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MODELLING THE INITIATION OF A HYDROTHERMAL ERUPTION – THE SHOCK TUBE MODEL

A THESIS PRESENTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

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Abstract

Modelling of hydrothermal eruption phenomenon has been a growing topic of research for the past 20 years. To date, models have focussed on the underground quasi-steady flow and on the above ground eruption jet including particle deposition. Very little has been able to be said about the first few seconds of an eruption, nor has much modelling work been done on eruption causes. In this thesis we develop a shock tube model for the initiation of a hydrothermal eruption which aims to answer some of the remaining questions about causality of a hydrothermal eruption.

The new shock tube model reported in this thesis is the first model able to simulate the initiation of a hydrothermal eruption. We take into account the three phases present during the eruption; in the geothermal reservoir below ground, liquid water and water vapour are present, in the above ground flow we also include air. The fact that the flow below ground is moving through a porous medium is accounted for, and new numerical methods using the finite volume framework are developed to solve the arising system of equations. Numerical simulations are described which simulate various eruptions, including ones with steam caps and rapidly developing cracks in the geothermal reservoir. Results from numerical simulations will be able to guide the design of future lab experiments of hydrothermal eruptions. The work of this thesis results in the first model able to simulate the initiation of a hydrothermal eruption.
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# Contents

1 Introduction 1

2 Literature Review: Eruption Modelling 5
   2.1 Observations of Hydrothermal Eruptions 5
   2.2 Hydrothermal Eruption Background - Causes and Thermodynamic Profiles 6
   2.3 Previous Mathematical Modelling of Hydrothermal Eruptions 9
   2.4 The Progressive Flashing Model for a Hydrothermal Eruption 11
      2.4.1 Conceptual Model 11
      2.4.2 Below Ground 12
      2.4.3 Above Ground 14
   2.5 Volcanic Modelling 15
      2.5.1 The Shock Tube Model and the Euler Equations 16

3 Literature Review: Basic Numerical Methods 19
   3.1 The Finite Volume Method 19
      3.1.1 Godunov’s Method (a first order method) 21
      3.1.2 Approximate Riemann Solver 23
   3.2 New Model Goals 25

4 Multi-Phase Flow Equations and Methods 27
   4.1 Multi-Phase Flow Model 27
      4.1.1 Energy/Entropy Equation 31
   4.2 Multi-Phase Flow Methods 31
   4.3 The Two-Phase Solver 33
      4.3.1 Calculating the Two Phase Nozzling terms, $F_{LAG}$ 36
# List of Figures

1.1 *A selection of areas of the world with a high amount of hydrothermal activity. From [70]* ........................................ 2

1.2 *Sites of hydrothermal eruptions in the Taupo Volcanic Zone, New Zealand. From [70]* ........................................ 3

2.1 *The boiling point with depth temperature profile obtained by integration of equation 2.1* ........................................ 9

2.2 *Yellowstone National Park temperature drilling curve. Modified from [22]* ........................................ 10

2.3 *Geometry of the above ground flow. From [49]* ........................................ 15

2.4 *Diagram of the shock tube. Two chambers at different pressures are separated by a diaphragm at position $x = 0$.* ........................................ 17

3.1 *A graphical representation of the Godunov scheme. The domain is split into discrete cells, then the value of the function in each cell is taken to be constant and to be the cell average. Modified from [73]* ........................................ 22

3.2 *A graphical representation of the domain in the HLLC method. Three waves separate four constant states. Modified from [73]* ........................................ 24

4.1 *Graphical representation of the stratified method for computing multiphase flows. In each cell each phase has weights according to the volume fraction in that cell and its neighbouring cell. Adapted from [19]* ........................................ 34

4.2 *The 1-3 common border.* ........................................ 39

4.3 *Interaction borders for fluids $x$, $y$, and $z$.* ........................................ 41
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.4</td>
<td>The moving contact discontinuity. We see the volume fractions of the phases advecting to the right hand side of the tube as expected.</td>
</tr>
<tr>
<td>4.5</td>
<td>Errors in the liquid flow variables. We see that the method preserves the uniformity condition on the pressure and velocity of the flow.</td>
</tr>
<tr>
<td>4.6</td>
<td>Plots of the liquid density and the gas (the 2-phase gas) velocity with the old two-phase solver and the new 3-phase solver.</td>
</tr>
<tr>
<td>4.7</td>
<td>Difference between the velocities and pressures of gas 1 and gas 2 in the new three-phase solver. The plots show that the velocities and pressures of the two gases are equal.</td>
</tr>
<tr>
<td>4.8</td>
<td>Plots of the fluid velocity, pressure, and density for various number of cells and also the exact solution.</td>
</tr>
<tr>
<td>4.9</td>
<td>A zoomed in view of the contact and shock waves for the fluid pressure and density.</td>
</tr>
<tr>
<td>4.10</td>
<td>A plot of the individual fluid pressures and velocities. Each fluid phase is able to evolve independently of the others.</td>
</tr>
<tr>
<td>4.11</td>
<td>A convergence plot of the velocity of gas 1 as the mesh is refined from 50 cells to 1600 cells. The figure shows convergence of the solution under mesh refinement.</td>
</tr>
<tr>
<td>5.1</td>
<td>The fluid begins at the point A. The three-phase solver is applied to move the state of the fluid to point B. Then the relaxation routine is applied causing the fluid state to move down line L and finish at point C. The evaporation solver is then applied which quantifies the amount of boiling in the system.</td>
</tr>
<tr>
<td>6.1</td>
<td>Adjoining cells of a fluid in a vertical state. The pressure is determined by Equation 6.3.</td>
</tr>
<tr>
<td>6.2</td>
<td>Error in the gas density and pressure. Output time is 5ms and each cell is 0.01m in length. The old middle wave speed does not conserve the static state.</td>
</tr>
</tbody>
</table>
6.3 Error in the gas velocity and a plot of the middle wave speed. Output time is 5ms and each cell is 0.01m in length. The old middle wave speed is greater than zero for the compressible and causes the gas velocity to increase. 

6.4 Error in the gas density and pressure. Output time is 5ms and each cell is 0.01m in length. The new middle wave speed conserves the static state.

6.5 Error in the gas velocity and a plot of the middle wave speed. Output time is 5ms and each cell is 0.01m in length. The new middle wave speed is exactly zero for the compressible case.

7.1 A schematic of the shock tube model. In the lower half of the column we have liquid water, while in the upper half we have air.

7.2 The position of the flashing front as a function of time for various $\Delta P^A$.

7.3 The position of the eruption jet as a function of time for various $\Delta P^A$.

7.4 The maximum fraction of liquid water boiled to water vapor at time 0.9007s as a function of $\Delta P^A$.

8.1 A schematic of the shock tube model. In the lower half of the column we have liquid water in a porous medium, while in the upper half we have air.

8.2 A flowchart summary of the numerical method used to solve the shock-tube equations.

9.1 The effect of varying porosity and cohesion.

9.2 The effect of varying permeability.

9.3 The amount of boiling for a given $\Delta P$.

9.4 The effect of $\phi_v0$ with $C_{pm} = 0$.

9.5 The effect of $\phi_v0$ with $C_{pm} = 5000$.

9.6 The amount of boiling for a given $\phi_v0$.

9.7 The erosion and eruption fronts for the solid phase.
9.8 A schematic of the shock tube model with a steam cap. In the lower part of the column we have liquid water in a porous medium, while in the upper part we have air.

9.9 Initially, the pressure profile in a liquid filled porous geothermal reservoir follows the boiling point with depth curve. The pressure at the ground surface (point B) is atmospheric.

9.10 A large vapour filled crack rapidly forms which transfers large pressures to shallow regions. The pressure at the top of the crack, the ground surface (point B), is equal to the pressure at the bottom of the crack (point A), and is greater than atmospheric.

9.11 Location of erosion front with no steam cap or crack. The front perpetually ‘mines’ into the geothermal reservoir.

9.12 Eruption jet and erosion front behaviour in the presence of a steam cap or crack. For small pressure differences the eruption is stifled due to lack of boiling.

9.13 Distance of ejection of solid material.

9.14 Distance of erosion into the geothermal reservoir by steam only.

9.15 The 2m steam cap and crack now propagates an erosion front into the geothermal reservoir.

9.16 The maximum erosion distance by steam erosion only is always less than the natural pressure difference due to liquid filled reservoir conditions.

10.1 A schematic of the shock tank setup. A large 2D liquid reservoir at boiling point conditions is separated from the atmosphere with some pressure jump $\Delta P = P_W - P_A$, $P_W > P_A$. At some weak point, A in the figure, the higher pressure liquid is exposed to the lower pressure atmosphere, triggering an eruption.
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>$L$ superscript</td>
<td>36</td>
</tr>
<tr>
<td>4.2</td>
<td>Conditions for a common border between phases</td>
<td>40</td>
</tr>
<tr>
<td>4.3</td>
<td>Test 1 parameters</td>
<td>47</td>
</tr>
<tr>
<td>4.4</td>
<td>Test 2 parameters</td>
<td>50</td>
</tr>
<tr>
<td>4.5</td>
<td>Test 3 parameters</td>
<td>52</td>
</tr>
<tr>
<td>4.6</td>
<td>Test 4 parameters</td>
<td>54</td>
</tr>
<tr>
<td>4.7</td>
<td>Test 4 convergence</td>
<td>56</td>
</tr>
<tr>
<td>7.1</td>
<td>Flashing front position, eruption jet position, and the amount of boiling at 0.9s.</td>
<td>81</td>
</tr>
<tr>
<td>8.1</td>
<td>Conditions for a common border between phases (three-phase)</td>
<td>92</td>
</tr>
<tr>
<td>9.1</td>
<td>Test parameters</td>
<td>100</td>
</tr>
<tr>
<td>9.2</td>
<td>95% confidence interval for the gradients of Figure 9.1 and their $R^2$ values.</td>
<td>102</td>
</tr>
<tr>
<td>9.3</td>
<td>95% confidence interval for the gradients of Figure 9.2 and their $R^2$ values.</td>
<td>104</td>
</tr>
<tr>
<td>9.4</td>
<td>95% confidence interval for the gradients of Figure 9.3 and their $R^2$ values.</td>
<td>105</td>
</tr>
<tr>
<td>9.5</td>
<td>Test parameters</td>
<td>113</td>
</tr>
</tbody>
</table>