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An initial-boundary value problem arising in cell population growth modelling

A thesis submitted in partial fulfillment of the requirement for the degree of Master of Science in Mathematics

at

Massey University, [Manawatu]
Institute of Fundamental Sciences
Mathematics

by

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2012

Abstract

A partial differential equation modelling cell populations undergoing growth and division characterised by size is studied. This equation is a special case of the fragmentation equation studied by Michel *et al*. [5] with no dispersion term, and the problem is of the initial-boundary value type.

Eigenfunction solutions are derived for separable solutions and related properties such as spectrum, uniqueness and unimodality are investigated. We show that the spectrum is continuous and that the decay of the eigenfunctions is exponential at a critical eigenvalue and algebraic otherwise.

The existence of a fast decay general solution $n(x, t)$ is then established. The problem can be solved analytically, and it is shown that the solution is unique and smooth. The solution properties are illustrated with some numerical simulations.

Finally, the role of exponential decaying eigenfunction solutions is interpreted from the standpoint of the general solution. The asymptotic behaviour as $t \to \infty$ of the general solution is examined. Slow decay eigenfunction solutions are briefly discussed, but their mathematical role remains to be explored.

Acknowledgements

It has been a privilege to be supervised by the associate professor Bruce van Brunt and Senior Lecturer Tammy Lynch. I acknowledge that this thesis could not be made possible without their help and support. I specially dedicate my thanks and acknowledgment of gratitude toward professor Bruce for his kindly contributions, despite his busy time, in reading my work and offering invaluable details and advice on organization, and the themes of the thesis.

Finally, I sincerely thank my parents, family, and friends, who always provide the advice and support. Thanks also go to my scholarship sponsor "King Abdullah Scholarships Programme" of Saudi Arabia that gave me the chance to pursue my study and get the best of education.

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Abbreviations

SSD	**S**teady **S**ize **D**istribution
PDE	**P**artial **D**ifferential **E**quation
PDF	**P**robability **D**ensity **F**unction
ODE	**O**rdinary **D**ifferential **E**quation

Symbols

$n(x,t)$	Number density	$\frac{1}{[l]}$
x	Cell size	$[l]$
t	Time	$[s]$
g	Cell growth rate	$\frac{[l]}{[s]}$
μ	Cell death rate	$\frac{1}{[s]}$
$W(\xi,x)$	Division kernel	$\frac{1}{[ls]}$

The notations l and s refer to size unit and second, respectively.

This thesis is dedicated to my parents with all of my love.