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A Mathematical Analysis of Reaction Diffusion Systems in Chemical and Biological Reactors with Macro and Micro Structures

A Thesis presented in partial fulfilment of the requirements for the degree of

Doctor of Philosophy

at

Massey University

by

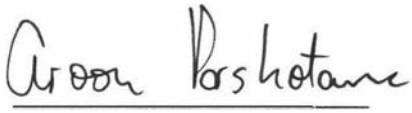
Aroon A. Parshotam, MSc(Hons), MPhil

1992

To my parents Kanti and Olga Parshotam and to my loving wife Kavita

Declaration

This thesis is submitted to Massey University and has not been submitted for a higher degree to any other University or Institution.



Aroon Parshotam

March, 1992

Acknowledgements

I wish to express my sincere gratitude to my supervisors, Prof. Graeme Wake, Dr. Alex McNabb and Assoc. Prof. Rao Bhamidimarri.

Thanks are due also to Dr. Vijay Bhaskar for his help during the numerical part of this thesis and to collegues Dr. Robert Sisson, Dr. Paul Bonnington, Robert Crawford and Mark Byrne for supporting me during this study.

Finally, I wish to thank my parents Kanti and Olga Parshotam for supporting us over all those years of study and my wife Kavita for showing such great love and understanding, particularly during the final stages of my PhD.

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Abstract

This thesis is concerned with generalised models for biological and chemical reactors such as the tubular, fluidised, fixed, packed, continuously stirred and trickle bed reactors.

Suppose n chemical components at concentrations C_i (i = 1,2,..,n) are "diffusing" and reacting in a homogeneous incompressible fluid with a known velocity profile $u(z)$ independent of C_i so that in the reactor region Λ, *div* u is zero. Immersed in the fluid may be a uniformly distributed population of particles which absorb these chemicals and act as local sites for reaction-diffusion phenomena. The particles are sources and sinks for the chemicals C_i in the fluid and these fluid concentrations govern the boundary conditions for the particle or local behaviour.

A system of equations is set up as a general model for these complex interactions. The principle limitations of this model are firstly that $u(t, z)$, the velocity profile in Λ is known and not coupled with the concentrations C_i in any way, and secondly the particles are assumed to be fixed relative to the coordinate system of z in Λ and sufficiently small so that a representative sample of them can be taken to be in a spatially constant concentration environment in Λ.

The objectives of this thesis are generalised comparison theorems for these systems which are used to prove uniqueness, existence, stability and other general qualitative features of such models. A number of examples from literature are examined.

Models conforming to the system described in this thesis have applications in biological wastewater treatment, biochemical manufacture, urea removal by the compact artificial kidney and industrial fermentation processes. Other potential modelling areas concern fertiliser or pollutants diffusing in soil moisture and reacting with soils, oxidation with product formation in waste deposits and industrial ore reduction processes. There are many other industrial and environmental problems with similar interacting macro and micro structures. These include the catalytic cracking and synthesis processes in chemical industries ranging from the making of synthesis gas from coal to oil refining.