

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

**Estimating and Evaluating the Archimedean-Copula-Based Models  
in Financial Risk Management**

A Dissertation Submitted in Fulfillment of the Requirements  
for the Degree of Doctor of Philosophy in  
Financial Economics

Massey University, Auckland,  
New Zealand

Qing Xu

2008

To my family

## Abstract

Copula is used to model multivariate data, as it accounts for the dependence structure and provides a flexible representation of the multivariate distribution. Recently a large number of Archimedean copulas have been proposed to deal with various dependence aspects in financial risk management, which invokes several new questions in some important yet under-researched areas. These questions, therefore, need further investigation.

This dissertation comprises three essays and probes into three untouched questions all involving the Archimedean-copula-based models. The first essay studies whether the Archimedean-copula-based portfolio value-at-risk (PVaR) model outperforms the Gaussian-copula-based PVaR model in out-of-sample forecasting. My empirical findings in this essay show that the Archimedean-copula-based PVaR model, especially the Clayton copula-based model, has better forecasting performance than the Gaussian-copula-based PVaR model in most cases in both the in-sample and out-of-sample periods. In addition, the data snooping problem (i.e., model risk) associated with the copula-based PVaR model is also explored.

The second essay examines the question of how to evaluate the non-Gaussian multivariate density forecasts. In this essay, I propose a test procedure, by using the likelihood ratio test based on the Kullback-Leibler information criterion, to evaluate the Archimedean-copula-based multivariate density forecasts, and apply the procedure to foreign exchange markets. The test procedure is not only conducive to fully ranking competing sophisticated models with the non-Gaussian-distributed multivariate densities, but also allows for model misspecification in both marginal and copula functions under the null and the alternative hypothesis.

The third essay focuses on this question: Will the PVaR estimation be improved if the Archimedean copula model takes into account conditional asymmetric tail dependence and time-varying investors' heterogeneous beliefs? I use the conditional skewed-t distribution (as the marginal function) to represent time-varying investors' heterogeneous beliefs, and employ three two-parameter Archimedean copulas to investigate dynamic asymmetric tail dependence between two of three Asian developed futures markets. My results provide strong evidence that such conditional copula models can improve the PVaR estimation and so a greater amount of diversification benefits can be reaped at a higher confidence level.

## Acknowledgements

I would like to express my heartfelt gratitude to my chief supervisor, Associate Professor Xiaoming Li, for his great encouragement and patience. I was fortunate to receive many perceptive and insightful comments from him, which had a significant impact on the final draft of the dissertation. I also wish to thank my co-supervisors, Dr. Abdullah Mamun and Dr. Ozughan Dincer, for their guidance, and other staff members of Commerce Department for their help and support. Meanwhile, my special gratitude goes to Professor Charles Corrado, who was my internal examiner, for his insightful comments. In addition, I owe my special thanks to Massey University Graduate Research School whose financial support (Doctoral Scholarship) has enabled me to complete this dissertation.

Several papers based on the work reported in this dissertation have been presented at international conferences and accepted by an academic journal for publication. Specifically, therefore, I am also indebted to the participants at the 13<sup>th</sup> Conference on the Theories and Practices of Securities and Financial Markets, the 14<sup>th</sup> Annual Symposium of the Society for Nonlinear Dynamics and Econometrics, and the 2007 International Symposium on Financial Engineering and Risk Management (FERM2007), and the referees of *Applied Financial Economics*, for their helpful suggestions.

Finally, as always, my greatest debts of gratitude are due to my family – my father Genlin Xu, my mother Mei Wang, and my wife Meili Sun– for their love and unfailing support. I would therefore like to dedicate this dissertation to them.

However, none of the above individuals hold responsibility for the opinions expressed, or for any remaining errors and omissions, which are my own.

# Table of Content

<b>Abstract</b>		I
<b>Acknowledgements</b>		III
<b>Tables</b>		VI
<b>Figures</b>		VIII
<b>Chapter 1</b>	<b>Introduction</b>	1
	1.1 Motivation	1
	1.2 Outline of the Dissertation	9
<b>Chapter 2</b>	<b>Copula, Rank Correlation, and Tail Dependence</b>	13
	2.1 Definition of Copula	13
	2.2 Elliptical Copulas	14
	2.3 Rank Correlations	19
	2.4 Archimedean Copulas	21
	2.5 Tail Dependence	25
<b>Chapter 3</b>	<b>Evaluating the Out-of-Sample Forecasting Performances of the Archimedean-Copula-Based Portfolio VaR Models</b>	27
	3.1 Introduction and Literature Review	27
	3.2 Copula-Based Portfolio VaR	34
	3.2.1 Portfolio VaR	34
	3.2.2 Estimating the standardized quantile by copula method	35
	3.2.2.1 Copula functions	35
	3.2.2.2 Standardized quantile estimation	40
	3.3 Hansen's (2005) SPA Test	40
	3.3.1 Loss function	41
	3.3.2 SPA test	41
	3.4 Empirical Results	43
	3.4.1 Data	43
	3.4.2 In-sample model estimation	51
	3.4.3 Copula-based PVaR estimation	58
	3.4.4 In-sample fitting performance	61
	3.4.5 Out-of-sample forecasting performance	65
	3.5 Summary and Conclusions	69
	Appendix 3.1	71

<b>Chapter 4</b>	<b>A Test for Evaluating the Archimedean-Copula-Based Multivariate Density Forecasts in Foreign Exchange Markets</b>	72
4.1	Introduction	72
	4.1.1 Motivation	72
	4.1.2 Literatures review	73
	4.1.3 Design of the test procedure	75
4.2	KLIC for Multivariate Density Forecast Model	80
4.3	Constructing the Copula-Based Multivariate Density	82
4.4	Hansen's (2005) SPA Test	83
4.5	Empirical Results	84
	4.5.1 Data	84
	4.5.2 In-sample ML estimation	88
	4.5.3 In-sample CBMD model performance	96
	4.5.4 Out-of-sample CBMD model forecast evaluation	98
4.6	The Economic Value of the Archimedean CBMD Forecast Evaluation	106
4.7	Conclusions	110
<b>Chapter 5</b>	<b>Estimating Dynamic Asymmetric Tail Dependences with Time-Varying Investors' Heterogeneous Beliefs in Asian Developed Futures Markets</b>	112
5.1	Introduction and Literature Review	112
5.2	Data	117
	5.2.1 Preliminary analysis	117
	5.2.2 Informal evidence of asymmetric tail dependence	118
5.3	Models	121
	5.3.1 Conditional Archimedean copula	121
	5.3.2 Conditional tail dependence	123
5.4	Estimation Method	124
	5.4.1 Two-stage maximum likelihood estimator	124
	5.4.2 Marginal model	125
5.5	Empirical Results	126
	5.5.1 Dynamic marginal distributions	126
	5.5.2 Dynamic asymmetric tail dependences	133
	5.5.3 Model evaluation	139
5.6	Portfolio VaR and Diversification Benefit	140
	5.6.1 Portfolio VaR	140
	5.6.2 Monte Carlo simulation for the standardized quantile	141
5.7	Concluding Remarks	149
<b>Chapter 6</b>	<b>Conclusions</b>	150
6.1	Summaries	150
6.2	Limitations and Further Research	155
<b>Bibliography</b>		157



## Tables

Table 2.1	Bivariate Copulas	16
Table 3.1	Summary Statistics of Price Index Returns	45
Table 3.2	Linear and Rank Correlations	46
Table 3.3	In-Sample ML Estimation of the TGARCH (1, 1) Model With Different Distributions of Standardized Returns	54
Table 3.4	Goodness-of-Fit Test Statistics for Different Distributional Restrictions	55
Table 3.5	In-Sample ML Estimation of Copulas with Different Marginals	56
Table 3.6	In-Sample Copula-Based PVaR Estimates	59
Table 3.7	Out-of-Sample Copula-Based PVaR Estimates	60
Table 3.8	Bootstrap $p$ -Values of the In-Sample SPA Test	64
Table 3.9	Bootstrap $p$ -Values of the Out-of-Sample SPA Test	68
Table 4.1	Summary Statistics of Daily Exchange Rate Returns	86
Table 4.2	In-Sample ML Estimation of the AR(1)-GARCH(1, 1) Model with Different Distributions of Standardized Errors	90
Table 4.3	Goodness-of-Fit Tests for Different Distributional Restrictions	91
Table 4.4	In-Sample ML Estimation of Copulas	93
Table 4.5	Results of the In-Sample SPA Test Based on the KLIC Loss Function	100
Table 4.6	Results of the Out-of-Sample SPA Test Based on the KLIC Loss Function	101
Table 5.1	Preliminary Analysis of Index Futures Returns	120
Table 5.2	Multivariate Normality Test for the Bivariate Returns	121
Table 5.3	Parameter Estimates of the TGARCH(1, 1) Model and the Conditional Skewness and Kurtosis	128

Table 5.4	Summary Statistics and Goodness-of-Fit Test for the Filtered Returns	129
Table 5.5	Parameter Estimates of the Unconditional Copulas	135
Table 5.6	Parameter Estimates of the Conditional Two-Parameter Archimedean Copulas	136
Table 5.7	Summaries of the Conditional Tail Dependence and the Time-Varying Parameters for the Conditional Two-Parameter Archimedean Copulas	138
Table 5.8	Results of the SPA Test	144
Table 5.9	Portfolio VaR and Diversification Benefit	144

## Figures

Figure 1.1	Relationship between the Marginal Distributions and the Multivariate Distribution	6
Figure 2.1	Contours of the Elliptical-Copula-Based Joint Density and the Three-Dimensional Plots of the Densities of the Elliptical Copulas	18
Figure 2.2	Relationships between Linear and Rank Correlations	20
Figure 2.3	Contours of the One-Parameter Archimedean-Copula-Based Joint Density and the Three-Dimensional Plots of the Densities of the One-Parameter Archimedean Copulas	23
Figure 2.4	Contours of the Two-Parameter Archimedean-Copula-Based Joint Density and the Three-Dimensional Plots of the Densities of the Two-Parameter Archimedean Copulas	24
Figure 3.1	Scatter Plots of the Simulated Copula-Based Random Variables	31
Figure 3.2	Efficient Frontier for Simulated Copula-Based Portfolio VaR	32
Figure 3.3	Density Plots and Contours of Copulas with Different Marginals	39
Figure 3.4	Return Plots and Scatter Plots of In- and Out-of-Sample periods for the FTSE 100-vs-Nikkei 225 Pair	48
Figure 3.5	Return Plots and Scatter Plots of In- and Out-of-Sample periods for the FTSE 100-vs-S&P 500 Pair	49
Figure 3.6	Return Plots and Scatter Plots of In- and Out-of-Sample periods for the Nikkei 225-vs-S&P 500 Pair	50
Figure 4.1	Relationship between Nonparametric and Parametric Joint Densities	77
Figure 4.2	Test Procedure	78
Figure 4.3	Three-Dimensional Plots and Contours of Bivariate Exchange Rate Return Pairs AD-BP, AD-JY, and AD-SF in In-Sample Period	102

Figure 4.4	Three-Dimensional Plots and Contours of Bivariate Exchange Rate Return Pairs BP-JY, BP-SF, and JY-SF in In-Sample Period	103
Figure 4.5	Three-Dimensional Plots and Contours of Bivariate Exchange Rate Return Pairs AD-BP, AD-JY, and AD-SF in Out-of-Sample Period	104
Figure 4.6	Three-Dimensional Plots and Contours of Bivariate Exchange Rate Return Pairs BP-JY, BP-SF, and JY-SF in Out-of-Sample Period	105
Figure 5.1	Plots of the Conditional Density and the Conditional Moments for the Filtered Index Futures Returns of Hang Seng	130
Figure 5.2	Plots of the Conditional Density and the Conditional Moments for the Filtered Index Futures Returns of Nikkei 225	131
Figure 5.3	Plots of the Conditional Density and the Conditional Moments for the Filtered Index Futures Returns of MSCI SIN	132
Figure 5.4	Three-Dimensional Plots and Contours of Bivariate Index Futures Return Pairs	145
Figure 5.5	Plots of Time-Varying Tail Dependences and Time-Varying Parameters of the Conditional BB7 Copula for the Hang Seng-MSCI SIN Pair	146
Figure 5.6	Plots of Time-Varying Tail Dependences and Time-Varying Parameters of the Conditional BB1 Copula for the Nikkei 225-MSCI SIN Pair	147
Figure 5.7	Scatter Plots of the Bivariate Simulated Random Variables Based on the Conditional Two-Parameter Archimedean Copulas with Conditional Marginals	148