EVAPORATIVE CRYSTALLIZATION OF ALPHA-LACTOSE MONOHYDRATE

A thesis presented in partial fulfilment of the requirements for the degree of

Doctor of Philosophy

In

Chemical Engineering

at Massey University, Manawatu
New Zealand.

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2012
Dedicated to

Rasikbihari....
**Abstract**

Evaporative crystallization has been used by Fonterra Cooperative Group (New Zealand) for producing lactose. It represents an important step during lactose manufacturing where control over crystal size can be obtained, a critical parameter governing the yield and end use. The art of operating these crystallizers has been developed by observation and not from scientific principles. This project was undertaken to understand the mechanisms controlling the crystal size in evaporative crystallizers.

A review of the existing literature showed that secondary nucleation is the major source of nuclei in industrial crystallizers. Based on the review, attrition, contact and fluid shear induced nucleation were identified as the probable secondary nucleation mechanisms in the studied system. Experimental investigation on each of the three mechanisms was carried out separately on a laboratory scale.

It was found that the crystal size had the most significant effect on attrition, followed by impeller speed, which together implies that the crystal collision energy intensity is the dominant factor producing new fragments. Contact nucleation was also found to be controlled by crystal-impeller collisions. It was found that at the studied supersaturation there exists a minimum kinetic energy of contact below which secondary nucleation would not occur. This threshold value was used as the basis to assess the contribution of various mechanisms at the industrial scale. Shear nucleation was found to be independent of shear above 5000 s$^{-1}$.

A mathematical model describing the operation of the industrial crystallizer was formulated. Sensitivity analysis was conducted by simulating the model for a range of operational and kinetic parameter values. It was found that the crystal size is affected most by secondary nucleation. The volume weighted mean size approximately halved with a 5.5 times increase in the secondary nucleation rate.
The model was refined to accommodate size dependent growth rate and growth rate dispersion. The kinetic parameters were fitted to match the measured size distribution from the industrial crystallizer. A range of simulations were conducted for various theoretical and empirical models and compared to that of plant measurements. Based on the results it was proposed that the majority of secondary nucleation is expected to occur in the pump and the boiling zone.
Acknowledgement

It’s not easy to move to an alien country to do a PhD. So my first thank to New Zealand and its people for making me feel comfortable and for being my home for close to four years. I will then like to thank my main supervisor, Prof. Tony Paterson for giving me the opportunity to work under him. Thanks Tony for showing constant faith in my abilities and being there whenever I needed advice, academic or personal. This PhD would not be possible without your guidance and encouragement.

Big thanks to my co-supervisors: Dr. Jeremy McLeod, Prof. Jim Jones and Prof. John Bronlund. Thanks Jeremy for proposing this PhD and getting me started into Fonterra during your last few days with them. Thanks for attending my weekly meeting from USA and taking a keen interest in the project. Jim, I learnt a lot about technical writing from you. I would be happy if I could become half as good as you. John, thanks for helping me with my model. That one small session with you on Matlab got me going!

My sincere thanks to my mentors at Fonterra: Tony Styles and Raymond Joe. It is not easy to carry out experimental studies in an industrial environment. Raymond, thanks for making sure that I have all the resources to complete my trials at Kapuni. Tony thanks for arranging the work contract with Fonterra for the last six months of the project when I ran out of funding. My special thanks to Karl Goble, who was the go to person during my stay at Kapuni; whether it be collecting data from the plant or for discussing my mathematical model or simply someone to talk to regarding the project. Karl, you have got the most amazing collection of theses and references on crystallization!! Thank you to Sharon, Michelle and all others at the Kapuni site who made my stay there enjoyable.

Financial security is a critical factor for any PhD student. Therefore, I will like to thank the Foundation for Research Science and Technology, Government of New Zealand for providing me with the scholarship to able me to undertake this project. I
would also like to thank Fonterra from the bottom of my heart for sponsoring my conference trips and university fees.

I spent most of my PhD study in Taranaki. So I will like to thank all the people who made my stay enjoyable at Hawera and helped me at some stage or other: The Turuturu cricket team and its members, Chandu Bhana and his family, Daniel Hovell and Jimmy. The cricket matches over the weekend were fun and allowed to take my mind off the PhD for little bit. Chandu, thanks for inviting us to your home and the volleyball games on Mondays. Thank you to Daniel and Jimmy for being great flatmates and providing an environment where I could return to in the evenings and relax. Daniel, I will always remember the round the mountain relay which I ran for your team. It was a great experience! Thanks.

I also need to thank people who helped me during my stay at Massey and in Palmy. First of all I will like to thank the PhD students at SEAT in the sustainable processing cluster: Horng, Ihli, Georg, Konrad, Fitri, for making my PhD experience memorable. My thanks to intern students: Amandine, Amit and Yohannes, for being part of my PhD journey at different stages. Thanks to Ann-Marie, Bruce, Anthony and John for helping me in different ways in my experimental work. Thanks to Gayle and Linda for sorting out all the administrative and financial issues during the length of my PhD.

My sincere thanks to Rajesh for being my first point of contact and helping me settle in New Zealand. Thanks to Mallesh for being a wonderful flatmate during my second term in Palmy. I will also like to thank Nilesh to whom I approached whenever I needed some advice. I will like to express my gratitude towards my friends in the badminton group for all the fun I had while playing with them. Palash, I still hope to beat you some day in singles! Special thanks to Ken Mercer for taking me on the most beautiful journeys and giving me an opportunity to see the real NZ. Ken, I will never forget our three day hiking trip to Nelson Lakes National Park!!

I now like to say thank and pay obeisance to my parents for their constant emotional and spiritual support. I cannot express in words my gratitude towards them for the sacrifices they have made. To my brother Saket, thanks for your belief in me. I also
express gratefulness to all my in-laws, especially parents-in-law for their kind words and love.

My hearty thanks to the most special person who became the part of my life during the latter half of my PhD: my wife Sneha. I need to first say sorry to you that you missed the festivities and occasions which are very special for a newly married Indian girl because I dragged you half away across the world to a completely new country. Thanks for all the love and support. Your smiling face is the biggest energy booster for me!!
# Table of Contents

ABSTRACT 1

ACKNOWLEDGEMENT III

LIST OF FIGURES XI

CHAPTER 1 PROJECT OVERVIEW 1

CHAPTER 2 CRYSTALLIZATION THEORY 5

2.1 Nucleation 5

2.2 Growth 7

2.3 Lactose 9

2.4 Industrial Crystallizer 10

2.4.1 Forced Circulation Evaporative Crystallizers 14

2.5 Modelling 15

2.5.1 Method of Moments (MOM) 18

2.5.2 Discretization 19

2.6 Conclusions 20

CHAPTER 3 LITERATURE REVIEW ON SECONDARY NUCLEATION 21

3.1 Collision Induced Secondary Nucleation 24

3.1.1 Attrition 24

3.1.2 Contact Nucleation 26

3.2 Shear Secondary Nucleation 29

3.3 Secondary Nucleation Studies on Lactose 33

3.4 Conclusions 35
CHAPTER 4  EXPERIMENTS ON SECONDARY NUCLEATION  37

4.1  Attrition  37
  4.1.1  Theory  38
  4.1.2  Materials and Methods  43
  4.1.3  Results and Discussions  44
  4.1.4  Conclusions  51

4.2  Contact Nucleation  52
  4.2.1  Theory  52
  4.2.2  Crystal-impeller Contacts  53
  4.2.3  Crystal-crystal Contacts  55
  4.2.4  Experimental  57
  4.2.5  Results and Discussions  60
  4.2.6  Conclusions  68

4.3  Shear Nucleation  69
  4.3.1  Results and Discussions  73
  4.3.2  Conclusions  74

CHAPTER 5  MODEL DEVELOPMENT AND SENSITIVITY ANALYSIS  75

5.1  Mathematical Model Development  75
  5.1.1  Mass Balance  76
  5.1.2  Population Balance  79
  5.1.3  Growth and Nucleation Kinetics  81
  5.1.4  Lactose Solubility  82
  5.1.5  Lactose Mutarotation Kinetics  82

5.2  A Brief Review on Lactose Crystallization Kinetics  83
  5.2.1  Growth  83
  5.2.2  Secondary Nucleation  84
  5.2.3  Primary Nucleation  86

5.3  Simulation and Sensitivity Analysis  87
  5.3.1  Effect of Growth Rate  88
  5.3.2  Effect of Primary Nucleation  92
  5.3.3  Effect of Secondary Nucleation  97
  5.3.4  Effect of Evaporation Rate  101
5.3.5 Effect of Residence Time 104
5.3.6 Effect of Temperature 108

5.4 Summary and Conclusions 111

CHAPTER 6 INDUSTRIAL CRYSTALLIZER 113

6.1 Hydrodynamics 113

6.2 Plant Data and Parameter Estimation 119
6.2.1 Sampling and analysis 119
6.2.2 Results 121
6.2.3 MSMPR Analysis 125
6.2.4 Simulation and Parameter Estimation 132

6.3 Simulations using Mechanistic Models for Secondary Nucleation 142
6.3.1 Attrition 142
6.3.2 Contact Nucleation due to Crystal-Impeller Collisions 147
6.3.3 Contact Nucleation due to Crystal-crystal Collisions 151
6.3.4 Plant trials 159

6.4 Conclusions 162

CHAPTER 7 PROJECT REVIEW 163

NOMENCLATURE 167

REFERENCES 171

APPENDIX I I

APPENDIX II V

APPENDIX III VII

APPENDIX IV XVII
Figure 1-1. Framework for describing the crystallizer behaviour. .......................... 2

Figure 2-1. Solubility diagram, supersaturation generation and crystallization kinetics [adapted from Mersmann (2001b) and Jones (2002)]. Anti-solvents lower the solubility of the solute. On anti-solvent addition (referred to as dilution in the figure), the excessive solute (defined by the anti-solvent solubility curve) precipitates out. Chemical reaction leading to high concentrations of the new generated species can also trigger precipitation. ............................................................................................................. 6

Figure 2-2. Nucleation mechanisms [adapted from Randolph & Larson (1988)]. ........ 7

Figure 2-3. Plot of crystal growth rate, G, vs. solution velocity (or slurry mixing) showing the mechanism governing crystal growth (Genck, 2003). .............................................................................................................. 8

Figure 2-4. Effect of agitation speed on lactose growth rate at a supersaturation of 5.23 g per 100 g of water at 30°C (McLeod, 2007). .............................................................................................................. 8

Figure 2-5. α-lactose tomahawk morphology [reproduced from van Kreveld & Michaels (1965)]. ................................................................................................................................. 9

Figure 2-6. Mean crystal size produced by different industrial crystallizers as a function of residence time and nucleation rate [reproduced from Wöhlk, Hofmann, & de Jong (1991)]. ......................................................................................................................... 11

Figure 2-7. Forced circulation evaporative crystallizer [reproduced from Bennet (2002)]. ................................................................................................................................. 12

Figure 2-8. DTB crystallizer [reproduced from Bennet (2002)]. ............................ 13

Figure 2-9. Fluidized bed crystallizer [reproduced from Bennet (2002)]. ............... 13

Figure 2-10. Feedback interactions between various factors effecting crystallization [adapted from Randolph & Larson (1988)]. ................................................................. 15

Figure 2-11. Semi logarithmic plot of population density versus crystal size for an MSMPR continuous crystallizer. ................................................................................................. 17

Figure 3-1. Secondary nucleation classifications as applicable to an industrial crystallizer ................................................................. 24

Figure 3-2. Contact nucleation photomicroscopic cell [reproduced from Garside, et al., (1979)] ................................................................................................................................. 27

Figure 3-3. Growth rate comparison of lactose under different experimental conditions. ▲present data from Shi, et al., (1989) for a photomicroscopic cell, × present data from Shi, et al., (1990) for a CSTR (continuous stirred tank reactor). Temperature ranged from 30-60 °C. Residence time for CSTR studies varied from 4.6-52 minutes and growth rates determined by MSMPR population balance
technique. In the photomicroscopy study, growth of a population of nuclei was followed in a single experiment for 30-500 minutes, with mean growth rate plotted.

Figure 3-4. Secondary nucleation studies on lactose. A: Spontaneous nucleation line/Supersolubility curve (Hunziker, 1949); B: Solubility curve; C: Forced crystallization (Hunziker, 1949) [refers to the optimum viscosity (a function of temperature) and supersaturation at which the solution was seeded and held under vigorous agitation for 60 minutes]; D: Critical supersaturation for growth (Shi, et al., 1989); E: SNT (Butler, 1998); F: Forced secondary nucleation (Shi, et al., 1989) and G: Nuclei first detection curve (Wong, Bund, Connelly, & Hartel, 2011).

Figure 4-1. Different attrition mechanisms [adapted from Barbosa-Cánovas, Ortega-Rivas, Juliano, & Yan (2005)].

Figure 4-2. Effect of different dominant attrition mechanisms on the CSD with time [reproduced from Malave-Lopez & Peleg (1986)].

Figure 4-3. Particle size distributions and difference plots for a range of attrition outcomes.

Figure 4-4. Normalized frequency CSD for a single HHH run. The trial name represents high levels of crystal size, impeller speed and concentration.

Figure 4-5. Cumulative size distribution at 0 and 60 minutes for selected experimental runs. The legend gives high/low levels of particle size, impeller speed and particle concentration respectively. The data shown is the average of four replicates.

Figure 4-6. Differential attrition index (standard error bars, n=4) measured between time, t=0 and t. Trial names represent high, medium and low levels of crystal size, impeller speed and concentration.

Figure 4-7. Importance of seed pretreatment for fines. The beaker on the left contained untreated settled seed crystals and would have led to an over-estimation of secondary nucleation, while that on the right are the washed seeds.

Figure 4-8. Secondary nucleation versus time (Supersaturation: 10.7 g of α-LMH per 100 g of water / Seed size: 357 μm / Seed loading: 2% (w/w) / Impeller speed: 550 rpm). Standard deviation error bars for 4 replicates are shown.

Figure 4-9. Effect of seed loading on secondary nucleation rate at constant supersaturation (8.4 g α-LMH per 100 g water) for different combinations of seed size and impeller speed. Standard deviation error bars for 4 replicates are shown.

Figure 4-10. Effect of seed loading on secondary nucleation from kinetic energy and contact frequency perspective at constant supersaturation (8.4 g of α-LMH per 100 g water).

Figure 4-11. Secondary nucleation at constant seed loading of 2% (w/w) at 10.7 and 6.7 s.s. (g α-LMH per 100 g water) from a kinetic energy and frequency of contacts perspective. Standard deviation error bars based on 4 replicates are shown. The 8
kinetic energies correspond to the same combinations of impeller speed and crystal size as shown in Figure 4-9.

Figure 4-12. Experimental setup for shear secondary nucleation.

Figure 4-13. Shear secondary nucleation: Effect of shear rate and supersaturation. The primary nucleation trials contain no immobilized lactose crystals. Supersaturations are g of α-LMH per 100 g of water.

Figure 5-1. A generalized schematic of an evaporative crystallizer.

Figure 5-2. Growth studies on lactose.

Figure 5-3. Secondary nucleation studies on lactose.

Figure 5-4. Lactose primary nucleation rate (McLeod, 2007).

Figure 5-5. Effect of growth rate kinetics on a), dissolved lactose; b), supersaturation; c), slurry density; d), crystal concentration; e), mean diameter; f), instantaneous growth rate; and g), coefficient of variation. Solid line represent profiles obtained by using growth parameters of Shi, et al.,(1990) and the broken line uses that of McLeod(2007).

Figure 5-6. Effect of primary nucleation rate constant on a), dissolved lactose; b), supersaturation; c), slurry density; d), crystal concentration; e), instantaneous primary nucleation rate; f), mean diameter and g), coefficient of variation.

Figure 5-7. Effect of secondary nucleation rate constant on a), dissolved lactose; b), supersaturation; c), slurry density; d), crystal concentration; e), instantaneous secondary nucleation rate; f), mean diameter; and g), coefficient of variation.

Figure 5-8. Effect of evaporation rate on a), dissolved lactose; b), supersaturation; c), slurry density; d), crystal concentration; e), mean diameter; and f), coefficient of variation.

Figure 5-9. Effect of residence time on a), dissolved lactose; b), supersaturation; c), slurry density; d), crystal concentration; e), mean diameter; and f), coefficient of variation.

Figure 5-10. Effect of temperature of operation (constant for an evaporative crystallizer) on a), dissolved lactose; b), supersaturation; c), slurry density; d), crystal concentration; e), mean diameter; and f), coefficient of variation.

Figure 6-1. Fluid dynamics scale in a crystallizer, x represents crystal (reproduced from Rielly & Marquis (2001)).

Figure 6-2. Compartment model for a forced circulation evaporative crystallizer (reproduced from Kramer, et al. (2000)).

Figure 6-3. Prediction of the height at which boiling starts inside the calandria tubes of the Kapuni FC evaporative crystallizer.
Figure 6-4. Slurry velocity in the boiling zone in the calandria tubes of the Kapuni FC evaporative crystallizer. .......................................................................................................................116

Figure 6-5. Compartmental model for the Kapuni FC evaporative crystallizer........117

Figure 6-6. Supersaturation decay transient profile in absence of evaporation i.e. no supersaturation generation, for prediction of t_{ac,1/2} .................................................................118

Figure 6-7. Residence time profile of the Kapuni FC evaporative crystallizer during the three sampling trials. .............................................................................................................121

Figure 6-8. Dissolved solids profile of the Kapuni FC evaporative crystallizer for the three sampling runs (MA: moisture analyzer and HRM: Handheld refractometer ). 122

Figure 6-9. Total solids profile of the Kapuni FC evaporative crystallizer for the three sampling runs (measured using the Metler Toledo moisture analyzer) ....122

Figure 6-10. Crystalline solids content profile of the Kapuni FC evaporative crystallizer for the three sampling runs (calculated from mass balance using total solids and dissolved solids). .......................................................................................123

Figure 6-11. Crystal size distribution profile of the Kapuni FC evaporative crystallizer for the three sampling runs a) Trial 1 b) Trial 2 c) Trial 3. 0 minute indicates the ..124

Figure 6-12. Volume weighted mean diameter profiles of the Kapuni FC evaporative crystallizer for the three sampling runs .................................................................125

Figure 6-13. Population density curve for lactose crystals obtained from the Kapuni FC evaporative crystallizer under steady state.................................................................126

Figure 6-14. Size dependent growth rate fitted to the population density curve in Figure 6-13 .........................................................................................................................128

Figure 6-15. Size dependent growth rate versus crystal size. Fitted values of the parameters a, c and G_e for the Mydlarz model are 0.0059, 0.0091 and 1.82 μm min^{-1}, respectively. ...............................................................................................................129

Figure 6-16. Various growth rate dispersion models fitted to plant population density. .................................................................................................................................131

Figure 6-17. Probability density distribution for various growth rate dispersion models (value of the fitted parameters is listed in Table 6-1). ....................................................132

Figure 6-18. Effect of grid size on simulation results. ................................................134

Figure 6-19. Dissolved lactose profile for Kapuni FC evaporative crystallizer for fitted growth, primary nucleation and secondary nucleation rate constants. The broken and continuous lines represent the model prediction during the semi-batch and the continuous mode respectively .................................................................136
Figure 6-20. Total solids profile for Kapuni FC evaporative crystallizer for fitted growth, primary nucleation and secondary nucleation rate constants. The broken and continuous lines represent the model prediction during the semi-batch and the continuous mode respectively.................................................................136

Figure 6-21. Crystal concentration profile for Kapuni FC evaporative crystallizer for fitted growth, primary nucleation and secondary nucleation rate constants. The broken and continuous lines represent the model prediction during the semi-batch and the continuous mode respectively.................................................................137

Figure 6-22. Volume weighted mean diameter profile for Kapuni FC evaporative crystallizer for fitted growth, primary nucleation and secondary nucleation rate constants. The broken and continuous lines represent the model prediction during the semi-batch and the continuous mode respectively. The line through the measured data points is drawn as a visual guide to compare with the model prediction. ............137

Figure 6-23. Steady state population density distribution for Kapuni FC evaporative crystallizer predicted by the model using optimized fitted parameters. ...............138

Figure 6-24. Dry crystal sieve analysis representative of the CSD dynamics of the Kapuni FC evaporative crystallizer. .................................................................138

Figure 6-25. Dissolved lactose profile for size independent growth.................139

Figure 6-26. Volume weighted mean diameter profile for size independent growth. .........................................................................................................................140

Figure 6-27. Population density curve for size independent growth...............140

Figure 6-28. Dissolved lactose profile using method of moments and parameters estimated by SIG (size independent growth). .........................................................141

Figure 6-29. Volume weighted mean diameter profile calculated using the method of moments.................................................................142

Figure 6-30. Effect of impeller blade discretization on simulation results. ........145

Figure 6-31. Dissolved lactose concentration profile with attrition (calculated by Mersmann model) as the source of secondary nucleation. Markers depict measured values. .................................................................146

Figure 6-32. Volume weighted mean diameter profile with attrition (calculated by Mersmann model) as the only source of secondary nucleation. Markers depict measured values.................................................................146

Figure 6-33. Crystals withdrawn from the crystallizer at steady state.............147

Figure 6-34. Secondary nucleation at constant seed loading of 2% (w/w) at 10.7 and 6.7 s.s. (g α-LMH per 100 g water) from a kinetic energy and frequency of contacts perspective. .........................................................................................................................149
Figure 6-35. Plot of ln of the intercept of the lines fitted in Figure 4-11 versus the ln of the supersaturations to determine $K_N$ and $b$ in Equation 4-12. ................................. 149

Figure 6-36. Dissolved lactose concentration profile with crystal-impeller contact nucleation at the operational circulation pump speed as the source of secondary nucleation. Markers depict measured values. .......................................................... 150

Figure 6-37. Volume weighted mean diameter profile with crystal-impeller contact nucleation at the standard operating circulation pump speed as the source of secondary nucleation. Markers depict measured values. ......................................................... 150

Figure 6-38. Solid liquid suspension (redrawn from (Eskin et al., 2004)).............. 151

Figure 6-39. Variation of flow quality with the tube height in the calandria of the Kapuni FC evaporative crystallizer. .................................................................................................. 156

Figure 6-40. The refractive index (dissolved solids in wt %) trends for the plant trial conducted at two temperatures. The figure on the left gives trend for the lower and to the right for the higher temperature. .................................................................................. 159

Figure 6-41. Observed crystal size distribution during unplanned one off trial. Blue line depicts the CSD with the lower feed concentration. Normal feed resumed around midday. Green and red lines depict CSD after approximately 3 and 4 hours of resumption of normal feed. ......................................................................................... 161
List of Tables

Table 2-1  Various Moments of Density Distribution and Mean Crystal Sizes ..........19

Table 3-1 Selected Data from Sung, et al., (1973) showing the Generation of Secondary Nuclei for Magnesium Sulphate at Different Levels of Mixing and Supersaturations. .................................................................30

Table 3-2 Data from Wang, M.L., et al., (1981) in Terms of Shear Rates at a Constant Absolute Supersaturation for Citric Acid (16.6 gm per 100gm of water) .................32

Table 3-3 Secondary Nucleation Studies in Literature on Lactose .........................35

Table 4-1 Factors Varied during the Attrition experiments. H = High, M= medium, L =Low .................................................................................................44

Table 4-2 Experimental Design Matrix (Size:Speed:Concentration) .................44

Table 4-3 Comparison of Calculated and Experimental Volumetric Attrition Ratios. To Remove the Effect of Concentration) the Mid Values for the Bands HH*, HL*, LH* and LL* After 60 Minutes are Used .................................................................50

Table 4-4 Run Time (t) for Different Trials Conducted for Each Experiment with Two Impeller Speeds of 400 and 550 rpm, and Four Particle Sizes of 106-212, 212-300, 300-425 and 425-600 Microns .................................................................60

Table 4-5 Values of Different Parameters for Impeller-Crystal Contact Induced Secondary Nucleation ........................................................................62

Table 4-6 Values of Different Hydrodynamic Parameters for Crystal-Crystal Contact for Different Impeller Speeds .....................................................62

Table 4-7 Inter-Particle Mean Space and the Kinetic Energy of Contact for Crystal-Crystal Contacts ........................................................................63

Table 4-8 Straight Line Fits Through the Data Points of Figure 4-9 And Kinetic Energy of Collisions for Various Combinations of Seed Size and Impeller Speed......64

Table 4-9 Comparison of the Experimental and Theoretical Slopes of the Secondary Nucleation Rate Line .................................................................66

Table 5-1 Secondary Nucleation Studies on Lactose .............................................86

Table 5-2 Values and Range of Parameters Studied During Simulation ...............88

Table 5-3 Values of the Crystallizer Outputs at Steady State at the Maximum and Minimum Values of the Various Kinetic and Operation Parameters Simulated ......112
Table 6-1 Growth Rate Dispersion Models .......................................................... 130

Table 6-2 Kinetic Parameters Estimated from Size Dependent and Size Independent Growth Models.......................................................... 142

Table 6-3 Values of Critical Parameters for Crystal-Crystal Contacts in the Circulation Loop Pipe.......................................................... 155

Table 6-4 Values of Critical Parameters for Crystal-Crystal Contacts in the Calandria Pipes .......................................................... 156
List of Equations

Equation 2-1 Population balance equation ................................................................. 16
Equation 2-2 Population balance equation for a batch process .................................... 16
Equation 2-3 Population balance equation for a continuous process .......................... 16
Equation 2-4 Population density for a steady state continuous process ................. 17
Equation 2-5 Nucleation rate calculation for MSMPR crystallizer ............................... 17
Equation 2-6 Discretized Method of lines for solving population balance equation ... 20
Equation 4-1 Secondary nucleation: empirical model ................................................. 37
Equation 4-2 Difference function for a frequency distribution .................................. 40
Equation 4-3 Differential attrition index definition .................................................... 40
Equation 4-4 Volumetric attrition rate due to impeller-crystal collisions ................. 48
Equation 4-5 Volumetric attrition rate due to crystal-crystal collisions .................... 48
Equation 4-6 Volumetric crystal hold up ................................................................. 49
Equation 4-7 Volumetric attrition rate due to crystal-crystal collisions in terms of crystal size and concentration ................................................................. 49
Equation 4-8 Relationship between target efficiency and crystal size ....................... 49
Equation 4-9 Volumetric attrition rate in terms of crystal size .................................. 49
Equation 4-10 Volumetric attrition rate ratio for a given impeller .............................. 50
Equation 4-11 Volumetric attrition rate ratio approximation by DAI slope ratio ....... 50
Equation 4-12 Collision secondary nucleation ....................................................... 52
Equation 4-13 Kinetic energy of impeller-crystal collision ........................................ 53
Equation 4-14 Mass of a single crystal .................................................................... 53
Equation 4-15 Crystal-impeller collision velocity .................................................... 53
Equation 4-16 Kinetic energy of contact for impeller-crystal collisions ................. 54
Equation 4-17 Impeller-crystal collision frequency .............................................. 54
Equation 4-18 Target efficiency as a function of Stokes number ................................ 54
Equation 4-19 Stokes number ...................................................................................... 54
Equation 4-20 Contact nucleation due to impeller-crystal collisions ....................... 55
Equation 4-21 Kinetic energy of crystal-crystal collision .............................................. 55
Equation 4-22 Inter-particle space in a slurry suspension .............................................. 55
Equation 4-23 Crystal-crystal collision velocity .......................................................... 55
Equation 4-24 Length of smallest eddy ....................................................................... 56
Equation 4-25 Energy dissipation rate in an agitated system ........................................ 56
Equation 4-26 Reynolds number for an agitated system .............................................. 56
Equation 4-27 Crystal-crystal collision frequency ...................................................... 56
Equation 4-28 Total crystal volume in Malvern presentation unit .............................. 58
Equation 4-29 Crystal volume per ml of the sample ................................................... 59
Equation 4-30 Number of crystals in size bin (i) .......................................................... 59
Equation 4-31 Total number of crystals across all size bins ........................................ 59
Equation 4-32 Theoretical ratio of the slopes of fitted lines in Figure 4-9 .............. 65
Equation 4-33 Approximate impeller-crystal collision frequency ............................... 66
Equation 4-34 Wall shear stress for a flow through a rough tube ............................... 69
Equation 4-35 Shear rate at the wall ............................................................................ 69
Equation 4-36 Pressure drop for a flow inside a tube ................................................. 70
Equation 4-37 Wall shear stress for a flow through a rough tube ............................... 70
Equation 4-38 Shear rate at the tube wall in terms of pressure drop across the tube .. 70
Equation 4-39 Pressure drop in a U-tube manometer ............................................... 70
Equation 5-1 Solute balance over crystallizer ............................................................ 76
Equation 5-2 Semi-batch solute balance ..................................................................... 77
Equation 5-3 Dissolved lactose concentration differential for semi-batch mode ...... 77
Equation 5-4 Overall mass balance ............................................................................ 77
Equation 5-5 Condition for constant operation. Volumetric rate of feed equal to volumetric rate of water removal ................................................................. 78

Equation 5-6 Dissolved lactose concentration differential for continuous mode ...... 78

Equation 5-7 Mean residence time ........................................................................ 78

Equation 5-8 Zeroth moment derivative ................................................................. 79

Equation 5-9 First moment derivative .................................................................... 79

Equation 5-10 Second moment derivative .............................................................. 79

Equation 5-11 Third moment derivative ................................................................. 79

Equation 5-12 Fourth moment derivative ............................................................... 79

Equation 5-13 Coefficient of variation of a size distribution .................................. 79

Equation 5-14 Slurry voidage .................................................................................. 80

Equation 5-15 Rate of change of slurry voidage ..................................................... 80

Equation 5-16 Volume weighted mean diameter .................................................... 80

Equation 5-17 Crystal content of the slurry ................................................................ 80

Equation 5-18 Total solids in the slurry ................................................................... 80

Equation 5-19 Slurry density-total solids relationship .......................................... 81

Equation 5-20 Primary nucleation rate ................................................................... 81

Equation 5-21 Secondary nucleation rate ............................................................... 81

Equation 5-22 Growth rate ..................................................................................... 81

Equation 5-23 Equilibrium $\alpha$-lactose solubility .................................................. 82

Equation 5-24 Correction factor for $\alpha$-LMH ....................................................... 82

Equation 5-25 Equilibrium lactose solubility .......................................................... 82

Equation 5-26 Total dissolved lactose ..................................................................... 82

Equation 5-27 Mutarotation rate constant .............................................................. 83

Equation 5-28 Dissolved $\alpha$-lactose concentration .............................................. 83

Equation 5-29 Equilibrium mutarotation rate constant as a function of temperature .. 83

Equation 5-30 Relationship between residence time and mean particle size .......... 104
Equation 6-1 Converting volumetric distribution to population density distribution 125
Equation 6-2 Size dependent growth rate ................................................................. 127
Equation 6-3 Population density with size dependent growth rate .......................... 127
Equation 6-4 Growth rate from population density distribution ............................. 129
Equation 6-5 Cumulative size distribution with growth rate distribution .................. 130
Equation 6-6 Converting cumulative number distribution to frequency distribution 131
Equation 6-7 Analytical solution for inverse gamma distribution function .......... 131
Equation 6-8 Secondary nucleation rate due to attrition .......................................... 143
Equation 6-9 Collision secondary nucleation .......................................................... 143
Equation 6-10 Modified Equation 4-20 for secondary nucleation due impeller-crystal collisions ................................................................. 148
Equation 6-11 Determination of collision nucleation rate constant and supersaturation dependence ..................................................................................... 148
Equation 6-12 Stokes number .................................................................................. 152
Equation 6-13 Particle relaxation time ..................................................................... 152
Equation 6-14 Turbulent eddy life time .................................................................. 152
Equation 6-15 Turbulent kinetic energy .................................................................. 153
Equation 6-16 Energy dissipation rate in a pipe flow .............................................. 153
Equation 6-17 Shear velocity .................................................................................. 153
Equation 6-18 Wall shear stress ............................................................................. 153
Equation 6-19 Crystal RMS velocity ...................................................................... 153
Equation 6-20 Constant in Equation 6-19 ............................................................... 153
Equation 6-21 Crystal-crystal collision frequency .................................................. 154
Equation 6-22 Kinetic energy of crystal-crystal contact ......................................... 154