Potential harm to the school was minimised by

the use of non-identifying information in the final report. In order for the research to be ethically relevant with the intention of benefit (Massey University, 2017) all the resources used in the research and a summary of the findings were made available to other teachers in the school at the conclusion of the study.

3.9 Summary

A qualitative research design was selected as the most appropriate research method for this study. In order to examine the teachers and students' perspectives when mathematical practices had been included in the teaching learning and assessment of mathematics a teaching experiment design was used. Data was obtained through interviews, observations and classroom artefacts. Reliability and validity were ensured through careful documentation, triangulation, and analysis of data. Ethical principles were maintained throughout the study. The findings of this study are documented in Chapter Four.

Chapter Four: Findings and Discussion

4.1 Introduction

The literature chapter drew attention to the argument that primary school aged children should be participating in mathematical discourses that are embedded within the mathematical practices in order to develop mathematical proficiency. Evidence has been presented bringing awareness to a shift in understanding regarding mathematics pedagogy occurring in many countries that is transforming the way that mathematics is taught and assessed. Furthermore, a correlation was shown between the developments of social interplays that nourish mathematical discussions with the flow on effect being individual conceptual development. Facilitating participation for all students to engage in social interplays involving mathematical reasoning is at the forefront of reforming primary school mathematics.

The aim of the study was to offer a glimpse inside a Years Five and Six classroom through the thoughts and actions of students and teachers as they embarked on a journey to develop skills in argumentation and reasoning alongside mathematical content. This chapter outlines findings of this study drawn from participant experiences through the analysis of student and teacher interviews and classroom observations. The findings are presented in four sections. Within each section, themes that emerged are explored relating to the three questions that underpin this research. A discussion examining and exploring the findings whilst making connections to research literature accompanies each section.

Firstly section 4.2 sets the scene in the classroom prior to the intervention. Initial student perceptions on the learning process in mathematics are explored and discussed. Secondly section 4.3 highlights teachers' pedagogical strategies used to support students' to participate and communicate mathematical discourse within a safe environment. The following section 4.4 describes changes in students' perceptions regarding their own learning process in mathematics as the classroom transformed to one of inquiry. Links between teacher actions that were perceived by both the students and teachers as being key reasons for changes in student thinking about their learning process are highlighted. Finally Section 4.5 examines the centrality of assessment in mathematics instructional

practice. This section analyses the effects assessment had on the teaching and learning process in a multidimensional mathematics class.

4.2 Students Initial Perceptions of the Mathematical Learning Processes

Prior to the intervention, students were learning geometry through the context of designing and developing a geometric city for a period of four weeks. From observations in the classroom, this unit was of high engagement as students were using nets to build cities. Whole class, small group, and individual teaching strategies were observed. Even though there was lots of talk in the classroom, the questioning resulted in teacher expected answers usually with the teacher in a more conventional role reiterating explanations. At this time, initial interviews were conducted with each child in the study ($n = 24$) exploring students' reflections of their mathematical learning process.

During the initial interviews, students were first asked: *What are you learning in mathematics at the moment?* As Boaler (2016) highlights, if students know what they are learning, this helps them know what success looks like and helps start a self-reflection process that is an invaluable tool for learning. In Figure 4.1, the data shows that 61% of all student responses included statements about *learning content*. Boaler describes a content driven approach to learning mathematics as being consistent with one-dimensional mathematics classrooms, where one practice is valued above all others and as a consequence, achievement is quantified. In this study, the mathematical learning process as identified by the students was focused mainly on content.

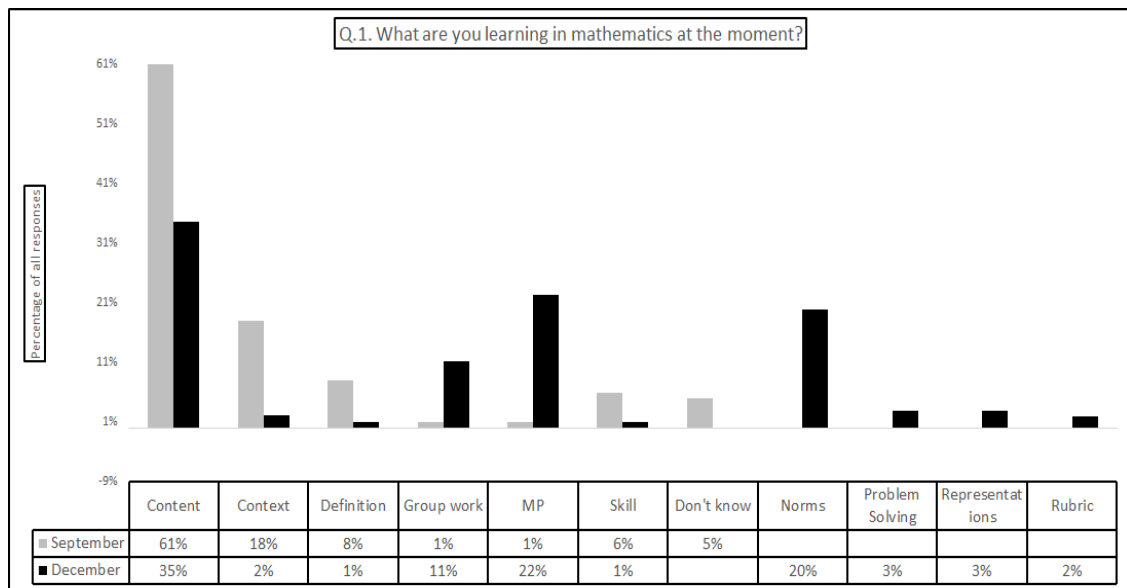


Figure 4.1: A comparison of the data received from initial student interviews in September and final interviews in December when asked: *What are you currently learning in mathematics?* ($n = 24$) students.

4.2.1 Importance of Context on the Mathematical Learning Process

Context was the next most common response in the initial interview with 18% of all student responses equating context with mathematics. In contrast, at the end of the intervention only 2% of all student responses mentioned context as synonymous with mathematics. Whilst many of the tasks during the intervention did contain a relatable context, the same context was not consistent throughout all the tasks as it was prior to the intervention. In order to assist teachers transitioning to mixed ability group teaching, the focus on task design during the intervention was on open tasks rather than tasks with a strong contextual basis. The intention of the open tasks was to enable access for students working at different achievement levels by having different entry and exit points. Even though context was not a strong component during the intervention, the findings in this study highlight the importance of it. More students, as shown by the responses prior to the intervention, view context as synonymous with learning mathematics when it is a strong component of pedagogy. Mathematics context is extremely important as it provides synchronicity between the real world and mathematics. Hunter (2007) highlights the importance of context by including it in the Communication and Participation Framework (CPF) where she describes the problem context necessary to make explanation experientially real.

Through the use of relevant contexts, teachers can provide more opportunities for students to explain and justify in authentic ways. By designing tasks that relate to the lives of diverse learners, teachers are able to carefully raise the status of certain students as these students can then offer relevant contributions to an intellectual discussion. It is of importance to note in this study, prior to the intervention, building a geometric city provided a relevant context to this group of students as the town they lived in was going through a building boom. In addition, the geometric city tasks had varying entry and exit pathways. However, what was missing from the mathematical learning process prior to the intervention was the explicit teaching of mathematical practices; facilitated through the use of classroom norms thus generating mathematical discussions involving argumentation within and across groups of students.

4.3 Transforming Teaching and Learning of Mathematics

Through regular meetings with the teachers and myself, resources for students to promote talk moves and a rubric based on the SOLO framework with mathematical statements referring to mathematical practices were developed, shared and discussed. Teachers drew on the CPF (Hunter, 2007) to establish the social and mathematical norms. Current research articles provided by the researcher supported teachers with practical ideas on how to establish a mathematical inquiry environment reflecting a multidimensional approach to the teaching, learning and assessment of mathematics.

4.3.1 Norms Established to Facilitate Group Work

In order to position students to be able to participate in mathematical discourse involving mathematical argumentation, it was essential that consideration was given to establishing a collaborative classroom environment. It was understood by the teachers that students needed to be working in groups that supported each other in order for the social conditions essential to grow the use of mathematical practices. However, as stated by Cohen and Lotan (2014) groups cannot just work together, they need to be taught how to interact or one or two students do all the work and make all the decisions for the group. Some students will take over and some will hide in the background and do nothing. Therefore, in order to create a safe environment where students felt safe to engage in group work, classroom norms were established. The teachers explained to the students that the roles for both the students and the teacher would be changing. As a consequence they would be learning how to work in groups. In order to help improve group

interactions the teachers drew on Boaler et al's. (2017) activities for building norms and Hunter's (2007) CPF. The norms turned out to be highly influential for the students in understanding their responsibility and roles when working in a group. This is similar to findings from Hunter's (2009) study exploring how a teacher used interactional strategies to facilitate students to engage in collaborative interaction and productive mathematical discourse. The study found that the norms shaped how students learnt as the norms guided student interactions.

To establish the norms in this current study, the teachers asked the students to reflect on 'things they didn't like people to say or do when working in a group'. Ideas were collated and displayed on the wall for reference (see Figure 4.2).

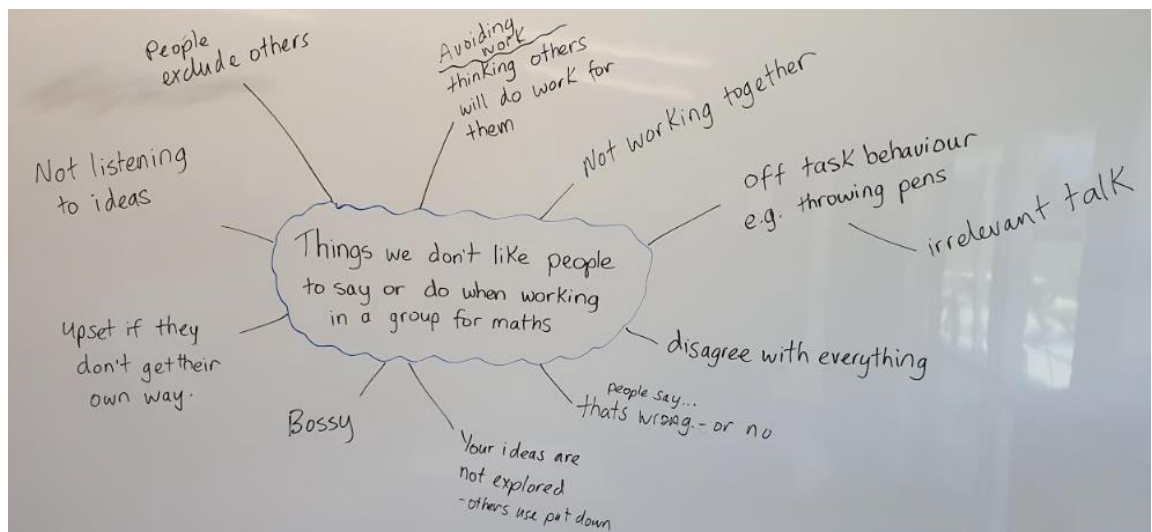


Figure 4.2. Group norms. 'Things we don't like people to say or do when working in a group at mathematics'.

The ideas generated turned out to be a powerful reflective tool for the students about how they normally behaved in mathematics class. It was observed that teachers were able to refer back to the poster later when positive group work behaviour needed to be reestablished.

Next, the teachers did the same activity for 'what we do like to happen when working in a group at maths time'. These became the agreed upon classroom norms that were used throughout the study (see Figure 4.3)

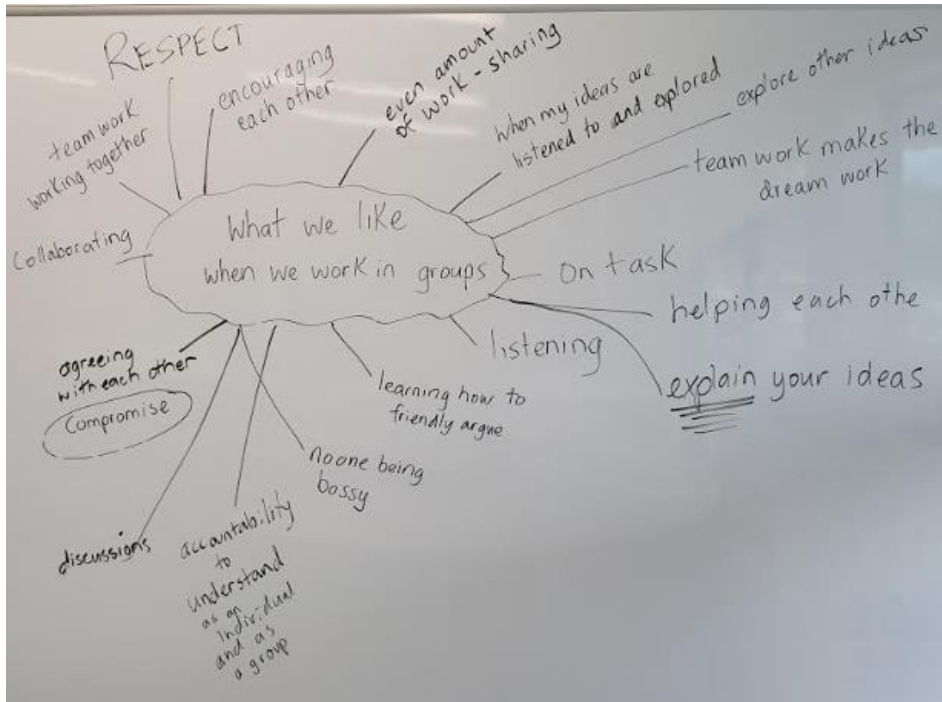


Figure 4.3 Agreed on classroom norms. ‘What we like when working in groups’.

Throughout the intervention more group norms were added to the list. The norms offered by the students became more sophisticated, as such, an increase in mathematical norms emerged. See Appendix E for the agreed upon norms as generated throughout the intervention.

In order to promote group work teachers made explicit the changes being made by commencing each mathematics session with a discussion about the group norms. As Boaler (2016) highlights, it helps students to enact positive norms knowing that the same norms are shared by their peers. Additionally, students were given the opportunity to communicate a norm they were focusing on at the start of the lesson. The following classroom observation shows how one of the teachers orchestrated discussions to develop proficiency in using group norms.

Teacher: In your groups I want each of you to say what group norm you are going to work on today. Don’t randomly pick one, pick one that you think is going to help you.

Student 1: Sharing ideas

Teacher: Why are you picking that one?

Student 1: I don't know, it is just something I don't normally do

Student 2 - It might just be friendly arguing instead of just normal arguing, if someone gives you an idea don't just chuck the idea out the window, take it on board as you might use it again and need it if needed

Teacher - Why might that be a good idea?

Student 2: Just so like you are contributing others ideas and not just having them all to yourself

Student 3: Collaboration

Teacher: And what is that going to look like in your group?

Student 3: Not sitting down, actually trying contributing, not lazing about actually

Teacher: So you are going to be really focused today?

Student 3 - Yes

Teacher: Why might that be important for you?

Student 3: Because I need to learn a wee bit more because I am struggling a bit in maths

Teacher: So you are finding it a wee bit hard so do you think if you make more of an effort that will help you with your learning?

Student 3: Yes

Student 4: Include other people

Teacher: What might you do if you notice that someone is not being included in your group?

Student 4: Not giving any ideas. Instead of doing the work for them you would ask them questions about what they think. You would ask them questions to get them involved.

The group norms were a key factor for the students to work towards collaborative participation as they gave students focus as can be seen in the following extract from the final interviews.

Researcher: What has been the major difference for you in the changes to your responses at the start of this study?

Student: The norms

Researcher: Why?

Student 1: They are basically like look at them every time before you start your problem so that you have a goal to work on. They are good cause we need to achieve a goal and if it wasn't there we wouldn't really be focusing on it.

Student 2: I used to be very quiet in maths and not do anything but now I am quite loud and give ideas.

It was observed through classroom observations that students became more adept at choosing relevant norms as by the time the teacher started the lesson most students had chosen a norm to focus on.

Teacher: The children really picked norms that were relative to them

As the students began to get a more complex view of the classroom norms, they began to offer justifications for choosing a norm without being prompted. In Teacher B's class, the students were invited to physically take a norm off the wall and place it in front of them while they were working. Unprompted, the groups started discussing amongst themselves why they chose the norm that they did before they started working on the day's problem.

Student 1 I have chosen *to include everyone* as normally I take over

Student 2 I am going to make the effort to contribute more

The impact of the norms and the talk moves were observed early on in the study in the second week as noted by one of the teachers.

Teacher: This student doesn't usually talk much during maths, his confidence has improved so much due to his focus on the norm of 'not giving up even when the work is too hard'. He reflected on the fact that his focus would drop off when he did not understand something someone has said. The talk moves question prompts have helped him to interrupt group and whole class discussions to question explanations given. He is now becoming excited to share and take risks.

As Boaler (2016) highlights, preparing students to work in groups is important. Similarly, Hunter (2009) states that students begin to think for themselves about positive and negative group discussions by developing norms. As a result they become more thoughtful about ways they interact in groups and the class starts to develop a more productive discourse community.

4.3.2 Group Work

As the students became more adept at choosing norms, changes were observed in the process of group interactions resulting in more equitable practices. Two inter-related examples that were observed are highlighted. These examples demonstrate improvements observed in intergroup relationships and intellectual and social learning goals as a result of heterogeneous grouping. These two examples demonstrate that carefully constructed mixed ability groupings have advantages for both, perceived high and low achievers. Firstly, two students, A and B perceived as high achieving students began to show more respect for other group members. Usually these two students struggled when working in groups as typically they dominated mathematical discussions. Secondly, it was observed that by changing how more confident students, such as Student D participated in groups afforded space for less confident students to contribute. For example, Student C who was normally quiet during mathematical discussions was allowed time to process mathematical concepts. As a direct consequence of the norms he was given time to revisit, process, and interact with a range of explanations for a task offered firstly within his small group and later when the teacher carefully orchestrated the groups to explain their thinking to other groups.

4.3.2.1 Disrupting Group Hierarchy

Often high achieving students can dominate the conversations in the group while others feel frustrated and become disengaged (Cohen & Lotan, 2014). For example, Student A,

one of the students who found it hard to work in groups as she was a very dominant character, chose to focus on a personal norm to *not to take over or be bossy*. During the first classroom observation there were tears in Student A's group as there was much disagreement as she dominated the task activity. During Student A's interview after the intervention she identified the norms as helping her to work in a group. She continued to demonstrate an understanding about how other students learn mathematics. In other words she was learning how to participate in a community of learners.

Student A: Sometimes I've got something in my head I am going to explain, probably they know but then they don't and sometimes I get a bit frustrated cause how do you not understand? But then I remember that some people learn differently and I try to explain it like in a different way for them, like draw it or say it or umm do it in a different way, so then some people need to see it, some people need to hear it, some people need to like feel it with like blocks or something. I have learnt that if somebody has a different idea that umm even though you might think it doesn't work you still try it cause sometimes it will work out or sometimes it won't work and then you have an idea and then they do it and then it works.

Similarly, learning to work in groups had a significant impact on Student B who usually preferred working alone and also demonstrated dominant traits when working in groups.

Student B
So umm we have learnt about group norms that we made ourselves and we're like everyone can have their turn, sometimes you have to compromise and I feel they (the norms) help the group work really well cause in our group we have a lot of strong characters and it is quite useful when we use the group norms because they help us stay on track and also get the work done

Both students A and B are high achieving in the traditional way of teaching procedures and computational strategy but struggle when working in groups, thus missing opportunities to develop their own argumentation and reasoning skills in a social context. Furthermore, these are the students who previously held a higher status in mathematics

as they were quick to answer and often the loudest, thus enjoying a higher proportion of the talk time at the expense of others. As Hunter and Hunter (2018) bring to our attention the lens of status serves as a tool for identifying patterns of inequitable classroom interactions.

Researcher: Which norms have you been concentrating on?

Student B: I have been working on there is no I in team because I quite like being a leader of a group but now I have learnt that it is time to like you can still give your ideas but let others share their ideas too.

Researcher: And has that been helpful?

Student B: Sometimes people didn't like to work in groups with me because I was that type of person that would like do it by myself but now with the new way of learning I feel more open to working with groups now and yea.

In the interview Student B identified group work as both the most influential effect for her changing how she interacts with mathematics and as something she needs to continue to improve on. Furthermore, she is also transferring her learning of how to work in groups in mathematics to other areas of the curriculum. The following vignette shows evidence of the value of creating a peer environment where students can feel safe to engage in constructive learning talk that is of relevance across the curriculum (Ministry of Education, 2013).

Researcher: Has this helped you in other curriculum areas?

Student B: Yes so like when we are doing book club and stuff I kind of like reading by myself and like talking to myself in my head but now like it is quite fun because I discuss with others which I wouldn't usually do about a book and stuff it's quite fun.

If given the choice Student B would now choose to work in groups. As is evident by her response, she is beginning to experience the joy of working with others.

Student B: It's kind of helped because I am more used to working in groups and it is more fun for me because I like working by myself so I feel like if now that we are working in groups more often it's more like helpful for me I am getting used to working in groups. I guess I would work in groups now if we have the choice.

These are both examples of high achieving students who are starting to learn to consider the ideas of others in the group. As a result of the norms and group work Student A and Student B are now beginning to learn to respect the ideas of others in their group. By changing how dominant group members participate within a group offers space for less confident students to contribute as was evident in the observation of Student C.

4.3.2.2 Opening Space for all Students to Access Mathematics

The following example shows how the three elements of Complex Instruction (CI) opened space for a quieter student to access a variety of mathematical discourses within a lesson. As a result of how the teacher constructed the lesson by using: a group worthy task with varying entry and exit tasks; promoting the status of Student C; and facilitating socio and mathematical norms, meaningful group interactions were observed.

Student C, a quiet student who like many students needed time to process information, was afforded space to think and contribute value to the discourse. Next is an excerpt from Student C's interview after the intervention which shows that since the norms had disrupted the status quo of how mathematics traditionally operated, space was opened up for him to participate. As a result his confidence had increased.

Student C: I just don't normally collaborate cause I normally just hide away and don't normally um collaborate as much I normally hide away and am scared if I get it wrong

Researcher: And how about now?

Student C: I feel a lot more confident collaborating

Researcher: Why is that?

Student C: Because I have just practiced a lot more of it from doing the group norms when we are in a group

Researcher: Do you feel a bit safer in your group now?

Student C: Yes yes, I feel a lot more confident now and not more hide away

Researcher: So in your group what have you been learning to do, what have you been practicing?

Student C: We have been trying to explain more of the problems we solve and also justify it

Researcher: What does that mean?

Student C: Justify means like, kind of means like protect um like and defend your answer.

The researcher observed Student C during a typical lesson. The students were working on a group worthy task involving illustrating multiplication (see Figure 4.4). The teacher used visual proofs from Boaler et al., (2017, p.185) in order to explore the connections between visual and numerical models for multiplication.



Figure 4.4 Visual Proofs, (Boaler, Munson & Williams, 2017, p. 185)

As complex instruction allows all students to contribute to multi-level tasks within a safe environment established as a result of the norms, Student C was afforded space to engage with mathematical discourse at a speed that worked for him. Student C first engaged with mathematical reasoning and argumentation within a group small enough that everyone's voice was heard. Following the small group discussion, he was then able to participate in the orchestrated sharing of ideas, along with three other groups which involved students engaging in classroom discussion that was facilitated by the teacher. Student C was able to participate with the reasoning of the problem for the duration of the many components of the lesson.

Whilst working on the visual proof in small groups, Student D (who like Students A and B was usually dominant in the group) got the right answer but his reasoning was flawed. This example demonstrates that sometimes students get the right answer with no conceptual understanding, hence the importance of teaching mathematical practices. Student C who is usually quiet had picked up on this misconception. Student C lacked confidence so the teacher supported him by picking up from his body language and how

he pointed to the visual proof that he had noticed the misconception. The teacher then prompted him to ask a question, thus developing his status as his questioning was intellectually valid. By prompting Student C to ask a relevant question is an example of the teacher drawing on the CPF. By using complex instruction in order to disrupt the status hierarchy she recognised that Student C needed support to access the mathematical discourse. As the teacher had reconstructed the expectations and obligations for how the students participated in collaborative discourse through the use of the norms the teacher opened the space for Student C to engage in mathematical practices. Students need to talk in order to learn maths (Askew, 2016) and the quality and quantity of talk matters for student learning (Shah & Crespo, 2018). The teacher noticed the disparity for Student C to have the opportunity to participate and addressed the problem.

The following vignette captures reasoning between these two students.

Student D: It is the area of both of these, They are the same. The same colour, these are different here as there are two bits here (looking at splitting the 24), half of it so you count $10 \times 24 = 140$, I mean 240 so half of it equal 112

Student C: But this one has two areas as well that one has two splits and that one has two splits

Student D but that one is a bigger number so it is split. They are not the same, that one is 12 and that one is 24 and that one is 10 so it will equal 112

Teacher: So do you understand how he worked it out can you explain it, Student C?

The teacher is probing Student C to see if he notices any mistakes with Student D's reasoning and computation skills.

Student D - So you times those two together and get 240 and then they halved it so it equals 120 (*this time he is correct*)

Teacher: Do you have a question here? Looking at Student C

Student C: No, it is just my question was just there are two bits here and there are two bits here as well. It is making me confused

Student C is trying to make sense of the two visual proofs. It appears that Student D understands the visual proof (doubling and halving example 10×24). Pointing to corresponding numbers as he explains that

Student D: It is the area of both of these, They are the same colour these are different here, they have 2 here half of it so $10 \times 24 = 240$ so half of it equal 112

Student D points to the writing underneath the visual proof, it is clear that student D is just reading off the numbers underneath the visual proof without understanding.

Teacher: So Student C can you explain this? What have they done here, what have they halved and doubled or what have they done.

Student C: They did 10 times 24 which equals 240 and then they did half of 240 which equals 120.

Unlike student D, Student C points to the picture of the visual proof to defend his explanation.

Teacher: So can you explain why have they done that

Student C: To make it easier

The teacher then brought the four groups of four to a sharing space to afford students the opportunity to share their thinking. Student C had the opportunity to listen to other group's explanations for the same problem, thus giving him further opportunities and time to think, process and start to develop conceptual understanding. Again the teacher is drawing on pedagogical practices through the use of the CPF where the small groups get the opportunity to offer explanations as a large group. The group were interacting and inquiring mathematically as a community.

Teacher facilitating groups for shared explanation: For some people, they looked at that and were finding it a little confusing so we are going to get a couple of groups to have a go see if they can explain one of these visual proofs. And then you start to think How did they get the answer? Did they all do it the same way?

Teacher: Group A, would you like to explain one of them please?

Student from Group A: So on it it says 24 times 10 with a dotted line in the middle because it is 24 times 5 so they doubled 10 as it is easier to find the answer and then they just halved it to get the answer which is 120.

During the share back time the researcher observed Student C actively listening. Hunter and Hunter (2018) noted active listening is key to sense making. Next the teacher posed similar questions with different numbers (see figure 4.5)

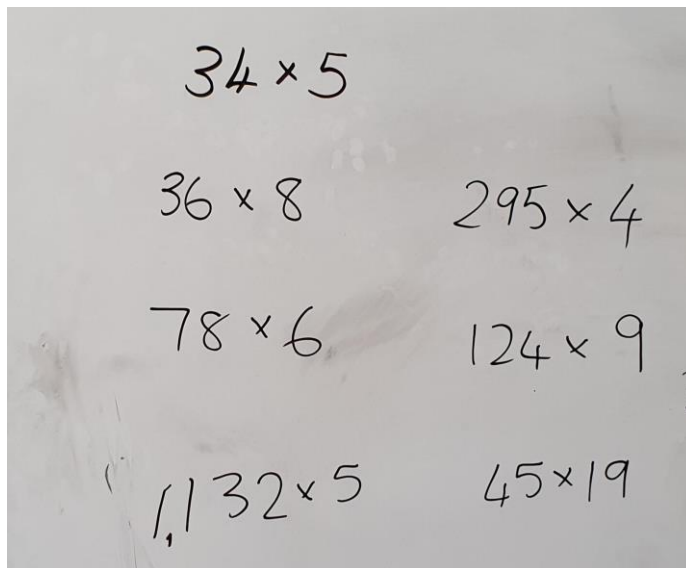


Figure 4.5 Similar problems for students to choose from.

Student C's group chose to solve 34×5 and decided to work on the whiteboard. Student D took it upon himself to take the pen as was reflective of his usual dominant role. However, there was a discussion with the group about how to solve the problem. Student D drew a representation of how they solved 34×5 (see Figure 4.6).

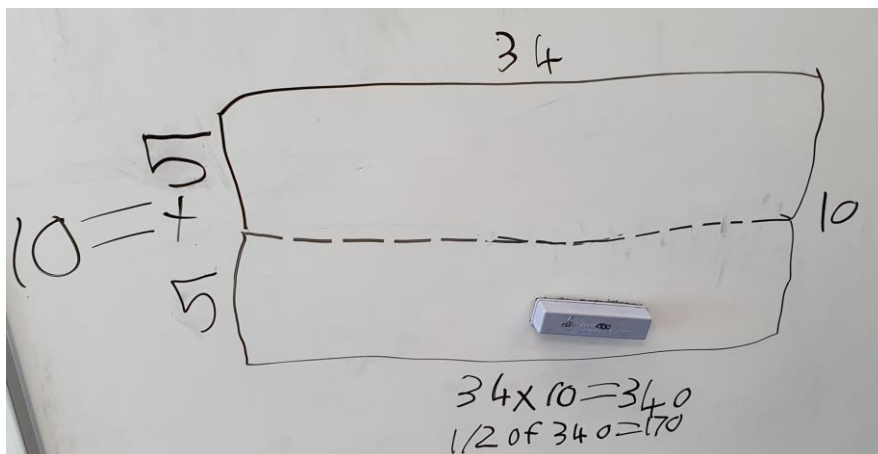


Figure 4.6. Representation of how the group solved 34×5

Next the teacher gave Student C the pen and asked the group to solve 36×8 (see Figure 4.7) to check for conceptual understanding for using visual proofs as the previous problem (34×5) was very similar to the first example (24×5). However, 36×8 required using a different visual proof.

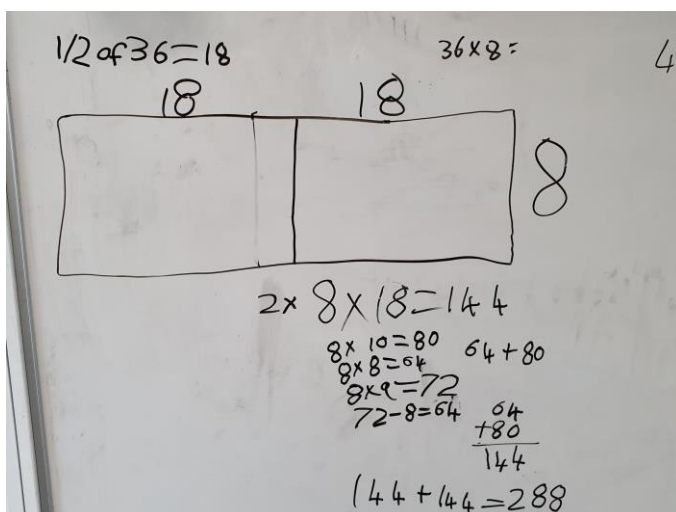


Figure 4.7. Student C holding the pen and contributing to solving the problem.

Student C was the most dominant voice in the group as they constructed a representation to solve 36×8 . It took the length of the entire lesson for him to begin to process the task conceptually. The pedagogy of enabling students to participate and communicate in mathematical discourse within small groups and later larger groups all discussing the same task, afforded the students time and space to think mathematically. This is an example of how facilitating mathematical practices alongside content allows students to experience the multidimensionality of mathematics, thus being able to think like

mathematicians. As Hunter and Anthony (2011) describe, for many students learning and becoming confident to construct, present and question mathematical explanations is a lengthy process, which requires teachers to continually press students to provide conceptual mathematical explanations. To achieve this, teachers needed to gradually build on and extend their own expectations for students to engage in justification and mathematical argumentation.

What these two examples demonstrate is that through learning how to work in groups by constructing shared norms, students can begin to appreciate the knowledge, expertise and experiences of other group members, thus raising the status of other students in the class. By disrupting the status quo where the loudest and quickest students have traditionally been given more space to engage with discourse; other students as demonstrated by Student C are just as capable of logical reasoning and argumentation when expectations through co-constructed norms are put in place. Boaler (2016) brings to our attention that by using mixed ability grouping does not mean that groups are lowered by the presence of perceived low achievers that in fact, group conversations rise to the level of the highest-thinking students.

By engaging with other students in argumentation regarding the conceptual understanding of one problem throughout the entire mathematics lesson created the environment for Student C to be able to experience resilience and perseverance. As a result, it was observed that he was able to offer his own explanations for similar tasks.

4.4 Changing Perceptions of the Mathematical Learning Process

At the end of the intervention, students were asked the same question as prior to the intervention: *What are you learning in mathematics at the moment?* The findings show that students were beginning to think of more ways to be mathematical. This can be seen by the wider spread of responses of ways to view mathematics (see Figure 4.1). Mathematical content dropped to 35% and mathematical practices 22%, norms 20%, and group work 11% were now included. Overall, after the intervention students demonstrated a more multidimensional understanding of what they were learning in mathematics. Within these themes, qualitative differences in complexity of thinking are apparent. For example, in the vignette below a student talked about what she was learning both prior to the intervention and after the intervention. This student was normally one

of the quieter students who lacked confidence. Prior to the intervention she talked about content and context. After the intervention she expanded her thinking about mathematics to include mathematical practices referring to explanations, justifications, and friendly arguing. She also described using norms such as contributing ideas, talking, and listening. As can be seen in this example she appears to have broadened her view on what mathematics is, and how to participate in mathematics.

Researcher (Initial interview): What have you been learning in maths?

Student: Geometric city, using our geometry skills learning angles, points, 3d shapes what a 2d shape is, making cities, we are making a tree house park, attraction, museum, we need to include points, angles and 3d shapes. I am learning to observe shapes and show what they are in 30 seconds, that is an octagon, or that is a cube

Researcher (After the intervention): What have you been learning in maths?

Student: We are learning to justify and explain our thinking. We needed to learn to explain and say that I think this is right because someone might come back and say... but what happens with this 5 wouldn't that make your answer wrong? And then you have to friendly argue and you have to try and find out the answer together without saying I am right and I am always right. Just like trying to say it in a nicer way like we have been working on friendly arguing and we have been using the norms. You choose one and you say like .. my one is contributing my ideas cause I don't usually say lots in an argument like yep umm but I do sometimes hang back and let others do it but now I have been trying to put in my opinion and say what I have been thinking even if it might be wrong.

Researcher: Has this made a difference for your learning in maths?

Student: Yes, instead of actually hearing, I am actually talking to them and saying oh but what about this and I am learning more by listening and not hanging back also.

Evident in her response, is her growing understanding of the need to explain her thinking and offer justification. This trajectory of learning mathematical practices is what Hunter and Hunter (2018) contend students need to do in order to develop mathematical reasoning. They need to learn to be confident to explain and represent their mathematical reasoning as well as justifying reasoning. Similar to investigating student beliefs about learning mathematics within a mathematical inquiry community in this study, Hunter, Hunter & Restani, (2020) report on student perceptions regarding mathematical learning. For example, students in their study showed shifts in the changing perceptions of their roles in mathematics. This included describing mathematical explanations as a form of justifying their thinking.

4.4.1 Changing Views on What Success Looks Like in Mathematics

Initial views of the students highlighted content and context as the main components of their mathematics learning process. After the intervention, students' views extended to include mathematical practices, norms, and group work. In order to determine changes in their views about what it means to be proficient in mathematics they were asked at both interviews: *How they were going with their learning?* (see Figure 4.8). The majority of all students' responses showed that they believed they were *going well* with their learning both before and after the intervention.

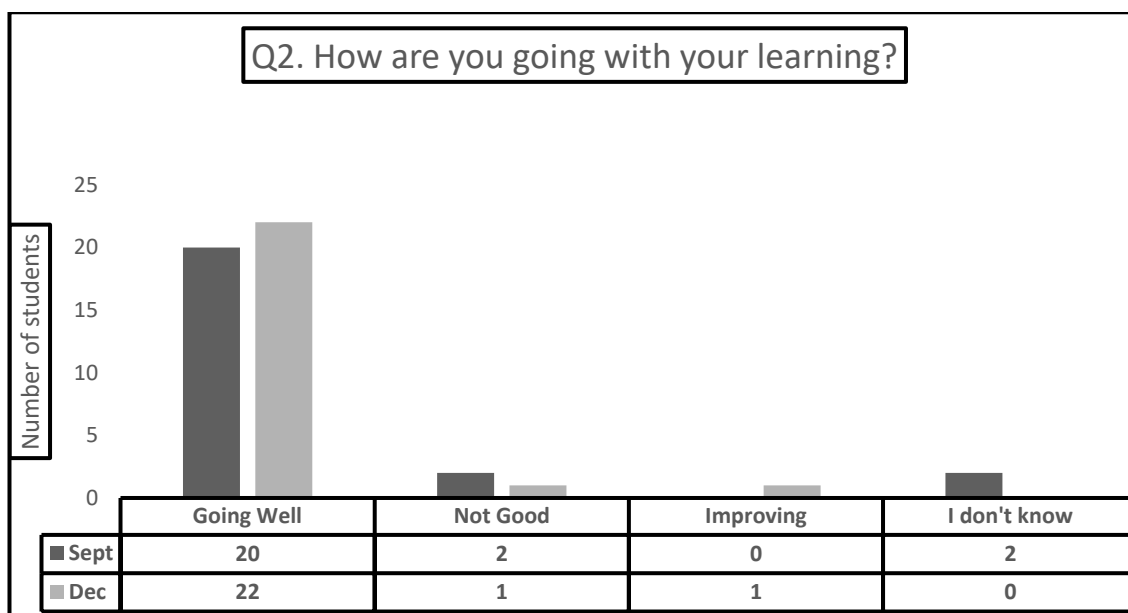


Figure 4.8: A comparison of the data received from initial student interviews in September and final interviews in December when asked: *How are you going with your learning?* ($n = 24$) students.

Students who responded that their learning was *going well* prior to the intervention ($n = 20$) and after the intervention ($n = 22$) were asked an additional question, *Why do you think your learning is going well?* (see Figure 4.9). All responses ($n = 20$) from students in the initial interviews could be categorised under two categories. Firstly, *I don't know* 54% and secondly *content learnt* 46%. Boaler (2016) reports that a major failing of traditional mathematics classes is that students rarely have much idea of what they are learning or where they are in the broader learning landscape. For those that gave an answer categorised as *content learnt*, viewed the procurement of content as synonymous with success and proficiency in mathematics. This is consistent with research studies (e.g., Boaler, 2016; Enyedy, 2003; Selling, 2016) that report student views in one-dimensional mathematics classrooms that encourage procedural mathematics instruction focusing on drilling skills, learning facts and following procedures.

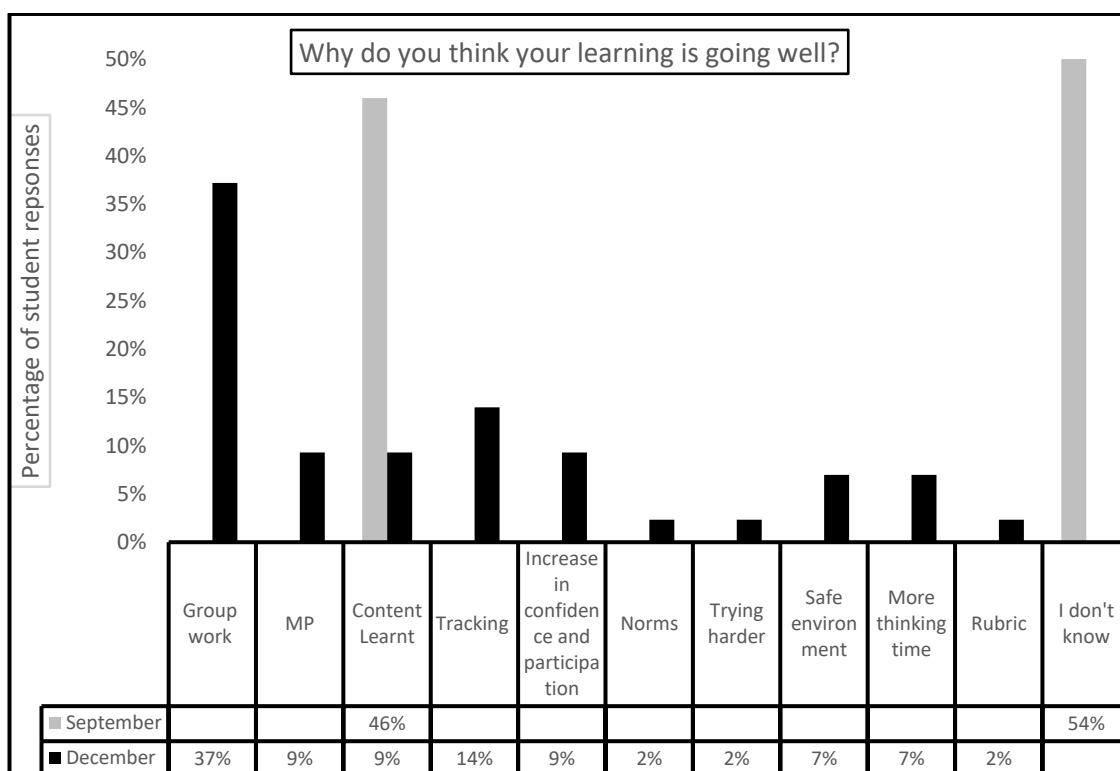


Figure 4.9: A comparison of all responses received from initial student interviews in September and final interviews in December when asked: *Why did you say your learning was going well?* Prior to the intervention.

The following are examples from two students in the initial interviews who didn't know why they said they were *going well* with their learning.

Researcher: So you said you were going well with your learning, how do you know you are going well with your learning?

Student 1 : I am on track to where the teacher wants us

Researcher: How do you know what the teacher wants?

Student 1: I am guessing, I don't know

Student 2: I just feel like I am going well.

Below are examples of two student responses from the initial interviews who thought they were *going well* with their learning because of content.

Researcher: How are you going with your learning?

Student 3: I'm pretty good, I'm going well, I'm searching up what they are so if I ever need the shapes again I will know what they are.

Student 4: Cause I think I needed to get three right

After the intervention, there was a notable shift in the responses of the students who believed that their learning was *going well*. Along with content, students began to see success through an increasingly multidimensional lens. Their responses included a broader view of mathematics whilst recognising the importance of: working in groups, using the rubric as a resource, engaging in mathematical practices, an increase in confidence and as a result of participation; they felt they were achieving as they felt safe, had more thinking time and were trying harder (see Figure 4.3).

After the intervention, a number of student responses (37%) attributed their success and improvement to working better in groups. This evidence shows that students were beginning to experience learning as involving the interconnection between themselves

and others in the environment. This highlights the positive outcomes that can happen when people work together on group goals as identified by Cohen and Lotan (2014). Understanding that success in mathematics includes how you work in groups shows a significant shift in how students think about their mathematical learning process. Enyedy's (2003) study investigating how students' mathematical understandings changed as a result of social participation demonstrated that cognitive and social aspects of intelligent activity are inseparable. As students work in groups their individual conceptual understanding is developed by providing public accounts to others as they need to articulate, explain, and defend a claim.

Evidence of students experiencing mathematics as a socio-cultural activity can be seen in the following four extracts from the study along with responses that draw on the values of reciprocity, relationships and inclusion. In the first extract the student believed they were *going well* with their learning because in the group they were learning from each other. This is evidence of learning about reciprocity. This student is also beginning to appreciate the different ways that other students learn mathematics.

Researcher: How are you going with your learning?

Student: I am doing better than I used to think?

Researcher: Tell me more about that

Student: Because we do more work in groups so I have learnt off other kids and also about how they learn.

In the next extract the student describes how the group started to collaborate even though at the start they didn't really like each other.

Researcher: Why did you say you are going well with your learning?

Student: Well because our group we didn't really collaborate together which is because

of the group that we didn't want to be in like we didn't want to be with those people but then at the end we were starting to collaborate a bit more, like talking a bit more and sharing our ideas.

These students are beginning to develop deeper relationships within their mathematical inquiry community.

In the following extract the student talks about a safer environment where they don't feel judged by others. As a consequence they are more confident to share their ideas.

Researcher: How are you going with your maths?

Student: Alright, I've learnt like I am a lot more confident in it now and sharing

Researcher: What is the main reason for you being more confident in maths now?

Student: because people are not judging you, like it is quite a small group it is not like a whole bunch of people..it gets you thinking a bit..if you don't know you ask your group and in your group you work it out and might talk with each other and stuff until you finally figure it out.

In this example, we see the students drawing on reciprocal relationships and collaboration: *You ask your group.... in your group you work it out.... you talk with each other*. They are including each other irrespective of their perceived achievement levels.

Finally, in this fourth extract the student connects *going well* with learning group responsibility and again an inclusive value is apparent in how this student perceives what *going well* with their learning means.

Student: Working collaboratively like making sure that everybody is on the same page and like everybody knows what they are doing and not just like rush ahead.

After the intervention students also attributed their perception of knowing they were going well with their learning to using the SOLO rubric (16%), engaging in mathematical practices (9%), content learnt (9%), an increase in participation and confidence (9%), and a safe environment (7%). All these responses demonstrate that the students were gaining a more holistic understanding of what success in mathematics means for them. Of note is that no student at the end of the intervention gave a response of *I don't know why I said I was going well*, contrasting with 54% of responses at the initial interview who could not give one reason why they stated that they were going well with their learning. If a student cannot reflect on their own learning process, then it needs to be built into the instructional practice otherwise students will not be learning to learn. If they only mention content learnt as a way to monitor progress then content is how they view the mathematical learning process.

4.4.2 Changes in Student Metacognition

The principle of learning to learn puts the student at the centre of teaching and learning. Asking students: *What they need to do in order to improve?* challenges students to think of the next steps in their learning process and offers opportunities for them to provide different ideas about mathematics. When students reflect on their own learning processes they are learning how to learn (Ministry of Education, 2007). In the initial interviews, 49% of the student responses indicated that they did not know what they needed to do in order to improve, this figure dropped to 5% of the responses after the intervention (see Figure 4.10). The initial student interview responses 42% showed that students believed that in order to improve they needed to learn more. This figure drops to 24% after the intervention.

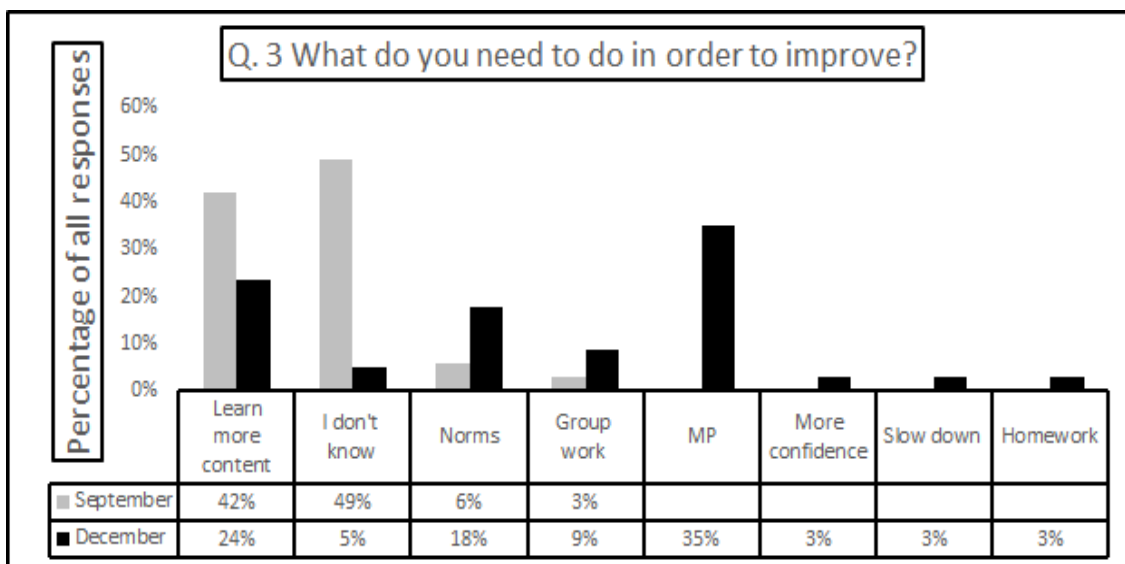


Figure 4.10: A comparison of the total responses from students received from initial student interviews in September and final interviews in December when students ($n=24$) were asked: *What do you need to do in order to improve?*

Following is a response from a student at the initial interview when asked *what you need to do in order to improve?*

I don't know. I would like to know, it would make me want to learn more if I know what I need to work on

The analysis from the initial interviews showed that students either do not have a view of the trajectory of a mathematical learning process or alternatively see the trajectory for success to mean learning more content. Importantly, analysis of the data from the final interviews showed an increased spread of responses reflecting that the students were considering more ways to be mathematical. The next steps to improve in mathematics identified by the students after the intervention include: mathematical practices 35%; content 24%; norms 18%; and group work 9% (see Figure 4.10). Similar to Boaler's (2016) perceptions of a multidimensional maths class we see evidence from students' perspectives of a more integrated approach to the mathematical learning process after the intervention. These results show both individual and collective goals which contrasted with the strong tendency for an individualistic goal: *to learn more* before the intervention.

4.4.3 Teacher Actions Responsible for Changes in Student Metacognition

During the final individual interviews, students were given the opportunity to read and compare their answers with their responses from their initial interviews. They were then asked for reasons for changing their thinking on how they viewed the learning process in mathematics. Results show (see Figure 4.11) that teacher actions (as identified by the students) with the most influence on the students' change in thinking about their mathematical learning process were the establishment of the group norms (31.5%) and participating in groups (31.5%).

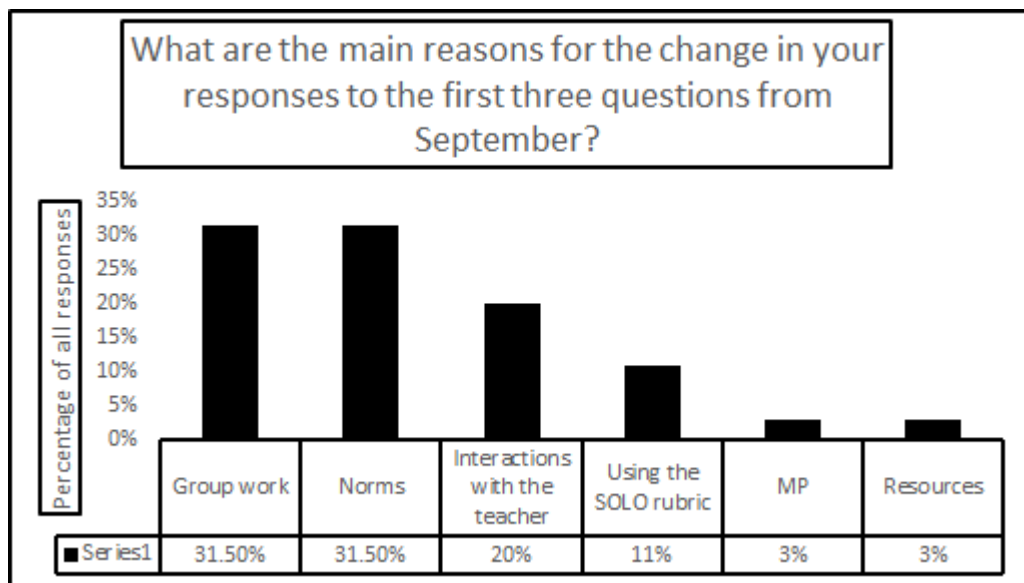


Figure 4.11: Total responses received from the final student interviews ($n = 24$) in December when asked: *What are the main reasons for the change in your responses to the first three questions from September?*

These findings are reflective of the extensive body of research by Cohen and Lotan (2014) regarding teacher actions for the successful use of cooperative learning to build equitable classrooms. As identified by the students in this study, the classroom norms facilitated by the teachers informed them about how they were expected to behave towards each other when working in a group.

The next section explores the role of assessment in a reform classroom.

4.5 The Centrality of Assessment in a Reform Classroom

As the school in this study had successfully used the SOLO framework for a number of years in other curriculum areas; it was deemed logical to trial using the SOLO framework to capture the multidimensionality of reform mathematics lessons that would be essentially preparing students for engagement in reasoning and argumentation for NCEA. Even though NCEA does not start for most students until Year 11 the idea of working towards this alignment seemed logical.

4.5.1 Students' Views on Using the SOLO Rubric

At the interviews the researcher asked students whether the rubrics were helpful. 56% of all students' responses found the rubric useful and 44% didn't find it useful. Following are excerpts from the students that stated that the rubrics were helpful.

Student : Because so when it says like *I can explain one of the steps I took to solve a problem*, it's like I understand it because it is saying that like how did I solve the problem and I was explaining how I solved it and it was helping me understand like helping me to be confident. It is like a reminder to see, it really gave me confidence to stand up and speak to the class about why I think that is how to solve it.

It has given some students a trajectory of learning as a pathway to improve

Researcher: Have the rubrics been helpful to you?

Student : Yes kind of

Researcher: In what way?

Student : So I know where I am

Researcher: Give me an example

Student: Probably the building knowledge stage

Researcher: Why?

Student: Like I explain my idea of a problem

Researcher: What do you have to do next?

Student: I have to justify it

Researcher: How will that look?

Student: Umm I will use because

In this case, the student showed an understanding of the trajectory of mathematical practices. It was not a prescriptive view as they knew at this stage that they were able to explain their idea of a problem. They knew that their next step was to use the word *because* to justify their explanation. As a result of the rubric, this student was able to reflect on their learning process and know that their next step was to justify their explanation.

4.5.2 Teachers' Views on Assessment

Initially, the two teachers used the rubrics in different ways as is highlighted in the following extract from the interview with both teachers that was held half way through the study. Thoughts behind how the teachers viewed assessment in a reform mathematics setting highlighted two key issues. Firstly, is it possible to assess mathematical practices without them becoming prescriptive (R. Hunter, personal communication, October, 20, 2019)? Therefore, should mathematical practices be included in assessment practices? This idea is reflected by Teacher A's perspective. Secondly, the view that assessment drives instruction (Barnes et al., 2000; Schoenfeld, 2015). If mathematical practices are not assessed will they get the same weighting in classroom instruction as content? This reflects Teacher B's perspective. Schoenfeld ideas suggest that although the content of knowledge is assessed, it is hard presently to argue that mathematical practices are assessed in any meaningful way. Furthermore, he continues, if assessments provide

meaningful opportunities to demonstrate proficiency in areas such as communicating reasoning the instructional focus will broaden in desirable ways.

Teacher A used the rubric as a snapshot of where the students were at any given point. She saw the rubric helpful as a learning trajectory.

Teacher A: The rubric shows a snapshot of where they are at and where they should go to next to take it further and it shows them that aspirationally it's to get to generalisation but they might not always get there but that is where they are always aiming. The rubric shows them where the next learning is for them and how they can take it further. It is a snapshot of where they are, on that particular day.

Researcher: So is it possible to assess mathematical practices?

Teacher A: Mathematical practices isn't about being assessed, it's about kids knowing how to deepen their mathematical practices and how to take it to the next step it's not about them being assessed as it can change on any given day with whatever topic they are on they will get to different levels within that. It's that the kids know what they've got to do to get to the next thing. But I don't see it as being something that you can actually assess as this is where they are working. I don't think you can because on a different topic they might be working here or a different day they might be here.

Teacher A's thoughts on assessment highlights the interrelationship of the mathematical practices, thus making it hard to assess each as a separate entity. Hunter and Hunter (2018) state that mathematical practices overlap and interrelate. Furthermore, they contend that students should be able to first explain and represent their thinking before they are expected to justify and generalise. Likewise Askew (2016) mentions that explaining and justifying has to be as much something that is happening in the moment.

In contrast, Teacher B felt that unless mathematical practices were included in the assessment schedule they wouldn't get the same weighting as content. Her opinions align with findings from research studies about the influence of assessment on instruction (e.g., Barnes et al., 2000; Schoenfeld, 2015). For example Barnes et al. found that the content

of assessment has been seen to exert a strong influence over what is taught. Moreover, the aforementioned researchers contend that assessment can lead to teaching based on a restricted set of goals, which misrepresents richer expectations framed in curriculum documents.

Teacher B: In schools there are marks that are required for the assessment overview that need to be done for an overall teacher judgement for reporting. If we are wanting to use the key competencies and 21st century skills and focus on that as well as content on the same level - schools are not going to jump onto that unless there is a way that you can assess mathematical practices because it is never going to have the same weighting as content otherwise.

It is interesting to note that Teacher B used the word schools instead of teachers being the major decision makers here. This highlights her perception of the influence that leadership teams have on assessments and as a result, classroom practice. Her opinion reflects Barnes et al's. (2000) statement that changes in assessment policies can be used as a powerful lever for reforming schools mathematical instructional practices. Teacher B also saw mathematical practices as 21st century, key competency skills. Her opinion shows evidence as suggested by Barnes et al. that assessment portrays "the messages that a school system sends to teachers through means by which success is measured in public terms for teachers and students, and by which teachers are held accountable. Such mandated assessment arrangements shape teachers' beliefs about what is important, and bear more directly on the implemented curriculum than documentation alone" (p. 627). As Pegg (2003) and Barnes et al. argue, asking teachers to reform their mathematics class without aligning assessment with the current curriculum and content is sending teachers contradictory messages about what the system expects of them in mathematics.

If mathematical assessments only measure success as being content and computational skills then a missed opportunity for support for teachers like Teacher B to include mathematical processes as part of mathematics is missed. Inclusion of mathematical practices in the assessment process doesn't mean that these need to become prescriptions. Rather as Selling (2016) suggests mathematical practices can be made explicit after they have been used in an authentic way. In the current study, the use of a rubric supported

teachers and students to make statements as to whether arguments and reasoning were included in the explanations, justifications and/or generalisations. Thus the quality of response and development of the complexity of thinking of students could be valued and measured.

The following vignette shows deeper concerns that emerged between the two teachers about the focus for assessment. The timing of the intervention played a part in this discussion as grades and reports were needed to be written as it was Term Four. The issue of teaching to the test naturally occurred as is indicative of research (e.g., Biggs, 1998; Schoenfeld, 2015) highlighted in the review of the literature in Chapter Two. Teacher A acknowledged that ‘it is just the way that it is’ that teachers teach to the test. She believed that if you didn’t teach to the test some students would not do so well in the test. As an experienced mathematics teacher she is comfortable with knowing that in the long run they are better off with learning mathematical practices even though success is not measured in this way. The problem with this reasoning is that teachers are accountable to stakeholders for student results. Assessments are high stakes for both students and teachers (Barnes et al., 2000), and the research shows assessment dictates classroom practice as a result. Conversely, Teacher B indicates that she needed the curriculum content and assessment to align. Otherwise for her including mathematical practices was abstract in relation to reporting on progress.

Teacher A: Teachers will teach to the test and it does come down to that and that is just the way it is and it may be to the detriment that the kids don’t do so well on some of those tests if you don’t teach to the test, but you would hope in the long run it is actually better for the students to learn mathematical practices because they have a deeper understanding of maths.

Teacher B: I struggle with that because I struggle to do a task like that and be able to gather the data I need to make an assessment from the class tasks. I am not a confident mathematician and for me having those assessments and having the tasks I can go yep they’ve got it, or no they don’t but if I have to abstractly think about it...if I have to say my kids are here or here on the curriculum I will struggle with it because I haven’t got the yes or no answers that I usually have in other areas of the curriculum and that is

hard for me and that is something that I struggle with if I am to explicitly include mathematical practices in my pedagogy..

Even though Teacher B was developing skills in eliciting mathematical practices and her students were engaging in increased mathematical discourse, she strongly felt a need to stop using a reform approach for a week during the intervention as the approach was not aligned to the school's assessment schedule. She wanted to prepare her students for the end of year tests and felt she needed to give the students practice in rote learning and procedural assessments that would be used to measure their progress. She was accountable for the results of the tests and the tests measured narrow, procedural mathematics presented with multiple choice answers, not the broad creative, and growth mathematics that is so important (Boaler, 2016). Her thoughts and actions reflect findings from the numerous research articles referred to in the literature review stating that assessments always define the actual curriculum (Barnes et al, 2000; Biggs, 1998; Ramsden, 1992; Schoenfeld, 2015). Is this type of high stakes assessment a good basis to make judgments regarding the ability of the students in Teacher B's class to deal with 21st century daily life? These findings reinforce Serow et al's. (2016) question. Does the school's assessment policy guide teachers to assess what really needs to be assessed?' The answer is complex as can be seen by this example. Instructional practice has included explicit teaching of mathematical practices. However, the school's assessment policy sends the message to the students, teachers, and parents about what the school values in terms of success with mathematics. Learning facts and procedural skills only or the inclusion of skills in argumentation.

4.6 SUMMARY

By comparing students' views from before and after the intervention it is clear to see that students were beginning to see mathematics as more than just procedural fluency. The students were beginning to value talking and sense making as an integral part of the learning process. As a result of the current pedagogy and assessment procedures in New Zealand many students have a narrow view of what success in mathematics looks like for them. In this study the students either didn't know, or thought of success in mathematics as *learning more content*. This narrow view of success means that some students achieve success and others do not (Boaler, 2016). After the intervention when mathematical practices had been facilitated through promoting discussions within social

groupings, the students started to develop a broader criterion for what success in mathematics looks like. By students being able to identify that the learning process in mathematics includes learning to justify and explain their thinking; along with learning to interact and listen to mathematical discussions in groups, they were beginning to develop skills needed for NCEA mathematics at the secondary level. By engaging mindfully with reasoning and argumentation at the primary level sets primary students up for success as they move through the school system. Grades in NCEA are awarded on the complexity of thinking of mathematical practices. However, even though some teachers in primary schools are beginning to believe in the breadth of mathematics and value multidimensional mathematics in the classroom they are currently assessing students only whether they get correct answers to procedural questions. In order to produce more equitable outcomes and change the instructional focus to one of proficiency and strengths based mathematics, students need to be assessed on the many ways of working mathematically as identified by the students in this study. As Boaler highlights, if teachers used students' mathematics work rather than test performance – recording for example, whether they reason, justify, ask questions and build on each other's thinking, they are then assessing the multidimensionality of mathematics. When students are assessed on the many ways of working mathematically, many more students are successful.

Chapter Five: Conclusion

5.1 Conclusion

This research drew on students' and teachers' views and perspectives regarding the teaching, learning and assessment in a reform classroom where the teachers changed the instructional focus from one of deficiency to one of proficiency and equity. Based on qualitative analysis this thesis shows changes in students' reflections regarding their own learning process in mathematics over a period of three months. Data from student responses from the individual interviews shows that the majority of students initially viewed the learning process in mathematics as learning content. Students' perspectives on how to progress further in mathematics was also to learn content, just more of it. This view of the mathematics learning process aligns with Boaler's (2016) description of a more traditional one-dimensional teaching approach to mathematics, whereby mathematics is seen as a set of facts and procedures to be learnt from the teacher. After the intervention some students started to shift their perceptions of the learning process in mathematics to reflect a broader view. Included in their reflections were discussions regarding speech acts that are embedded within the mathematical practices. Articulating that being able to explain, justify and argue shows shifts in students' understanding of what it means to think mathematically. Additionally, students started to talk about the importance of the socio-cultural aspects of being part of a community of learners where they were beginning to value and enjoy sharing ideas and helping others to understand. Similarly, Enyedy (2003) brings to our attention that social interactions and intelligent activity are inseparable.

Hunter's (2009) study found that specific teacher actions led to shifts in the nature of students' participation. She found that students moved to become more critical active participants who engaged in productive discourse. Examined in this current study were the changes in teacher pedagogical actions identified by the students as being instrumental to shifting views of the mathematics learning process through an increasingly multidimensional lens. Clear examples are provided showing the pivotal role of the teacher in addressing how the students interacted collaboratively. For this group of students the sense of safety, trust and respect developed as a result of the social and socio-mathematical norms. The norms were identified by the students as an important factor to be able to participate in group work. Evident throughout the analysis

of observations of group work, shifts in both how students communicated mathematical thinking and the more equitable spread of mathematical discourse was noted. The pedagogy drawn from the Communication and Participation Framework (CPF) (Hunter, 2007) allowed small groups time to work on a task. Followed by combining to a larger inquiry group working on the same task changed the instructional focus from a more traditional pedagogy of getting the right answer to a focus on affording all students time to construct conceptual understandings. These observations align with Shah and Crespo (2018) views on pedagogy that compels students to collaborate. With this rearrangement supporting non-competitive individual and collaborative work, all the students have a role to play in a group worthy task.

The learning process in mathematics has traditionally reflected procedural understanding focused on number and computation. As a consequence, assessments typically reflect the testing of students' ability to regurgitate knowledge and facts. The pedagogy in this study sought to explicitly include mathematical practices along with number and computation as part of the learning process in mathematics. Therefore, it was deemed necessary to examine assessment in this reform intervention. This view aligns with the National Governors Association Center for Best Practices and Council of Chief State School Officers (2010) understanding that mathematical practices and procedural skills are equally important, and both should be assessed using mathematical tasks of sufficient richness. Some students found using the rubric helpful as a reminder to offer explanations and justify their claims during mathematics class. The rubrics were also identified as giving students confidence that the mathematical practices were a valued part of the learning process. Throughout the intervention the teachers became aware of the necessity to allow mathematical practices to develop naturally as part of the learning process rather than tease them apart prescriptively. Importantly, the results of this study in regard to assessment support the contentions made by Barnes et al. (2000) and Schoenfeld (2015) that assessment drives instructional practice. During the intervention in one of the classrooms, the reform pedagogy reverted back to traditional teaching of procedures and facts in preparation for mandated end of year high-stakes assessment. This finding is similar to observations highlighted in the recent New Zealand report on Trends in Assessment (Hipkins and Cameron, 2018). The report showed that teachers narrowed the focus of teaching mathematics in order to focus instead on technical points needed to lift

achievement to specific curriculum levels. Conclusions drawn from Hipkins and Cameron's study were that assessment has a direct influence on pedagogy.

5.2 Including Mathematical Practices in Primary Schools

Currently mathematical practices are subsumed in the assessment of content and often remain implicit and invisible in classrooms throughout most primary classrooms in New Zealand. Consequently, the narrowness by which success is measured means that some students rise to the top of classes, receive teacher praise and gain good grades, as others sink to the bottom, with most students knowing where they are in the hierarchy that this environment creates (Boaler, 2006). By the time students reach NCEA level at secondary school they have already been 'sorted' into 'who is good at mathematics' and 'who is not'. Stinson (2004) highlights the fact that 'sorting' students as young as eight years old can determine future opportunities and a higher education. The introduction of reform mathematics that draws out student's potential rather than highlights deficiencies in New Zealand primary schools represents a significant shift in educational discourse with the intention to narrow the attainment gap. Stinson (2004) describes these sentiments in his article where he brings to our attention how traditionally mathematics is used for gatekeeping to determine ability groups. Stinson suggests alternatively that gatekeeping in mathematics could be used for empowerment. Students just need the key to the gate. Results from this current study suggest that by disrupting the normal way mathematics is traditionally taught by co-constructing new norms and teaching students how to respect each other whilst developing skills in mathematical discourse in social groups, is a good start to giving students the key that Stinson refers to. Numerous researchers both in New Zealand (e.g., Anthony & Hunter, 2011, 2017; Hunter, 2007; 2009; Hunter & Hunter, 2018; Hunter et al., 2016) and internationally (e.g., Boaler, 2016; Lepak, 2014; Moschkovich, 2013; Serow et al., 2016; Shah & Crespo, 2018; Spelling, 2016; Whitenack & Yackel, 2002) suggest ways to reform mathematics pedagogy to reflect the multidimensionality of mathematics, thus giving more students the key to success in mathematics.

5.3 Including Mathematical Practices in Assessment

Mathematics reform-oriented curricula promoting equity under investigation in this study clearly demonstrates the inclusion of mathematical practices is an effective way of catering for diverse needs and drives achievement gains for all students. As discussed

current support for the inclusion of mathematical practices in classroom programmes in primary schools is beginning to emerge in New Zealand. However, reforming pedagogy without corresponding assessments denies students the ability to reflect on their learning process and in addition, severely hinders the implementation of reforming mathematics in primary schools.

Within this study attempts were made to incorporate mathematical practices as well as content in an assessment schedule. The Structure for Observable Learning Outcomes (SOLO) taxonomy was used in order to add a qualitative dimension sensitive to student cognitive development. This study found that for some students the SOLO rubric provided a shared language that offered support to recognise the subtle differences between the mathematical practices. These views align with how researchers (e.g., Boaler, 2016; Lepak, 2014; Loong et al., 2018; Van de Walle et al., 2019) used rubrics to support and measure the quality of the mathematical conversations that students participate in.

5.4 Implications and Opportunities for Future Research.

This study illustrates the significant shift in how Years Five and Six students viewed the mathematical learning process to include conceptual understanding and adaptive reasoning. By going beyond specifying content expectations by including mathematical practices embodies the principle in the New Zealand Curriculum Document of learning to learn mathematics for understanding. Many of the students in this study were able to identify the pedagogical actions that led to them being able to participate and communicate mathematically in an inquiry community. It would be worthwhile to examine the changing views of a larger group of students and teachers over a longer period of time for two reasons. Firstly, in regards to students. As this study was relatively short some of the students still held on to the view of mathematics as a narrow set of facts and procedures. It would be beneficial to gauge perceptions after a full school year. Moreover, it would be expected that after engaging with reform mathematics for a longer period students would develop a more complex view of mathematical practices and as a result develop deeper content knowledge. For example, engaging in more robust debates and developing skills in generalisation, Secondly, in regards to teachers. In this study the teachers were just beginning to implement a philosophy that was drawn from the Developing Mathematical Inquiry Communities (DMIC) approach. However, it would

be worthwhile to examine teacher actions and views after participation in the DMIC Professional Learning Development (PLD) project over a period of time. DMIC PLD is currently gaining momentum in New Zealand schools. The PLD offered through a DMIC approach encourages teachers to include mathematical practices as part of complex instruction. In addition, it would be highly informative to gauge perspectives from these teachers who have become skilled in implementing reform pedagogy regarding the role of assessment by using rubrics to support the development of mathematical practices. Further investigations into exploring aligning reform pedagogy and reform assessment is an important area of research.

The researcher could find no New Zealand research that examined reforming mathematics summative and formative assessments to support reform pedagogy. Serow et al.'s (2016) review of 21st century mathematics assessment found that research literature in mathematics assessment is notable by its absence. This may explain why even though there is a revolution currently occurring in New Zealand primary schools to reform pedagogy no such revolution has occurred yet with the alignment of assessment. However, some international researchers (e.g. Barners et al., 2000; Boaler, 2016; Hodge, 2008; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; Rand, 2004; Schoenfeld, 2015; Van de Walle et al., 2019) are highlighting the importance of aligning reform pedagogy with reform assessment. Moreover, Serow et al. state in their review of mathematics assessment that any material that has been published has been conducted by researchers outside the mathematics education community. One such research article that refers to mathematics assessment is from New Zealand researchers Hipkins and Cameron (2018). The aforementioned researchers state in their report of the 'Trends in New Zealand Assessment' regarding assessments across the curriculum areas, the fact that pedagogy and assessment should align and that assessments should be used for learning.

The Common Core State Standards include standards for mathematical practices in their guiding document for curriculum implementation. The inclusion of these proficiencies are seen as an integral component of all mathematics teaching, learning and assessment (Van do Walle et al., 2019). Further research would be warranted in New Zealand for supporting teachers to focus on equity and discourses by the inclusion of mathematical

practices in the Mathematics Achievement Objectives within the New Zealand Curriculum

5.5 Concluding Thoughts

The focus of this research was to offer a window inside a classroom that was embarking on a journey to change how mathematics was taught. The intention was to draw on research that shows that all students can learn mathematics. The data generated in this collaborative design study provided evidence that many students embraced using mathematical practices and felt more confident to participate when the narrow focus was opened to include the multidimensionality of mathematics. For this moment in time competition changed to allow time and space for the mathematical practices to grow within a collaborative environment. Changes need to be occurring in New Zealand Primary schools to address the problem of high numbers of students leaving the school system at the secondary level without an understanding of basic mathematical concepts. Currently mathematics becomes too difficult for students in the latter years of high school as they need to be able to offer explanations, justifications and skills in questioning and argumentation. Currently within New Zealand primary classrooms momentum is gaining for opportunities for teachers and students to engage with reform mathematics pedagogy. However, taking up space in mathematics classrooms are assessments that do not align with pedagogy taught. Researchers in Australia and the United States of America are addressing the issue of the centrality of assessments. Similarly, The New Zealand Centre for Educational Research has identified that alignments need to be made with pedagogy and assessment (Hipkins & Cameron, 2018).

This research thesis adds to the research literature focusing on including mathematical practices in primary schools and makes a contribution to the discussion regarding the centrality of assessment in a mathematics classroom.

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Appendices

Appendix A: Communication and Participation Framework

Communication and Participation Framework: Teacher actions to engage students in mathematical practices

<i>Developing conceptual explanations including using the problem context to make explanation experientially real</i>			
Model providing a mathematical explanation. Use the context of the problem not just the numbers.	Re-voice, and extend an explanation using the problem context. Expect mathematical reasons not "tidying" $19+7=20+6$ because $6+1=7$ and $19+1=20$	Question to scaffold students to extend their explanations to include the problem context and what they did to the numbers mathematically.	Model and support the use of questions which clarify an explanation. What do you mean by? What did you do in that bit? Can you show us what you mean by? Could you draw a picture of what you are thinking?
Have the students develop two or more ways to explain a strategy solution which may include using materials	Compare explanations and develop the norm of what makes an acceptable explanation. Reinforce what makes it mathematical	Launch the problem and have the students read it as a group, discuss, interpret, reinterpret collectively using student voice.	Shortly after the small groups begin to solve a problem as a large group have them describe their different starting point. Reinforce acceptability of multiple ways. Support them to make connections to other or previous problems.
Ask the students in small groups to examine their explanations and explore ways to revise, extend and elaborate on sections they think others might not understand			Have students examine their explanation, predict the questions they will be asked and prepare explanations.

<i>Active listening and questioning for sense-making of a mathematical explanation</i>				
Discuss and role-play active listening. Use inclusive language "show us", "we want to know", "tell us".	Structure the students explaining and sense making section by section.	Emphasise need for individual responsibility for sense-making	Provide space in explanations for thinking and questioning	Affirm models of students actively engaged and questioning to clarify sections or gain further information

<i>Collaborative support and responsibility for the reasoning of all group members: Use core Pasifika values</i>				
Provide students with problem and think-time then discussion and sharing before recording	Establish use of one piece of paper one pen. Explore family structures where everyone participates	Establish expectation that students agree on construction of a solution strategy that all members can explain.	Explore ways to support students indicating need to ask a question during large group sharing. Use no hands up or the use of <i>koosh balls etc.</i>	
Explore ways for the students to support each other using a range of cultural models e.g. all in the same waka paddling together, or a <i>kapa</i> haka group which requires the expert to be responsible to bring the group up to their level of expertise	Select a different member of the small group to explain	During large group sharing change the explainer mid explanation	When questions are asked of the small group select different members to respond	

<i>Developing justification and mathematical argumentation</i>				
Require that students indicate agreement or disagreement with part of an explanation or a whole explanation	Ask the students to provide mathematical reasons for agreeing or disagreeing with an explanation. Vary when this is required so that the students consider situations when the answer is either right or wrong.	Model and support the use of questions which lead to justification like 'How do you know it works?', 'Can you convince us', 'Why would that tell you to', 'why does that work like that', 'so what happens if you go like that', 'are you sure it's', 'so what happens if', 'what about if you say... does that still work', 'so if we		
Ask the students to be prepared to justify sections of their solution strategy in response to questions.	Require that the students analyse their explanations and prepare collaborative responses to sections they are going to need to justify	Model ways to justify an explanation "I know $3 + 4 = 7$ because $3 + 3 = 6$ and one more is 7".	Structure activity which strengthens student ability to respond to challenge	Encourage the use of 'so if', 'then', 'because' to make justifications. Use this format to validate an explanation
Expect that group members will support each other when explaining and justifying to a larger group	Explicitly use wait time or think time before requiring students to respond to questions or challenge	Require that the students prepare ways to re-explain in a different way an explanation to justify it.	Provide wait time to allow students to prepare questions which lead to justification	

<i>Developing representing as part of exploring and making connections (How can I/we make sense of this for my/ourselves) Communication and justification (How can I explain, show, convince other people)</i>			
Expect the use of a range of representations including acting it out, drawing a picture or diagram, visualising, making a model, using symbols, verbalising or putting into words, using materials.	Expect the students to explain and justify using the representation as actions on quantities not manipulation of symbols (use context)	Require that the students compare and contrast representations and evaluate for efficiency	Ask students to re-represent their thinking in different forms in response to questions or for clarification

<i>Developing the use of mathematical language</i>			
Expect the use of mathematical language to describe actions while making mathematical explanations	Expect the use of correct mathematical terms. Ask questions to clarify terms and actions on symbols (using the context).	Require the use of mathematical words to describe actions. Reword or re-explain mathematical terms and mathematical explanations. Use other examples to illustrate meaning.	Require students that the students pose questions using appropriate mathematical language.

<p align="center">Developing generalisations: Representing a mathematical relationship in more general terms. Looking for rules and relationships. Connecting, extending, reconciling.</p>		
<p>Ask the students to consider what steps they are doing over and over again and begin to make predictions about what is changing and what is staying the same.</p>	<p>Ask the students to consider if the rule or solution strategy they have used will work for other numbers. Consider if they can use the same process for a more general case. (e.g. what happens if you multiply any number by 2)</p>	<p>Model and support the use of questions which lead to generalisations:</p> <p><i>Does it always work?</i> <i>Can you make connections between...</i> <i>Can you see any patterns?</i> <i>Can you make connections between...</i> <i>How is this the same or different to what we did before?</i> <i>Would that work with all numbers?</i></p>

Appendix B: Principal and Board of Trustee information sheet and consent

Dear xxxxxx Primary School Principal and Board of Trustees,

As you know I am currently on study leave to complete a thesis for a Master of Education (Mathematics Education) at Massey University. I would like to undertake a research project at xxxxxx Primary School starting on 26th August 2019 to run for approximately ten weeks. I would like to ask your permission to work on this project.

My thesis will examine how using SOLO taxonomy as a learning and assessment tool supports children's development of mathematical content and mathematical practices such as explaining, reasoning, representing, justifying and generalising. The activities the children will be doing will be part of their normal classroom instruction that will bring together pedagogies that the school is currently using, trying to establish and wanting to sustain, such as: New Pedagogies for Deep Learning, SOLO taxonomy, Developing Mathematical Inquiry Communities, and visible learning.

Teacher 1 and Teacher 2 have expressed interest in the project and with your permission; I would like to invite the children in their classes to be involved with the consent of their parents/caregivers. The project will involve surveys, interviews and observations. I would also like to collect evidence of the children using language showing their development of the mathematical practices using a recording device. I will be working alongside the teachers for most mathematics classes for the ten week duration and the three of us will be planning the teaching sessions together. The children will be using journals for reflection purposes as is good practice and excerpts from the journals will be used as evidence of progress of the development of their mathematical practices.

All data (electronic files and copies of children's work) will be stored in a secure location, with no public access and used only for this research. In order to maintain anonymity the school name and names of all children/teachers will be assigned pseudonyms in any publications arising from this research. At the end of the year, a summary of the study will be provided to the school and made available for you to read.

If you have any further questions about this project you are welcome to discuss them with me personally or contact me through my email: jaynef@xxxxxx.school.nz

If you are happy for this project to be undertaken at xxxx Primary School, please complete the attached consent form.

Kind regards

Jayne Fitzgerald 027 XXXX XXX

Massey University

"This project has been evaluated by peer review and judged to be low risk. Consequently it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named in this document are responsible for the ethical conduct of this research. If you have any concerns about the conduct of this research that you want to raise with someone other than the researcher(s), please contact Professor Craig Johnson, Director (Research Ethics), email humanethics@massey.ac.nz".

BOT and Principal Consent Form

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to the study taking place at XXXXX Primary School

Signature:

Date:

.....

**Full Name -
printed**

.....

APPENDIX C : Teacher Information Sheet and Consent Form

Dear Teacher A and Teacher B,

As you know I am currently on study leave to complete a research project for a Master of Education (Mathematics Education) at Massey University. My thesis is a study examining how using SOLO Taxonomy can support students to access and assess mathematical practices.

Together we have discussed the need for the children to be able to monitor their learning in mathematics. This study seeks to align pedagogy with assessment in order for children to be able to use 'assessment for learning'. Now I am formally inviting you to be part of a collaborative teaching design experiment research process, in which we investigate using SOLO as an 'assessment for learning' tool. We will also be examining how the instructional environment and tasks support children to develop the discourse inherent when using mathematical practices.

Permission to participate in the study will be sought from both the children in your class and their parents/caregivers. The children and their parents/caregivers will be given full information about the study, and consent forms will be requested.

The interview and observations will take place in the classroom and be part of the normal mathematics programme. Two individual interviews will explore the children's knowledge of their learning and progress in mathematics. One interview will take place next week, (week starting 26th August) at the beginning of the project and the second in the middle of Term 4. The time involved for each interview will be no more than 10 minutes. The interview with each child will be audio recorded.

We will plan a unit of work together to start the following week that will involve using SOLO taxonomy as a teaching and learning tool for students to monitor their progress in mathematics. The lessons will form the basis of the classroom mathematics programme for the duration of the study.

Small group discussions that involve participating children, may be audio recorded. If you are playing a part in these discussions e.g. talk moves, then permission will be sought from you before your comments are included. You may at any time ask that the audio recorder be turned off and any comments you made deleted.

A collection of copies of the children's mathematics reflections, written work and charts may be observed or video recorded to support evidence of student use of mathematical

practices. The mathematics activities they do in the class will be the same whether they agree to be in the study or not as discussed with you in the ethics peer review stage of the research. No evaluation of the instructional programme will occur other than that which is grounded in the context of the study.

All data (electronic files and copies of children's work) will be stored in a secure location, with no public access and used only for this research. In order to maintain anonymity the school name and names of all children/teachers will be assigned pseudonyms in any publications arising from this research. All efforts will be taken to maximize your confidentiality and anonymity which means your name will not be used in this study and only non-identifying information will be used in reporting. However, total anonymity cannot be guaranteed due to my position as both a researcher and a current member of staff. Near the end of the study a summary will be presented to you to verify accuracy, and following any necessary adjustments, a final summary will be provided to the school and teachers involved.

Please note that you have the following rights in response to the request to participate in this study:

- decline to participate;
- in any lesson, you have the right to ask for the audio tape to be turned off at any time;
- withdraw from the study at any point;
- ask any questions about the study at any time during participation;
- provide information on the understanding that your name will not be used unless you give permission to the researcher;
- be given access to a summary of the project findings when it is concluded.

If you have further questions about this project you are welcome to discuss them with me personally:

Jayne Fitzgerald. Email. jaynef@xxxx.school.nz Cell Phone: 027 XXXX XXX

or contact my chief supervisor at Massey University

- Dr Jodie Hunter: Senior Lecturer. Institute of Education. Massey University. Phone (09) 4140800 Extension 43518. Email. j.hunter1@massey.ac.nz

Kind regards

Jayne Fitzgerald

Massey University

"This project has been evaluated by peer review and judged to be low risk. Consequently it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named in this document are responsible for the ethical conduct of this research. If you have any concerns about the conduct of this research that you want to raise with someone other than the researcher(s), please contact Professor Craig Johnson, Director (Research Ethics), email humanethics@massey.ac.nz".

Teacher Participants Consent Form

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to being audio-taped

I agree/do not agree to participate in this study under the conditions set out in the Information Sheet.

Signature: Date :

Full Name – printed

Appendix D : Student and Parent Information Sheet and Consent Form

Dear _____ & _____

I am doing a research project for a Master of Education at Massey University. I am going to look at how children can use assessment to help them to be able to explain, justify and generalize when solving problems in mathematics.

I would like to invite you with your parent's permission to be involved in this study. Mrs xxxx, your teacher has also agreed to participate in the study. The Board of Trustees has also given their approval for me to invite you to participate, and for me to do this research from 26th August to the middle of Term 4 2019.

If you agree to be involved. I will interview you about your knowledge of your learning and progress in mathematics. There will be two interviews one next week and one in the middle of next term. The time involved in the interview will be no longer than ten minutes. The interview will be tape-recorded and you can ask that the tape-recorder be turned off and that any comments you have made be deleted if you change your mind or are not happy about what you have said.

Your teachers and I will plan a unit of work to start the following week that will involve you using SOLO taxonomy as a way for you to monitor your progress in mathematics. The lessons will be taught in your classroom and some small group discussions that you have, may be audio or video recorded. You may at any time ask that the audio or video recorder be turned off and any comments you made deleted. With your permission I might sometimes collect copies of your mathematics reflections, written work and charts you make to support your explanations to the group. You have the right to refuse to allow the copies to be taken.

The mathematics activities you do in the class will be the same whether you agree to be in the study or not. The interview and observations will take place in the classroom and be part of the normal mathematics programme. It is possible that talking about your learning may help you clarify what you know about your mathematics learning and progress in learning to explain, justify and generalize your thinking.

All data (electronic files and copies of children's work) will be stored in a secure location, with no public access and used only for this research. In order to maintain anonymity the school name and names of all children/teachers will be assigned pseudonyms in any publications arising from this research. All efforts will be taken to maximize your confidentiality and anonymity which means your name will not be used in this study and only non-identifying information will be used in reporting. However total anonymity

cannot be guaranteed due to my position as both a researcher, and (even though I am on study leave this year) a current member of staff.

I ask that you discuss all the information in this letter fully with your parents before you give your consent to participate.

Please note that you and your parents have the following rights in response to the request to participate in this study:

- to say that you do not want to participate in the study;
- withdraw from the study at any point;
- to ask for the audio or video recorder to be turned off and any comments you may have made be deleted;
- to refuse to allow written copies of your work to be taken;
- to ask any questions about the study at any time during participation;
- to participate knowing that you will not be identified at any time;
- be given access to a summary of the project findings when it is concluded.

If you have further questions about this project you are welcome to discuss them with me personally:

Jayne Fitzgerald. Cell: 027 xxxxxx. Email jaynef@xxxxxx.school.nz

or contact my chief supervisor at Massey University

- Dr Jodie Hunter

Kind regards

Jayne Fitzgerald

Massey University

"This project has been evaluated by peer review and judged to be low risk. Consequently it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named in this document are responsible for the ethical conduct of this research. If you have any concerns about the conduct of this research that you want to raise with someone other than the researcher(s), please contact Professor Craig Johnson, Director (Research Ethics), email humanethics@massey.ac.nz. "

Student Participants Consent Form

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to being audio-taped

I agree/do not agree to being videotaped

I agree/do not agree to participate in this study under the conditions set out in the Information Sheet.

Child’s Signature: Date :

Full Name – printed

Parents of Student Participants Consent Form

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to being audio-taped

I agree/do not agree tobeing videotaped

I agree/do not agree toparticipate in this study under the conditions set out in the Information Sheet.

Parent’s Signature: Date :

Full Name – printed

Appendix E Agreed Upon Norms as Generated Throughout the Intervention

