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**ANALYSIS OF A DYNAMICAL SYSTEM OF ANIMAL GROWTH  
AND COMPOSITION.**

**A thesis submitted in partial fulfilment of the requirements for the degree of**

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**in**

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## **ABSTRACT**

This thesis investigates the analysis of the extended model of animal growth proposed by Oliviera *et al* (personal communication, July 2009). This mechanistic model of animal growth based on a detailed representation of energy dynamics focussing on the interaction between four compartment of body composition; nutrient level, fat content, visceral protein and non-visceral protein. The model is mathematically analysed and the behaviour of the model for different feeding level is examined. The animal growth model exhibits thresholds typical of nonlinear systems and multiple stable steady states which have distinct basins of stability which depend on the value of the large number of physiologically-determined parameters. These have not been previously explored theoretically and these are done in this thesis. The model demonstrates richer behaviour where path-following techniques are used to explore the distribution in parameter space of the varying phenomenology.

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