Mathematical Model of the Forced Cooling of Anodes used in the Aluminium Industry

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ABSTRACT

The aluminium industry consumes large amounts of electrodes, especially anodes, to operate the smelters. These anodes must be baked at high temperatures in order to give them certain mechanical and electrical properties, after which they are cooled. Baking is done in large furnaces made up of pits inside which the anodes are placed in layers and surrounded by packing coke. The furnaces are of two types - open and closed. In a closed furnace, the pits are lined with refractory bricks inside which flues run vertically and large covers are used to close over parts of the furnace.

This thesis presents a mathematical model of part of a forced cooling section of a closed furnace, where air is being sucked or blown through the flues by fans, so that the anodes cool more rapidly. Both one- and two-dimensional models are developed in order to calculate the transient temperature distribution in the anodes, packing coke and side flue wall. For the two-dimensional model, the transient temperature and pressure distributions of the air in the side wall flues and fire shafts are also calculated. After exploring an analytical method for the one-dimensional case, numerical techniques are used thereafter.

Given initial block and air temperatures, the two-dimensional model allows calculation of the appropriate temperature and pressure distributions for various mass flows of air in the side wall flues and fire shafts. The results show that for a sufficiently high mass flow, the anodes can be cooled enough so that they can be safely removed from the pits after three fire cycles (the length of time the anodes are exposed to forced cooling).

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CONTENTS

NOMENCLATURE

Alternative uses are separated by a semi-colon. Dimensions are in square brackets.

A, B	defined functions
A_n , B_n, C_n, D_n	constants
a, b, c, d, e, f	lengths associated with a flue and a flue wall [m]
Bi	Biot number [–]
c_m, m = 1 to 9	constants
c_p, cf_p	specific heat capacity at constant pressure of the block and the fluid or air [J/kg K]
ca_p, cp_p, cw_p	specific heat capacity at constant pressure of the anode, packing coke and flue wall [J/kg K]
E, e	total and internal energy per unit mass of the block [J/kg]
ea, ep, ew	internal energy per unit mass of the anode, packing coke and flue wall [J/kg]
estTf	estimated fluid temperature [K]
F, f	defined functions
Fo, $\hat{Fo}$	Fourier [–] and 'hatted' Fourier number of the block [1/K]
Foa, Fop, Fow	Fourier number of the anode, packing coke and flue wall [–]
G	defined function
h	enthalpy per unit mass of the block [J/kg]; heat transfer coefficient [W/m^{2}K]
hc, hr	heat transfer coefficient for convection and radiation [W/m^{2}K]
k	thermal conductivity of the block [W/mK]; real constant
ka, kp, kw	thermal conductivity of the anode, packing coke and flue wall [W/mK]
kf	thermal conductivity of the fluid [W/mK]

ℓ	characteristic length [m]
L_x	width of the block [m]
L_y	depth of the block [m]
$\dot{m}$	mass flow of the fluid [kg/s]
max.	maximum
min.	minimum
Nu	Nusselt number [–]
p	pressure in the block; pressure of the fluid or air [N/m^2]
percentdiff	percentage difference between fluid temperatures
pfl, pfs	pressure of the fluid in a flue and a fire shaft [N/m^2]
Pr	Prandtl number [–]
$\dot{Q}$	rate of heat flow per unit area of the block [J/sm^2]
$\dot{Q}$w	rate of heat flow per unit area at the flue wall [J/sm^2]
Re	Reynolds number [–]
S	surface of an elemental volume in the block [m^2]
T	temperature of the block [K]
t	time [s]
Ta, Tp, Tw	temperature of the anode, packing coke and flue wall [K]
Tf	fluid or air temperature [K]
V	elemental volume in the block or a flue [m^3]
v	velocity of the fluid [m/s]
W	width of a 'large' flue [m]
X	defined function
x, y, z	spatial coordinates

Greek

α, $\hat{\alpha}$	thermal diffusivity [m^2/s] and 'hatted' thermal diffusivity of the block [m^2/sK]
αa, αp, αw	thermal diffusivity of the anode, packing coke and flue wall [m^2/s]
Γ_1, Γ_2	defined functions
Δp	change in the pressure of the fluid; pressure difference across a fan [N/m^2]
ΔQ	heat transferred from the block in time Δt [J]
ΔQa, ΔQp, ΔQw	heat transferred from the anode, packing coke and flue wall in time Δt [J]
ΔQf	heat gained by the fluid or air in time Δt [J]
Δt	overall time step length [s]
Δta, Δtp, Δtw	length of time step in the anode, packing coke and flue wall [s]
Δx	distance between mesh points in the x-direction [m]
Δxa, Δxp, Δxw	distance between mesh points in the anode, packing coke and flue wall [m]
Δy	distance between mesh points in the y-direction [m]
ε	emissivity of the flue wall [–]; roughness of the flue wall [m]
Θ_1, Θ_2, Θ_3, θ	defined functions
Λ_1, Λ_2, Λ_3	defined functions
λ	real constant; defined function; friction factor [–]
μ	dynamic viscosity of the fluid [kg/ms]
ρ	density of the block [kg/m^3]
ρa, ρp, ρw	density of the anode, packing coke and flue wall [kg/m^3]
ρf	density of the fluid [kg/m^3]

σ	Stefan-Boltzmann constant [W/m^2K^4]
τ	integration variable
ψ	defined function

Subscripts

i, i = 1 to Nx	mesh points in the x-direction
j, j = 1 to Ny	mesh points in the y-direction
p	constant pressure [N/m^2]
q, q = 1 to Nxa	mesh points in the x-direction in the anode
r, r = 1 to Nxp	mesh points in the x-direction in the packing coke
s, s = 1 to Nxw	mesh points in the x-direction in the flue wall
w	flue wall
n	natural number

Superscripts

n	time steps, n = 1 to Nt

CHAPTER 1 INTRODUCTION

1.1 Background

The aluminium industry consumes large amounts of electrodes, especially anodes, to operate the smelters. These carbon anodes are made of petroleum coke held together by a pitch binder. They must be baked to a given temperature, approximately 1200 ^{o}C, following a given temperature profile (no more than 10 -15 ^{o}C/hour) in order to end up with the required mechanical and electrical properties. Baking is done in large furnaces made up of pits inside which the unbaked anodes are placed in layers and surrounded by packing coke. These furnaces are of two types, one of which is the Riedhammer (vertical ring, or closed) furnace, a schematic of which is shown in Figure 1.1.

In the Riedhammer furnace, the pits are lined with refractory bricks inside which flues run vertically and large covers are used to close over parts of the furnace. A typical Riedhammer furnace consists of two or three fire trains grouped together on a rectangular shaped ring. As shown in Figure 1.1, each fire train comprises about fourteen sections, or sets of pits, and consists of three zones - preheat, fire and cooling zone. Hot combustion gases flow through the flues in the fire and preheat zones, whilst air flows through the flues in the cooling zone.

The cooling zone is divided into two parts - natural and forced cooling. In the natural cooling part, the anodes are just left to cool. The forced cooling sections have big fans which either blow or suck air through the flues to increase the rate of cooling of the anodes (see Figure 1.1). There is one fan per section. The rate of cooling is not constrained and may be done as quickly as possible. The fans, fire ramps and exhaust manifold are moved in the fire direction by one section every 32 or 36 hours. This time period is called the fire cycle.

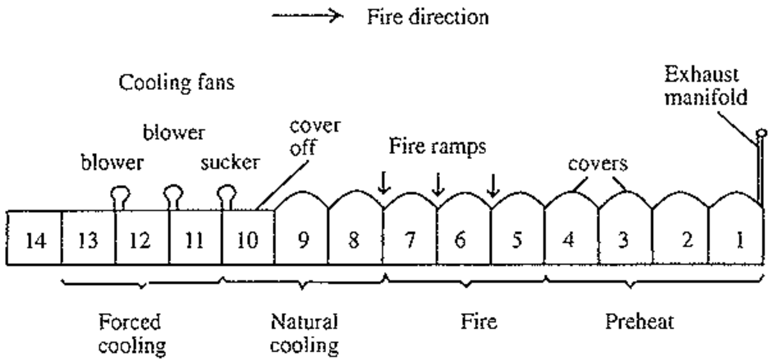

Figure 1.1
A schematic longitudinal view of a typical fire train arrangement in a Riedhammer furnace at NZ Aluminium Smelters Ltd (see Bourgeois *et al.*, 1990).

1.2 Previous work

Mathematical modelling of ring furnaces started seriously in 1980 with Furman and Martirena (1980). Since then others have developed different aspects of ring furnace (open and closed) mathematical modelling. The advent of computational fluid dynamics packages has enabled the development of more elaborate models. Due to the ring furnace's large dimensions and time constant ($2\frac{1}{2}$ - 3 weeks), experimentation on a real furnace is not only impractical, but also risky, lengthy and costly. Hence the need for mathematical models in order to analyse and predict performance.

Furman and Martirena (1980) used a three-dimensional finite difference model. The total duration of the baking cycle was simulated. Therefore the time period was long, 300 - 400 hours. In order to save computation time, most of the time steps used were correspondingly long. The first 10 hours were divided in steps of 0.1, 0.2, 0.5, 2.2, 3.0 and 4.0 hours, all further steps were 7.0 hours long. To ensure the stability of the calculations with these long temporal steps, an implicit Crank-Nicholson scheme was used. The equations were solved using successive relaxation. 960 nodes were used - 4 in the y-direction (x-direction in this thesis), 20 in the z-direction (y-direction in this thesis) and 12 in the x-direction (z-direction in this thesis). A 'sensitivity' analysis was performed. This involved introducing a significant variation of a given property and observing the corresponding temperature calculations. It was found that the thermal conductivity of the packing coke and the vertical gradient of the gases' temperature along the flues were the parameters which decisively influenced the calculations. It was claimed, therefore, that these were the only parameters which needed to be known accurately. The temperature-dependent thermal properties of the anodes, packing coke, flue walls and gases were not adjusted as the temperature varied.

The paper by de Fernández *et al.* (1983) used the identical set of nodes as used in the paper of the previous paragraph, so it was a three-dimensional model. However, no mention was made of the solution method. Initially the temperature difference between the top and bottom layer of anodes in a pit was calculated. It was found that this difference was in much closer agreement with the experimental difference, when the thermal properties of the anodes, packing coke and flue walls were adjusted according to the temperature reached at the end of the last time step. The 'negative' image of the heating temperature distribution was used to try and model the cooling temperature distribution. It did not work and this was attributed to two factors, one of which was the occurrence of natural convection. It was suggested that a battery of fans be used on uncovered sections in order to overcome this natural convection. Clearly, fans were not being used at this particular smelter when this paper was written.

The transient two-dimensional model presented by Bourgeois *et al.* (1990) neglected the heat transfer in the longitudinal direction, that is, from the fire shaft to head wall. This helped to simplify the model and keep CPU time down. It was claimed that experimental studies had shown that this longitudinal temperature variation was small compared with the flue wall to anode-centre and vertical variation. Unlike the models from the earlier papers, this one incorporated pressure measurements, namely the draught profile along a fire train. One of the limitations of the model was that it could not determine the temperatures in the forced cooling sections (they were considered disconnected from the main fire train). There was fairly good agreement between calculated and experimental results.

Bui *et al.* (1992) divided a furnace section into four zones - fire shaft, under-lid, pit and under-pit. For gas flow distribution, it was stated that the underlid zone was the most important; whereas for heat transfer to, and therefore presumably from, the anodes, the pit zone was the most important. A three-dimensional model of heat transfer and fluid flow for the under-lid zone of any section of the fire train was developed. For the purposes of validation, the under-lid zone of the first covered cooling section was simulated. The solution procedure was not discussed in detail, but the general purpose computational fluid dynamics PHOENICS code was used as a solver. A larger number of nodes was used, namely 23180. The calculated results followed reasonably well the trend of the measured ones.

1.3 This work - an outline

A heat transfer and pressure distribution model of part of a forced cooling section (from now on called section) of the Riedhammer furnace is presented in this thesis. The cover has been removed, the packing coke is still in place and a blowing or sucking fan (blower or sucker) is positioned over the fire shafts - see Figure 1.2. Each uncovered section is a separate entity disconnected from the main fire train.

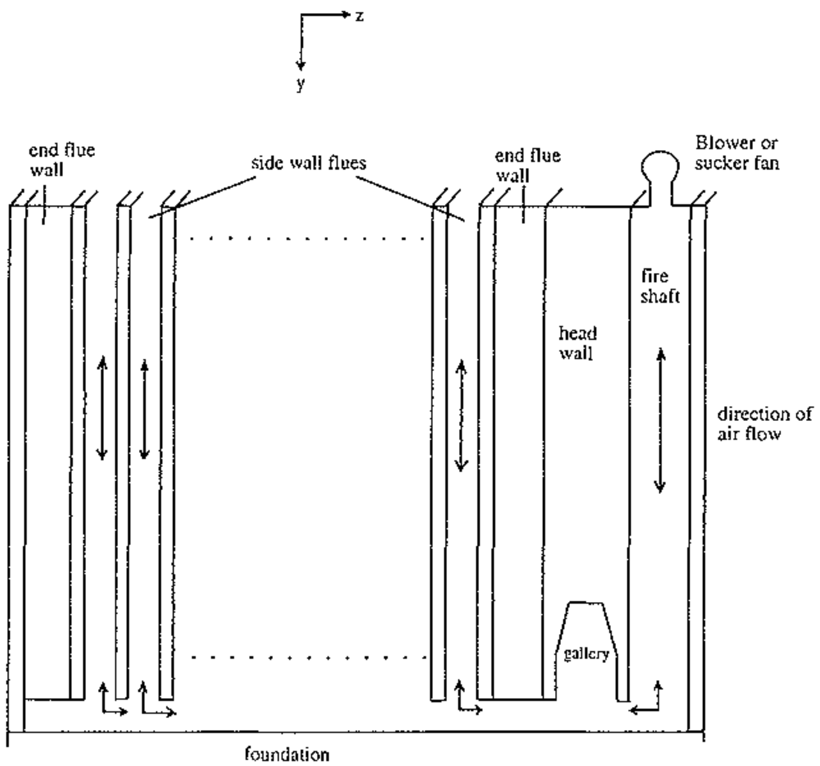

Figure 1.2
Schematic longitudinal view of a flue wall in a forced cooling section

The model presented here is used to determine the effect that different mass flows of air have on:

(a) the block (anodes, packing coke and side flue wall) temperature,
(b) the air temperature in the side wall flues, and
(c) the air pressure in the side wall flues and the fire shafts.

Heat is transferred from the anodes into the air in the side wall flues via the packing coke and the side flue wall. The transfer of heat is taken to be by conduction in the anodes and flue wall, and is assumed to be by conduction in the packing coke. The heat is then transferred into the air by convection and radiation from the surface of the flue wall.

The section is three-dimensional. Simplified one-dimensional and two-dimensional models are studied in Chapter 2. This is done by concentrating on the anodes, packing coke, side flue walls and side wall flues part of the section. Simplifications involved in developing the models are justified by a result from Bui *et al.* (1992) and dimensional considerations.

In Chapter 3, the heat conduction equation is derived in the present context, to give a partial differential equation. The consequences of assuming constant thermal conductivities or otherwise is examined. Using the models from Chapter 2, boundary conditions are then added to the partial differential equations to give boundary value problems.

The thermal properties of the anodes, packing coke and side flue walls are discussed and calculated in Chapter 4.

The one-dimensional heat equation is solved analytically for three different sets of boundary conditions in Chapter 5. The last set are those of the model developed in Chapter 2.

In Chapter 6, explicit numerical methods are used to solve the boundary value problem. Using these, the problem is solved for the case where the thermal conductivities are constant within the anodes, packing coke and side flue wall, to give the transient temperature distribution in the block. In this constant thermal conductivities case, the thermal conductivities are the only thermal property that is not being adjusted as the temperature varies with time; whereas in the non-constant case all thermal properties are being adjusted as the temperature varies with time (it is assumed these properties are dependent on temperature). For the non-constant thermal conductivities case, the boundary value problem is set up, but not solved due to the introduction of non-linear terms. For this one-dimensional case, the air temperature in the side wall flues is assumed to be constant.

The two-dimensional model is developed in Chapter 7. Only the constant thermal conductivities case is considered. This builds on the work done in the previous chapter on the one-dimensional model. Unlike the one-dimensional model, the temperature of the air in the side wall flues is changing with time and space as heat is transferred into it from the block. This is modelled using an implicit numerical method and combined with the two-dimensional heat equation for the block to give the transient one-dimensional temperature distribution of air along the flues and the transient temperature distribution in the block. As a check on the working, the heat given out by the block and the heat gained by the air in the flues is calculated. Because of the set-up of the model, these should be approximately equal if the calculations are done correctly.

The transient one-dimensional pressure distribution in the side wall flues and fire shafts is calculated in Chapter 8.