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**Characterization and Flow of Food and Mineral  
Powders**

**A thesis presented in partial fulfillment of the  
requirements for the degree of**

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**Hornng Yuan Saw**

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## Abstract

Powders are important commodities across different industries, such as the food and pharmaceutical industries. In these industries, powders are usually made, mixed, milled, packaged, and stored; these operations require the powders to move and flow under desired conditions and different stress levels. Failure to flow will cause hindrances to production; therefore knowledge of powder flow or flowability is important. There is a constant demand for accurate, reliable, and robust measurement and characterization methods for powder flowability.

Powders behave differently under varying conditions; the behaviour of a powder is influenced by particle size distribution, and powder handling and processing conditions. There is to date no one “standard” method to characterize powder flowability; it is common to use a variety of methods and devices to measure flow properties and provide insight into the behaviour and flow characteristics of powders under different conditions.

The flow properties of model food and mineral powders were measured and assessed by shear testing, compression via tapping, fluidization, and powder tumbling. Shear testing was done with an annular shear cell following Jenike (1964) and Berry, Bradley and McGregor (2014). Compression via tapping was performed according to a procedure in the dairy industry (Niro, 1978) and the European Pharmacopoeia (Schüssele & Bauer-Brandl, 2003). Fluidization was used to measure powder bed expansion and bed collapse following the powder classification framework provided by Geldart and co-workers (Geldart, 1973; Geldart, Harnby, & Wong, 1984; Geldart & Wong, 1984, 1985). Powder tumbling was performed in a novel *Gravitational Displacement Rheometer*, GDR, which measured the motion and avalanche activity of powders that moved under their own weight when rotated in a cylinder at different drum speed levels. The flow data from each characterization method were evaluated individually with regards to particle size distribution and then assessed collectively. The findings presented and discussed include the i) demonstration of the dominant influence of surface-volume mean particle diameter on powder flow properties, ii) characterization of flowability based on Jenike’s arbitrary flow divisions, iii) development of new correlations for the estimation of powder cohesion and bulk density at low preconsolidation stresses, iv) demonstration of hopper outlet diameter as a measure of flowability, v) demonstration of the limited utility of Hausner ratio as a flowability index, vi) substantiation of von Neumann ratio as a sensitive and useful indicator for identifying the onset of bubbling in fluidized beds using bed pressure fluctuation data, and vii) demonstration of the utility of standard deviation of the GDR load cell signal as an indicator of powder avalanche activity. These findings provide improved understanding and knowledge of powder flowability; they can be used to assist and facilitate the development of new techniques and solutions relevant to the handling and processing of powders especially in the food and pharmaceutical industries.

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## Nomenclature

$A_p$	Surface area of material [m <sup>2</sup> ]
$a_s, b_s$	Fitting parameters of Equations 3.8 and 3.9 [units according to usage]
$a_t, b_t$	Fitting parameters of Equations 4.1 [units according to usage]
$B$	Minimum width of outlet required for mass flow from a hopper [m]
$C$	Cohesion [Pa]
$C^*$	Estimated cohesion [Pa]
$C_0$	Cohesion at zero preconsolidation stress [Pa]
$c_1, c_2$	Fitting parameters of Equation 4.9 [units according to usage]
$d_{pi}$	Incremental mean particle diameter [m] which is the mean of the sum of upper and lower nominal apertures in sieve analysis
$d_{10}$	Particle diameter at 10% in a cumulative size distribution [m]
$d_{32}$	Surface-volume mean particle diameter [m]
$d_{32,M}$	Surface-volume mean particle diameter measured with Mastersizer 2000 [m]
$d_{32,S}$	Surface-volume mean particle diameter measured with sieve analysis [m]
$d^*_{32}$	Surface-volume mean particle diameter calculated with Mastersizer data using bins equivalent to a full sieve analysis according to BS 410; powder in the range of 0–38 $\mu\text{m}$ has been grouped together and assigned a mean particle diameter of 19 $\mu\text{m}$ in the calculation [m]
$d_{50}$	Particle diameter at 50% in a cumulative size distribution [m]
$d_{90}$	Particle diameter at 90% in a cumulative size distribution [m]
FF	Powder flow function [-]
$F_{10}$	Fraction of fines smaller than 10 $\mu\text{m}$ calculated with Mastersizer data [-]
$F_{20}$	Fraction of fines smaller than 20 $\mu\text{m}$ calculated with Mastersizer data [-]
$F_{30}$	Fraction of fines smaller than 30 $\mu\text{m}$ calculated with Mastersizer data [-]
$F_{38}$	Fraction of fines smaller than 38 $\mu\text{m}$ calculated with Mastersizer data [-]
$F_{45}$	Fraction of fines smaller than 45 $\mu\text{m}$ calculated with Mastersizer data [-]
$ff$	Jenike hopper flow factor [-]
$g$	Gravitational acceleration [m s <sup>-2</sup> ]
$H$	Powder bed height [m]
$H_{mf}$	Bed height at incipient fluidization [m]
$H_R$	Hausner ratio [-]
$H_{R,1250}$	Hausner ratio at 1250 taps [-]
$H^*_{R,1250}$	Hausner ratio estimated with Equation 4.9 and $c_1$ and $c_2$ values in Table 4.2 [-]
$i$	Label for data point [-]
$k_1, k_2$	Fitting parameters of Equation 3.7 [units according to usage]

$k_{C1}, k_{C2}$	Fitting parameters of Equation 4.10 [units according to usage]
$k_{F1}, k_{F2}$	Fitting parameters of Equation 4.15 [units according to usage]
$k_{G1}, k_{G2}$	Fitting parameters of Equations 3.14 and 3.15 [units according to usage]
$k_{J1}, k_{J2}$	Fitting parameters of Equations 3.12 and 3.13 [units according to usage]
$k_{N1}, k_{N2}$	Fitting parameters of Equations 3.10 and 3.11 [units according to usage]
$k_{s,M1}, k_{s,M2}$	Fitting parameters of Equation 3.6 [units according to usage]
$k_{t,M1}, k_{t,M2}$	Fitting parameters of Equation 4.2 [units according to usage]
$m_0$	Powder mass in the loose poured state [g]
$m_{\text{tap}}$	Powder mass after $N^{\text{th}}$ taps [g]
$N$	Number of taps [-]
$n$ (of Chapter 3)	Shear index of Warren-Spring equation [-]
$n$ (of Chapter 5)	Number of data points [-]
$n_1, n_2$	Fitting parameters of Equation 4.5 [units according to usage]
$P$	Applied pressure [Pa]
$T$ (of Chapter 3)	Powder tensile strength [Pa]
$T^{-1}$ (of Chapter 5)	Inverse of von Neumann ratio, Equation 5.2 [-]
$t_c$	Time required for hindered settling, Equation 5.3 [s]
$U$	Superficial gas velocity [ $\text{m s}^{-1}$ ]
$U_{\text{bv}}$	Minimum vigorous bubbling velocity [ $\text{m s}^{-1}$ ]
$U_{\text{mb}}$	Minimum bubbling velocity [ $\text{m s}^{-1}$ ]
$U_{\text{mb,v}}$	Experimental minimum bubbling velocity detected by visual inspection of bed surface [ $\text{m s}^{-1}$ ]
$U_{\text{mb},\sigma}$	Minimum bubbling velocity estimated using the plot of $\sigma:U$ and determining $U$ for $\sigma=0$ by extrapolation [ $\text{m s}^{-1}$ ]
$U_{\text{mf}}$	Minimum fluidizing velocity [ $\text{m s}^{-1}$ ]
$V_{\text{B}}$	Bulk volume of material [ $\text{m}^3$ ]
$V_{\text{Initial}}$	Initial volume of powder bed in the GDR after the powder was shaken horizontally and vertically for an unreported fixed number of times and allowed to settle under its own weight [ $\text{m}^3$ ]
$V_{\text{New}}$	Volume of powder in GDR measured at the first 11 revolutions [ $\text{m}^3$ ]
$V_{\text{p}}$	Volume of material [ $\text{m}^3$ ]
$x$ (of Chapter 5)	Sample variable; bed pressure drop [Pa]
$x_i$ (of Chapter 2)	Volume fraction of particles in $i^{\text{th}}$ mean particle diameter range in sieve analysis [-]
<i>Greek letters</i>	
$\Delta P_{\text{b}}$	Bed pressure drop [Pa]

$\Delta P_d$	Distributor pressure drop [Pa]
$\Phi_w$	Kinematic angle of wall friction [°]
$\delta_e$	Effective angle of internal friction [°]
$\varepsilon_{mb}$	Bed voidage at bubbling onset [-]
$\varepsilon_{mf}$	Bed voidage at incipient fluidization [-]
$\mu$ (of Chapter 3)	Coefficient of friction [-]
$\mu$ (of Chapter 5)	Gas viscosity, Equation 5.3 [N s m <sup>-2</sup> ]
$\theta_a$	Minimum angle for which avalanches are observed [°]
$\theta_p$	Semi-included angle of the conical section of a hopper [°]
$\theta_s$	Minimum angle which triggers powders to slip [°]
$\rho_0$	Initial or loose poured bulk density [kg m <sup>-3</sup> ]
$\rho_{0,mNZS3111}$	Loose poured bulk density measured by the modified NZS3111 method [kg m <sup>-3</sup> ]
$\rho_B$	Bulk density [kg m <sup>-3</sup> ]
$\rho_g$	Gas density [kg m <sup>-3</sup> ]
$\rho_p$	Particle density [kg m <sup>-3</sup> ]
$\rho_{tap}$	Tapped density [kg m <sup>-3</sup> ]
$\rho^*_{0,1}$	Loose poured bulk density estimated with Equation 4.1 and $a_t$ and $b_t$ values in Appendix 4.1 [kg m <sup>-3</sup> ]
$\rho^*_{0,2}$	Loose poured bulk density estimated with Equation 4.1, $a_t$ values in Appendix 4.1 and $b_t=0.0427$ , the average value determined with the data of milled and spray-dried lactose powders, sand, and refractory dust [kg m <sup>-3</sup> ]
$\rho^*_{0,3}$	Loose poured bulk density estimated with Equation 4.2 and $k_{t,M1}$ and $k_{t,M2}$ values in Appendix 4.1 [kg m <sup>-3</sup> ]
$\sigma$ (of Chapter 3)	Consolidation stress [Pa]
$\sigma$ (of Chapter 5)	Standard deviation, Equation 5.1 [Pa]
$\sigma_c$	Major consolidation stress [Pa]
$\sigma_{crit}$	Critical stress developed in an arch surface [Pa]
$\sigma^*/\sigma^*_y$	Estimated ratio of major consolidation stress to unconfined yield stress [-]
$\sigma_D$	Major stress developed in a dome or pipe [Pa]
$\sigma_{pre}$	Preconsolidation stress [Pa]
$\sigma_{pre,min}$	Minimum preconsolidation stress [Pa]
$\sigma_{ws}$	Standard deviation of GDR drum weight shift [kg]
$\sigma_{ws,5RPM}$	Standard deviation of GDR drum weight shift at 5 RPM [kg]
$\sigma_{ws,10RPM}$	Standard deviation of GDR drum weight shift at 10 RPM [kg]
$\sigma_{ws,15RPM}$	Standard deviation of GDR drum weight shift at 15 RPM [kg]
$\sigma_{ws,20RPM}$	Standard deviation of GDR drum weight shift at 20 RPM [kg]
$\sigma_y$	Unconfined yield stress [Pa]

$\tau$	Shear stress [Pa]
$\tau_{\text{pre}}$	Constant shear stress at preshear [Pa]
$\tau_{\text{ss}}$	Steady-state shear stress [Pa]
$\omega$	Angular velocity [ $\text{rad s}^{-1}$ ]