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On the zero-point energy of elliptic-cylindrical and spheroidal boundaries

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Abstract

Zero-point energy is the energy of the vacuum. Disturbing the vacuum results in a change in the zero-point energy. In 1948, Casimir considered the change in the zero-point energy when the vacuum is disturbed by two parallel metal plates. The plates disturb the vacuum by restricting the quantum fluctuations of the electromagnetic field. Casimir found that the change in the zero-point energy implies that the plates are attracted to each other. With the recent advances made in the experimental verification of this remarkable result, theoretical interest has been rekindled. In addition to the original parallel plate configuration, several other boundaries have been studied. In this thesis, two novel boundaries are considered: elliptic-cylindrical and spheroidal. The results for these boundaries lead to the conjecture that zero-point energy does not change for small deformations of the boundary that preserve volume. Assuming the conjecture, it is shown that zero-point energy plays a stabilizing role in quantum chromodynamics, the leading theory of the strong interaction.

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I dedicate this thesis to my daughter, Delta.

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Nomenclature

In this thesis, Dirac's constant \hbar and the speed of light c are both unity. The signature of the Minkowski metric is taken to be $(1, -1, -1, -1)$.