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Resolving Decomposition By Blowing Up Points And Quasiconformal Harmonic Extensions

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Samuel Adam Kuakini Dillon

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Abstract

In this thesis we consider two problems regarding mappings between various two-dimensional spaces with some constraint on their distortion.

The first question concerns the use of mappings of finite distortion that blow up a point where the distortion is in some L^p class; in particular, we are interested in minimal solutions to the appropriate functional. We first prove some results concerning these minimal solutions for a given radially symmetric metric (in particular the Euclidean and hyperbolic metrics) by proving a theorem which states the conditions under which a minimizer exists, as well as providing lower bounds on the L^p-norm of the function. We then apply these results to the problem of resolving decompositions that arise in the study of Kleinian groups and the iteration of rational maps. Here we prove a result concerning for which values of p we can find a mapping of a particular form which shrinks the unit interval and whose inverse has distortion in the L^p space.

The second is in regards to the Schoen conjecture, which expresses the hope that every quasisymmetric self-mapping of the unit circle extends to a homeomorphism of the disk which is both quasiconformal and harmonic with respect to the hyperbolic metric. The equation for a harmonic map between Riemann surfaces with given conformal structures is a nonlinear second order equation; one wishes to solve the associated boundary value problem. We show here that the existence question can be related to a nonlinear inhomogeneous Beltrami equation and discuss some of the consequences; this result holds in more generality for other conformal metrics as well.

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