MATHEMATICAL MODELLING OF UNDERGROUND FLOW PROCESSES IN HYDROTHERMAL ERUPTIONS

A THESIS PRESENTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY
IN
MATHEMATICS

AT MASSEY UNIVERSITY, PALMERSTON NORTH
NEW ZEALAND.

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October 2000

Abstract

This thesis reports on a study of underground fluid flow and boiling processes which take place in hydrothermal eruptions. A conceptual model is presented for the eruptive process and a laboratory scale physical model confirming the effectiveness of this process is described. A mathematical formulation of the underground flow problem is given for two fluid flow regimes: two-phase homogeneous mixture (HM) flow and separable two-phase (SP) flow. Solutions to the system of equations obtained are solved under the simplifying assumptions of two-dimensional steady isothermal flow and transient non-isothermal horizontal flow. The main contribution of the study on steady isothermal flows is a description of how the ground flow may recover following a hydrothermal eruption. A numerical technique developed for plotting the streamlines in this case (and verified against analytic results) may also have applications in solving the steady non-isothermal flow problem. The main contribution of the study on the transient horizontal flow problem is a comparison of the differing predictions of HM and SP flow. The rate at which a boiling front progresses through a porous medium and the degree of boiling which occurs is described for each fluid flow regime. A set of horizontal physical experiments and numerical simulations have also been carried out for comparison with the mathematical model. Qualitative results for these three models agree. Suggestions given for improvements to the design of the physical experiment provide a basis for future study into the type of flow which occurs in hydrothermal eruptions.

Acknowledgements

- I would like to express my special gratitude to my chief supervisor Professor Robert McKibbin for providing me with a topic to explore that was both exciting and provided a range of modelling experiences (conceptual, mathematical and experimental). I highly valued his guidance, patience, and encouragement throughout this work and leave with respect for his contribution to this work and to the modelling of hydrothermal eruptions in general.

- I would like to thank the staff in the Institute of Fundamental Sciences, particularly those in the Mathematics Discipline, for creating a friendly and supportive environment in which to work.

- I am also grateful to my fellow mathematics graduate students for their invaluable contributions to the sanity of the author. In particular, I would like to thank Chris Palliser, Nick Allsop, Anton Raviraj, Mary Day, Mark Johnston, Tracey Flux, and Deborah Kelly. It's been fun.

- Thanks to Mark Johnston for the help he has given me in all things LaTeX.

- The physical experiments detailed in Sections 2.2 and 5.3 were done with funds provided by the Massey University Graduate Research Fund, the contribution of which is gratefully appreciated.

- Thanks are due to Sarah Codd for her assistance with the NMR experiments and interpretation. I would also like to thank the Institute of Fundamental Sciences workshop staff, in particular Noel Foote and Grant Platt for their help with the design and construction of experimental work.

- Thank you to Massey University for the Doctoral Scholarship and to the Mathematics Department and later the Institute of Fundamental Sciences for the Graduate Assistant and Casual Assistant work. This financial assistance and teaching experience is very much appreciated.
- Thanks Mom and Dad for starting me on this journey many years ago and for the innumerable kindesses (love, support, unwavering belief in me, financial aid in times of need) that have always sustained me.
- Finally, this thesis is dedicated with infinite appreciation and love to my husband, Brian. Thank you for bringing your smile into my life.

Nomenclature

All constants and variables used in this thesis are defined when they first appear in the text. Commonly used notation is summarized here. Units are as given below unless otherwise stated within the text.

Roman

A_e	energy per unit volume of formation, Jm^{-3}
A_m	fluid mass per unit volume of formation, $kg\ m^{-3}$
c	heat capacity, $Jkg^{-1}K^{-1}$
c_F	Forchheimer coefficient, *dimensionless*
$\mathbf{e}_r$, $\mathbf{e}_\theta$	unit vectors in the r, θ directions respectively, *dimensionless*
$\mathbf{g}$, g	acceleration due to gravity, ms^{-2}
h	specific enthalpy, Jkg^{-1}
k	permeability, m^2
k_{rg}, k_{rl}	relative permeability of the gas, liquid phases respectively, *dimensionless*
K	thermal conductivity, $Wm^{-1}K^{-1}$
p	pressure, Pa
$\mathbf{Q}_e$	energy flux per unit area, $Jm^{-2}s^{-1}$
$\mathbf{Q}_m$	mass flux per unit area, $kg\ m^{-2}s^{-1}$
r	radial coordinate, m
S	liquid saturation, *dimensionless*
t	time, s

T	temperature, K
u	specific internal energy, Jkg^{-1}
$\mathbf{u}$	pore-averaged velocity, ms^{-1}
$\mathbf{v}$	volume flux per unit area (Darcy velocity), ms^{-1}
$\mathbf{w}$	phase-averaged velocity, ms^{-1}
x	horizontal coordinate, m
z	vertical coordinate, m

Greek

$\eta = x/\sqrt{t}$	similarity variable, $ms^{-1/2}$
θ	angular coordinate, *radians*
μ	dynamic viscosity, $kg\ m^{-1}s^{-1}$
$\nu = \mu/\rho$	kinematic viscosity, m^2s^{-1}
ρ	density, $kg\ m^{-3}$
ϕ	porosity, *dimensionless*
Φ	velocity potential function, *units dependent on coordinate system*
Φ_m	specific mass flux potential function, *units dependent on coordinate system*
Ψ	velocity stream function, *units dependent on coordinate system*
Ψ_m	specific mass flux stream function, *units dependent on coordinate system*

Subscripts

f	fluid mixture
f	condition placed on the boundary $\eta = 0$ in the numerical experiment described in Section 5.2
i	condition placed on the boundary $\eta \to \infty$ in the numerical experiment described in Section 5.2
l	liquid phase
r	rock matrix
v	vapour phase
t	partial derivative with respect to time t
x	partial derivative with respect to horizontal distance x

Superscripts

$','' $	first and second derivatives with respect to similarity variable η

Table of Contents