Analysis and Application of the Spectral Warping Transform to Digital Signal Processing

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Abstract

This thesis provides a thorough analysis of the theoretical foundations and properties of the *Spectral Warping Transform.* The spectral warping transform is defined as a time-domain-to-time-domain digital signal processing transform that shifts the frequency components of a signal along the frequency axis. The z-transform coefficients of a warped signal correspond to z-domain 'samples' of the original signal that are unevenly spaced along the unit circle (equivalently, frequency-domain coefficients of the warped signal correspond to frequency-domain samples of the original signal that are unevenly spaced along the frequency axis). The location of these unevenly spaced frequency-domain samples is determined by a z-domain mapping function. This function may be arbitrary, except that it must map the unit circle to the unit circle.

It is shown that, in addition to the frequency location, the bandwidth, duration and amplitude of each frequency component of a signal are affected by spectral warping. Specifically, frequency components within bands that are expanded in frequency have shortened durations and larger amplitudes (conversely, components in compressed frequency bands become longer with smaller amplitudes).

A property related to the expansion and compression of the duration of frequency components is that if a signal is time delayed (its digital sequence is prepended with zeroes) then each of the frequency components will have a different delay after warping. This time-domain separation phenomenon is useful for separating in time the frequency components of a signal. Such

separation is employed in the generation of spectrally flat chirp signals. Because spectral warping will generally expand the duration of some frequency components within a signal, the transform must produce more output samples than there are (non-zero) input samples in order to avoid time-domain aliasing. A discussion of the necessary output signal length is presented.

Particular attention is given to spectral warping using all-pass mapping function, which can be realised as a cascade of all-pass filters. There exists an efficient hardware implementation for this all-pass SW realisation [1, 2]. A proof-of-concept application-specific integrated circuit that performs the core operations required by this algorithm was developed.

Another focus of the presented research is spectral warping using a piece-wise-linear mapping function. This type of spectral warping has the advantage that the changes in frequency, duration and amplitude between the non-warped and warped signals are constant factors over fixed frequency bands.

A matrix formulation of the spectral warping transformation is developed. It presents the spectral warping transform as a single matrix multiplication. The transform matrix is the product of the three matrices that represent three conceptual steps. The first step is to apply a discrete Fourier transform to the time-domain signal, providing the frequency-domain representation. Step two is an interpolation to produce the signal content at the desired new frequency samples. This interpolation effectively provides the frequency warping. The final step is an inverse DFT to transform the signal back into the time domain. A special case of the spectral warping transform matrix has the same result as a linear (finite-impulse-response) filter, showing that spectral warping is a generalisation of linear filtering. The conditions for the invertibility of the spectral warping transformation are derived.

Several possible realisation of the SW transform are discussed. These include two realisation using parallel finite-impulse-response filter banks and a realisation that uses a cascade of infinite-impulse-response filters.

Finally, examples of applications for the spectral warping transform are

given. These include: non-uniform spectral analysis (and signal generation), approximate spectral analysis in the time domain, and filter design.

This thesis concludes that the SW transform is a useful tool for the manipulation of the frequency content of digital signals, and is particularly useful when the frequency content of a signal (or the frequency response of a system) over a limited band is of interest. It is also claimed that the SW transform may have valuable applications for embedded mixed-signal testing.

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Nomenclature

The following is an index of symbols used in this thesis, with the page number of their first occurrence.

a	Spectral warping factor. May be complex, but is real for most practical situations.	23
a^*	Complex conjugate of a.	23
A, A_0, θ_0	Parameters for the chirp z-transform.	103
A_1, A_2	Constants.	13
a_n	n^{th} zero of a high-order all-pass mapping function.	30
a_I	The imaginary component of a.	25
a_R	The real component of a.	25
$B(\omega_{\max}, \omega_{\min})$	Unwarped signal's bandwidth.	34
$B(\hat{z})$	An M^{th}-order all-pass function.	72
$\hat{B}(\hat{\omega}_{\min}, \hat{\omega}_{\max})$	Warped signal's bandwidth.	34
$B_{hanning}$	Width of main lobe of a Hanning window function.	36
$\mathbf{C}_{M,N}$	The $N \times M$ frequency interpolation matrix.	49
$c_{m,n}$	The n^{th} element of the m^{th} row of $\mathbf{C}_{M,N}$.	59
$\mathbf{D}_M$	The M-point DFT matrix.	46
$\delta(t)$	Dirac-delta (impulse) function.	6
$\delta[n]$	Unit impulse at $n = 0$.	7
$\mathcal{E}(x, \tilde{x})$	Error ratio between some function x and its approximation $\tilde{x}$, defined as $\lvert x-\tilde{x}\rvert / x$.	35
$\mathcal{F}\{\}$	The Fourier transform operator.	15
$F(\omega)$	Discrete-time Fourier transform of $f[n]$.	36
$f[n]$	Discrete-time input (unwarped) sequence.	20

$\mathcal{F}^{-1}\{\}$	The inverse Fourier transform operator.	16
$\hat{f}[m]$	Discrete-time output (warped) sequence.	20
$\mathbf{f}, \mathbf{f}_N$	Input vector.	47
$G[m]$	Discrete Fourier transform of $g[m]$.	45
$G[m]\Big\vert_{m=i}$	A single sample in the warped frequency domain.	49
$g[m]$	Discrete-time output (warped) sequence.	46
$\mathbf{g}, \mathbf{g}_M$	Warped output vector.	47
$\mathbf{H}_{M,N}$	The $M \times N$ SW z-transform matrix.	46
$\|H_n(\omega)\|$	Magnitude gain response for N^{th}-order warping; $N = 1$ if omitted.	44
i, k	Integers.	54
$\mathbf{I}_N$	The $N \times N$ identity matrix.	68
L	Length of window sequence.	36
M	Integer number of output samples.	29
$\mathbf{m}$	Vector of row (output) indices.	47
N	Integer number of input samples.	47
n	Integer used as a discrete time domain index; typically used for input (unwarped) sequences.	20
N_0, N_1, N_2	Specific sample locations of the input sequence.	36
$\hat{N}_0, \hat{N}_1, \hat{N}_2$	Specific sample locations of the output sequence.	40
$\hat{N}_{\text{first}}$	The location of the first non-zero sample of the output sequence.	42
$\hat{N}_{\text{last}}$	The location of the last non-zero sample of the output sequence.	42
ω	Angular frequency in the z-domain.	20
ω_0	Centre frequency of a narrow-band signal.	35
$\omega_m, \hat{\omega}_m$	The m^{th} element of the vector $\boldsymbol{\omega}$ or $\hat{\boldsymbol{\omega}}$, respectively.	30
$\omega[m]$	The sequence of frequencies at which the Fourier transform of the input signal is evaluated.	45
$\hat{\omega}$	Angular frequency in the $\hat{z}$-domain.	20
$\omega_{\text{min}}, \omega_{\text{max}}$	Lowest and highest frequency components of a unwarped signal.	34
$\hat{\omega}[m]$	The sequence of warped frequencies at which the Fourier transform of the output sequence is evaluated.	59
$\hat{\omega}_{\text{min}}, \hat{\omega}_{\text{max}}$	Lowest and highest frequency components of a warped signal.	34
$\boldsymbol{\omega}, \hat{\boldsymbol{\omega}}$	Vector representation of the sequences $\omega[n]$ and $\hat{\omega}[m]$.	29
$S(\omega_{\text{max}}, \omega_{\text{min}})$	Bandwidth distortion function for first-order real warping.	34

$S(\omega_{\max}, \omega_{\min}, \hat{\omega}_{\max}, \hat{\omega}_{\min})$	Bandwidth distortion function.	34
$\mathbf{S}_{M,N}$	The $N \times M$ spectral warping transform matrix.	47
$\mathbf{s}_{M,n}$	The n^{th} column of $\mathbf{S}_{M,N}$.	76
$\mathbf{s}_{m,N}$	The m^{th} row of $\mathbf{S}_{M,N}$.	79
$s_N(\omega_{\min}, \omega_{\max})$	N^{th}-order duration distortion function; $N = 1$ if omitted.	39
$\tilde{S}(\omega_0)$	Approximate bandwidth distortion function for first-order real warping.	35
$\tilde{s}_N(\omega_0)$	Approximation of $s_N(\omega_{\min}, \omega_{\max})$.	39
τ	Time shift.	13
$T, \hat{T}$	The non-zero duration of unwarped and warped signals, respectively.	39
T_s	The sample period.	52
$\Theta_N(\omega)$	N^{th}-order z-domain mapping (warping) function; $N = 1$ if omitted.	20
$\theta_N(\omega)$	N^{th}-order frequency mapping (warping) function; $N = 1$ if omitted.	20
$\mathbf{U}$	The non-uniform DFT matrix.	72
W, W_0, ϕ_0	Parameters for the chirp z-transform.	103
W_M	$e^{-j2\pi/M}$.	46
$\mathbf{X}$	Matrix representation of $X[z_k]$.	72
$\mathbf{x}$	Matrix representation of $x[n]$.	72
$x(t)$	Continuous-time (output) signal.	13
$X(z)$	Fourier transform of $x[n]$.	72
$X(\hat{z})$	Warped Fourier transform of $x[n]$.	72
$x[n]$	A discrete-time sequence.	71
$X[z_k]$	The non-uniform DFT of $x[n]$.	71
$x^{\mathbf{y}}, x^{\mathbf{Y}}$	Element-wise vector/matrix exponent operation.	29
$\hat{X}[k]$	Warped DFT of $x[n]$.	73
$y(t)$	Continuous-time (input) signal.	13
$y[m]$	A discrete-time sequence.	13
$\mathbb{Z}$	The set of all complex numbers.	45
z	The complex frequency variable.	20
$\hat{z}$	The warped complex frequency variable.	20
$\otimes$	The convolution operator.	9